2-D $\vec{a}$ = constant motion $\vec{a} \Leftrightarrow (a_x, a_y)$ constant

$t = 0$ $\quad \vec{v}_0 \Leftrightarrow (v_{0x}, v_{0y})$ $\quad \vec{r}_0 \Leftrightarrow (x_0, y_0)$

Most General Form (actually rarely necessary)

- $v_x = v_{0x} + a_x t$
- $v_y = v_{0y} + a_y t$
- $x = x_0 + v_{0x} t + a_x \frac{t^2}{2}$
- $y = y_0 + v_{0y} t + a_y \frac{t^2}{2}$
- $\Delta x = \bar{v}_x t = \frac{(v_{0x} + v_{0x})}{2} t$
- $\Delta y = \bar{v}_y t = \frac{(v_{0y} + v_y)}{2} t$
- $v_x^2 - v_{ox}^2 = 2a_x (x - x_o)$
- $v_y^2 - v_{o_y}^2 = 2a_y (y - y_o)$

For special case (all objects projected near the surface of the earth)

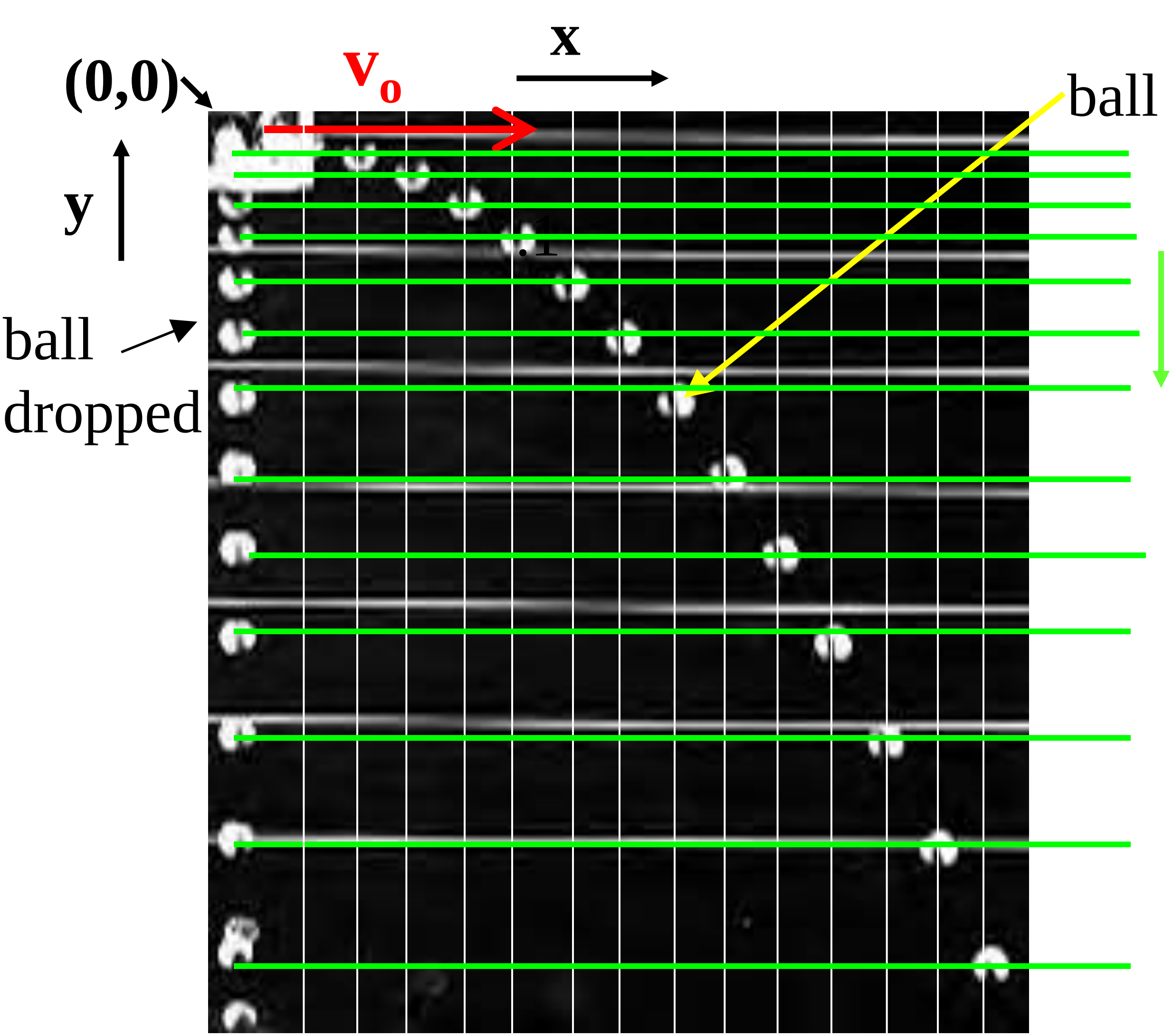
$$\vec{a} \Leftrightarrow \begin{matrix} a_x = 0 \\ a_y = -g \end{matrix} \quad \downarrow g \quad \uparrow +y$$

$$t = 0 \quad \vec{r}_0 \Leftrightarrow (r_{0x}, r_{0y}) \quad \vec{v}_0 \Leftrightarrow (v_{0x}, v_{0y})$$

Initial conditions

- $v_x = v_{0x}$
- $x = x_0 + v_{0x} t$
- $v_y = v_{0y} - g\,t$
- $y = y_0 + v_{0y} t - g \dfrac{t^2}{2}$
- $(y - y_0) = \dfrac{(v_{0y} + v_y)}{2} t = \bar{v}_y t$
- $v_y{}^2 - v_{o_y}{}^2 = 2(-g)(y - y_o)$

Begin with experiment



- increasing displacement in = times ($\sim t^2$)
- acceleration g in y direction

y-dir

projected ball motion = dropped ball motion

$$v_y = -gt$$
$$y = -gt^2/2$$

x-dir

equal displacements in equal times

constant velocity in x-dir

$$v_x = v_{ox}$$
$$x = v_{ox}t$$

http://paer.rutgers.edu/pt3/movies/simdrop.mov

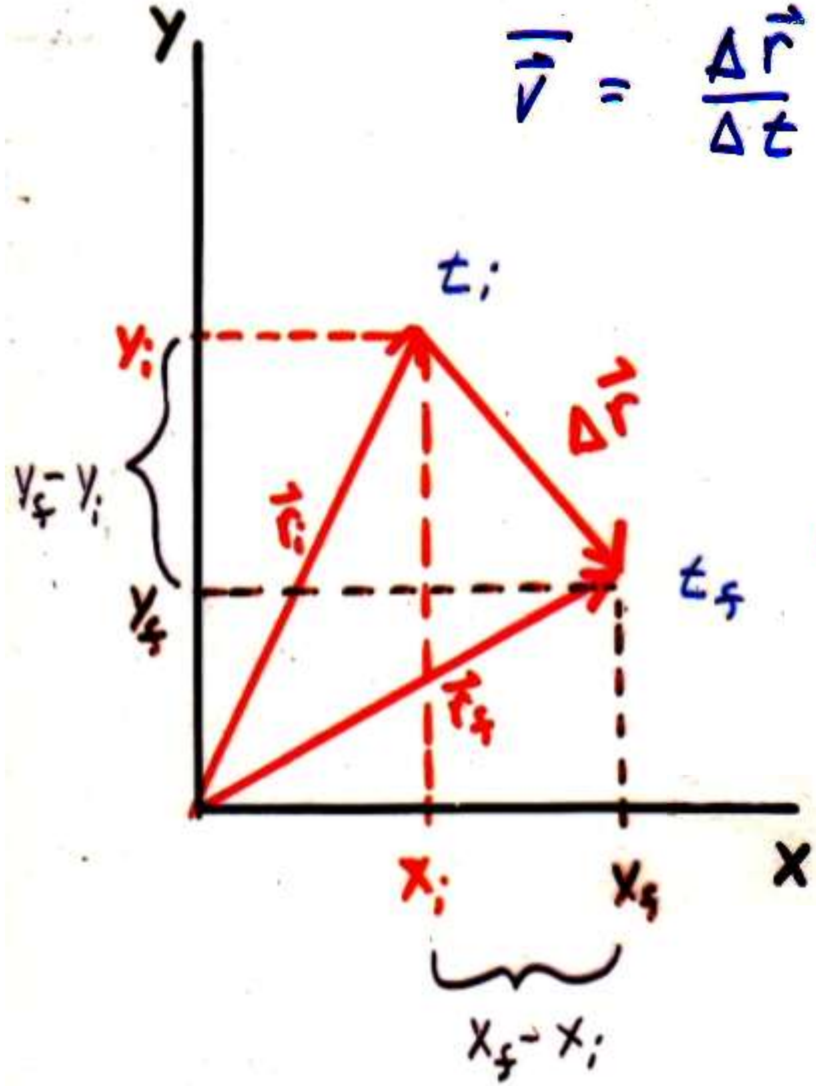
For special case (all objects projected near the surface of the earth)

$$\vec{a} \Leftrightarrow \begin{matrix} a_x = 0 \\ a_y = -g \end{matrix} \quad \downarrow g \quad \uparrow +y$$

$$t = 0 \quad \vec{r}_0 \Leftrightarrow (r_{0x}, r_{0y}) \quad \vec{v}_0 \Leftrightarrow (v_{0x}, v_{0y})$$

