$$\vec{F} = m\vec{a}$$

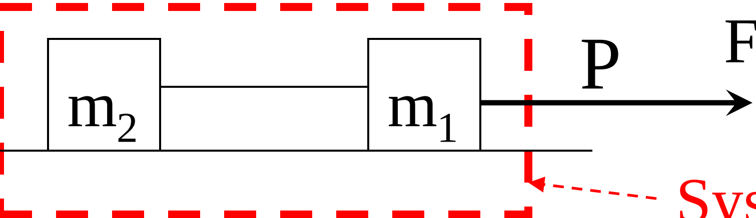
total

$$F_x = ma_x \qquad F_y = ma_y$$

$$\vec{F}_{total} = \sum \vec{F}_{individual}$$

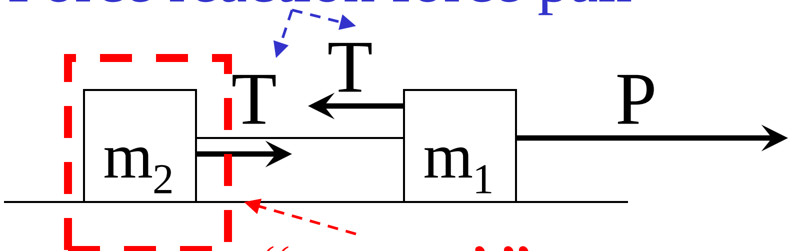
$\xrightarrow{+}$ Sign convention choice!!

Free body diagram

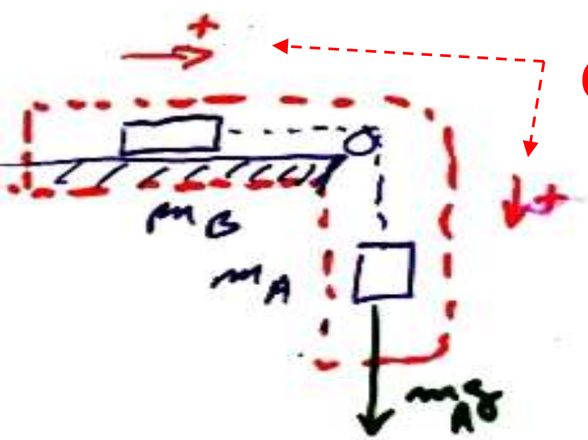


System choice

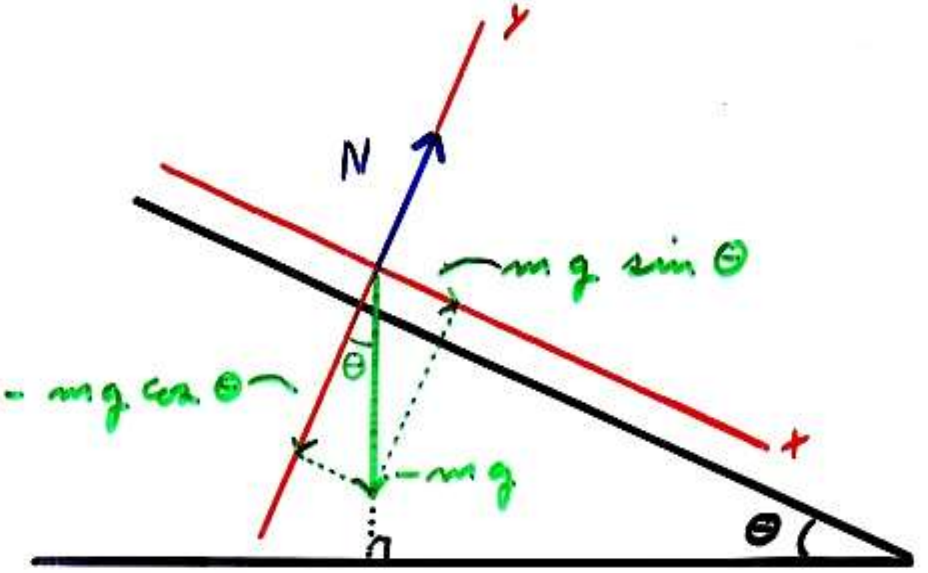
Force reaction force pair



"system' " cuts at point of internal force action



CONSISTANT sign convention choice around corners!!



$F_{f\,(max)} = \mu_s\, N$

$0 \le \mu_s < 1$

$F_f = \mu_k\, N$

Newton's 1st Law (law of inertia)

"A body at rest, or in uniform motion, tends to stay at rest or in uniform motion unless an unbalanced force acts on it."

Newton's 2nd Law

"An object is accelerated when an unbalanced force is applied to it. The rate of acceleration is proportional to the force and inversely proportional to the objects mass"

$$a = \frac{F}{m} \quad \text{or} \quad F = ma$$

Newton's 3rd Law

"For every action (force) there is an equal and opposite reaction (force)."

Newton's gravitational force law

1) F acts along the line connecting the centers of the objects (for spherical objects)
" central force"
2) F acts along the line connecting the centers of the objects (for spherical objects)
" central force"
3) F magnitude

$$F = \frac{G\,Mm}{r^2} \qquad G = 6.67\,(10)^{-11}\ [m^3/kg\ s^2]$$

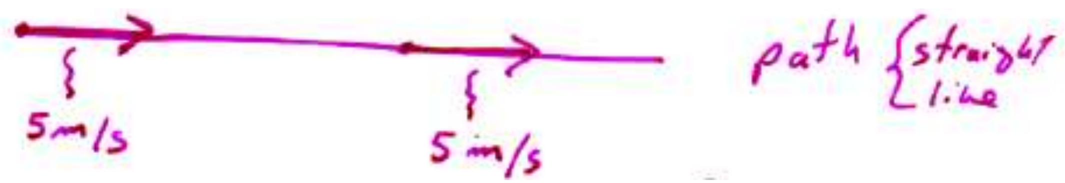
A few basic principles allows quantitative (mathematical) understanding of a tremendous body of phenomena. What physics is all about!!! [all fits on 1 page!!]

Newton's 1st Law (law of inertia)

"A body at rest, or in uniform motion, tends to stay at rest or in uniform motion unless an unbalanced force acts on it."

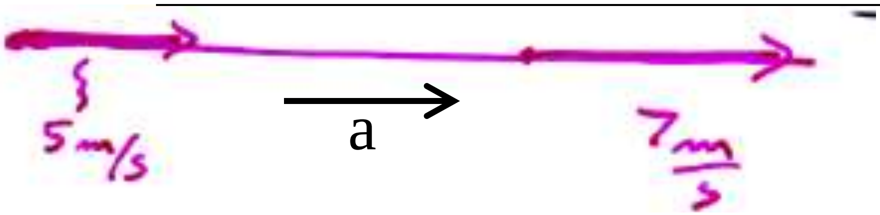
A bodies inertia is proportional to its mass [mass proportional to (but not=) weight]
1 kilogram = mass of 1 liter of water --- 1 gram = mass of 1 milliliter of of water

uniform motion = constant velocity motion= constant speed in a straight line motion



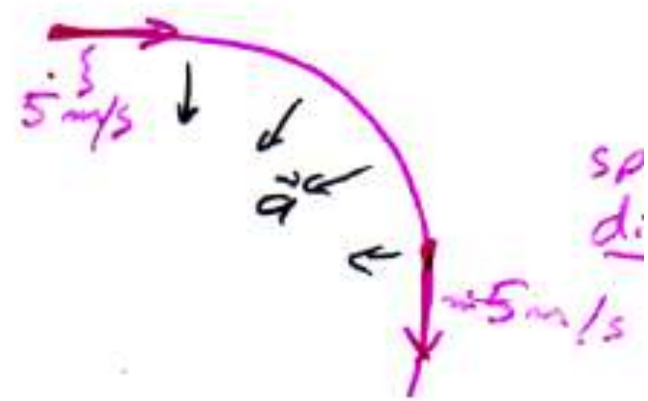
Need to measure deviation from constant velocity motion (uniform motion)!

acceleration $$\vec{a} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{\Delta t}$$



straight line but speed changes

speed constant but direction changes



Newton's 2nd Law

"An object is accelerated when an unbalanced force is applied to it. The rate of acceleration is proportional to the force and inversely proportional to the objects mass"

$$a = \frac{F}{m}$$ **or** $$F=ma$$

Could call "mass" objects tendency to resist acceleration "ottra".

units

$$F = m\ a$$

$$N = kg\ \frac{m}{s^2}$$

For a given force, the bigger the mass the smaller the acceleration

examples

$$F = m a \longrightarrow a = \frac{F}{m}$$

$2\ N$, $\frac{1}{2}\ kg$ $\longrightarrow a = \frac{2}{\left(\frac{1}{2}\right)} = 4\ m/s^2$

$\frac{kg\,m}{s^2}$

$t=0$, $v=0$; $t = 2s$, $v = at$ } $v_{2s} = 4 \cdot 2 = 8\ m/s$

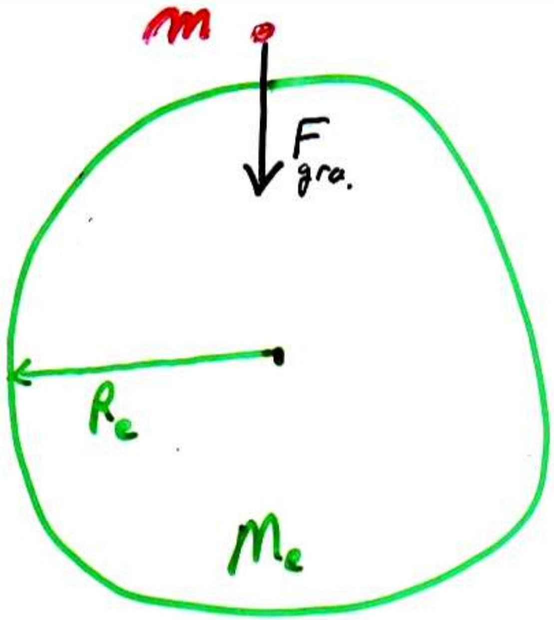
$2\ N$, $1\ kg$ $\longrightarrow a = \frac{2}{1} = 2\ m/s^2$

$v = at$ } $v_{2s} = 2 \cdot 2 = 4\ m/s$

Force of Gravity at Earth's Surface toward the Earth's center

$F = mg$ $\quad g = 9.8\ m/s^2$

Object of mass M at the surface of the earth.



a ↓

$$F_{gra} = \frac{G M_e m}{R_e^2}$$ N. U. L. G.

$$F_{2nd} = ma$$ N. 2nd Law

$$F_{gra} = F_{2nd}$$

$$\frac{G M_e \not{m}}{R_e^2} = \not{m} a$$

mass of object cancels ! !

