1D Motion

in general

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \Rightarrow \Delta x = \bar{v}\,\Delta t$$

$$\bar{a} = \frac{\Delta v}{\Delta t} \qquad \Delta v = \bar{a}\,\Delta t$$

Δx = area under $v(t)$ curve

$v(t)$ = slope of $x(t)$ curve

$a(t)$ = slope of $v(t)$ curve

For constant acceleration (a=constant) [Important special case]

ACTUALLY WILL OFTEN USE "y" NOT "x"

Const. a only → $\bar{v} = \frac{(v_f + v_i)}{2}$

$$t = 0,\ x = x_0\ ,\ v = v_0$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + a\frac{t^2}{2}$$

$$v^2 - v_o^{\,2} = 2a\,(x - x_o)$$

equivalent relations to attack problems

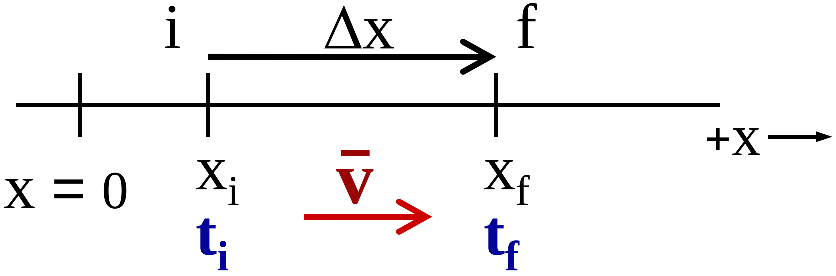
$$\bar{v} = \frac{(v + v_0)}{2}$$

$$\Delta x = \bar{v}\,\Delta t$$

Will derive later

- 1st experiment and problems

1 Dimensional Kinematics



- **displacement**
  - -how far
  - -what direction

$$\boxed{\Delta x = x_f - x_i}$$

units [m]

- **time interval**

$$\Delta t = t_f - t_i$$

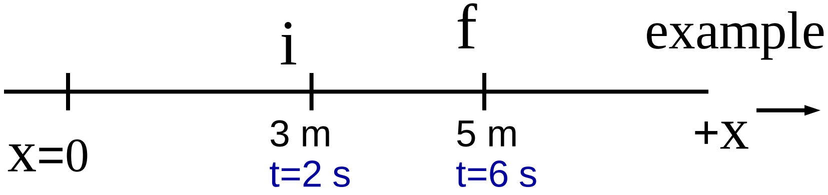
units [s]

- **average velocity** **units** [m/s]

$$\boxed{\bar{v} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v}\,\Delta t}$$

-how fast
-what direction

example 1



$$\Delta x = x_f - x_i = 5 - 3 = +2\text{ m}$$

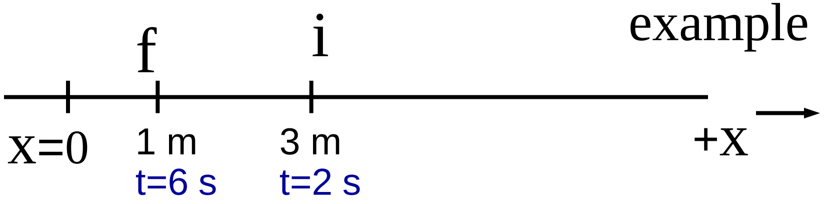
positive (+) => to the right!

$$\Delta t = 6 - 3 = 3\text{s}$$

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{5-3}{6-2} = +0.5\text{ m/s}$$

positive (+) => to the right!

example 2



$$\Delta x = x_f - x_i = 1 - 3 = -2\text{ m}$$

negative (-) => to the left!

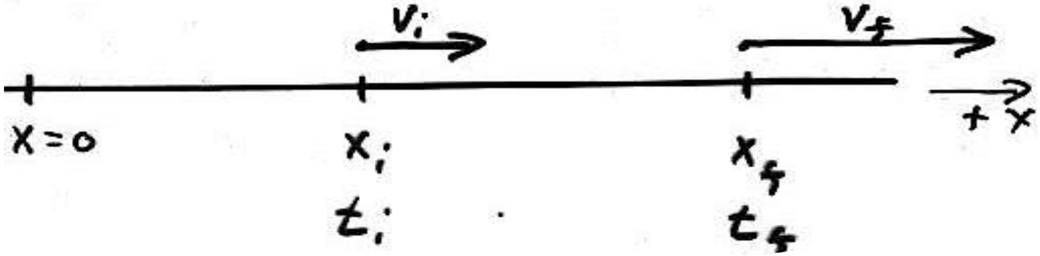
$$\Delta t = 6 - 2 = 4\text{s}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-2}{4} = -0.5\text{ m/s}$$

negative (-) => to the left!

Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \boxed{= \frac{dx}{dt}}$$



Instantaneous Acceleration

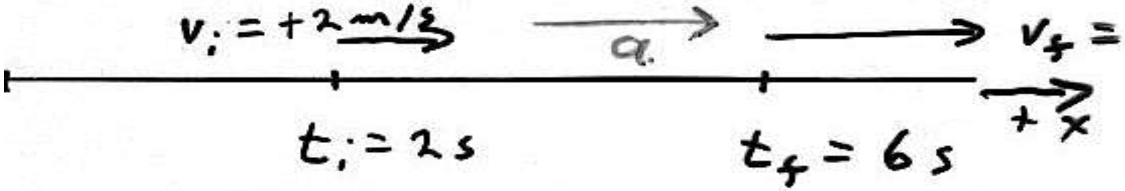
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Average acceleration

$$\boxed{\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}}$$

[Units]

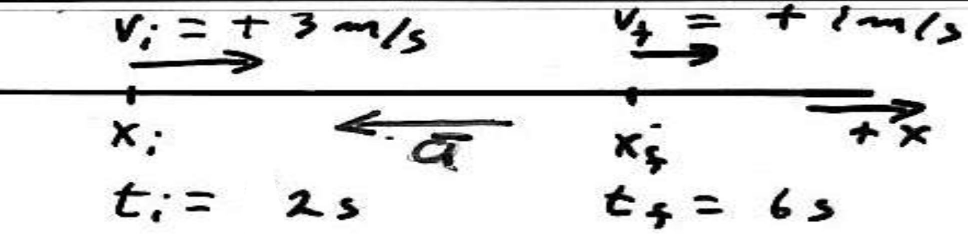
$$\left[\frac{\frac{m}{s}}{s} = \frac{m}{s^2}\right]$$



$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{5-2}{6-2} \frac{\text{m/s}}{\text{s}} = \frac{3}{4} \frac{m}{s^2}$$

$$\bar{a} = +0.75 \frac{m}{s^2}$$

to right



$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{1-3}{6-2}\,\frac{m/s}{s} = -\frac{2}{4}\,\frac{m}{s^2}$$

$$\bar{a} = -0.5\,\frac{m}{s^2}$$

↑

– means "to left"