Initial conditions

- $v_x = v_{0x}$
- $x = x_0 + v_{0x} t$
- $v_y = v_{0y} - g\,t$
- $y = y_0 + v_{0y} t - g \dfrac{t^2}{2}$
- $(y - y_0) = \dfrac{(v_{0y} + v_y)}{2} t = \bar{v}_y t$
- $v_y^{\,2} - v_{o_y}^{\,2} = 2(-g)(y - y_o)$



$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} \quad \rightarrow \bar{v}_x = \frac{\Delta x}{\Delta t} \quad \rightarrow \bar{v}_y = \frac{\Delta y}{\Delta t}$$

In general

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$\Delta\vec{r}$ x comp = $x_f - x_i$

$\Delta\vec{r}$ y comp = $y_f - y_i$

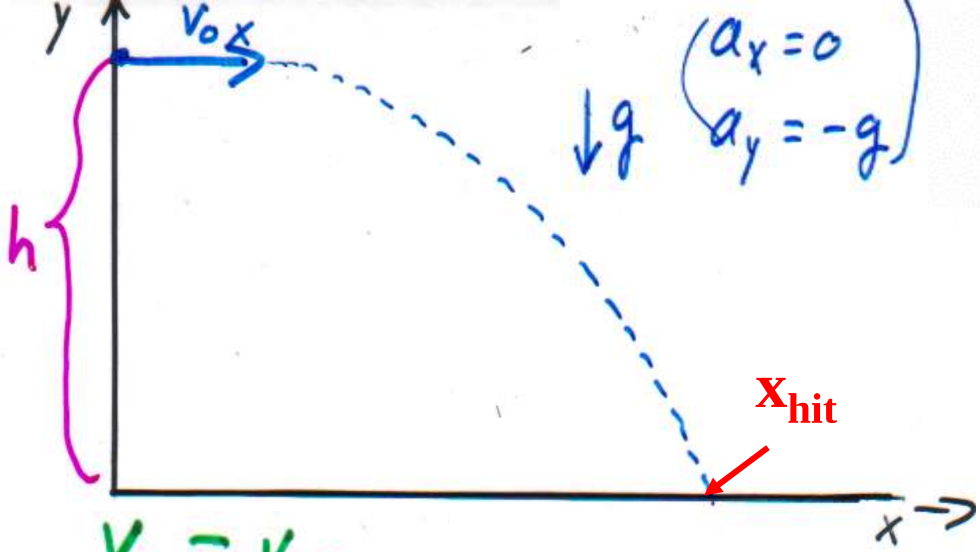
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

Special Case again
$\vec{a}$ = constant magnitude & direction
- usually choose $\vec{a}$ along y axis for convience

Object projected horizontally



$v_x = v_{ox}$

$x - x_0 = v_{ox} t$

$v_y = -gt$

$y = h - \frac{g t^2}{2}$

Use x_{hit} and h to find v_o

$$x_{hit} = v_{ox}\sqrt{\frac{2h}{g}}$$

$$v_{ox} = x_{hit}\sqrt{\frac{g}{2h}}$$

① $t_{hit\ ground}$ & where (x_{hit}, y_{hit})

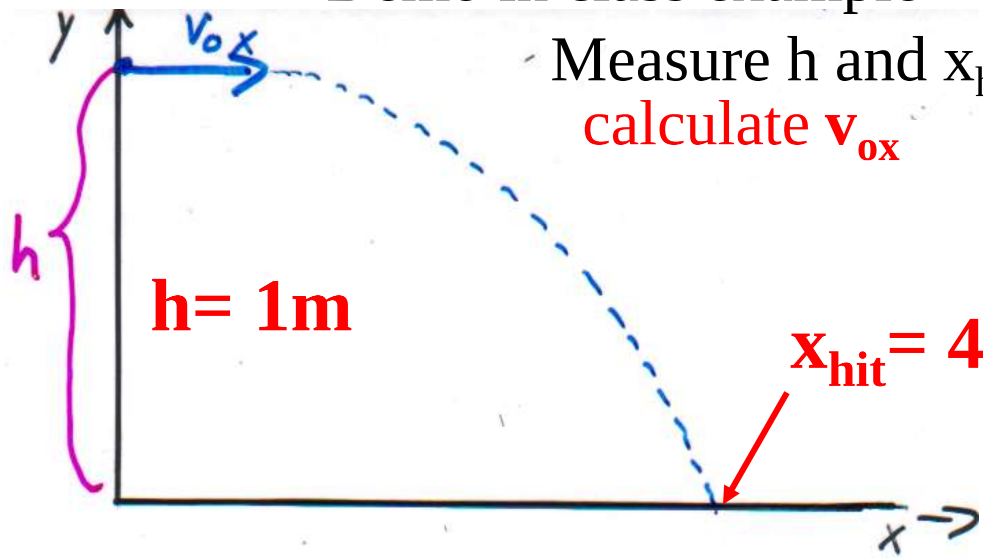
$\underline{\underline{y = 0}}\,! = h - \frac{g t^2}{2} \Rightarrow t_{hit} = \pm\sqrt{\frac{2h}{g}}$

$x = v_{ox} t = \underline{v_{ox}\sqrt{\frac{2h}{g}} = x_{hit}}$; $\underline{y_{hit} \equiv 0}$

Demo in class example

Measure h and x_{hit}

calculate $\mathbf{v_{ox}}$



recall

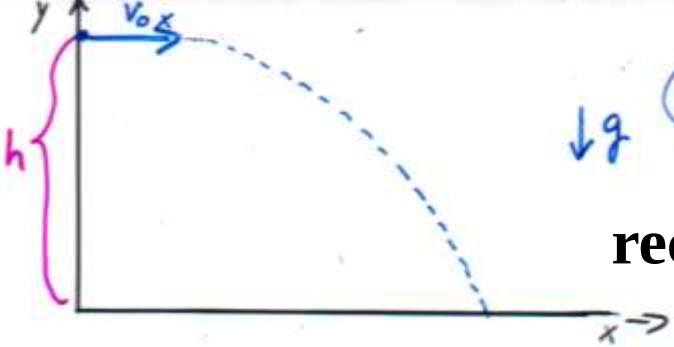
$$t_{hit} = \pm\sqrt{\frac{2h}{g}}$$

$$v_{ox}\,t = v_{ox}\sqrt{\frac{2h}{g}} = x_{hit}$$

$$v_{ox} = \frac{x_{hit}}{t_{hit}} = x_{hit}\sqrt{\frac{g}{2h}} = 4.5\,(m)\sqrt{\frac{9.8\ (m/s^2)}{2\ \ 1\,(m)}} = 11 m/s = v_{ox}$$

$$\sqrt{\frac{g}{2h}} = \frac{1}{t_{hit}}$$

calculate $\vec{v}_{hit}$



$\downarrow g \quad \begin{pmatrix} a_x = 0 \\ a_y = -g \end{pmatrix}$

recall

$v_x = v_{0x}$

$x - x_0 = v_{0x} t \quad (x_0 = 0)$

$y = h - \frac{g t^2}{2}$

$v_y = -g t$

$y = 0 = h - \frac{g t^2}{2} \Rightarrow t_{hit} = \pm\sqrt{\frac{2h}{g}}$

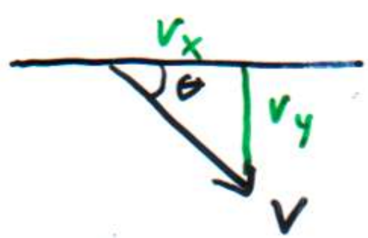
(2) $\vec{v}_{hit}$?

$$v_{y_{hit}} = -g\, t_{hit} = -g\sqrt{\frac{2h}{g}} = -\sqrt{2hg} = v_{y_{hit}}$$