Weight of an object, near the surface of the earth, = the gravitational force of the earth on the object

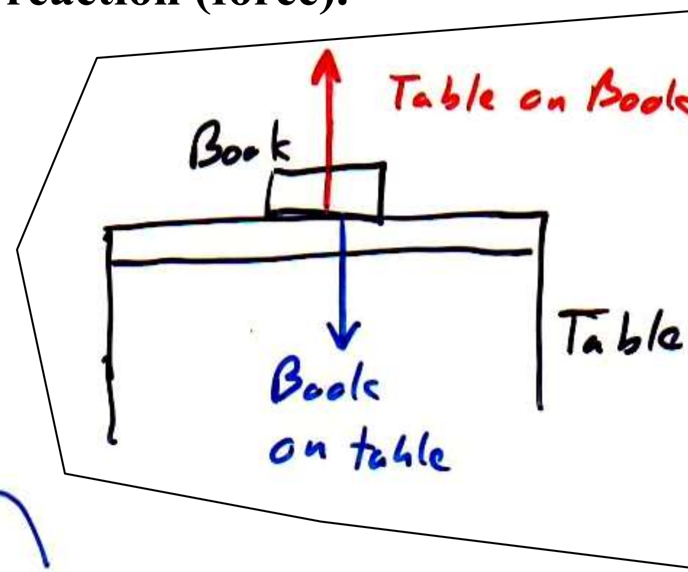
= mg

$$a = g = \frac{G M_e}{R_e^2}$$

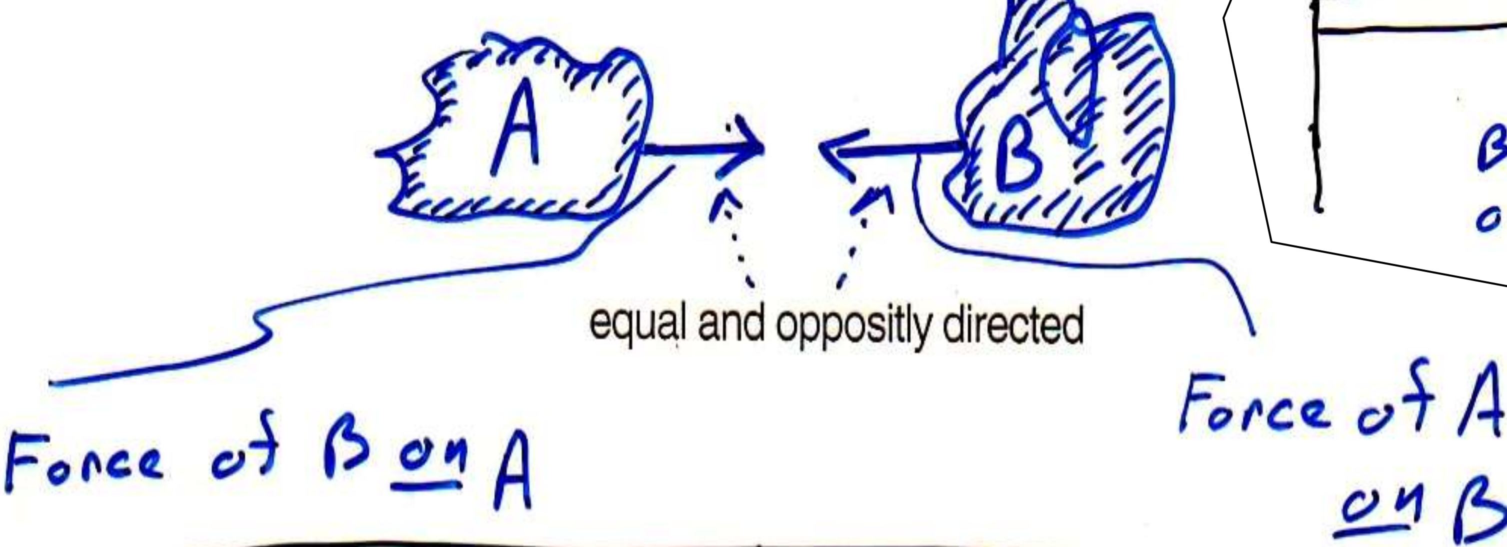
all objects accelerate near earth's surface with $g = 9.8\ m/s^2$ $(32\ ft/s^2)$

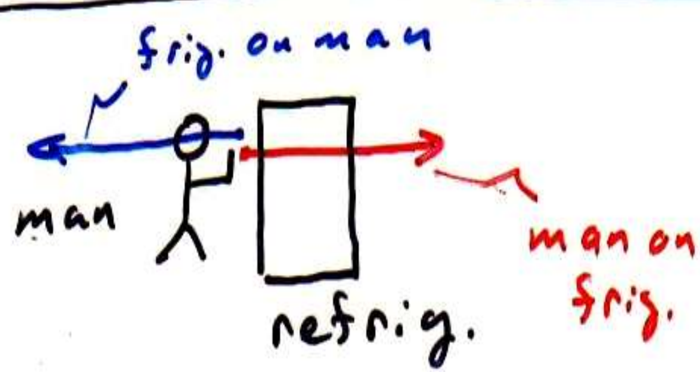
Newton's 3rd Law

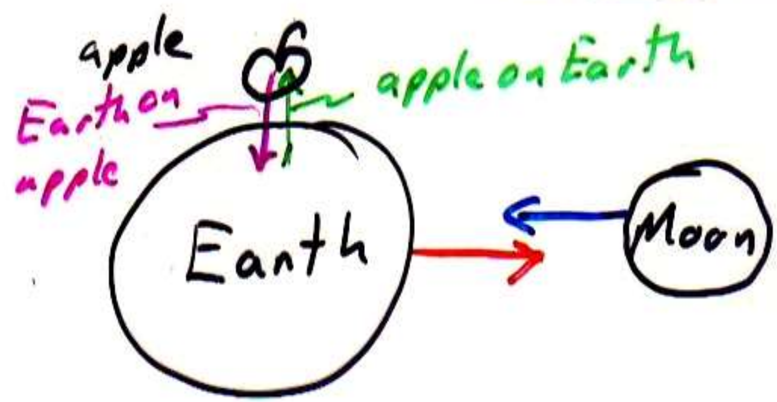
"For every action (force) there is an equal and opposite reaction (force)."



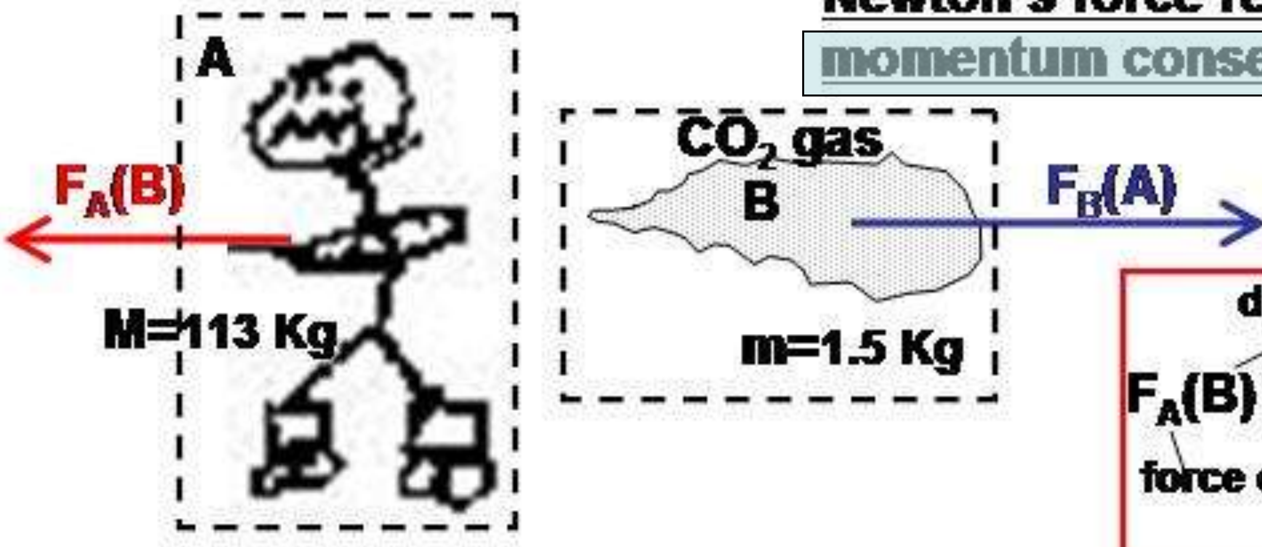
2 bodies (A & B) interact







Newton's force reaction force (3rd) Law & momentum conservation in rocket demo



Newton's 3rd Law

$$F_A(B) = -F_B(A)$$

($F_A(B)$: force on A, due to B; $F_B(A)$: force on B, due to A)

(simple but powerful !!)

Newton's 2rd Law $F = ma$ or $F = \frac{\Delta p}{\Delta t}$ where $p = mv$

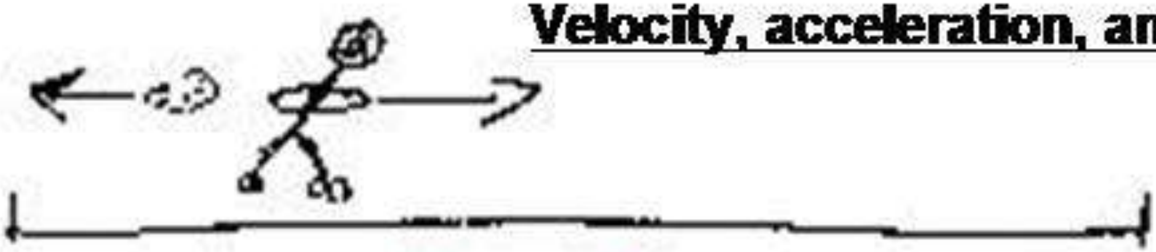
A+B system: no external forces $\Rightarrow$ **momentum conservation**

$$F_{tot} = F_A + F_B = 0 \Rightarrow \frac{\Delta p_{tot}}{\Delta t} = 0 \Rightarrow \Delta p_{tot} = \Delta p_A + \Delta p_B = 0$$

$$\Downarrow$$

$$\Delta p_A = -\Delta p_B$$

Velocity, acceleration, and force in rocket demo



$t=0$
$v_i=0$
$x=0$

$t=4s$
$x=10m$

observation

$\bar{v}=?$ $\quad \bar{v}=\dfrac{\Delta x}{\Delta t}=\dfrac{10m}{4s}=2.5\dfrac{m}{s}$

one way

$a=?$ (assume const a)

$$\Delta x=\frac{1}{2}at^2 \Rightarrow a=\frac{2(\Delta x)}{t^2}$$

$$a=\frac{2(10m)}{(4s)^2}=1.25\frac{m}{s^2}$$

$v_f=?$ $\quad v_f=at=\left(1.25\dfrac{m}{s^2}\right)(4s)=5\ \dfrac{m}{s}$

another way

$v_f=?$ (assume a=const) $\quad \bar{v}=\dfrac{v_f+v_i}{2}$

$$2.5\ \frac{m}{s}=\frac{v_f+0}{2}\Rightarrow v_f=5\ \frac{m}{s}$$

$a=?$ $\quad a=\dfrac{v_f-v_i}{\Delta t}$

$$a=\frac{(5\text{-}0)\frac{m}{s}}{4s}=1.25\frac{m}{s^2}$$

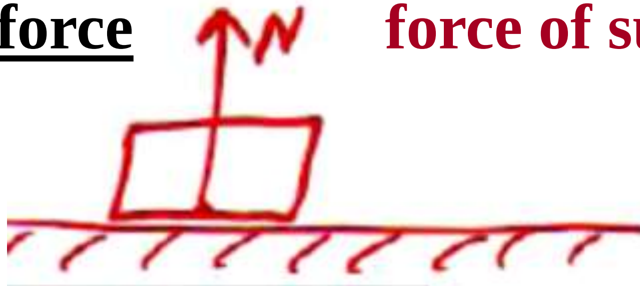
$F=?$ $\quad F=ma$

$F=(113kg)(1.25m/s^2)=141\ N$ (about 32 lb)

$$F_{external\ on\ system} = 0 \Rightarrow a_{system} = 0$$

Internal forces only

Normal force force of surface on the object in contact with it



⊥ to surface

Consider object of mass m on a surface

Free body diagram (of all forces on the object)

N-2nd Law

[for total force= sum of individual forces]

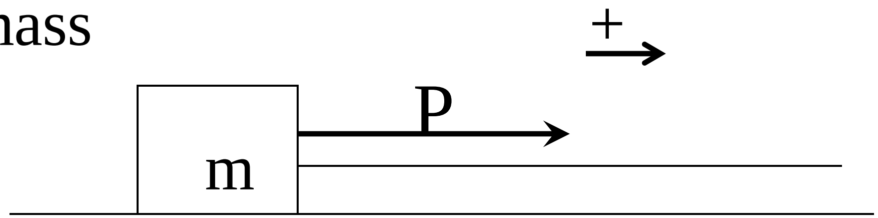
$$\sum_i F_{y_i} = N - mg = m a_y = 0$$

Object does "hop" or "burrow" so a_y=0

The sum of individual forces in y-direction are "balanced" so that the net force (in y-direction) zero.