Note! $\bar{a} = \frac{\Delta v}{\Delta t} \Longrightarrow \boxed{\Delta v = \bar{a}\,\Delta t}$

example

$$\bar{a} = -0.5\ m/s^2$$

$$\Delta t = 4\ s$$

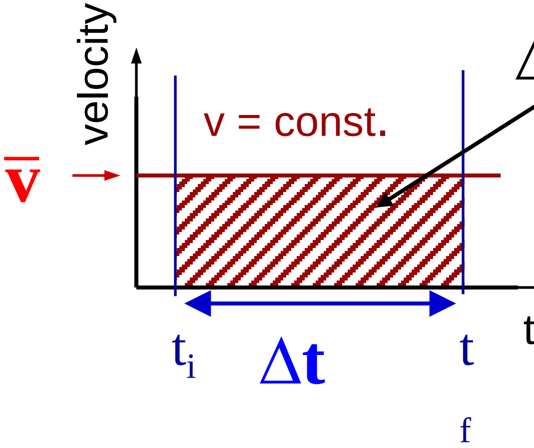
$$\Delta v = \left(-0.5\,\frac{m}{s^2}\right)(4\ s) = \bar{a}\,\Delta t$$

$$\boxed{\Delta v = -2\,\frac{m}{s}}$$

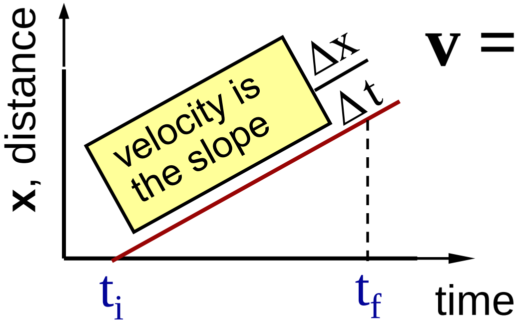
Graphical concepts

$$\mathbf{v = constant = \bar{v}\,,\ a = 0}$$

$$\Delta x = \bar{v}\,\Delta t$$



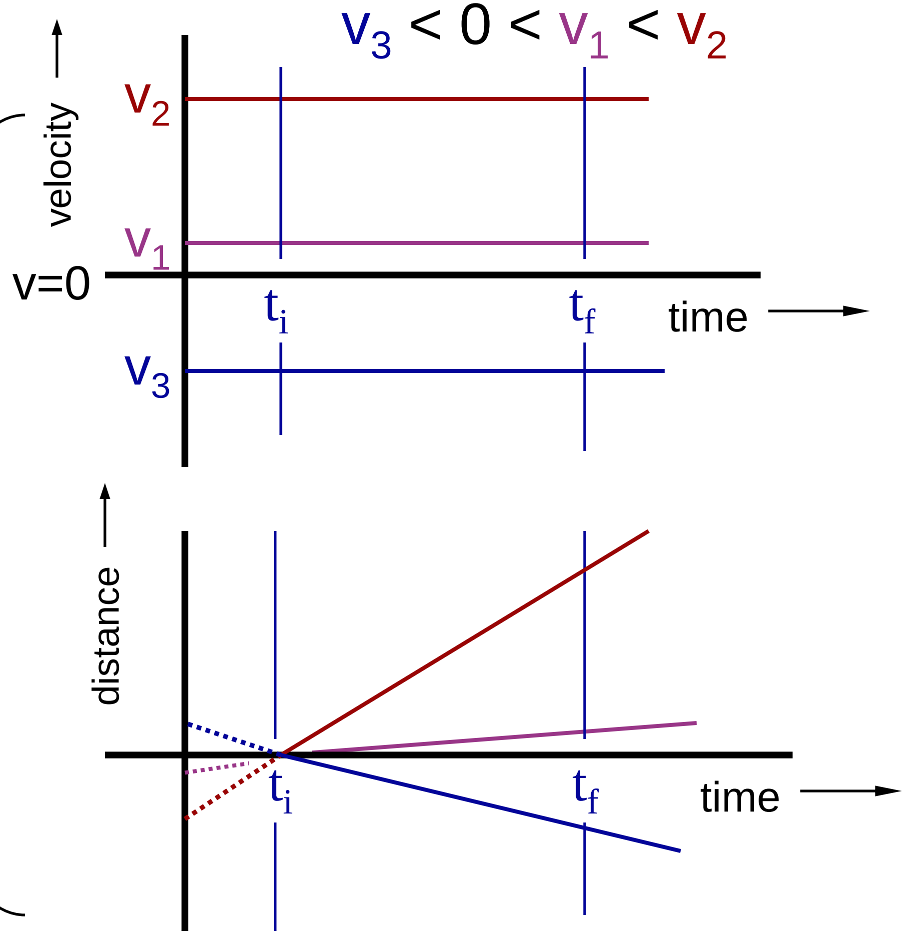
Displacement = integrated area under the velocity curve



$$v = \frac{\Delta x}{\Delta t}$$

Example - 3 Velocities:

$v_3 < 0 < v_1 < v_2$



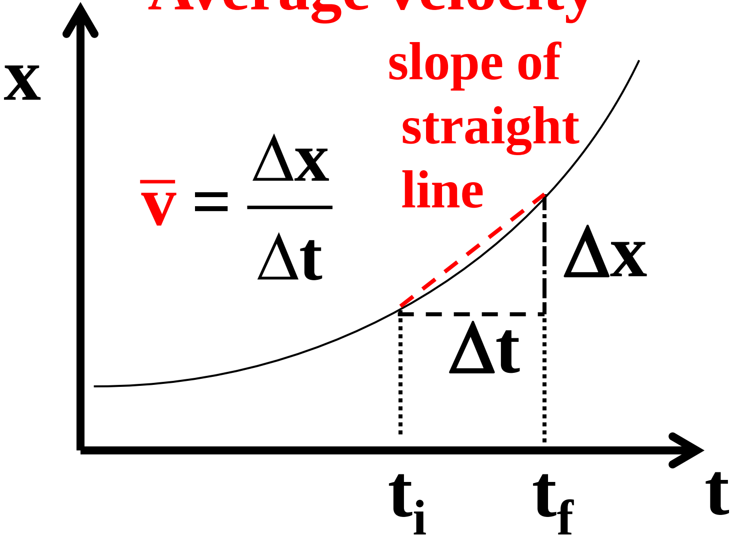
For v_1, v_2, and v_3 :

Which velocity is the slowest?

Which velocity is moving to the left?