(or) $v_y^2 - 0^2 = 2(-g)(0 - h)$ ← $v_y^2 - v_{o_y}^2 = 2a_y(y - y_o)$

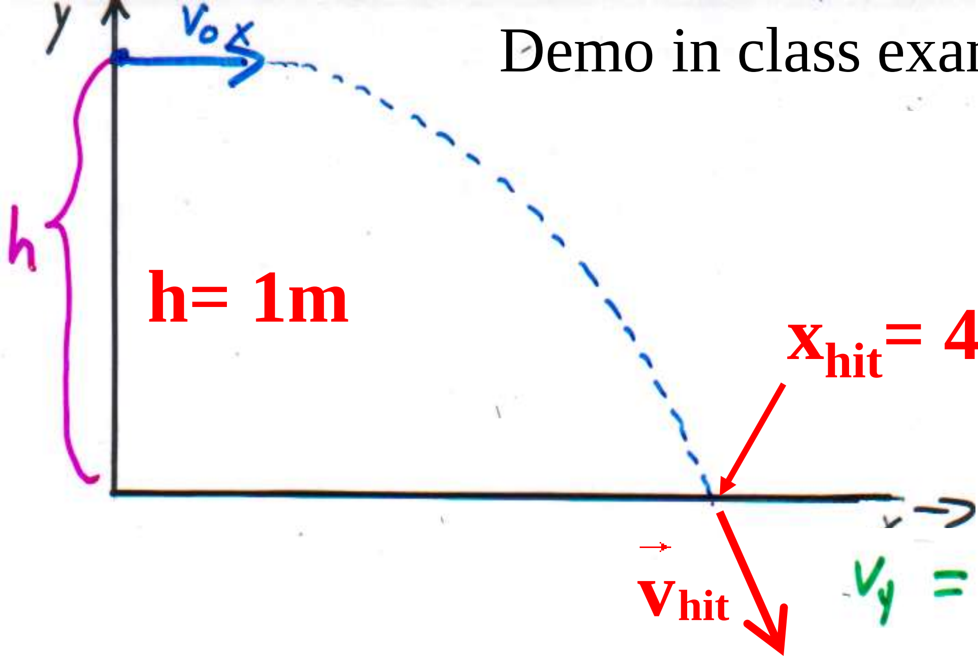
$v_{x_{hit}} = v_{0x}$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + (2gh)}$$



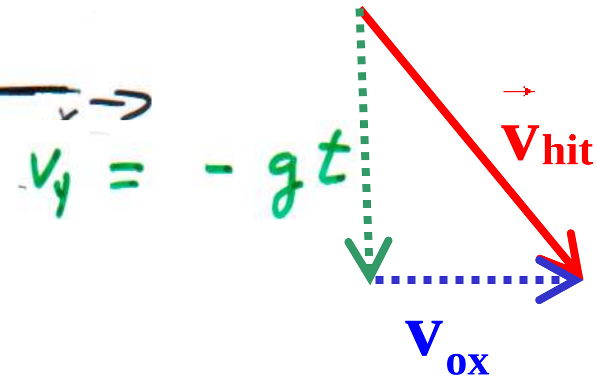
$$\theta = \tan^{-1}\left(\frac{-\sqrt{2hg}}{v_{0x}}\right)$$

Demo in class example



recall

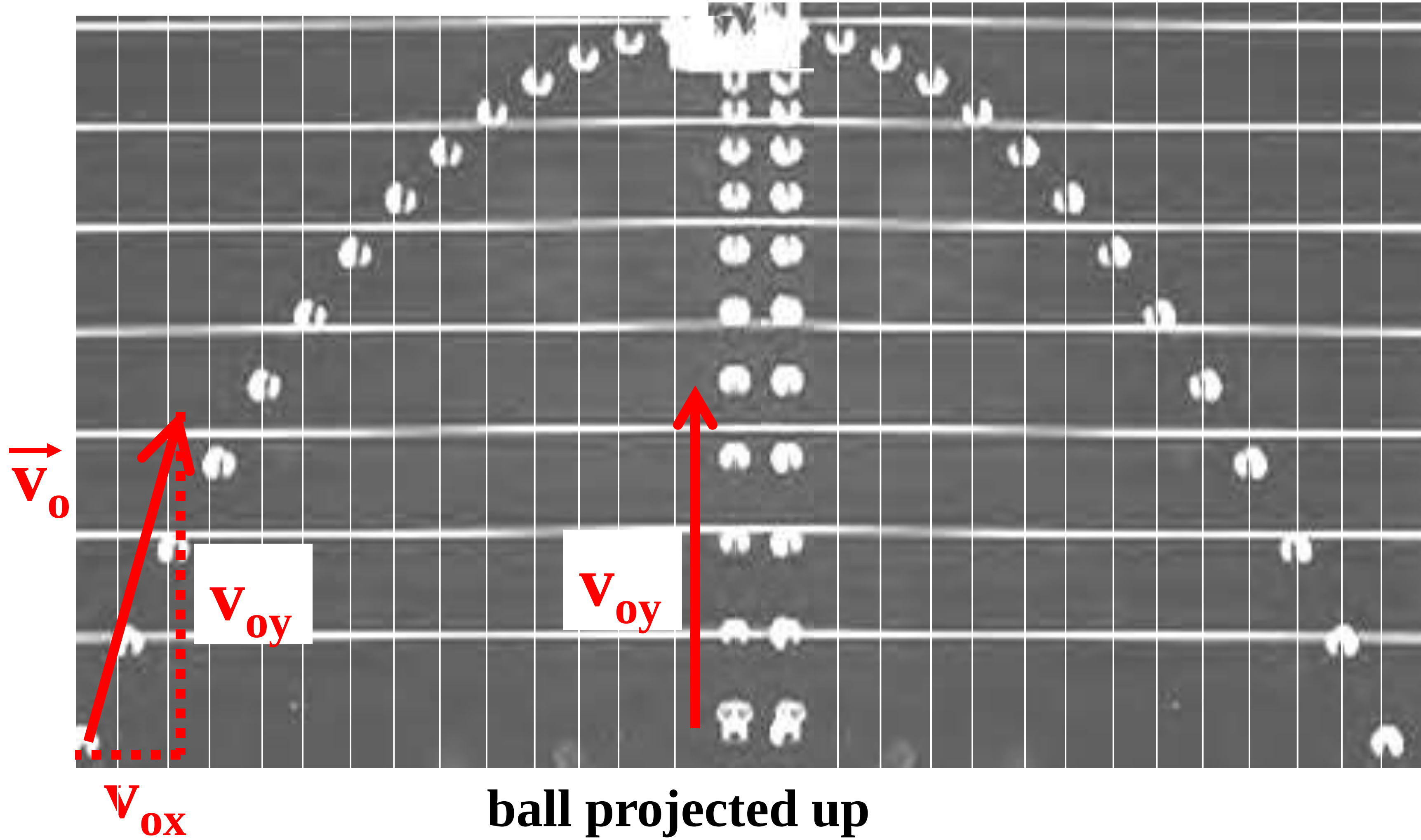
$$v_{ox} = \frac{x_{hit}}{\sqrt{\frac{2h}{g}}} = 11\ \frac{m}{s}$$



$$v_y = -g\,t_{hit} = -\sqrt{2hg} = -\sqrt{18.6\,(m/s)^2} = -4.3\ m/s$$

$$|\vec{v}_{hit}| = \sqrt{v_x^2 + v_y^2} = \sqrt{11^2 + 4.3^2} = \sqrt{121 + 18.5} = 11.8\ m/s$$

ball projected at angle



3-6

ball projected up

ball projected at angle

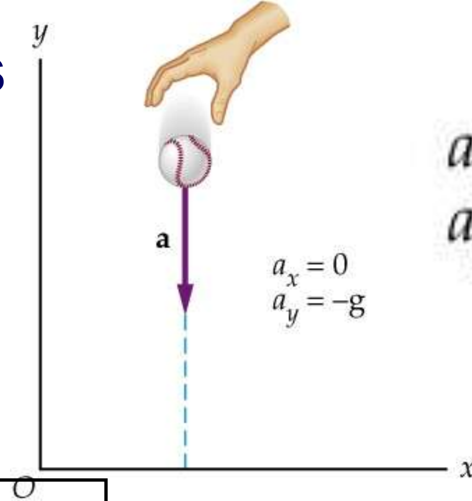
$$v_x = v_{ox}$$
$$x = v_{ox}t$$

constant velocity v_{0x}

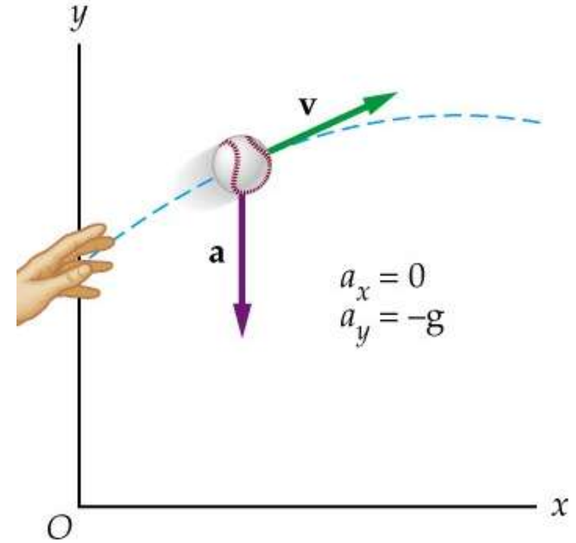
$$y = v_{oy}t - gt^2/2$$
$$v_y = v_{oy} - gt$$