There could be "unbalanced" force in x-direction.

1 mass



Free body diagram

N-2nd Law

$P = m\,a$

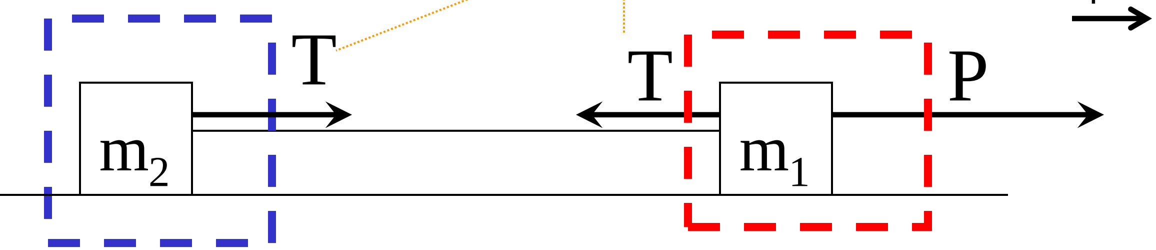
$a = \frac{P}{m}$

Ask a = ?

---

2 masses (coupled)

3rd Law (=)



N-2nd +3rd Law
Free body diagrams

Ask a = ?

T=?

N-2nd Laws

$$T = m_2\, a$$

$$P - T = m_1 a$$

$$P - m_2\, a = m_1 a$$

$$P = m_1 a + m_2\, a = (m_1 + m_2)\, a$$

$$a = \frac{P}{(m_1 + m_2)}$$

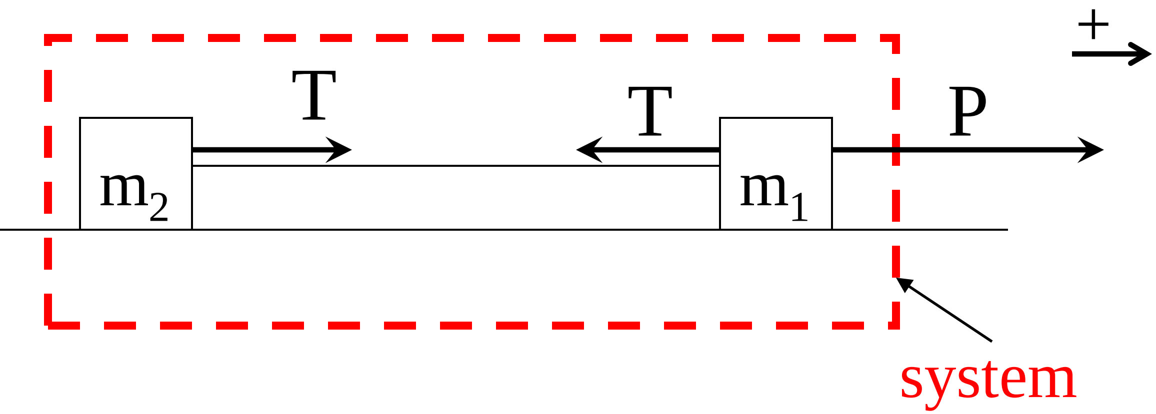
$$T = m_2 \frac{P}{(m_1 + m_2)}$$

Note: balanced forces in y-direction are assumed and ignored.

Now define system carefully
to solve specific question asked !!!

Internal forces (e.g. T) do not appear- only outside to inside forces.

2 masses



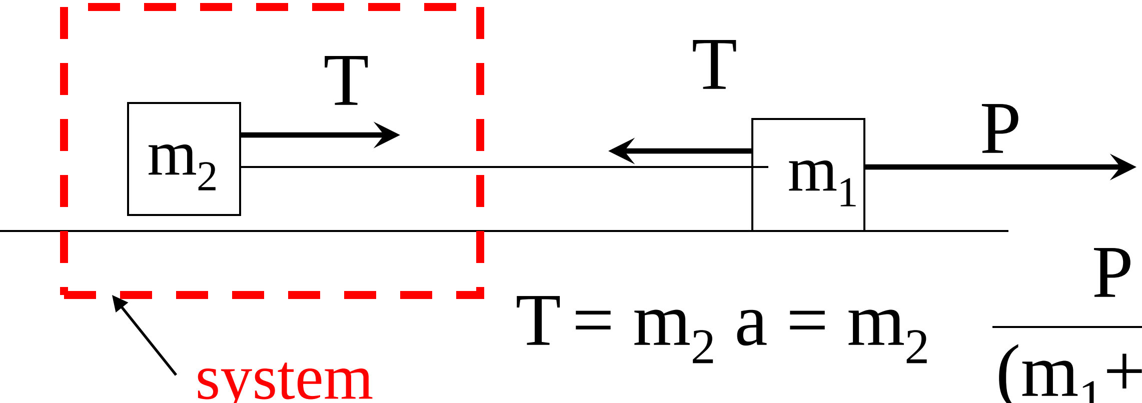
Ask a = ?

$$P = (m_1+m_2)\ a$$

$$a = \frac{P}{(m_1+m_2)}$$

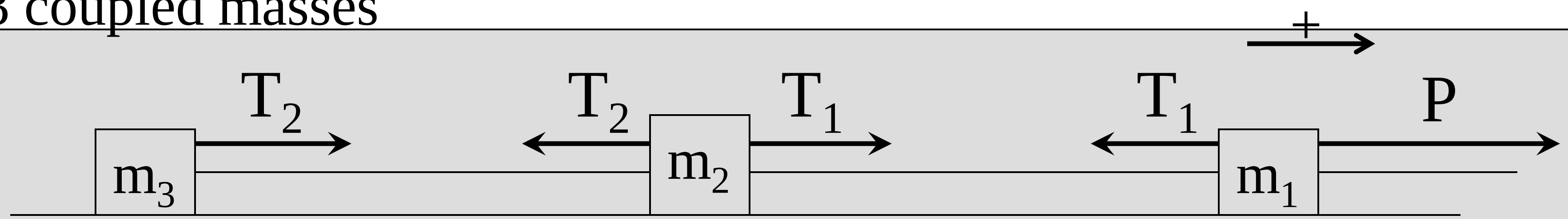
"system" cut at point of internal force action

Ask T= ?



$$T = m_2\ a = m_2 \frac{P}{(m_1+m_2)}$$

3 coupled masses

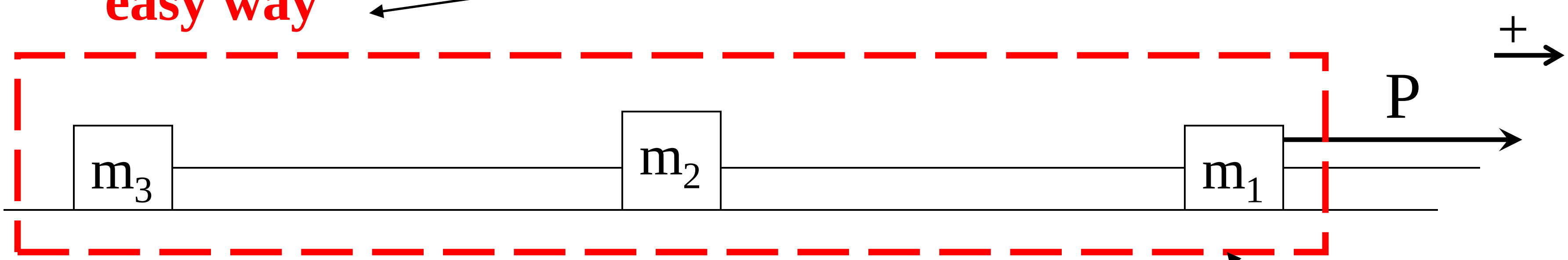


(3) $T_2 = m_3\, a$ (2) $T_1 - T_2 = m_2\, a$ (1) $P - T_1 = m_1\, a$

very hard way add (1)+(2)+(3) $\Rightarrow$ $P = (m_1 + m_2 + m_3)a$

Ask a = ?

easy way



$$P = (m_1 + m_2 + m_3)\, a = Ma$$

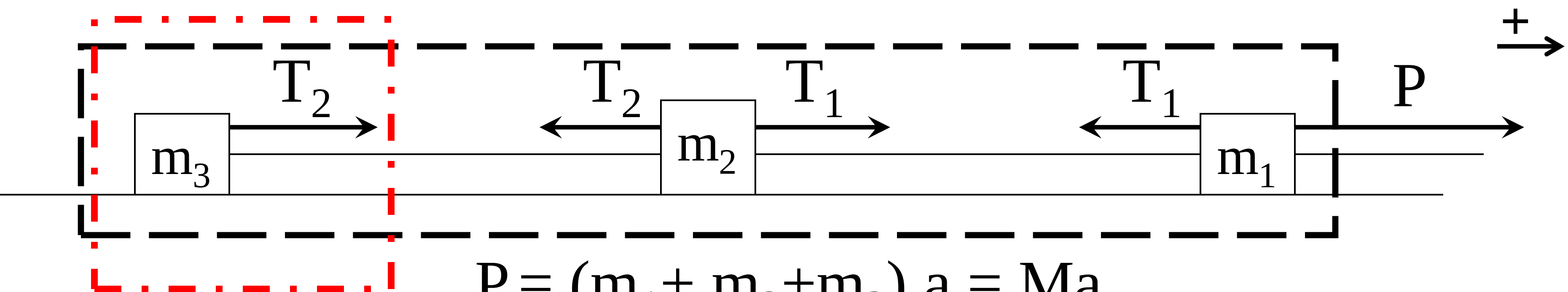
$$a = \frac{P}{M}$$

system

carefully define "system" to attack question asked !!

“system” <u>cut</u> at point of internal force action

Ask T_2=?



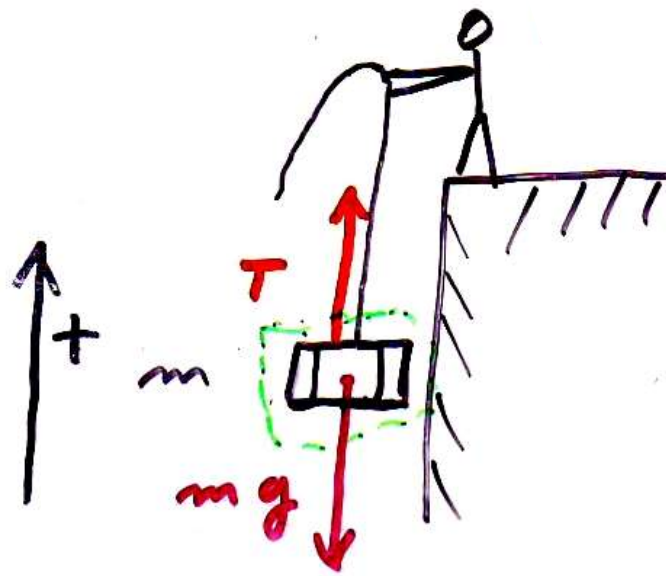
$$P = (m_1 + m_2 + m_3)\, a = Ma$$

$$T_2 = m_3\, a \leftarrow a = \frac{P}{M}$$

$$T_2 = m_3 \frac{P}{M}$$

Man holding rope attached to box

[Note: It is total F that = ma]



$$\underbrace{F}_{\text{total}} = ma$$

$$T - mg = ma$$

$T > mg$ $\quad a$ +#

$T < mg$ $\quad a$ −#

$T = mg$ $\quad \begin{cases} a = 0 \\ \vec{v} = \text{const.} \end{cases}$

note!