General graphical concepts

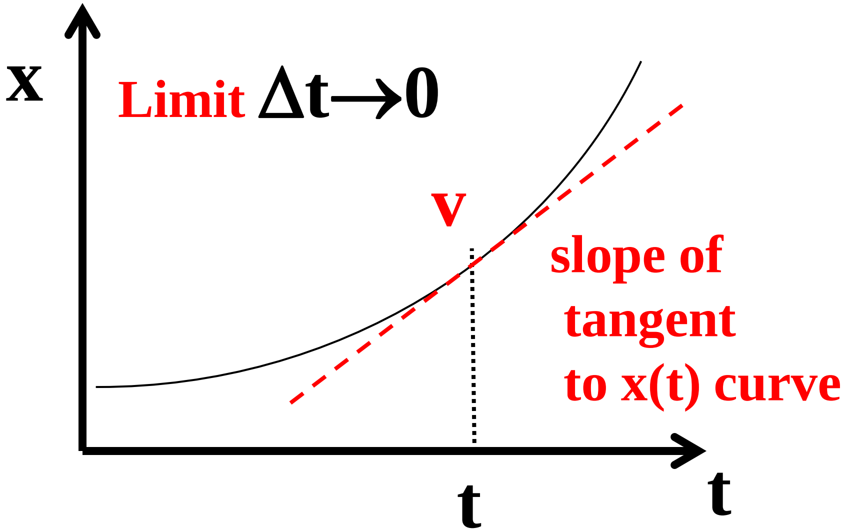
Average velocity



$$\bar{v} = \frac{\Delta x}{\Delta t}$$

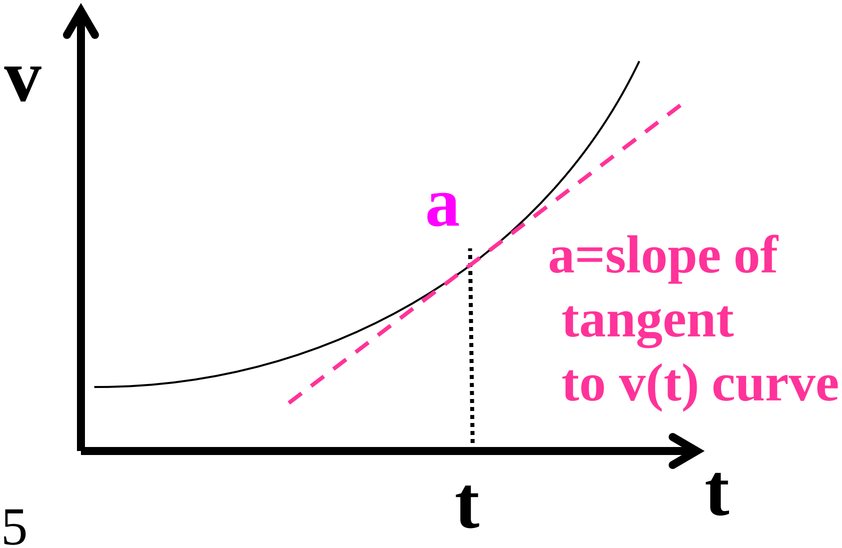
slope of straight line

Instantaneous velocity

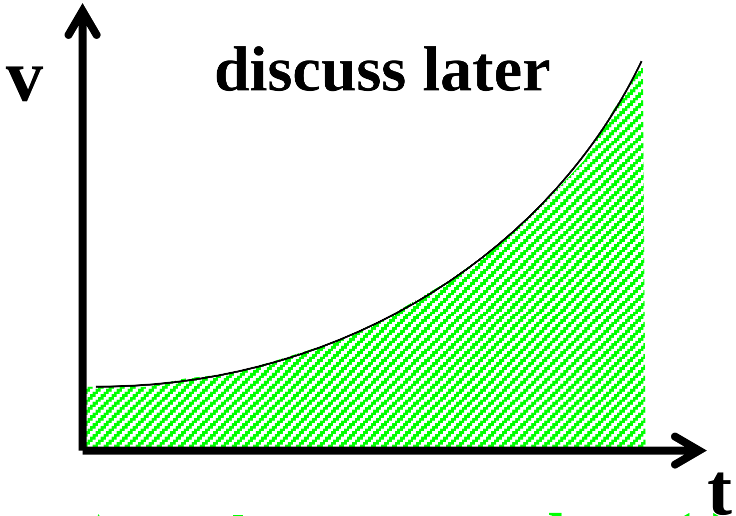


Limit $\Delta t \to 0$

slope of tangent to x(t) curve



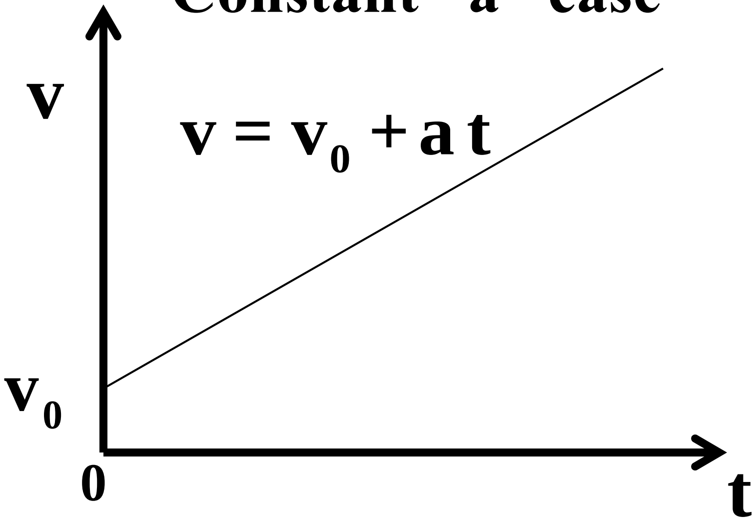
a=slope of tangent to v(t) curve



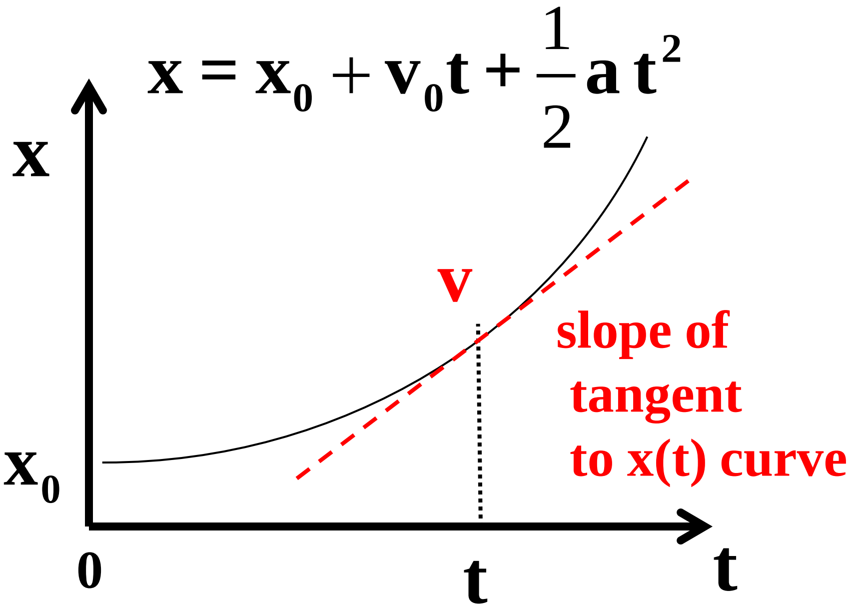
discuss later

$\Delta x = A =$ area under v(t) curve

Constant “a” case



$$v = v_0 + a\,t$$



$$x = x_0 + v_0 t + \frac{1}{2} a\,t^2$$

slope of tangent to x(t) curve

Graphical concepts

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in general

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equivalent relations to attack problems

$$\bar{v} = \frac{(v + v_0)}{2}$$

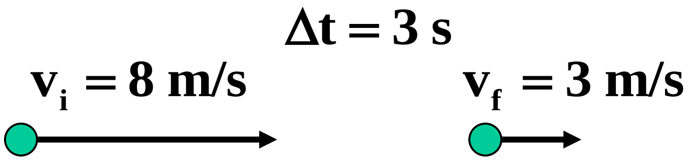
$$\Delta x = \bar{v}\,\Delta t$$

Will derive later

- 1st experiment and problems

problem

An object slows, with a constant acceleration, from 8 m/s to 3 m/s in a time interval of 3 s. How far has the object moved in this process?



$\Delta t = 3\ \mathrm{s}$

$v_i = 8\ \mathrm{m/s}$

$v_f = 3\ \mathrm{m/s}$

$+ \rightarrow$

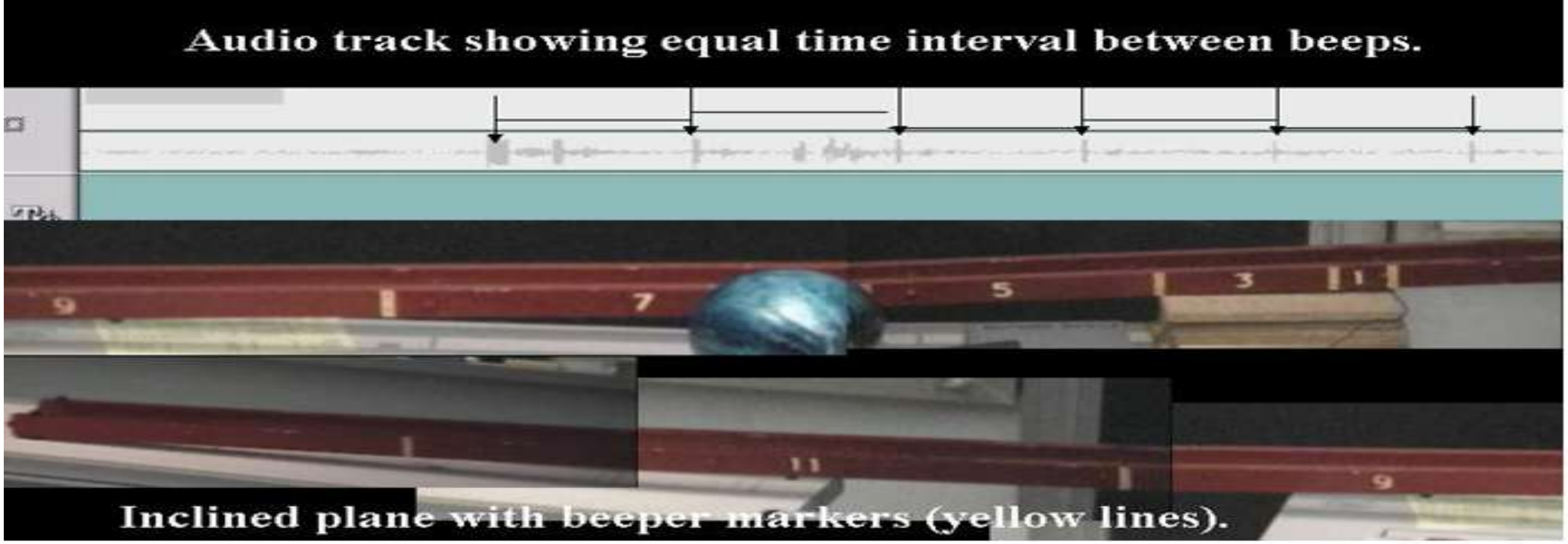
$$\boxed{\begin{gathered}\Delta x = \bar{v}\ \Delta t \\ \bar{v} = \frac{v_i + v_f}{2}\end{gathered}}$$

$$\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t = \left(\frac{8+3}{2}\right) 3\ [\mathrm{s\ m/s}] = 16.5\ \mathrm{m}$$