just like ball projected up with v_{0y}

Some Observations



$$a_x = 0$$
$$a_y = -g$$



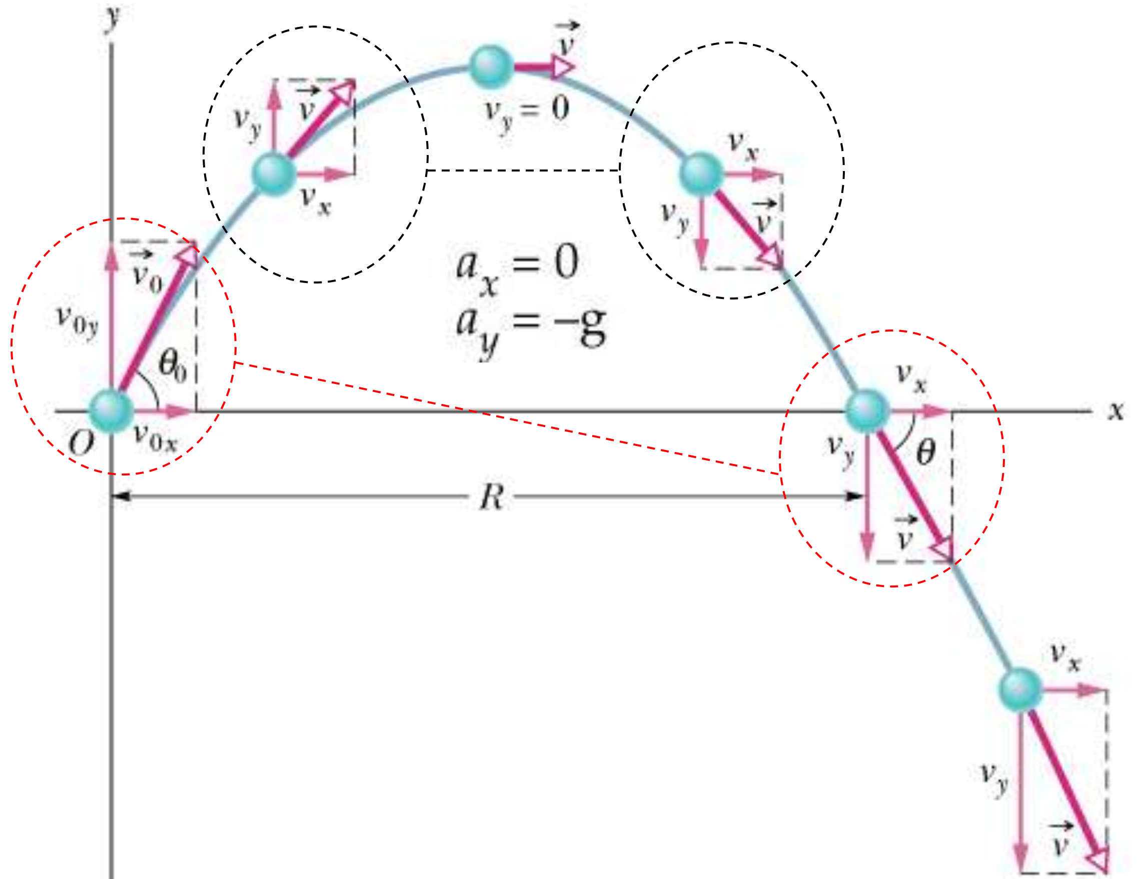
<u>Note the following:</u>

V_X and V_Y independent

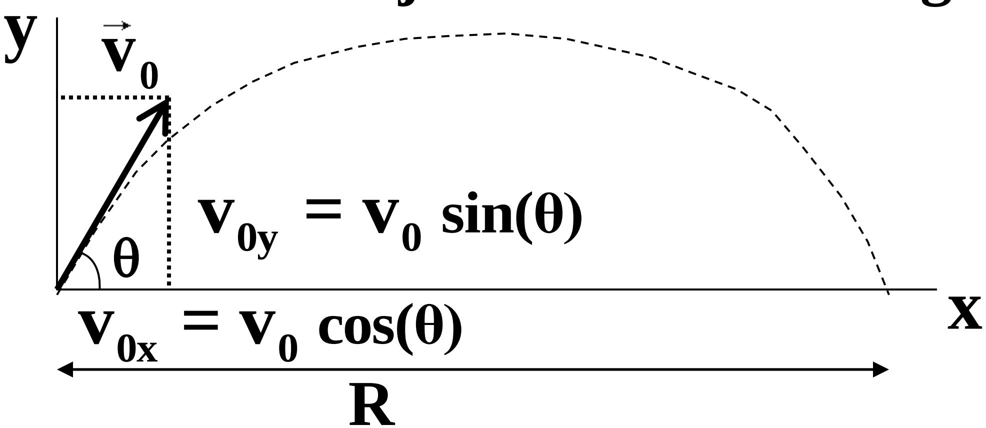
V_X remains constant

The symmetries in V_Y
The signs differ, but magnitudes are same

Vector analysis ⇒ V is the slope of the curve.



Projectile motion: Range



$$v_{0y} = v_0 \sin(\theta)$$

$$v_{0x} = v_0 \cos(\theta)$$

x-dir. const. vel.

$$v_x = v_{0x}$$

$$x = v_{0x} t$$

y-dir. const. acc.

$$v_y = v_{0y} - g\,t$$

$$y = v_{0y} t - g\frac{t^2}{2}$$

Range, R: when it hits the ground (y=0)

$$\therefore \quad y = 0 = v_{0y} t - g\frac{t^2}{2}$$

key !

$$0 = [t][v_{0y} - g\frac{t}{2}]$$

$$t = 0$$

t=0 shot from y=0

$$0 = [v_{0y} - g\frac{t}{2}]$$

$$t_{hit} = \frac{2v_{0y}}{g}$$

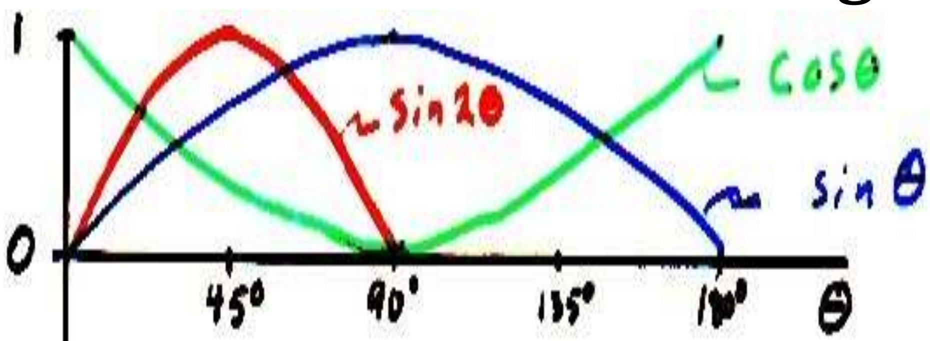
t_{hit} hits the ground at y=0

$$R = x(t_{hit}) = v_{0x} t_{hit} = v_{0x}\frac{2v_{0y}}{g}$$

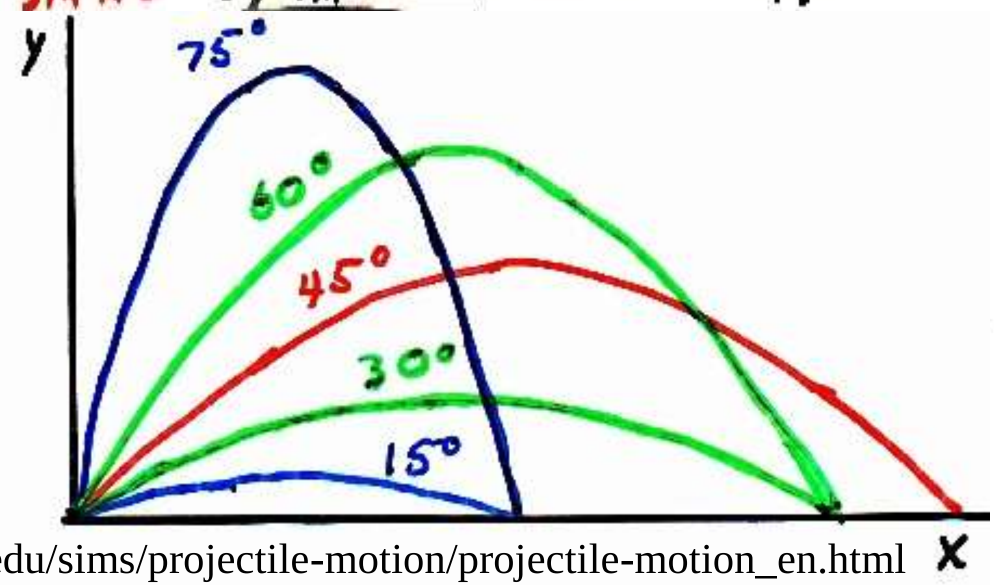
$$R = \frac{2v_{0x} v_{0y}}{g}$$

so $$R = \frac{2 v_{0x} v_{0y}}{g}$$ but $$v_{0y} = v_0 \sin(\theta)$$ $$v_{0x} = v_0 \cos(\theta)$$

$$\therefore \quad R = \frac{v_0^2}{g} 2 \sin(\theta)\cos(\theta) = \frac{v_0^2}{g} \sin(2\theta)$$