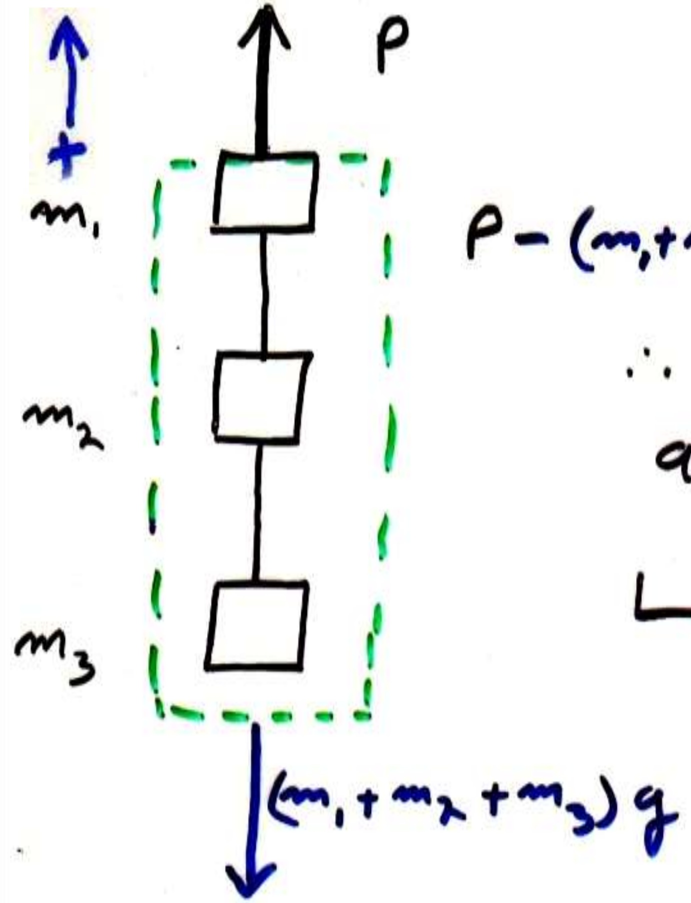
- 2 individual forces T and mg combine to give total force

Total force = ma !!

Also

$$T = m(a+g)$$

Big a
Big T !!



Q. What is acceleration?

$$\Sigma F = ma$$

$$P-(m_1+m_2+m_3)g = (m_1+m_2+m_3)a$$

use $g = 10\,m/s^2$

$\therefore$

$$a = \frac{P-(m_1+m_2+m_3)g}{(m_1+m_2+m_3)}$$

Example

$m_1 = 1.2$ kg $m_2 = 0.8$ kg

$m_3 = 1.0$ kg $P = 36$ N

$\therefore$

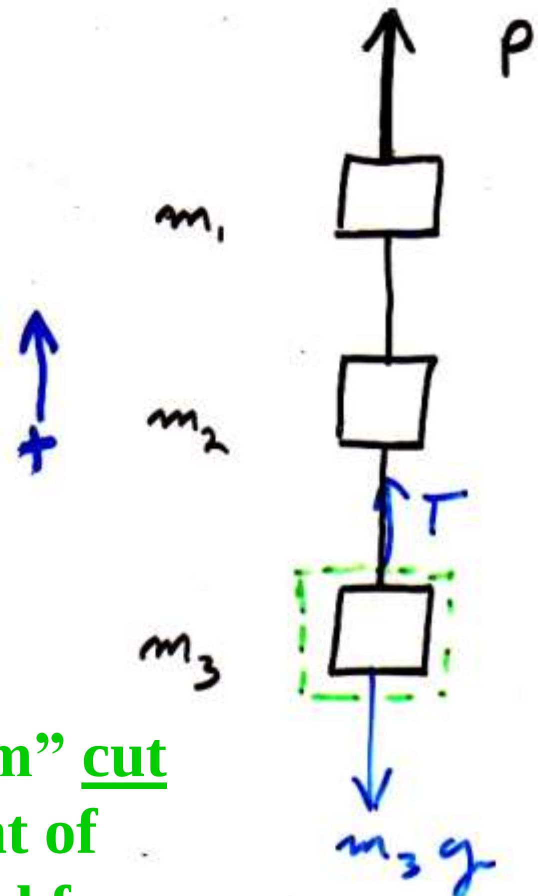
$$a = \frac{\{36 - [12 + 8 + 10]\}\ N}{[1.2 + .8 + 1.0]\ kg}$$

$$= \frac{36-30}{3.0}\ \frac{N}{kg} = +2\ \frac{m}{s^2}$$

$a = +2\ m/s^2$

up

Q. What is T between m_2 and m_3



$$T - m_3 g = m_3 a$$

$$\Downarrow$$

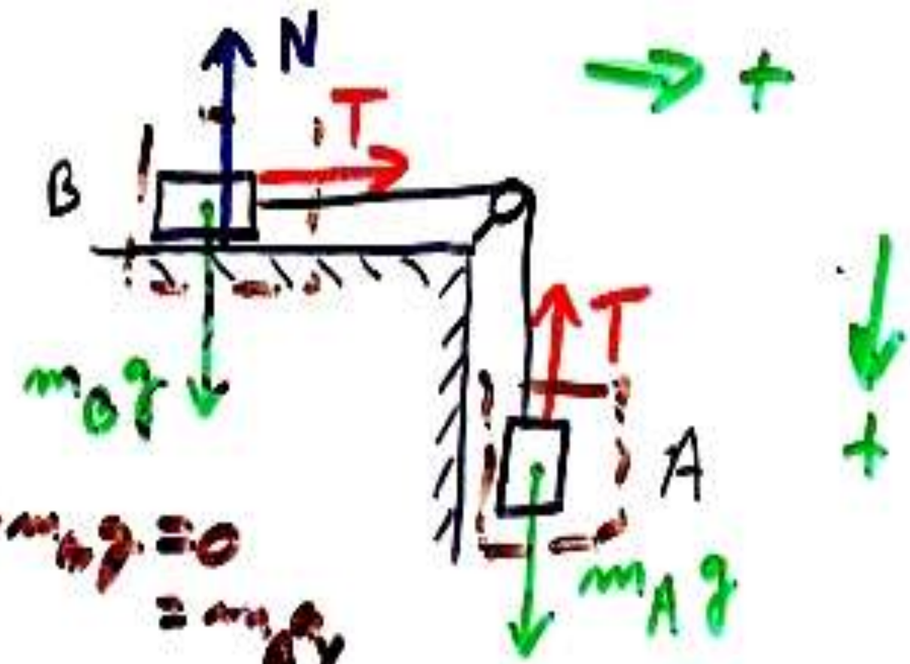
$$T = m_3 \left[a + g \right]$$

recall $a = \dfrac{P - (m_1 + m_2 + m_3) g}{m_1 + m_2 + m_3}$

"system" <u>cut</u> at point of internal force action

$$T = (1.0)(\text{kg}) \left[+2\,\text{m/s}^2 + 10\,\text{m/s}^2 \right]$$

$$= 12\ \text{kg}\,\frac{\text{m}}{\text{s}^2} = \underline{12\,\text{N}}$$



$N - m_B g = 0$
$= m_B a_y$

object B

$T = m_B a$

object A

$-T + m_A g = m_A a$

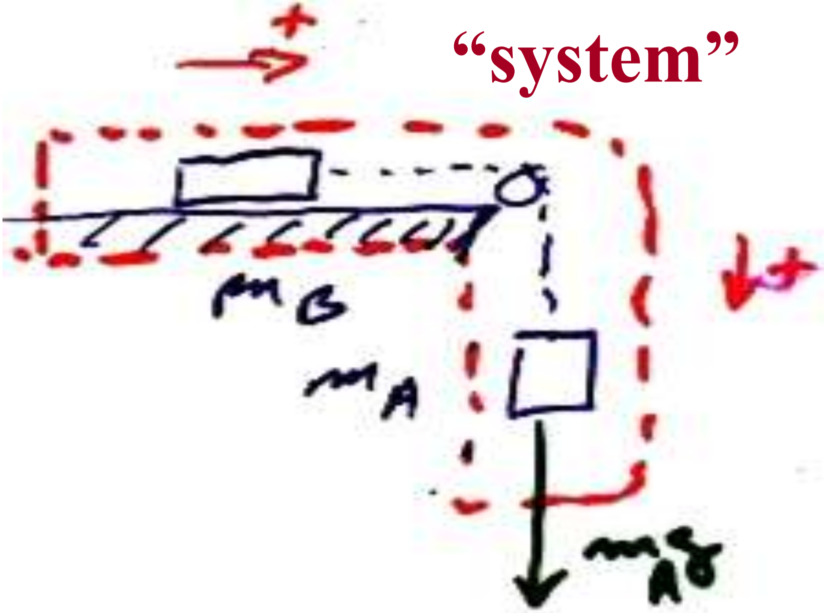
what is a

$-m_B a + m_A g = m_A a$

Going around corner, carefully choose sign convention consistently !!!