What was the acceleration of the object in this process?

$$a = \left(\frac{v_f - v_i}{\Delta t}\right) = \left(\frac{3-8}{3}\right)\ \left[\frac{\mathrm{m/s}}{\mathrm{s}}\right] = -\frac{5}{3}\ \frac{\mathrm{m}}{\mathrm{s}^2} = -1.66\frac{\mathrm{m}}{\mathrm{s}^2}$$

Galileo’s experiment:
balls rolling on inclined plane $\Delta x = at^2/2$


Audio track showing equal time interval between beeps.

Inclined plane with beeper markers (yellow lines).

(Response to error pointed out by A. Yankov 1/2005)

$$\Delta x = \frac{1}{2} a t^2 \qquad t = n\tau$$

divide time into equal intervals = τ
n = 0,1,2,3,4,5,6

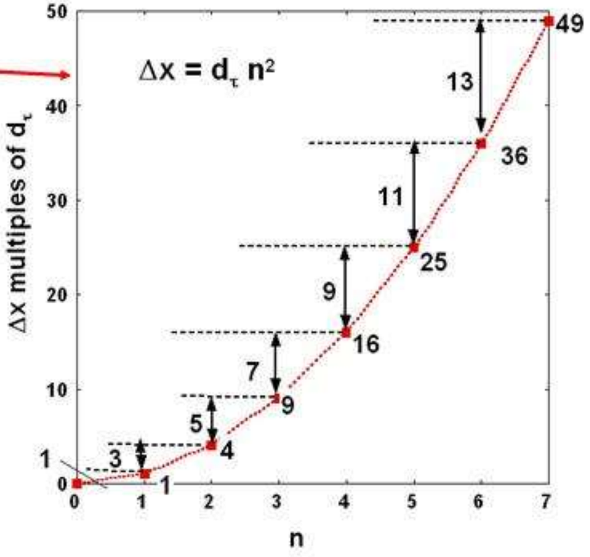
$$\Delta x = \frac{1}{2} a (n\tau)^2$$

define distance covered in time = τ

$$d_\tau = \frac{1}{2} a (\tau)^2$$

$$\Delta x = \left[\frac{1}{2} a (\tau)^2\right] n^2$$

$$\boxed{\Delta x = [d_\tau]\ n^2}$$

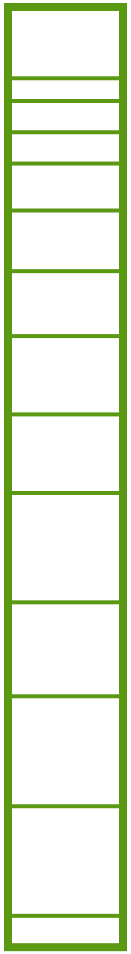


note if motion starts before t=0 more complicated

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

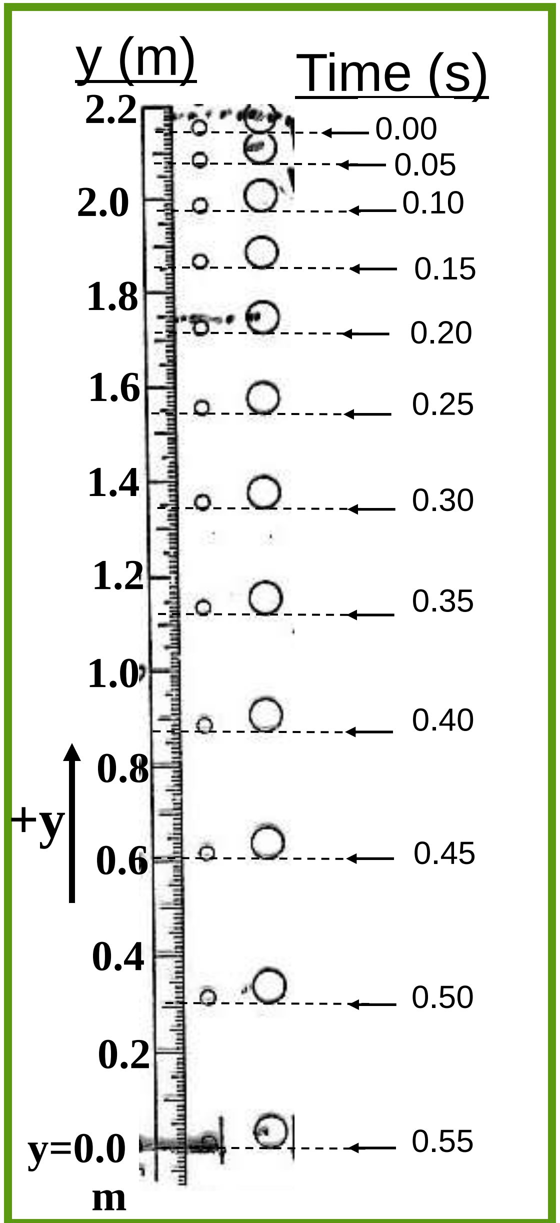
$$\Delta x = [v_0 \tau]\, n + [d_\tau]\ n^2$$

Experiment: ball (or anything) drop in Earth's Gravity (near surface)



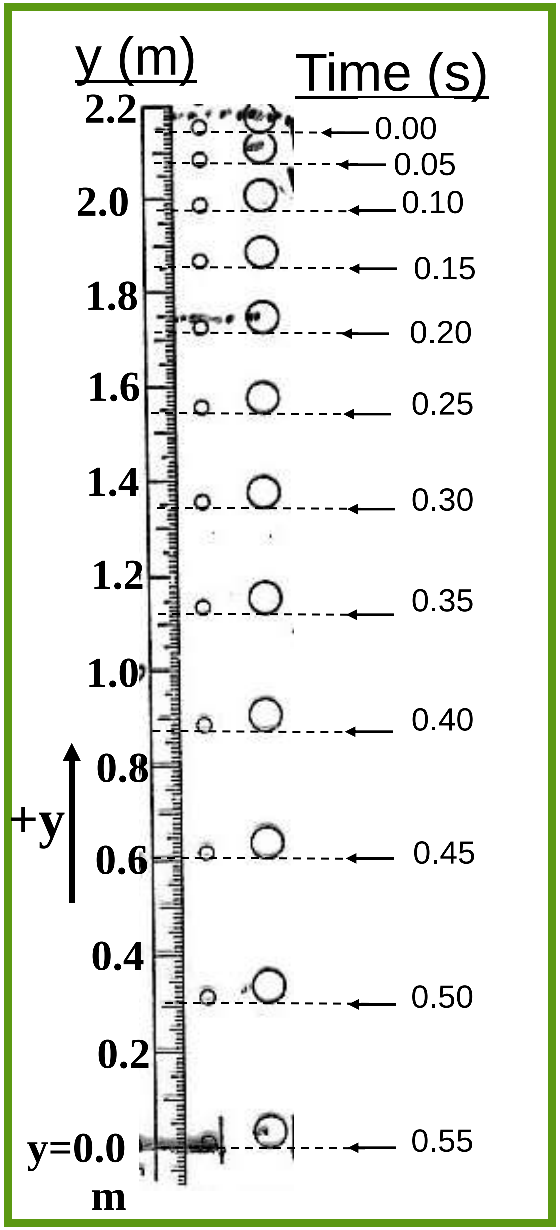
Right: a graph of a marble & ball falling as a function of time.

