

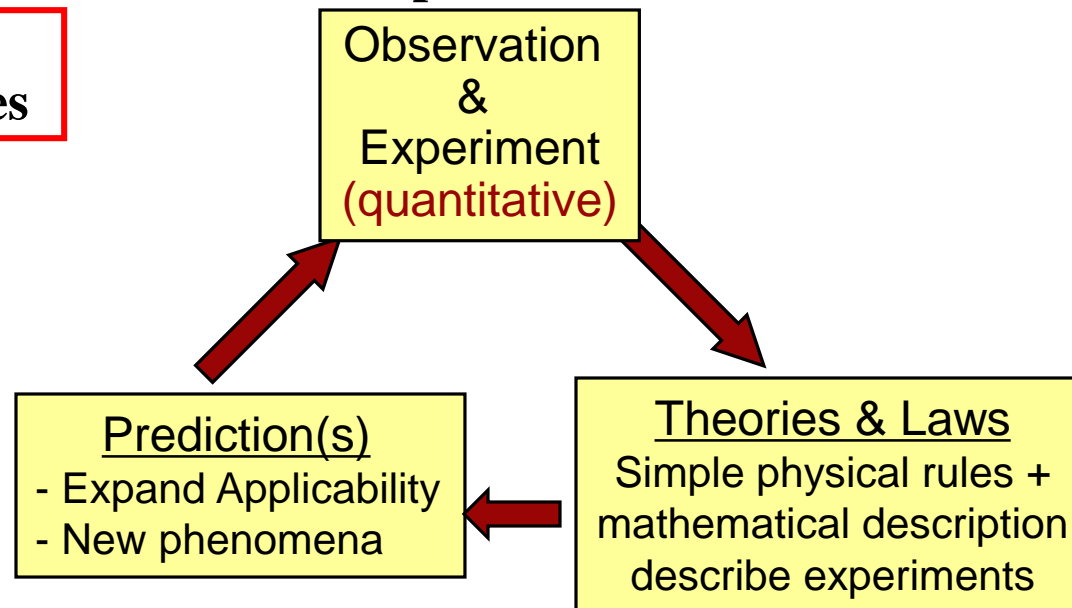
Physics 203

for syllabus & important messages see www.physics.rutgers.edu/~croft

Prof. Mark Croft – W113: 848-445-8746: croft AT physics.rutgers.edu

- **Physics – nature of things moving – Aristotle 350 BC**
- **Spirit of physics --e.g. Pythagorean tradition ~ 550 BC**
 - **musical tones described by mathematics (exactly!)**
 - **all nature can be described by mathematics**
(approximations and limitations must be recognized)
- **careful observation of natural phenomena essential**

• **Physics
underlies all sciences**



**'Beauty is truth, truth beauty,—that is all
Ye know on earth, and all ye need to know.'** (Keats)

measure: Space, Time, & Matter

Units: SI or MKS System

Distance	Mass	Time
meter	kilogram	second
m	kg	s or sec

Originally:

1 m = 1/(10 Millionth) of distance
from equator to North Pole

Now:

1 m = 1,650,763.7321 wavelengths
of orange light emitted from Kr.

Derived units

speed = distance/time: m/sec

later will see

Weight (force of gravity on mass) = mass distance/ time²

Newton = N= kg m/s² (unit of force)

Weight (in English units) = slug ft/s²=pound=lb

unit conversion- mole method

$$60 \text{ mi/hr} = ? \text{ m/s}$$

$$60 \frac{\text{mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ mi}}{60 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} = ? \text{ m/s}$$

cancel

$$\frac{60 \cdot 5280 \cdot 12 \cdot 2.54}{60 \cdot 60 \cdot 100} = \frac{5280 \cdot (12)^2 \cdot 2.54}{1000000} = \frac{26822.4}{1000}$$

$$\frac{60 \text{ mi}}{\text{hr}} \approx 26.8 \text{ m/s}$$

$$\left[\frac{1 \text{ mi}}{\text{hr}} = 0.446 \text{ m/s} \right]$$

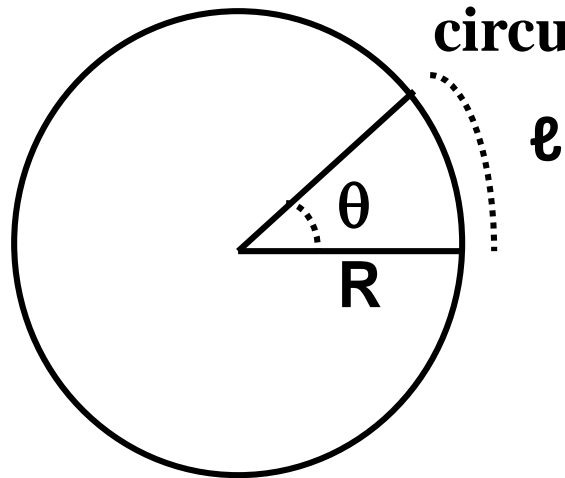
$$30 \text{ mph} \sim 13.4 \text{ m/s}$$

Note: the **units** start and finish with distance/time. All conversions are unity conversions $1 \text{ hr} = 60 \text{ min}$ e.g. $1 \text{ hr} / 60 \text{ min} = 1$.