$(m_A + m_B) a = m_A g$

$a = \left[\frac{m_A}{m_A + m_B} \right] g$



what is a

$$Ma = \Sigma F$$

$$(m_A + m_B)(a) = m_A g$$

$$a = \left[\frac{m_A}{m_A + m_B}\right] g$$

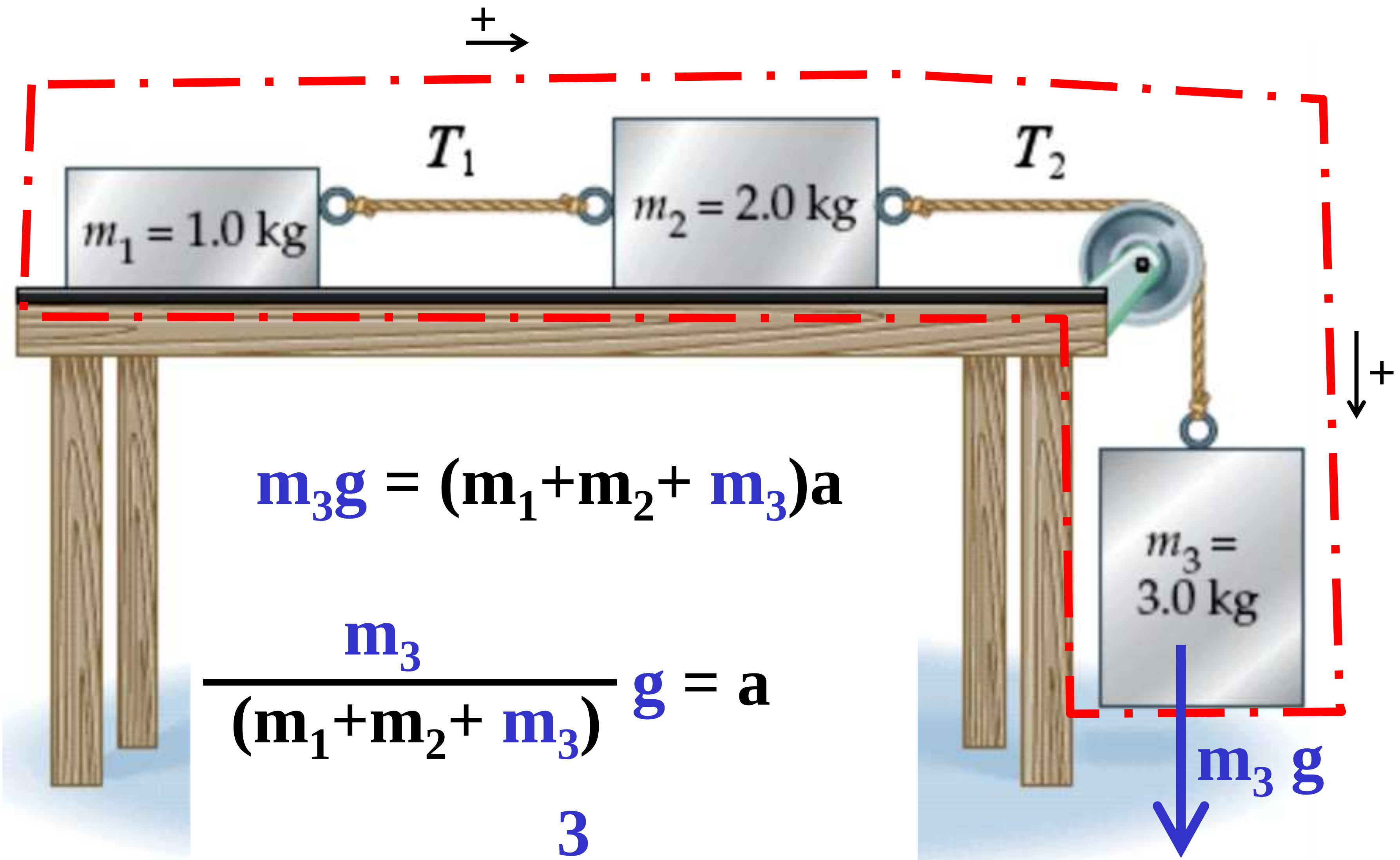
$m_B \approx 500\ g$ (.5 kg)

$m_A \approx 60\ g$ (0.06 kg)

$$\frac{m_A}{m_A + m_B} \approx \frac{60}{560} \approx 0.1$$

$$a \approx \frac{1}{10} g \approx 1\ m/s^2$$

Choosing "system" to answer question!!!



$$m_3 g = (m_1 + m_2 + m_3)a$$

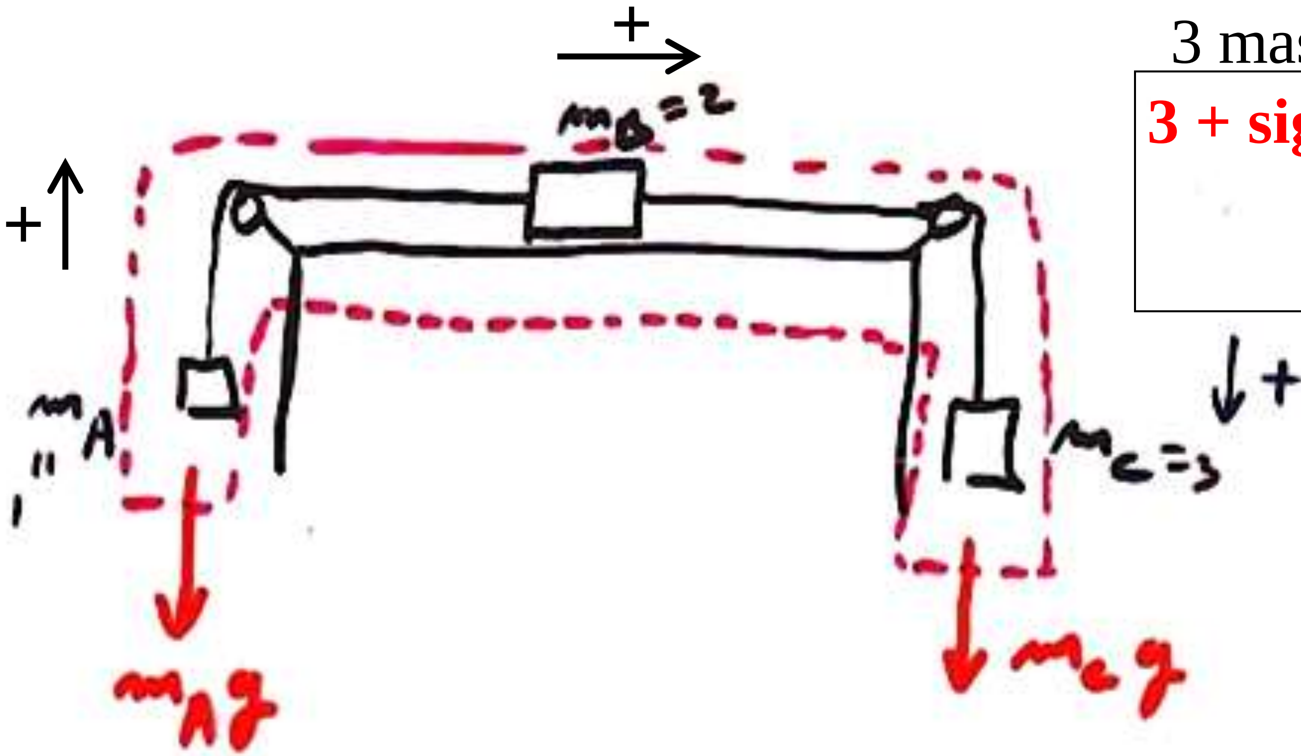
$$\frac{m_3}{(m_1 + m_2 + m_3)}\,g = a$$

$$[\mathrm{m/m}]\;[\mathrm{m/s^2}]\;\frac{3}{(1+2+3)}\,9.8 = a$$

$$[\mathrm{m/s^2}]\;4.9 = a$$

3 mass problem

3 + sign convention choices
Choosing "system"
to answer question!!!



Forces from outside box (system) acting to inside = $M_{\text{inside box}}\, a_{\text{box}}$

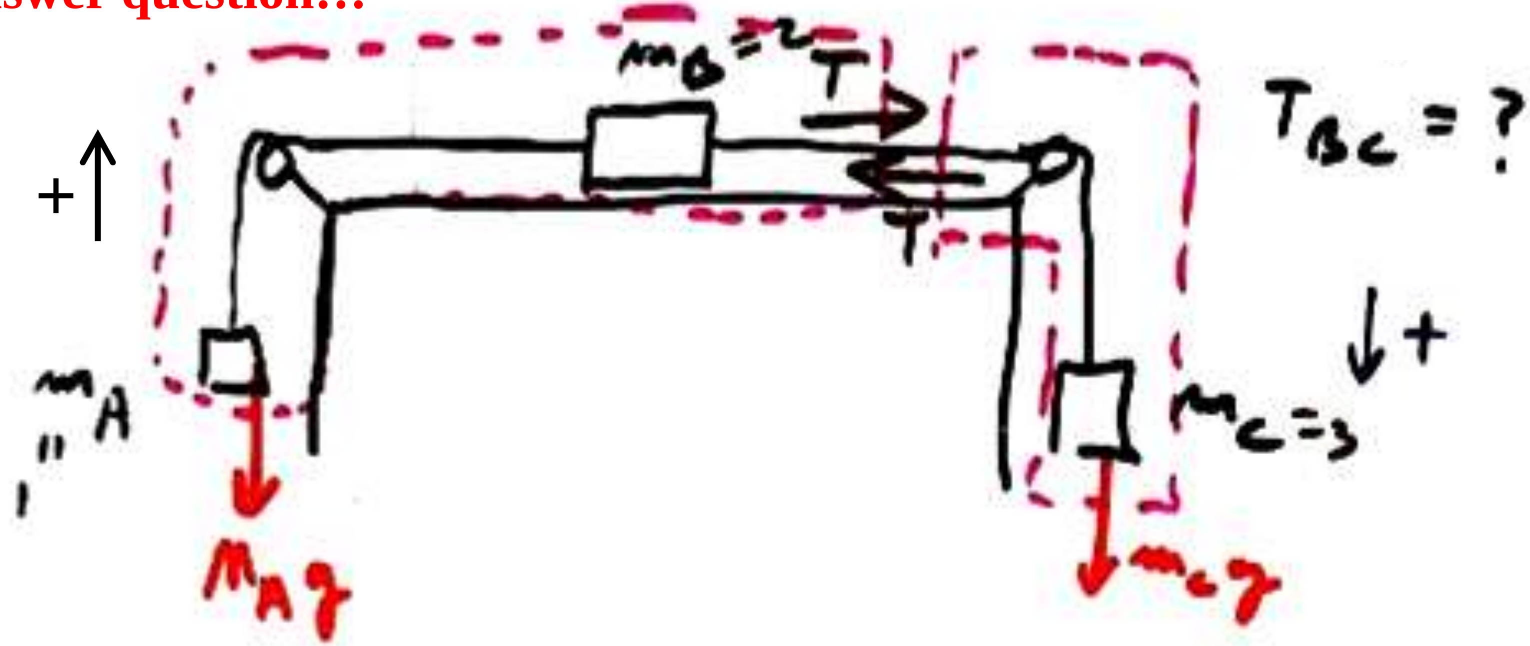
$$-m_A g + m_C g = (m_A + m_B + m_C)\, a$$

$$a = \frac{(m_C - m_A)}{(m_A + m_B + m_C)}\, g = \frac{(3-1)\,[\text{kg}]}{(1+2+3)\,[\text{kg}]}\, 9.8\,[\text{m/s}^2]$$

$$a \cong \frac{10}{3}\,\text{m/s}^2$$

Choosing "systems" to answer question!!!