Left: individual frames from marbles falling demo in class. (See web-based movie.)

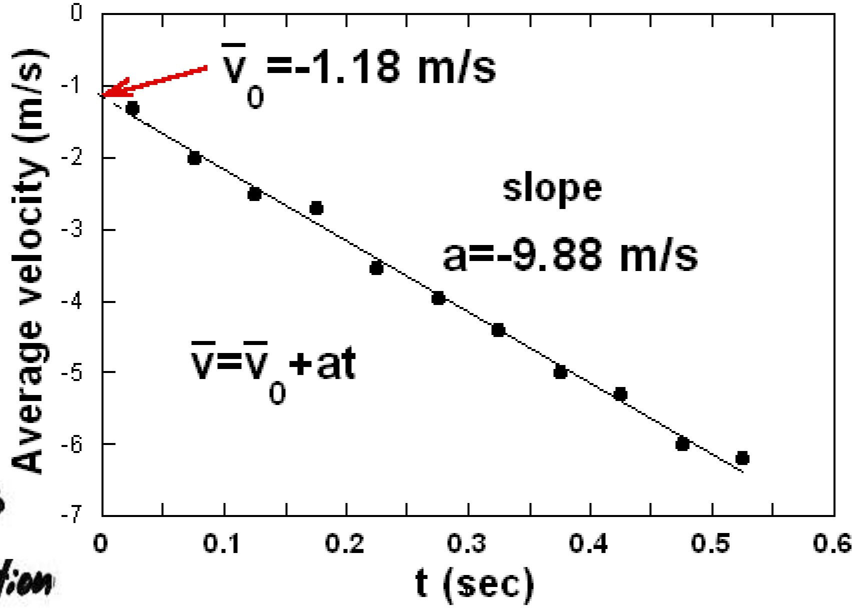


http://paer.rutgers.edu/pt3/movies/simdrop.mov

Objects near surface of earth fall with the same constant acceleration !!!



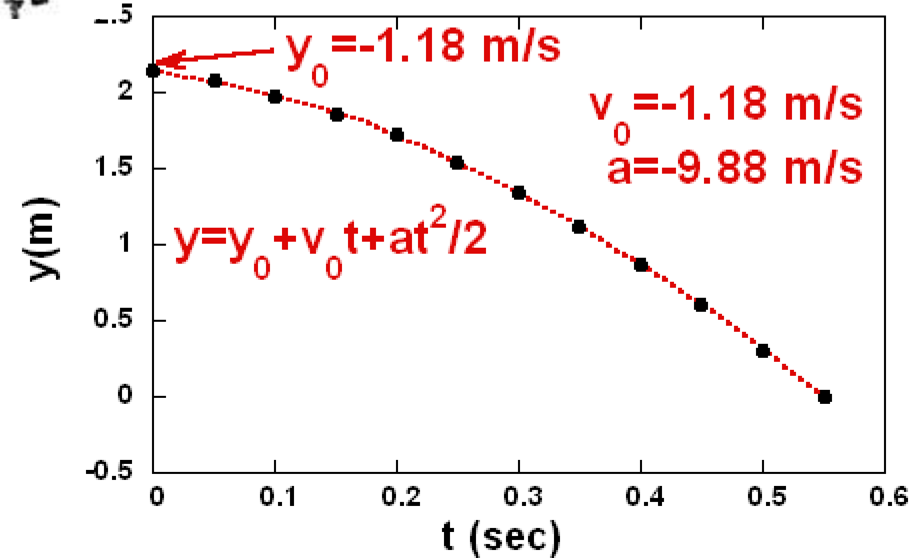
Experiment : Falling Object



Falling object thus constant acceleration

$a = -g = -9.8\,m/s^2$ (actual)

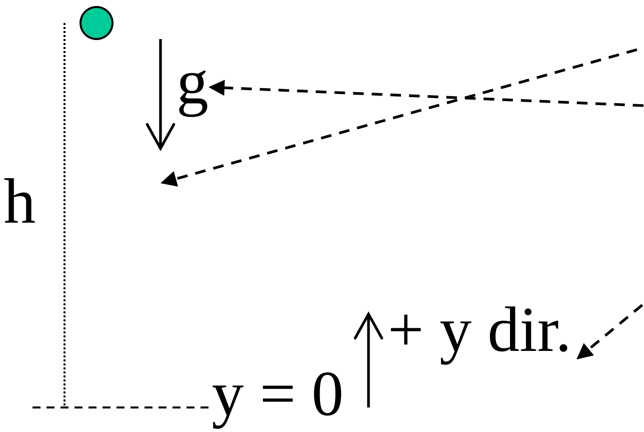
$g = 9.8\,m/s^2 = 32\,ft/s^2$



ball dropped (from rest)

Solve step by step

t = 0 : y=h : v_0=0



Q1- time it hits ground?

- identify time-position asked

- draw picture
- fix coordinate system
- identify given conditions
- recall general 1D, a=const relations

$$\mathbf{v = v_0 + at}$$

$$\mathbf{y = y_0 + v_0 t + a\frac{t^2}{2}}$$

$$\boxed{\mathbf{v^2 - v_o^2 = 2a(y - y_o)}}$$

$$\mathbf{v = \underset{0}{\cancel{v_0}} + at}$$

$$\mathbf{y = \underset{h}{y_0} + \underset{0}{\cancel{v_0}} t + \underset{-g}{a}\frac{t^2}{2}}$$

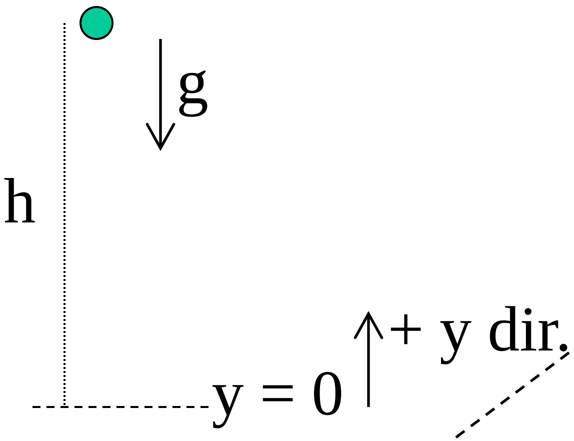
$$\boxed{\mathbf{v^2 - v_o^2 = 2a(y - y_o)}}$$

- input problem conditions into 1D relations (needed)

- Result of problem setup

ball dropped (from rest)

$t = 0 : y=h : v_0=0$



Result of problem setup

$$\mathbf{v = -gt} \quad (1)$$

$$\mathbf{y = h - g\frac{t^2}{2}} \quad (2)$$

Suppose h=2 m

$$\mathbf{t = \sqrt{\frac{2(2)}{9.8}}\sqrt{\frac{m}{m/s^2}}}$$

$$\mathbf{t = 0.64\ s}$$

Q1- time it hits ground?

$$\mathbf{y = 0 = h - g\frac{t^2}{2}} \quad \Rightarrow \quad \mathbf{t = \underset{-}{\textcircled{+}}\sqrt{\frac{2h}{g}}}$$

correct time

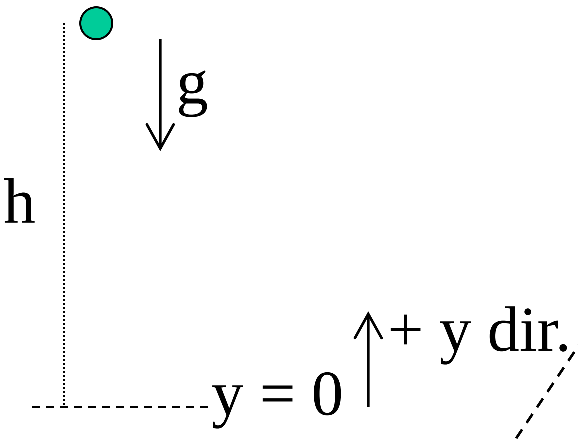
(- time before ball dropped...
will come back to this)