Note → sin 2θ symmetrical about θ = 45°



$R = \frac{v_0^2}{g} \sin 2\theta$

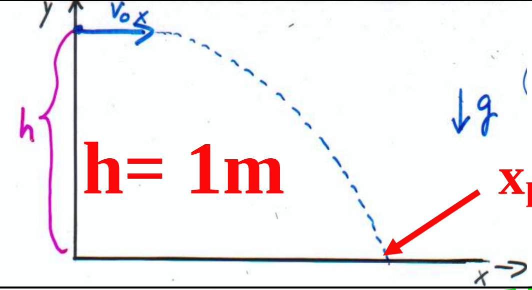
symmetrical about θ = 45°

- 45° max. range
- 45° ± φ have the same R

different max h & flight time

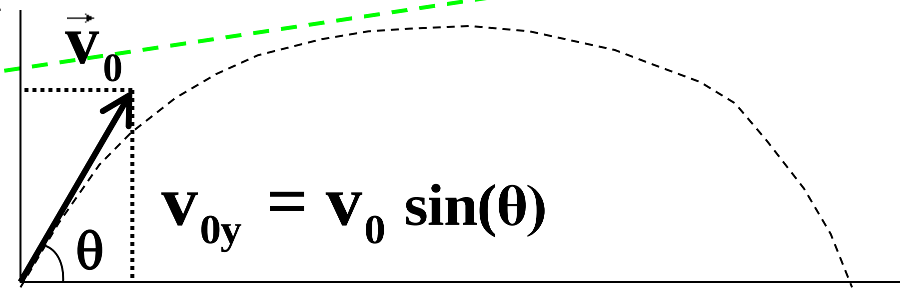
http://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html

2^{nd} class Demo : range same “gun” =same v_0



Recall Demo in class

$$v_{ox} = x_{hit}\sqrt{\frac{g}{2h}} = 11 \ \frac{m}{s}$$



$$\mathbf{v_{0y} = v_0 \sin(\theta)}$$

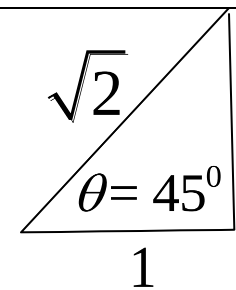
$$\mathbf{v_{0x} = v_0 \cos(\theta)}$$

$$\mathbf{R = \frac{2\, v_{ox}\, v_{oy}}{g}}$$

Recall

$$\theta = 45^0 \Rightarrow R_{max}$$

$$\mathbf{R_{max} = \frac{2\, v_o^{\,2}}{g(\sqrt{2})^2} = \frac{v_o^{\,2}}{g}}$$



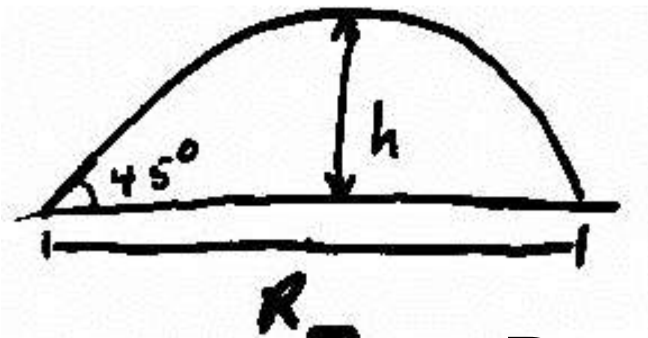
$$\sin(45^0) = \cos(45^0) = \frac{1}{\sqrt{2}}$$

$$R_{max}\,(\text{class demo}) = \frac{11^2}{9.8}\left[\frac{(m/s)^2}{m/s^2}\right] = 12.3m$$

good agreement

Scud missiles Iraq to Israel 1991 [~most of trajectory is that of a projectile]

Recall range problem max range θ= 45° clearly θ= 45° used to get max range



R_m=? v_0=?
h=?

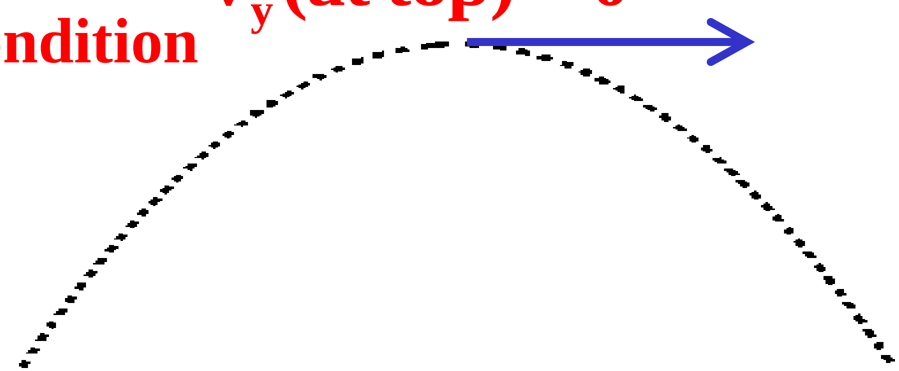
Time to get to max height (h)

$$R_{max} = \frac{2\,v_o^{\,2}}{g(\sqrt{2})^2} = \frac{v_o^{\,2}}{g} \qquad \theta = 45^o \qquad \sin(\theta) = \frac{1}{\sqrt{2}}$$

Time to get to max height (h)