3 mass problem



$$-m_A g + T = (m_A + m_B) a$$

$$-T + m_C g = m_C a$$

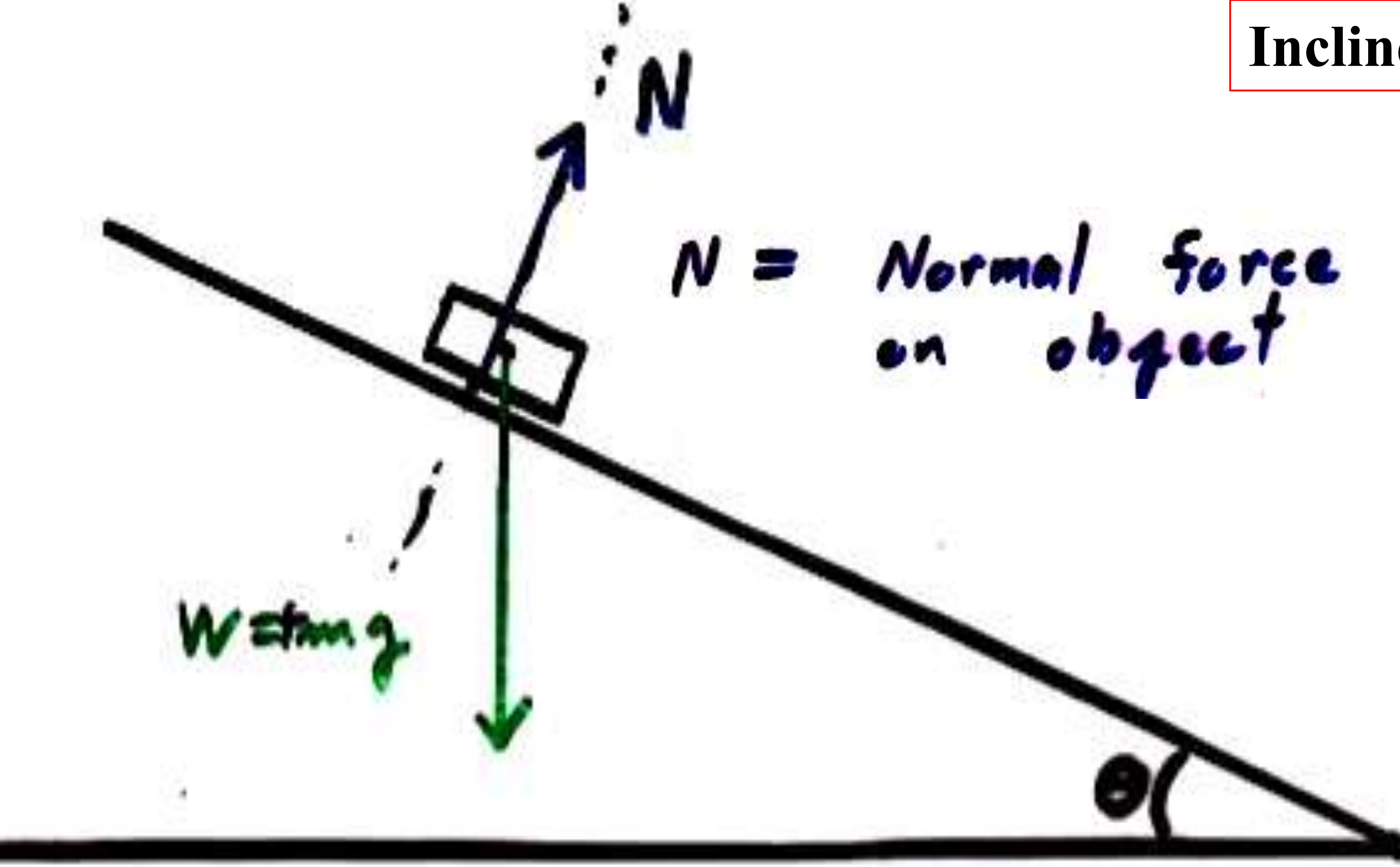
$$-T + 3 \cdot 10 = 3 \cdot \frac{10}{3}$$

$$T = +20\ N$$

recall

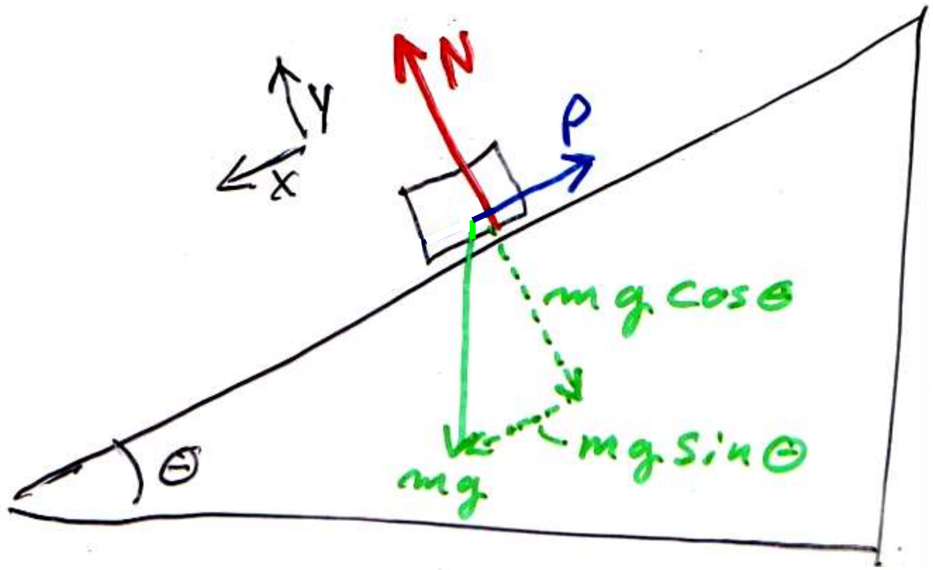
$$a = \frac{(m_C - m_A)}{(m_A + m_B + m_C)} g = \frac{10}{3}$$

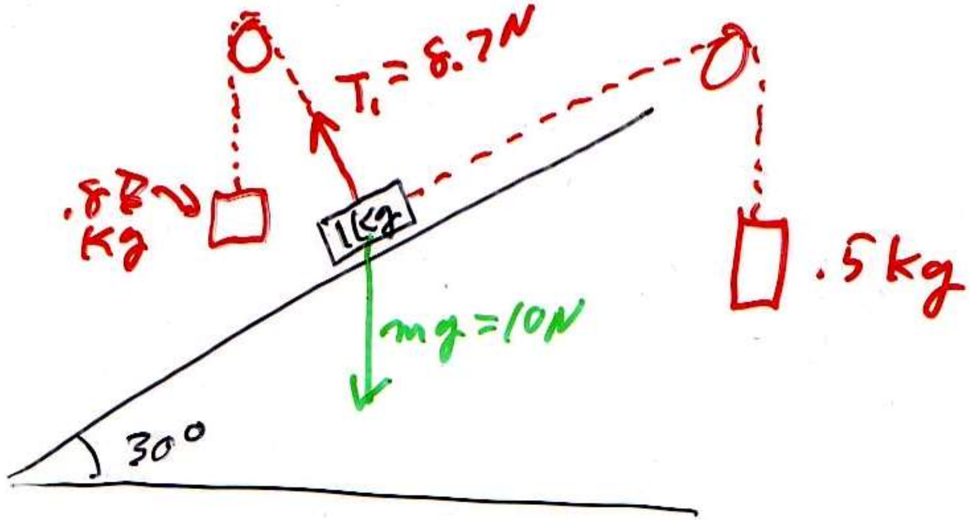
Inclined plane abc's



N = Normal force of plane on object

W = +mg = Force of gravity on object (Pull of earth on object)





Demo
Forces add like vectors
$\mathbf{F_{tot}=0 \Leftrightarrow a=0}$

x] $-P + mg\sin\Theta = F_x = 0$

y] $+N - mg\cos\Theta = F_y = 0$

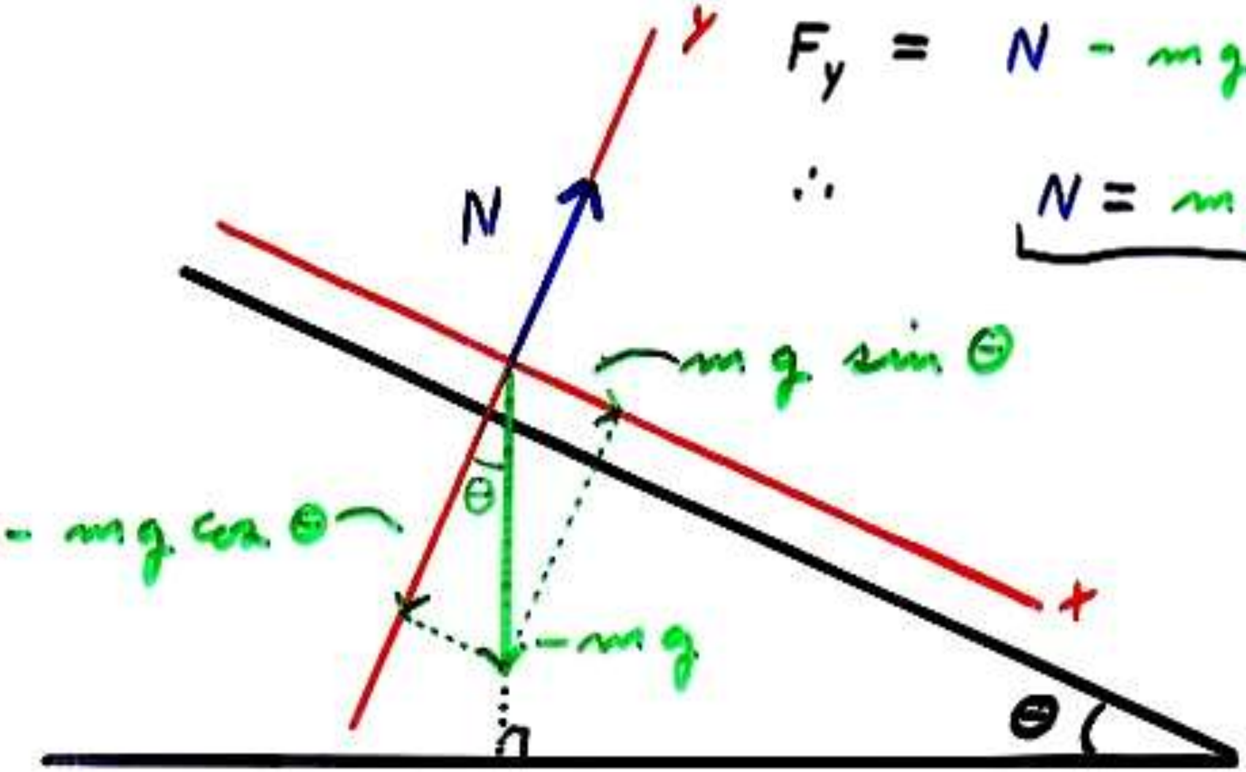
$\Rightarrow$ $P = mg\sin\Theta$ $N = mg\cos\Theta$

$m = 1\ \text{kg}$ $\Theta = 30^\circ$ $\begin{cases}\sin\Theta = \frac{1}{2} \\ \cos\Theta = .866\end{cases}$

$P = 1(\text{kg})\ 9.8\ \frac{\text{m}}{\text{s}^2}\ \frac{1}{2}$ $N = 1(9.8)\ .866$