- identify mathematical condition being asked !!!

hit ground→ ground defined at y=0 → y(t)=0 solve for t

ball dropped (from rest)

$t = 0 : y=h : v_0=0$



Result of problem setup

$$\mathbf{v = -gt} \quad (1)$$

$$\mathbf{y = h - g\frac{t^2}{2}} \quad (2)$$

If h=2 m

$$\mathbf{v = -\sqrt{2(9.8)\,2}\;\sqrt{(m/s^2)\,m}}$$

$$\mathbf{v = -6.3\ m/s}$$

Q1- time it hits ground?

$$\mathbf{y = 0 = h - g\frac{t^2}{2}} \Rightarrow \mathbf{t = \pm\sqrt{\frac{2h}{g}}}$$

correct time

Q2- velocity when it hits ground?

$$(1)\ \mathbf{v = -gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}}$$

one way

Plug time hit into (1)

another way use $\mathbf{v^2 - v_o^2 = 2a\,(y - y_o)}$

$$\mathbf{v^2 - \underset{0}{v_o^2} = 2\underset{-g}{a}\,(\underset{(0-h)}{\Delta y})}$$

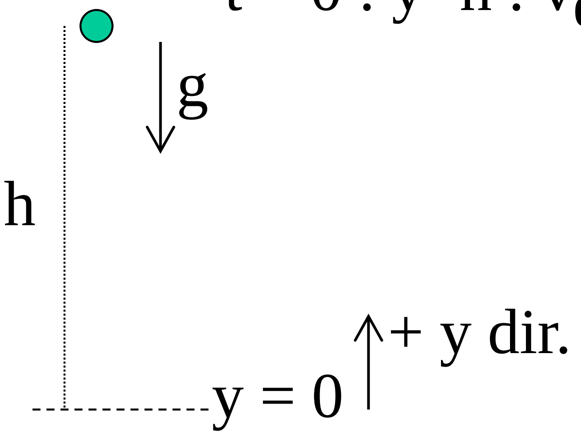
$$\mathbf{v^2 = 2gh}$$

$$\mathbf{v = \pm\sqrt{2gh}}$$

correct vel. - = down

ball dropped (from rest) **what about “– “time !!**

t = 0 : y=h : v_0=0



$$y = h - g\frac{t^2}{2} \quad (1)$$

$$v = -gt \quad (2)$$

Q1- time it hits ground?

$$y = 0 = h - g\frac{t^2}{2} \Rightarrow t = \pm\sqrt{\frac{2h}{g}}$$

correct time

what about “– “time !!

$y = 0$ at $t = -\sqrt{\frac{2h}{g}}$

and $v = +\sqrt{2gh}$

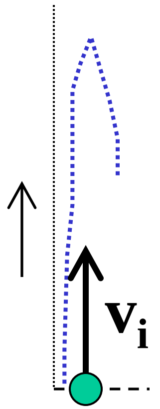
- ball could have been thrown upward at this –t
- from t=0 on problem the same
- **another possible t<0 history !!!**

$$v^2 - v_o^2 = 2a(\Delta y)$$

$$v = \pm\sqrt{2gh}$$

another way

ball thrown upward



$$v = v_0 - gt \qquad (1)$$

$$y = v_0 t - g\frac{t^2}{2} \qquad (2)$$

$\downarrow g = 9.8\ \text{m/s}^2$

$$\boxed{t = 0 : y = 0 : v_i = v_0}$$

y = 0

+ y dir.

- time to get to top ? **at top v=0 !!!**

(1)

$$\therefore\ 0 = v_0 - gt \Rightarrow t = \frac{v_0}{g}$$

at top

- maximum height ? **put t_{top} into (2)**

$$y_{max} = v_0 t_{top} - g\frac{t_{top}^2}{2}$$

$$\Downarrow$$

$$y_{max} = v_0\left(\frac{v_0}{g}\right) - \frac{g}{2}\left(\frac{v_0}{g}\right)^2 \Rightarrow y_{max} = \frac{v_0^2}{2g}$$

- How long till ground ?

at ground y=0 !!!

$$\therefore\quad 0 = y = v_0 t - g\frac{t^2}{2}$$

$$0 = \left[v_0 - g\frac{t}{2}\right][t]$$

$$0 = \left[v_0 - g\frac{t}{2}\right] \qquad t = 0$$

$$t = \frac{2v_0}{g}$$

back at ground

Started at ground t=0

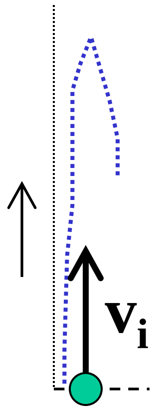
or another way

$$\overset{0}{v}{}^2 - v_o^2 = 2\overset{-g}{a}(\Delta y)$$

$$0^2 - v_o^2 = 2(-g)(\Delta y)$$

$$\Delta y = \frac{v_o^2}{2g}$$

ball thrown upward



$\downarrow$ g=9.8 m/s^2

$$v = v_0 - gt \qquad (1)$$

$$\mathbf{y = v_0 t - g\frac{t^2}{2}} \qquad (2)$$

t = 0 : y=0 : v_i=v_0

y = 0

+ y dir.

- time to get to top ? **at top v=0 !!!**

(1)

$$\therefore\ 0 = v_0 - gt \Rightarrow t = \frac{v_0}{g} \quad \textbf{at top}$$

$$v_0 = 6.3\,\mathrm{m/s}$$

$$t = \frac{v_0}{g} = \frac{6.3}{9.8}\frac{\mathrm{m/s}}{\mathrm{m/s^2}}$$

$$t = 0.64\,\mathrm{s}$$

- maximum height ? **put t_{top} into (2)**

$$\mathbf{y_{max} = v_0 t_{top} - g\frac{t_{top}^{\ 2}}{2}}$$

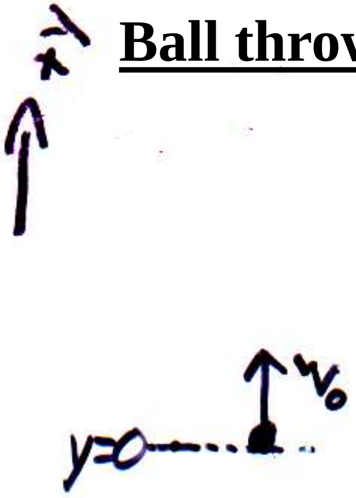
$$\Downarrow$$

$$\mathbf{y_{max} = v_0\left(\frac{v_0}{g}\right) - \frac{g}{2}\left(\frac{v_0}{g}\right)^2} \Rightarrow \mathbf{y_{max} = \frac{v_0^{\ 2}}{2g}}$$