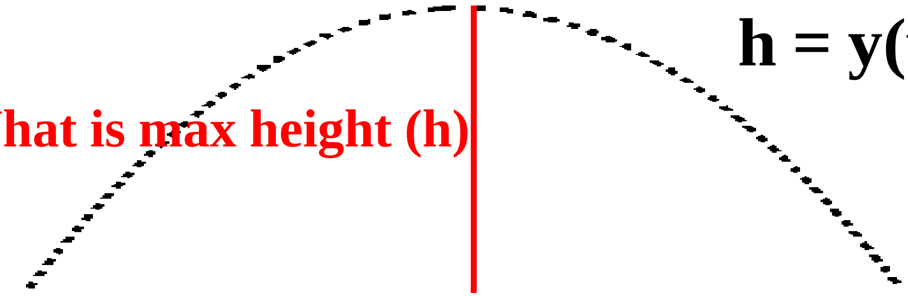
$$v_y = v_{oy} - gt = 0$$

crucial condition v_y **(at top) = 0**



$$\Rightarrow \quad t_{top} = \frac{v_{oy}}{g} = \frac{v_o \sin(\theta)}{g}$$

$$\theta = 45^o \quad t_{top} = \frac{v_o}{g\sqrt{2}}$$



$$h = y(t_{top}) = v_{oy}\, t - \frac{g}{2} t^2$$

$$t_{top} = \frac{v_{oy}}{g}$$

$$h = v_{oy} \frac{v_{oy}}{g} - \frac{g}{2}\left(\frac{v_{oy}}{g}\right)^2$$

$$h = \frac{[v_{oy}{}^2]}{2g}$$

$\theta = 45^o$

$$v_{oy} = \frac{v_o}{\sqrt{2}}$$

$\theta = 45^o$

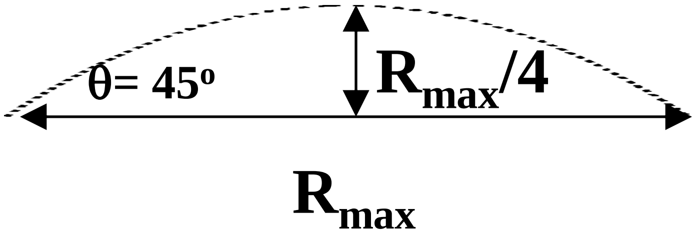
$$h = [\frac{v_o{}^2}{2}]\frac{1}{2g}$$

$$h = \frac{v_o{}^2}{4g}$$

But recall

$$R_{max} = \frac{v_o{}^2}{g}$$

$$h = \frac{1}{4} R_{max}$$

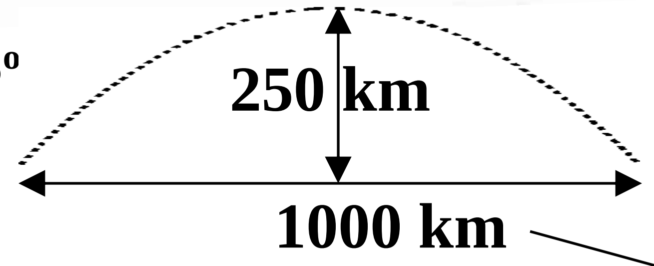


Scud missiles case 1991

$R_{max} \approx 1000\,\text{km} = 10^6\,\text{m}$ — Israel ≈ 1000 km Iraq

• $h = \frac{1}{4} R_m \approx 250\,\text{km} = 2.5(10)^5\,\text{m}$ θ= 45º

θ= 45º

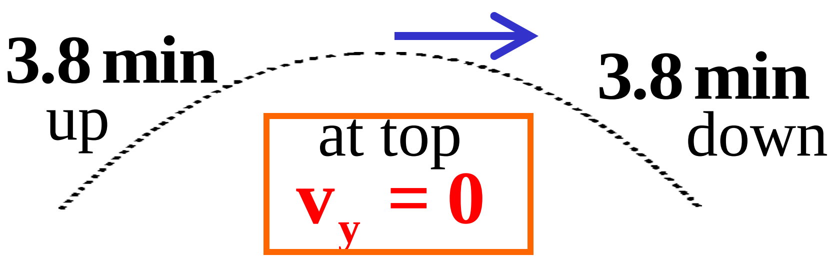


$$R_m = \frac{v_o^2}{g} \Rightarrow v_o = \sqrt{R_m g}$$

$$v_o = \sqrt{10^6\,[\text{m}]\,9.8\,[\text{m/s}^2]} = 3200\,\text{m/s}$$

θ= 45º $t_{top} = \frac{v_o}{g\sqrt{2}}$

$$t_{top} = \frac{3200\,[\text{m/s}]}{9.8\,[\text{m/s}^2]\sqrt{2}} = 229\,\text{s} = 3.8\,\text{min}$$



7.6 min total warning time

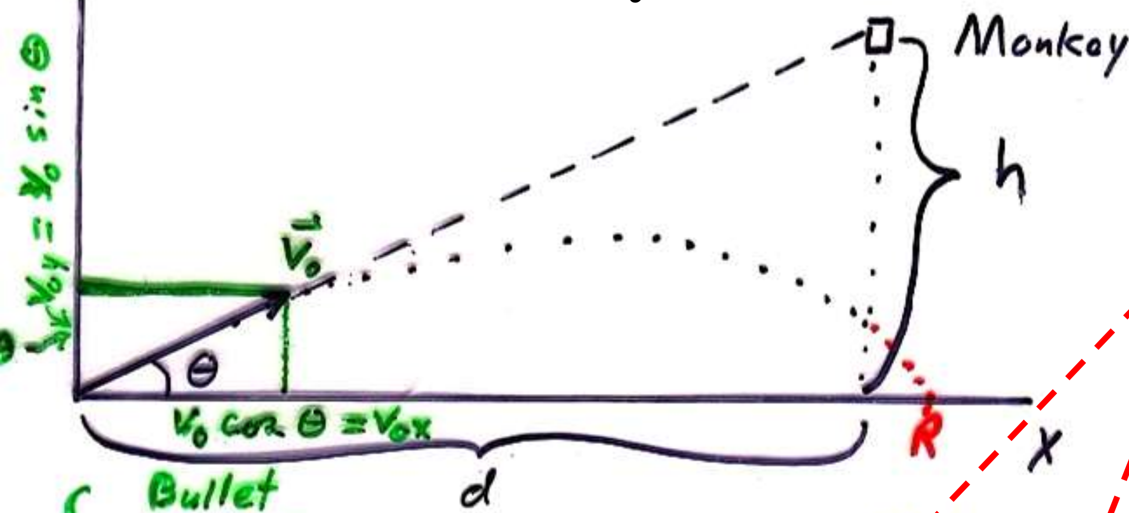
θ= 45º

$$v_x = v_{0x} = \frac{v_0}{\sqrt{2}} = 2300\,\text{m/s}$$

At top slowest

– try to intercept at top

"shoot the monkey" demo



Bullet

$$\begin{cases} x = v_{0x} t \\ y = v_{0y} t - \frac{g}{2} t^2 \end{cases}$$

Monkey

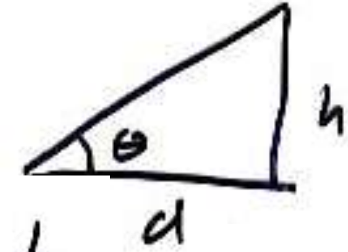
$$\begin{cases} y_m = h - \frac{g}{2} t^2 \\ x_m = d \end{cases}$$

To hit monkey

$$\left.\begin{aligned} x &= x_m \\ y &= y_m \end{aligned}\right\} \text{ at same time } t$$

$$x = x_m$$
$$v_{0x} t = d$$
$$\Downarrow$$
$$t = d/v_{0x}$$

$$y = y_m$$
$$v_{0y} t - g\frac{t^2}{2} = h - g\frac{t^2}{2}$$
$$\Downarrow$$
$$v_{0y} t = h$$



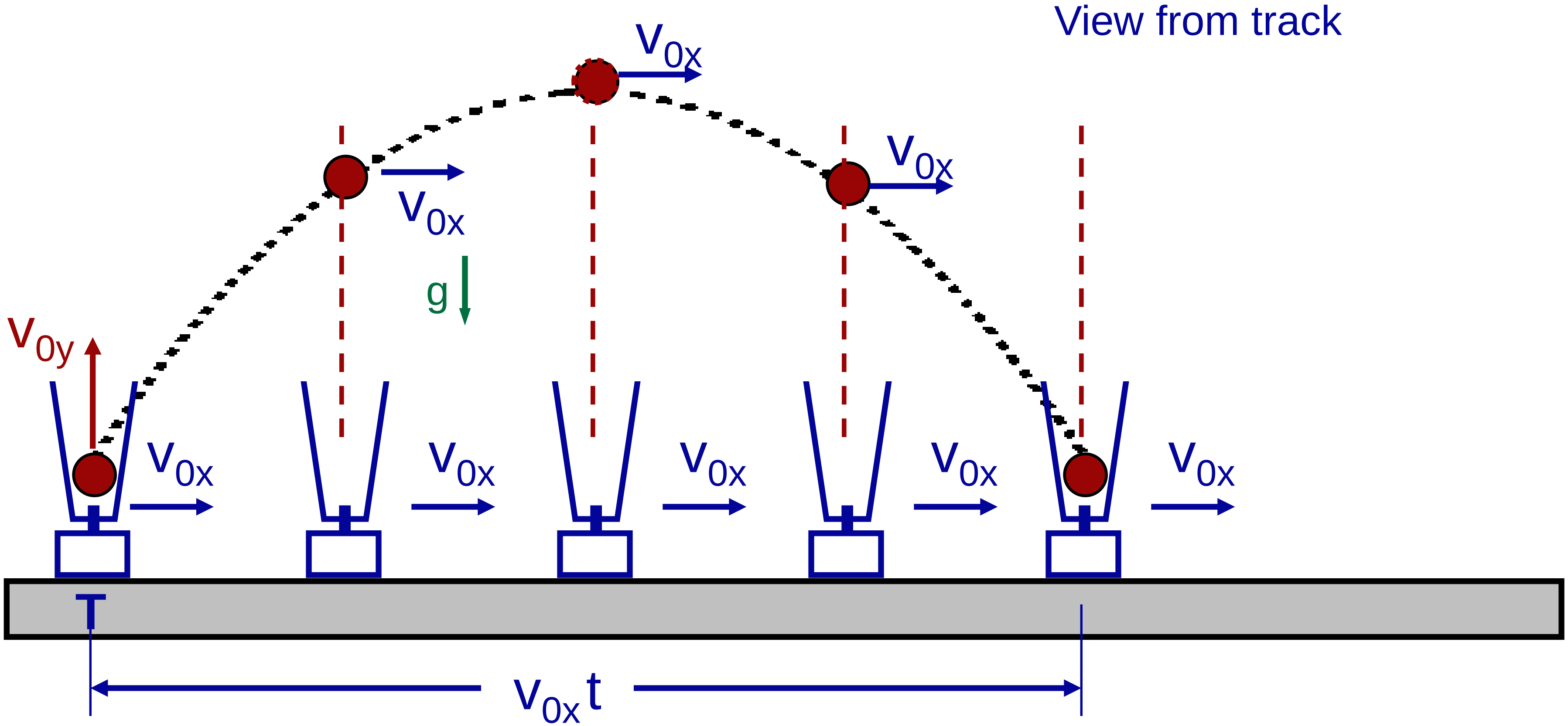
$$v_{0y} \frac{d}{v_{0x}} = h$$

$$\frac{v_0 \sin\theta \, d}{v_0 \cos\theta} = h$$

$$\boxed{\tan\theta = \frac{h}{d}}$$

Aim at monkey!!