$P \simeq 5\text{N}$ $N \simeq 8.7\ \text{N}$

Inclined plane abc's



$$F_y = N - mg\cos\theta = 0 = a_y$$

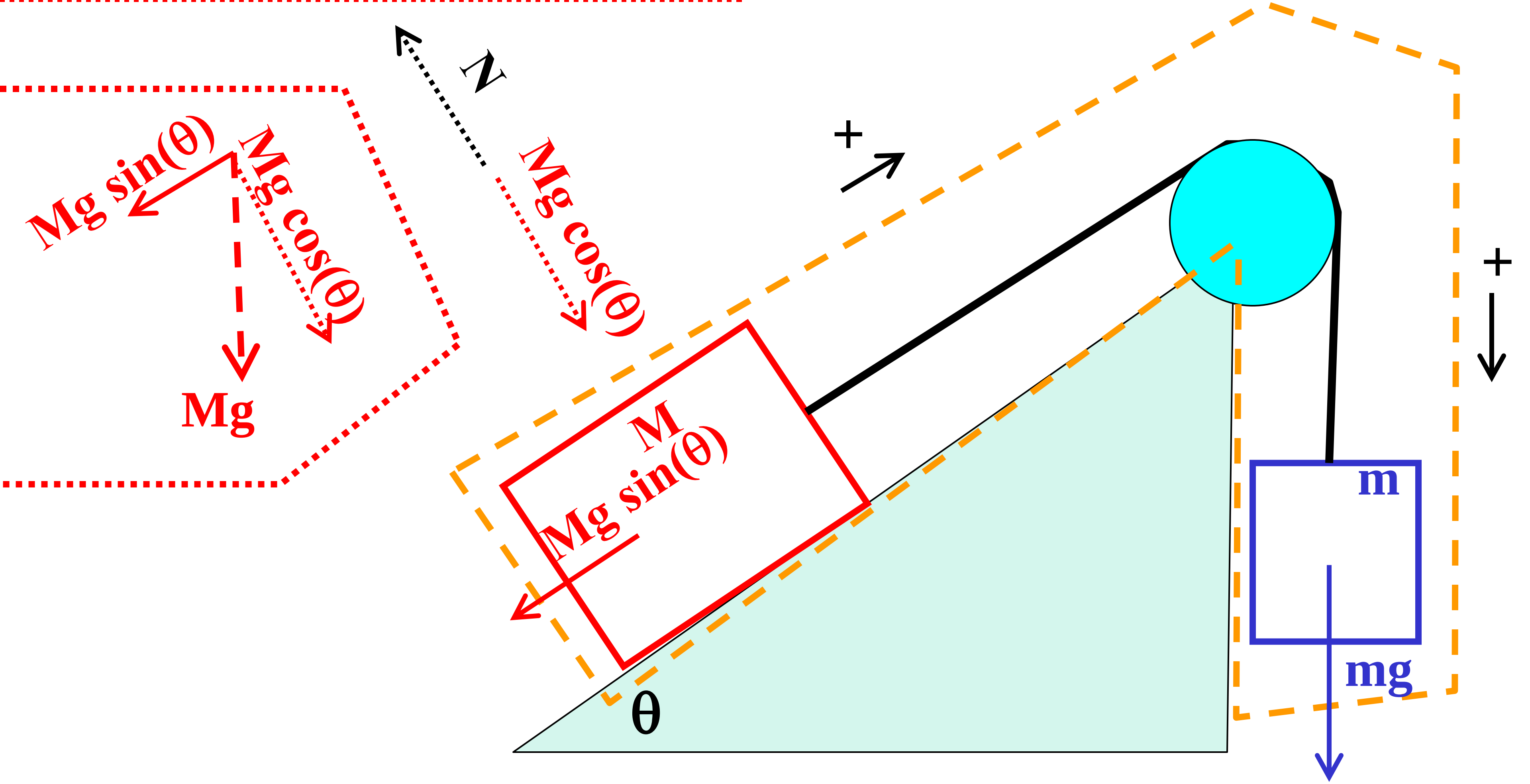
(no motion in y dir)

$$\therefore \quad N = mg\cos\theta$$

$$F_x = mg\sin\theta = m a_x$$

$$\therefore \quad a_x = g\sin\theta$$

Balanced force components in perp. Dir. $N - Mg\cos(\theta) = 0$



$$-Mg\sin(\theta) + mg = (M+m)a$$

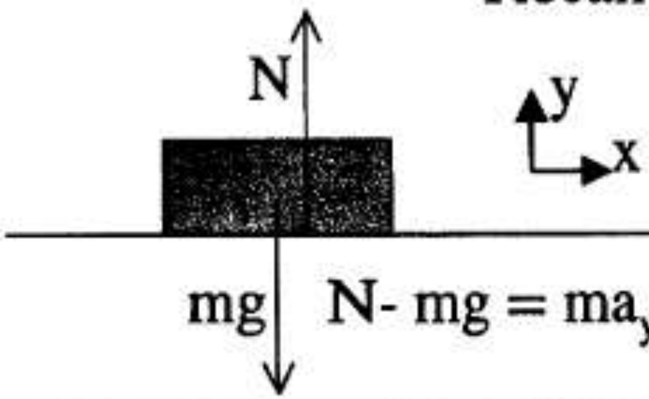
$$\frac{[-M\sin(\theta) + m]\,g}{(M+m)} = a$$

Friction

Two Cases

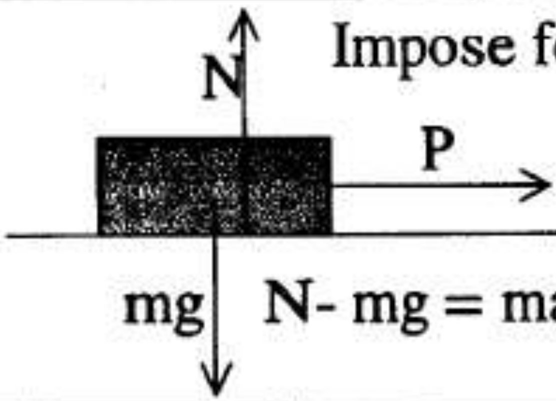
- Static Friction (no motion)
- Kinetic Friction (moving)

Recall object on table



$N - mg = ma_y = 0$

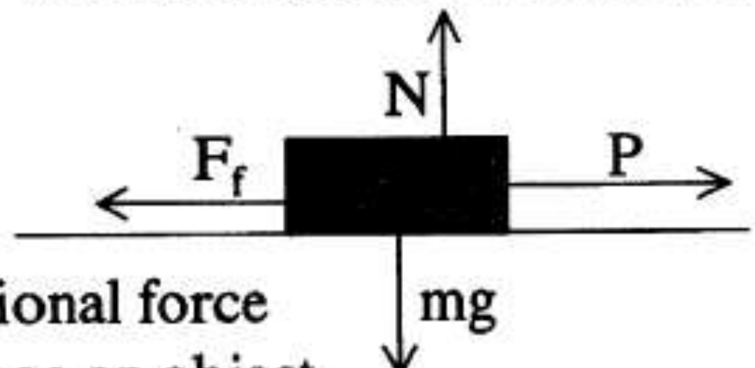
Impose force // to surface (no friction)



$P = ma_x$

$N - mg = ma_y = 0$

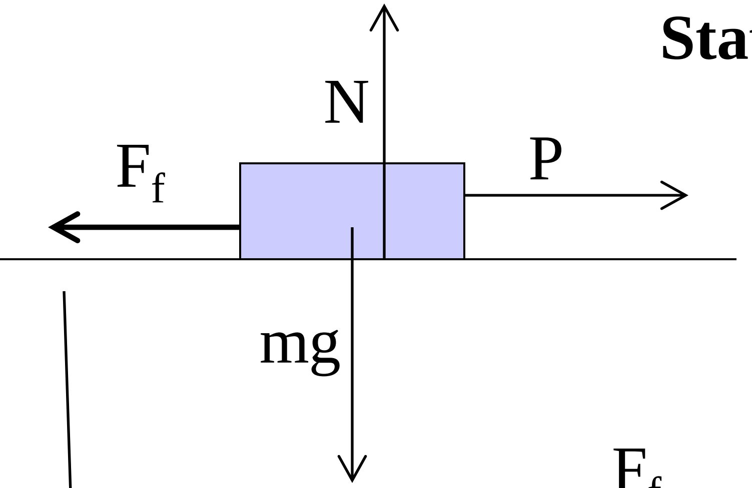
Now add frictional force // to surface



- F_f frictional force of surface on object
- F_f opposes motion

Stationary Object: Static Friction

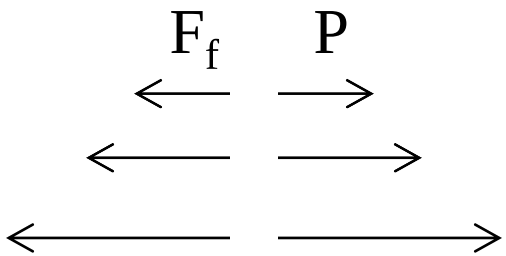
(pulled with force P)



$$-F_f + P = ma_x = 0 \quad !!!$$

For static friction ("always")

$$F_f = P$$



P increases $\Rightarrow$ F_f increases

There is a **largest value** of F_f for which the object remains at rest.

$$F_{f\,(max)} = \mu_s N$$

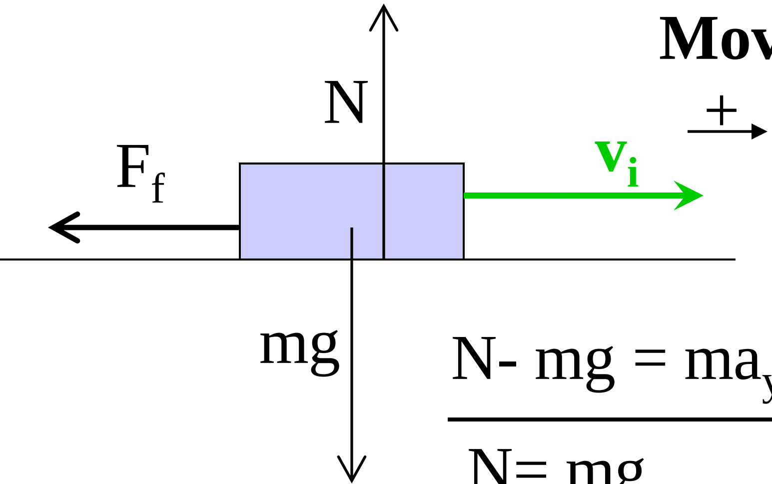
$$0 \le \mu_s < 1$$

μ_s = coefficient of static friction

In this simple case N= mg

Moving Object: Kinetic Friction (sliding)

(projected at velocity v_i)



$$-F_f = ma_x$$

$$N - mg = ma_y = 0$$

$$N = mg$$

For kinetic friction ("always")

$$F_f = \mu_k N$$

$$ma_x = -\mu_k N$$

Note v_i shown in figure IS NOT A FORCE !!!!

$$ma_x = -\mu_k mg$$

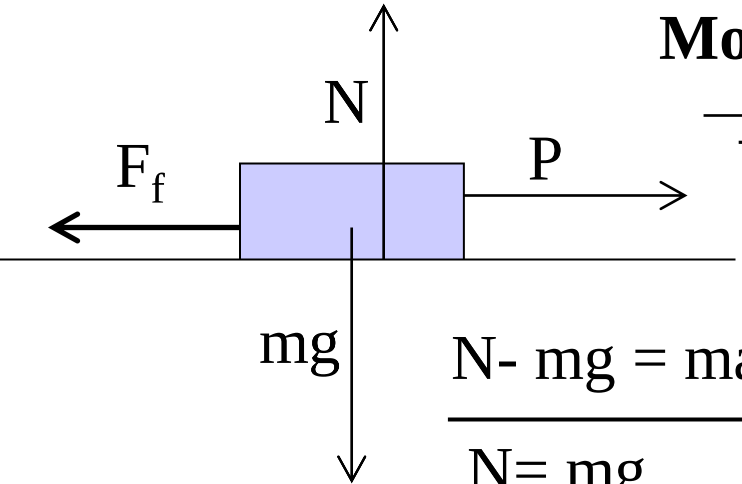
$$a_x = -\mu_k g$$

$$v = v_i - \mu_k g t$$

Friction slows object down

Moving Object: Kinetic Friction (sliding)

(pulled with force P)



$\xrightarrow{+}$

$$-F_f + P = ma_x$$

$$N - mg = ma_y = 0$$

$$N = mg$$

For kinetic friction ("always")

$$F_f = \mu_k N$$

$$ma_x = P - \mu_k N$$

$$ma_x = P - \mu_k mg$$

$\mu_k mg$ reduces force

Note: There are two possibilities in this kinetic case.