$$y_{max} = \frac{(6.3\,\mathrm{m/s})^2}{2(9.8\mathrm{m/s})}$$

$$y_{max} = 2\,\mathrm{m}$$

Ball thrown upward – graphical interpretation



$\downarrow g$

$$a = -g$$

$$y_i = 0$$

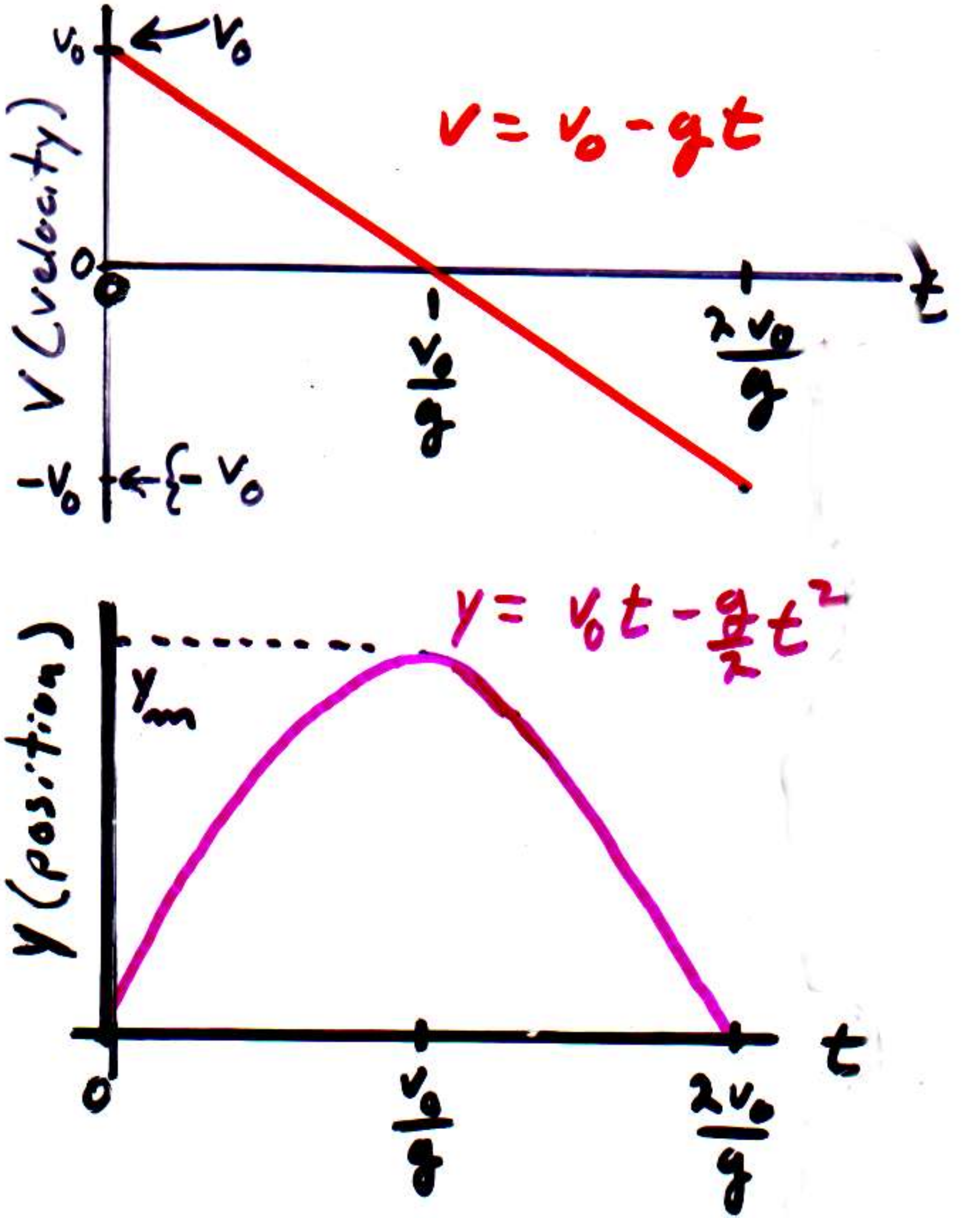
In general

$$v = v_0 + at.$$

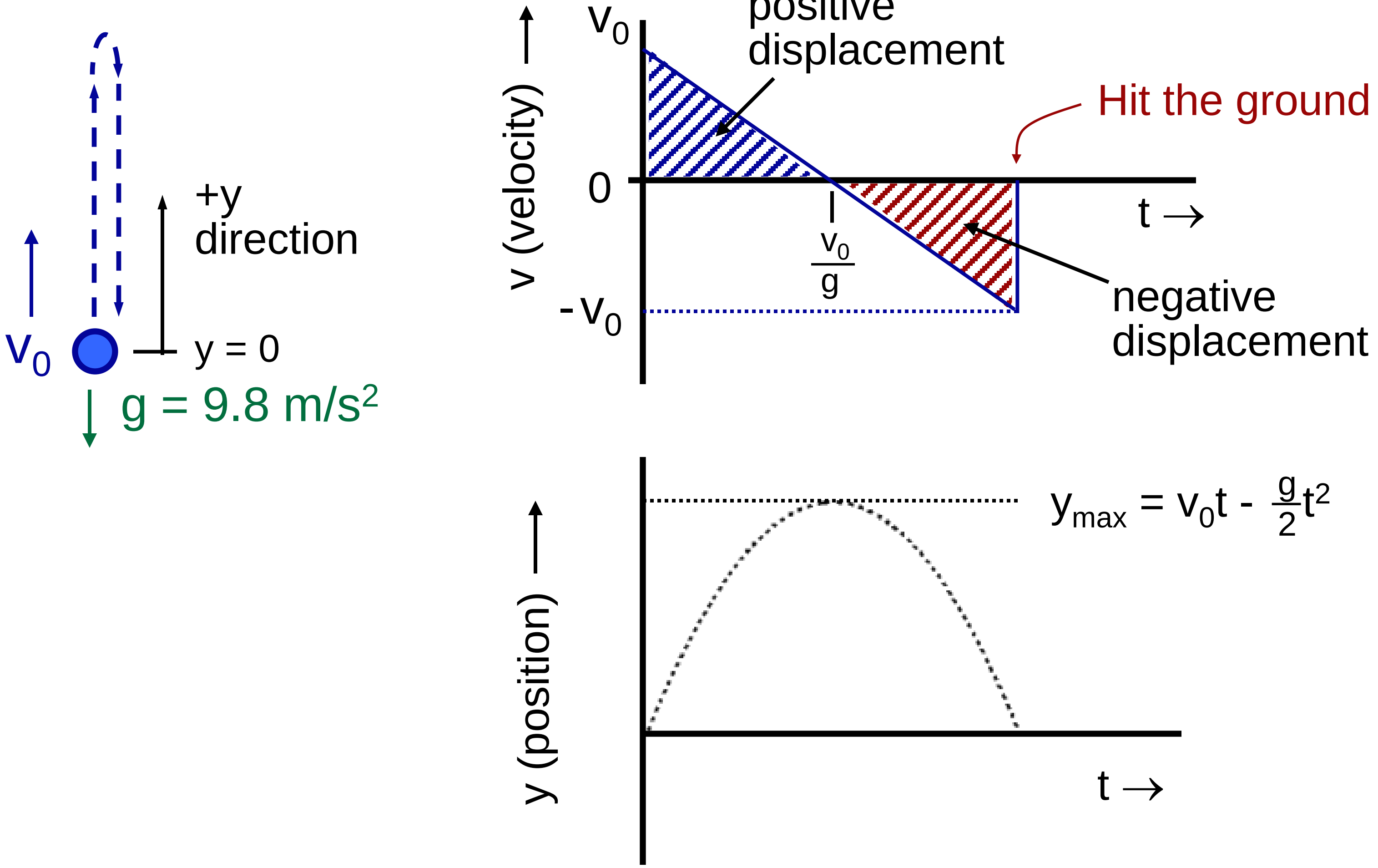
$$\cancel{y} = \cancel{y_0} + v_0 t + \tfrac{1}{2} a t^2.$$

In this case

$$\begin{cases} v = v_0 - gt & (1) \\ y = v_0 t - \frac{g}{2} t^2 & (2) \end{cases}$$