Howitzer Cart



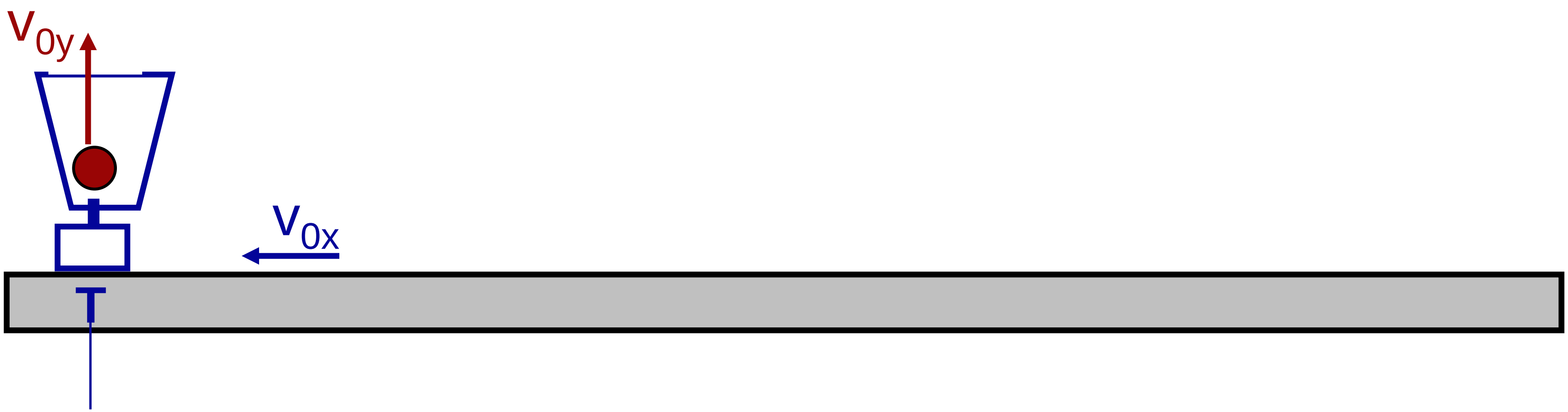
Cart moves at constant velocity. As the cart passes point "T" a firing pin is triggered, shooting the ball straight up. Where will the ball land?

x – motion =constant velocity for both

A. BACK IN CART

Howitzer Cart

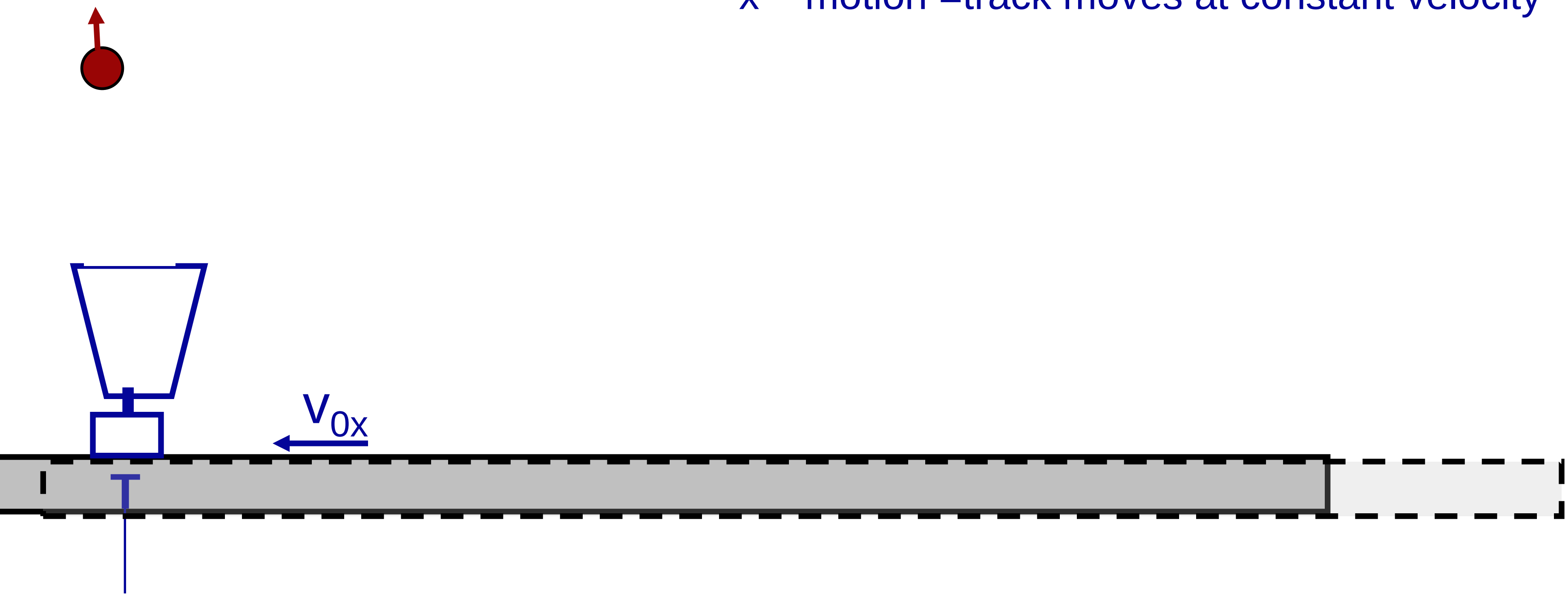
View from cart
x – motion =track moves at constant velocity



A. OBVIOUSLY BACK IN CART

Howitzer Cart

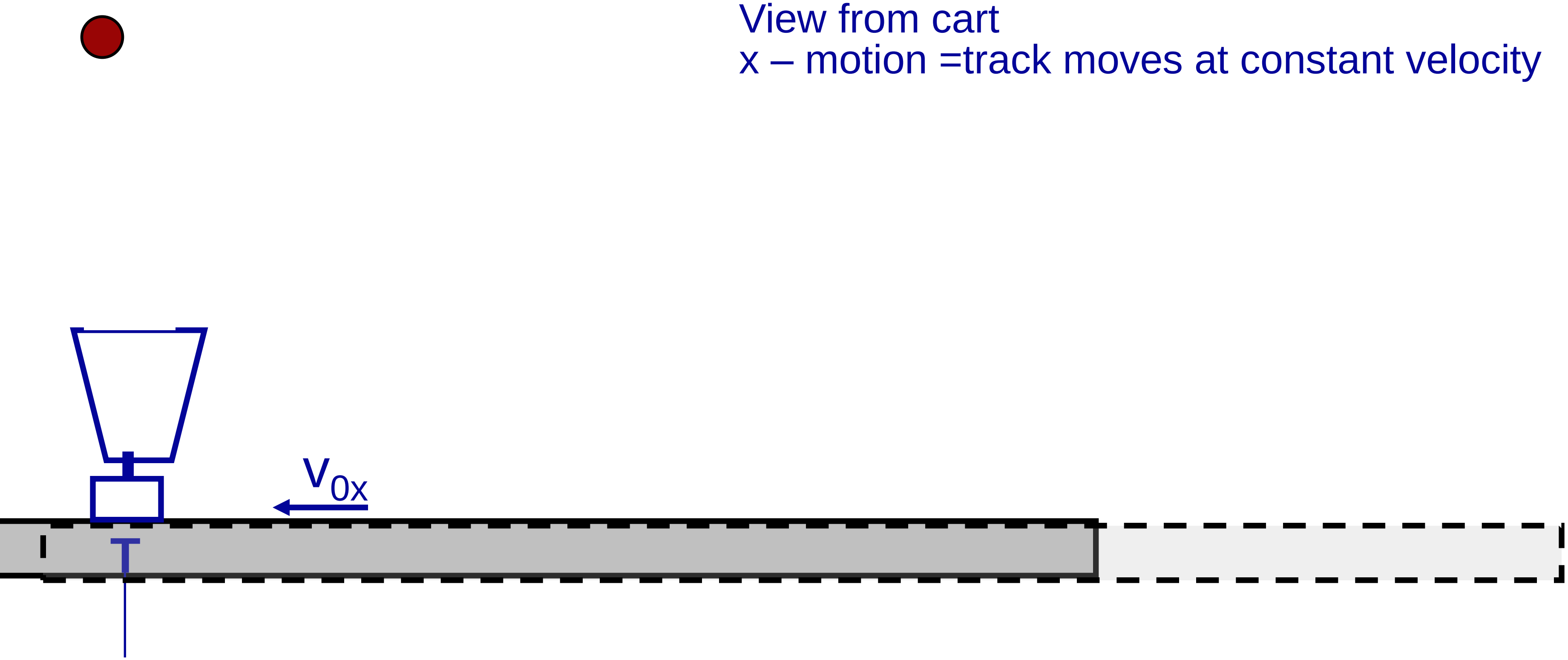
View from cart
x – motion =track moves at constant velocity