a) non zero acceleration

$$a_x = \frac{P - \mu_k N}{m}$$

b) zero acceleration

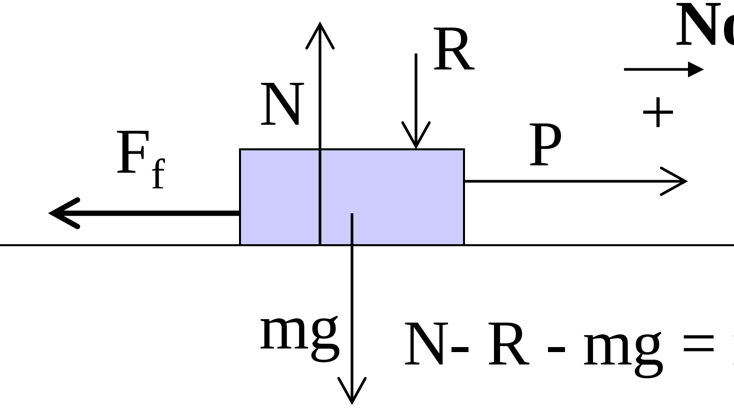
$$P = \mu_k N$$

$$a_x = 0$$

object moving

$$v = \text{constant}$$

Note: normal force is not always mg



$$-F_f + P = ma_x$$

$$N - R - mg = ma_y = 0$$

For kinetic friction ("always")

$$F_f = \mu_k N$$

$$N = mg + R$$

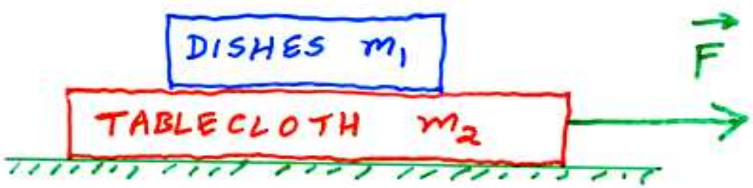
$$ma_x = P - \mu_k N$$

$$ma_x = P - \mu_k (mg + R)$$

vertical force contributes to retarding horizontal force

4-20

PULLING A TABLECLOTH WITH DISHES ON IT



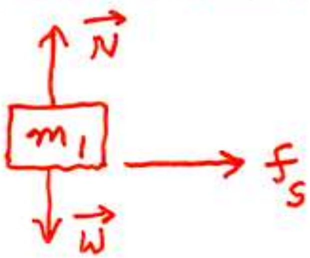
ASSUME NO FRICTION BETWEEN TABLECLOTH AND TABLE.

FIRST CONSIDER DISHES + TABLECLOTH AS "SYSTEM"

$$F = (m_1 + m_2)\, a$$

$$\Rightarrow a = \frac{F}{m_1 + m_2} . \qquad (1)$$

NOW CONSIDER DISHES AS SYSTEM:



$$f_s = m_1 a = \frac{m_1}{m_1 + m_2} F \quad \text{USING (1).}$$

BUT WE ALSO KNOW THAT

$$N - W = 0 \Rightarrow N = W = m_1 g .$$

AND $f_s \leq \mu_s N = \mu_s m_1 g$.

SO $\frac{m_1}{m_1 + m_2} F \leq \mu_s m_1 g \Rightarrow$ $F \leq \mu_s\, g(m_1+m_2)$

M. Kalelkar

IF F BECOMES GREATER THAN THIS VALUE, TABLECLOTH STARTS SLIDING UNDER DISHES !

- **if a smaller than this value the tablecloth + dishes move together**
- **if a is larger than this value the tablecloth starts sliding under the dishes !!!!**

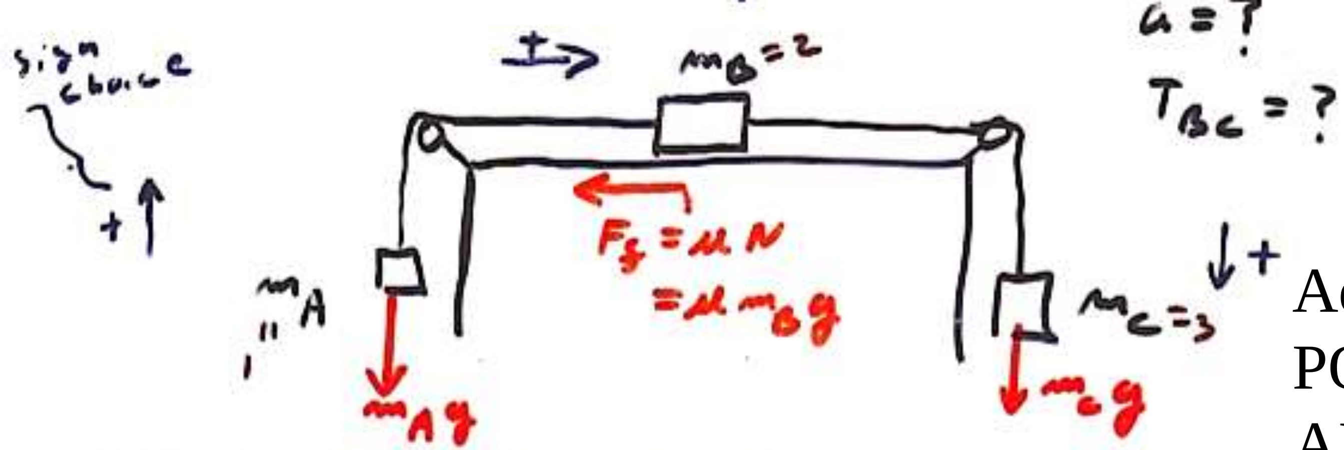
so pull it with big a !!!

$a \leq \mu_s g$!!!!!

$a\,(m_1+m_2) \leq \mu_s\, g\,(m_1+m_2)$

M. Croft

4-21



Add friction to problem
POSTPONED TILL
AFTER FRICTION

Suppose there is a frictional force on m_B.

Tot. external force = (tot. mass) a

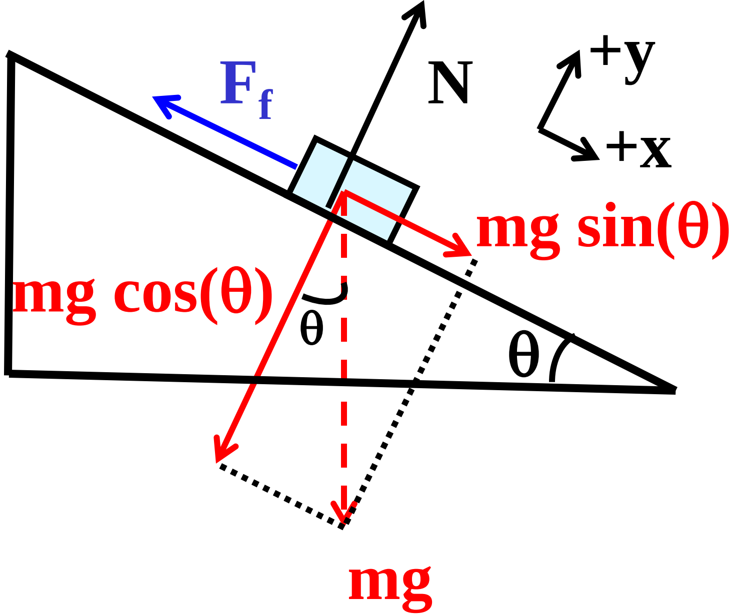
$$-m_A g + m_C g - \mu m_B g = (m_A + m_B + m_C)\, a$$

$$a = \frac{g\,[m_C - m_A - \mu m_B]}{m_A + m_B + m_C}$$

let $\mu = .5$

$$a = \frac{10\,[3 - 1 - (.5)2]}{1 + 2 + 3} = \frac{10}{6}(1) = 1.66\,m/s^2$$

$$a = [\underbrace{3.33}_{\text{no frict}} - \underbrace{1.66}_{\text{due to frict}}] = 1.66\,m/s^2$$



μ_S static friction coefficient determination

⇓

y, ⊥ to plane: $N - mg\cos(\theta) = ma_y = 0$

$N = mg\cos(\theta)$

x, || to plane: $-F_f + mg\sin(\theta) = ma_x$

$-F_f + mg\sin(\theta) = 0$ for static case

⇓

$F_f = mg\sin(\theta)$

experiment: increase θ until F_f reaches maximum

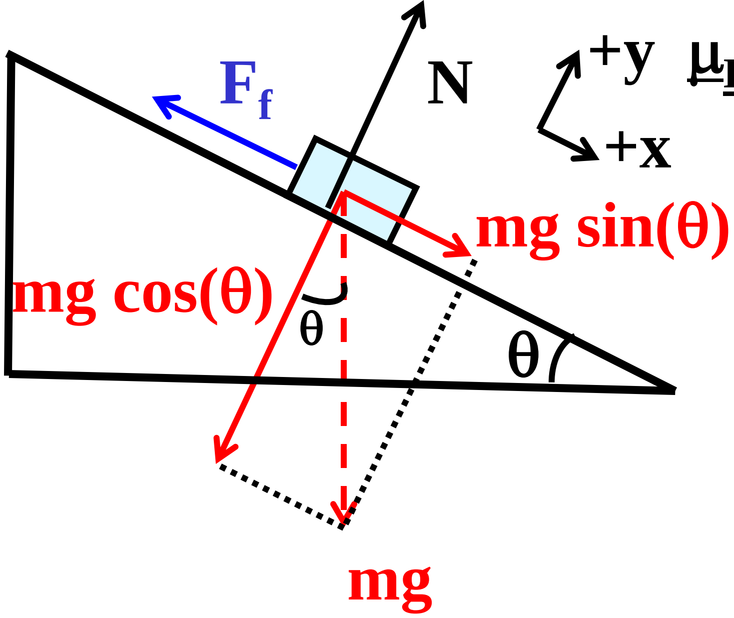
then $F_f = F_S = \mu_S N = \mu_S\, mg\cos(\theta)$

$\mu_S\, mg\cos(\theta) = mg\sin(\theta)$

$$\mu_S = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

observe $\theta = 21^o \Rightarrow \tan(\theta) = 0.38 = \mu_S$

4-23



μ_K kinetic friction coefficient determination

y, ⊥ to plane: $N - mg\cos(\theta) = ma_y = 0$

⇓

$N = mg\cos(\theta)$

x, || to plane: $-F_f + mg\sin(\theta) = ma_x$

for kinetic case

then $F_f = F_K = \mu_K N = \mu_K\, mg\cos(\theta)$

⇓

$-\mu_K\, mg\cos(\theta) + mg\sin(\theta) = ma_x$

$a_x = g\,[\sin(\theta) - \mu_K \cos(\theta)]$

experiment: observe a_x at given θ

Air/Fluid Resistance to Moving Object

Case object moving fast (golf ball...)

v_{rel} = speed relative to air (air at rest $v_{rel}=v$)

v_t = terminal velocity (air moving at v_a / $v_{rel}=v+v_a$)

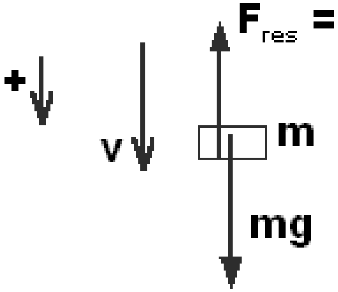
for falling object of mass m

A = cross-sectional area of object

ρ = density of gas(fluid)

C = drag coeff.

$F_{res} = [C\rho A/2]\, v_{rel}^2$



$$ma = F$$

$$ma = mg - \frac{C\rho A}{2} v^2$$

if $a=0$ $v=v_t$

$$0 = g - \frac{C\rho A}{2m} v_t^2$$

$$v_t = \sqrt{\frac{2mg}{CA\rho}} = \sqrt{gL} \qquad L = \frac{2m}{C\rho A}$$

sky diver example m=70 Kg; A = 0.7 m^2

C = 0.4

ρ(air) = 1.2 Kg/m^3

v_t= 64 m/s (140 mph) L = 417 m