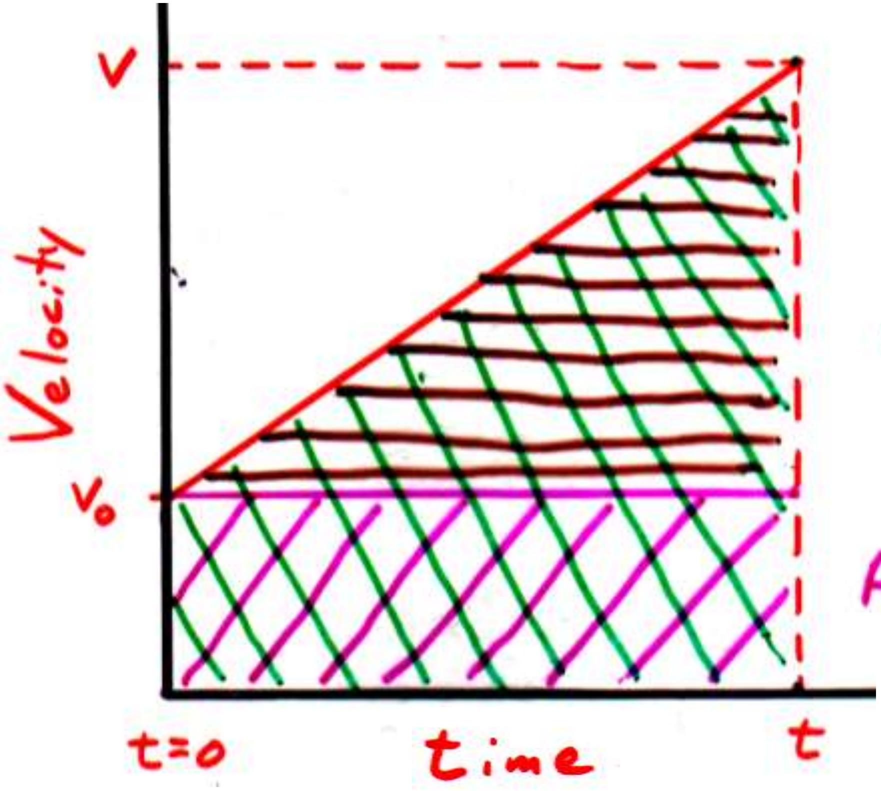


Ball thrown upward – graphical interpretation (cont)



$y_{max} = v_0 t - \frac{g}{2} t^2$

Appendix: Formulas and Proofs



Constant Acceleration

$v = v_0 + at$

Area = Δx

proof of $\bar{v} = \frac{(v_0 + v)}{2}$

for a = constant

Area = $\frac{1}{2}\, t\, (v - v_0)$

Area = $v_0 \quad t$

$A = \quad A + \quad A$

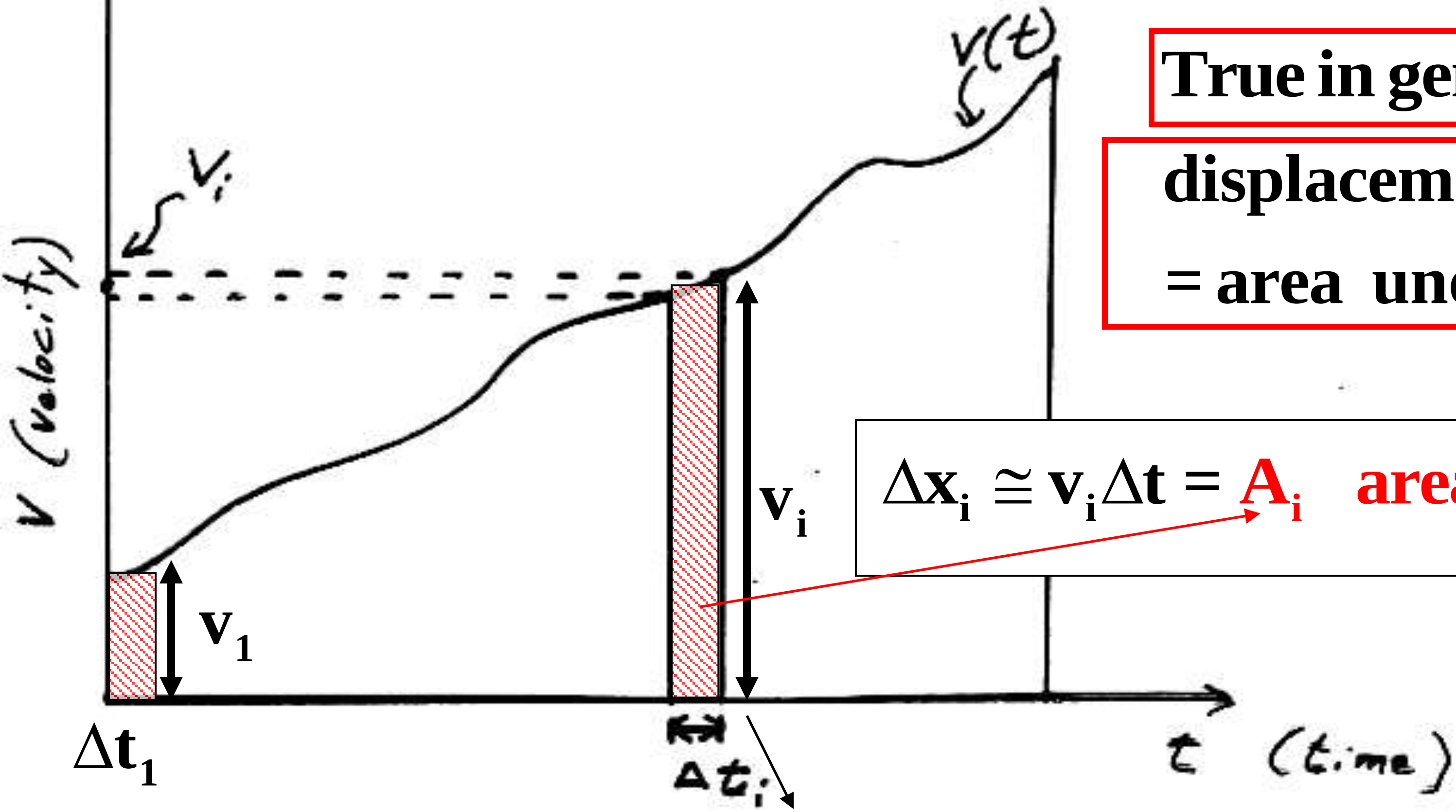
$$\Delta x = \frac{t}{2}(v - v_0) + \quad v_0 t$$

$$\Delta x = \underbrace{\frac{(v + v_0)}{2}}_{''}\, t$$

Bradwardine's Merton College Rule Oxford~1300's

note $\Delta x \equiv \bar{v} \quad t \Rightarrow \bar{v} = \frac{v + v_0}{2}$

a = constant



True in general !

displacement

= area under v(t) curve

$\Delta x_i \cong v_i \Delta t = A_i$ **area**

- $$\Delta x = \Delta x_1 + \Delta x_2 + \ldots + \Delta x_i + \ldots \Delta x_N$$

$$\Delta x \cong \sum_{i=1}^{N} \Delta x_i = \sum_{i=1}^{N} v_i \Delta t = \sum_{i=1}^{N} A_i \qquad \therefore \qquad \Delta x \cong \sum_{i=1}^{N} A_i$$

$$\lim_{N\to\infty} \Delta x = A \qquad \therefore \; A = \text{area under } v(t) \text{ curve}$$

back to proof of $\bar{v} = \frac{v_0 + v}{2}$ for $a =$ constant

Derivation of 1D a= const. relations

$$\Delta x = \bar{v}\,\Delta t$$

so $$\Delta x = \frac{[v + v_0]}{2}\, t$$

but $$v = v_0 + a t$$ $\underline{\Delta v = at}$

$$\therefore \quad \Delta x = \frac{[(v_0 + at) + v_0]}{2}\, t$$

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

or $\Delta x = x - x_0 \Rightarrow$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

at $t=0$

position $\underline{x_0}$

velocity $\underline{v_0}$

const. acc. $\underline{a}$

(1) $v = v_0 + a t$

(2) $x = x_0 + v_0 t + \frac{1}{2} a t^2$

Solve (1) for $t = \dfrac{v - v_0}{a}$

put this into (2)

$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{a}{2}\left(\frac{v - v_0}{a}\right)^2$$

$$= x_0 + \frac{v_0 v - v_0^2}{a} + \frac{v^2 - 2v_0 v + v_0^2}{2a}$$

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

$$\boxed{x - x_0 = \frac{v^2 - v_0^2}{2a}}$$

$\frac{m^2}{s^2}$ or $\underline{v^2 - v_0^2 = 2a(x - x_0)}$ * m $\frac{m}{s}$

* Later in course we will see this result follows from the work-energy Theorem or from conservation of energy (kinetic + potential)