A. OBVIOUSLY BACK IN CART

Howitzer Cart

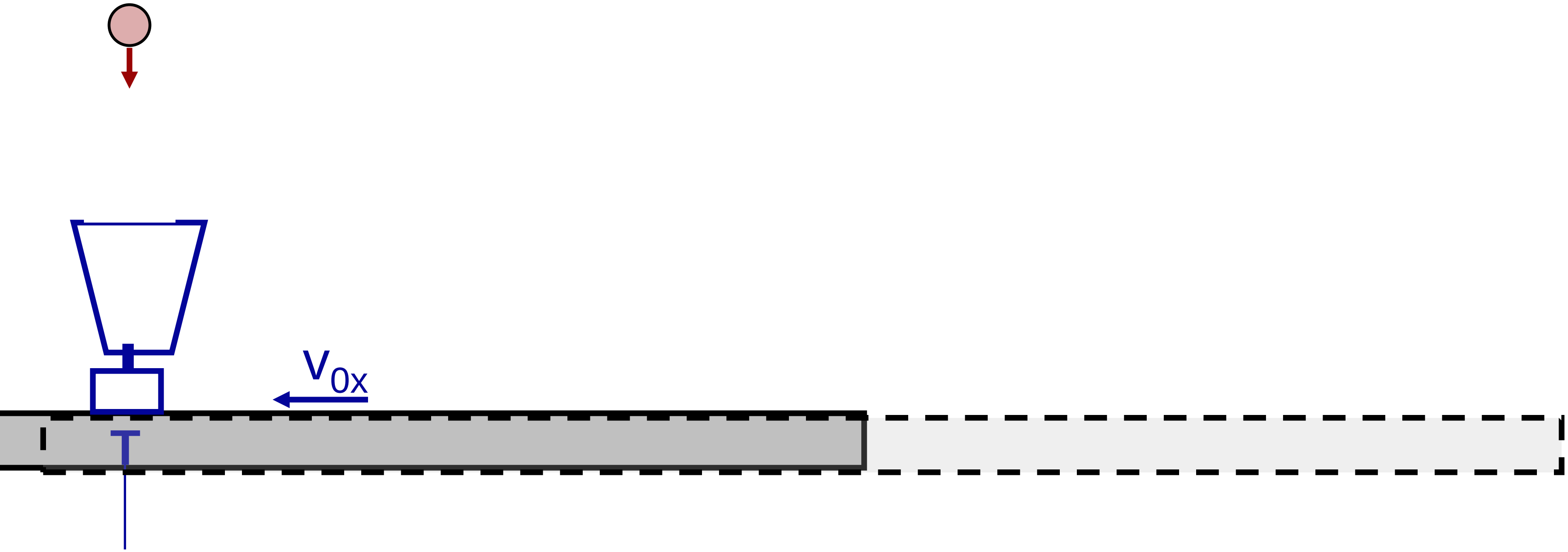
View from cart
x – motion =track moves at constant velocity



A. OBVIOUSLY BACK IN CART

Howitzer Cart

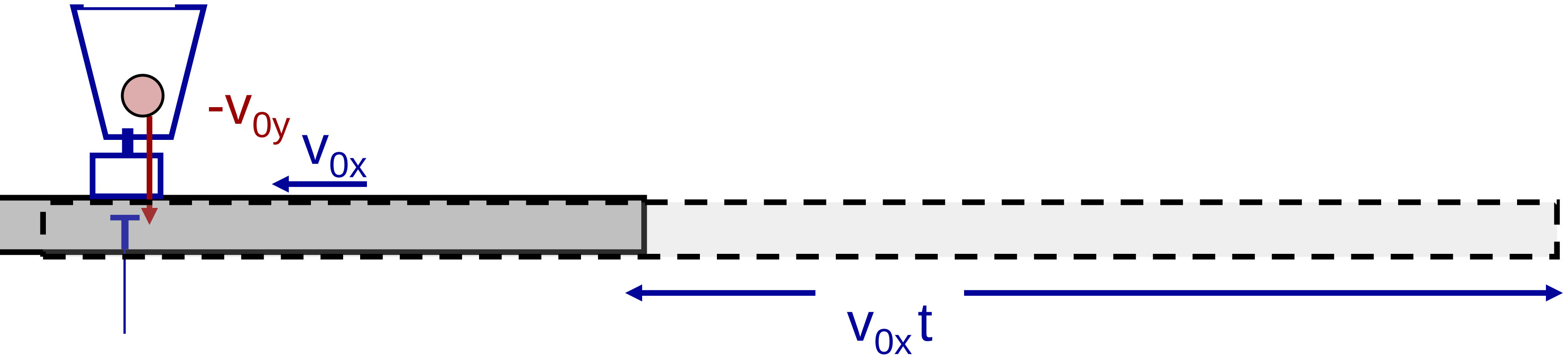
View from cart
x – motion =track moves at constant velocity



A. OBVIOUSLY BACK IN CART

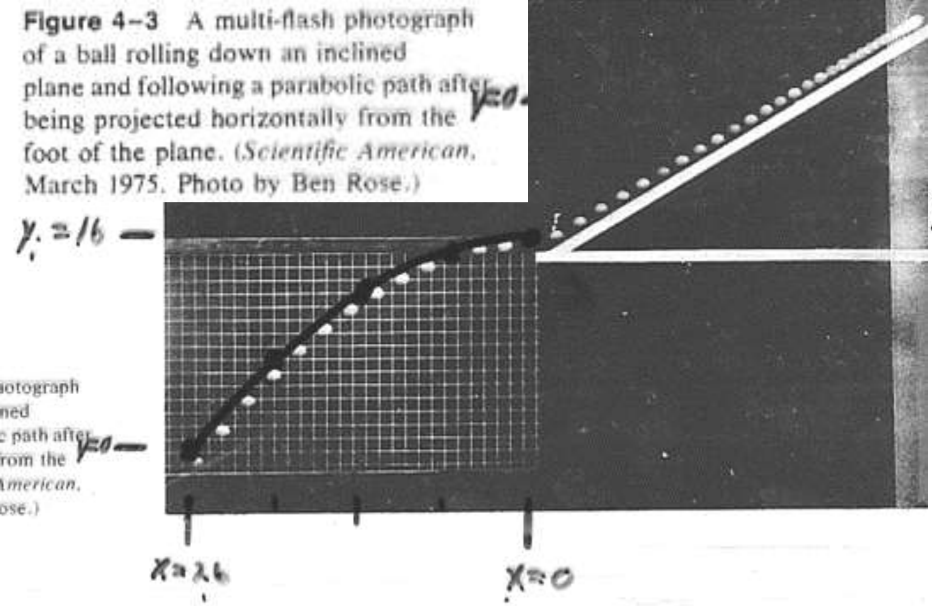
Howitzer Cart

View from cart
x – motion =track moves at constant velocity



A. OBVIOUSLY BACK IN CART

Figure 4–3 A multi-flash photograph of a ball rolling down an inclined plane and following a parabolic path after being projected horizontally from the foot of the plane. (*Scientific American*, March 1975. Photo by Ben Rose.)



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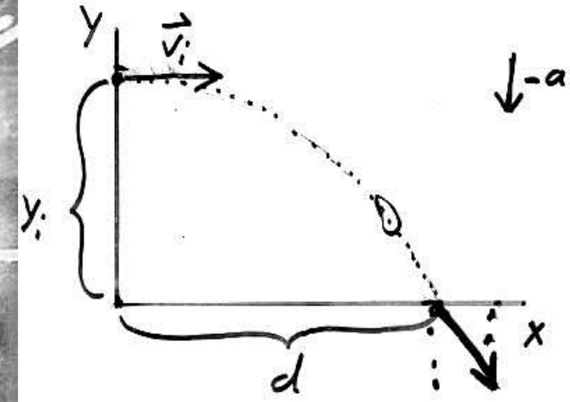

$a_x = 0$

$a_y = -a$

$t = 0$ $\left\{ \begin{array}{l} v_{ix} = v_i \\ v_{iy} = 0 \\ x_i = 0 \\ y_i = y_i \end{array} \right.$

$v_x = v_i$	$x = v_i t$ (1)
$v_y = -at$	$y = y_i - \frac{a}{2} t^2$ (2)

x	x exp	y	y exp
0	0	$y_i = 16$	16
$d/4$	6.5	$y_i\ 15/16 = 15$	15
$d/2$	13	$y_i\ 3/4 = 12$	12
$3d/4$	19.5	$y_i\ 7/16 = 7$	7
d	26	0	0

↕

$y = y_i \left[1 - \frac{x^2}{d^2}\right]$

- What is path in x, y plane

solve (1) for t $\quad t = x/v_i$

put into (2) $\quad y = y_i - \frac{a}{2}\left(\frac{x}{v_i}\right)^2$

use the fact $x = d$ when $y = 0$

$$0 = y_i - \frac{a}{2}\frac{d^2}{v_i^2}$$

$$\Rightarrow \quad \frac{a}{2 v_i^2} = y_i / d^2$$

$$\therefore \quad y = y_i \left[1 - x^2/d^2\right]$$