Problem Solving

1. Read problem carefully, reread
2. Draw a diagram and label
3. Write question in symbols
4. Find (better derive) relevant mathematical relation
5. Solve Equation
6. Plug in numbers
7. Check whether answer is reasonable
(Numbers & Units) (Should it be + OR - ?)
8. Talk to someone about the problem and its solution

Measuring Angles

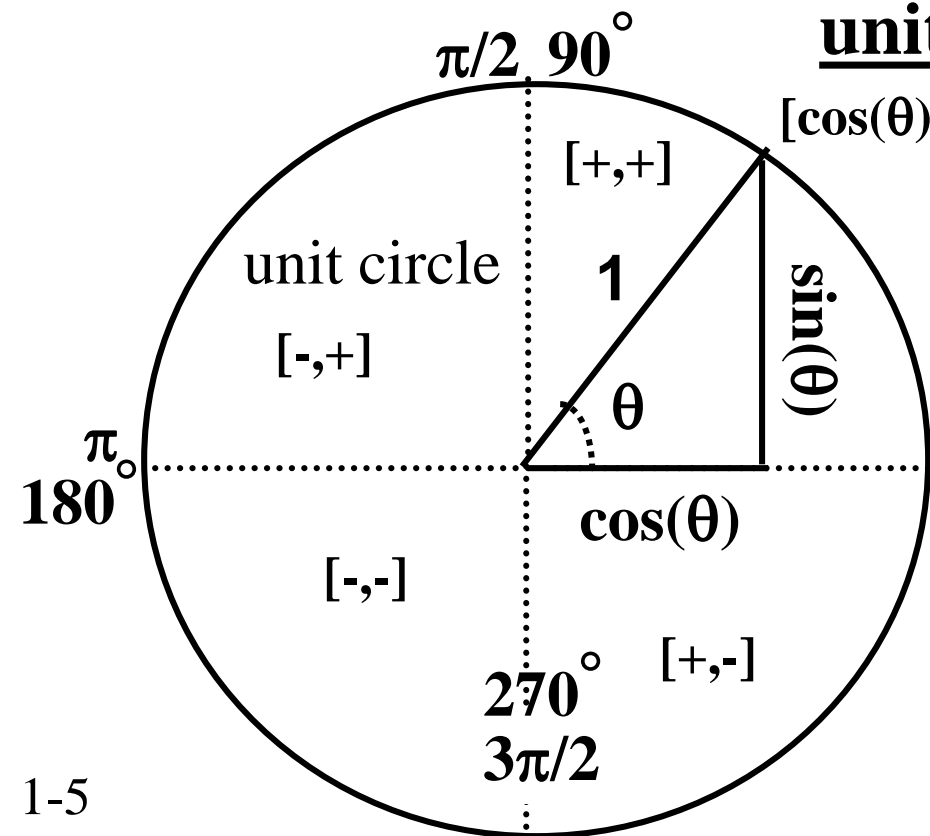


$$\text{circumference} = 2\pi R$$

$$\theta(\text{radians}) = \frac{l}{R}$$

$$\theta(\text{degrees}) = \theta(\text{radians}) \frac{l}{2\pi R} \cdot 360$$

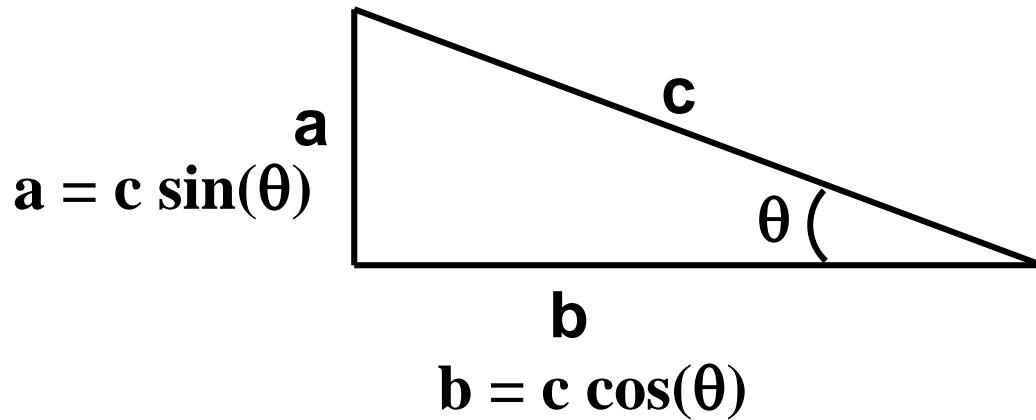
unit circle



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$0 \quad 2\pi \quad 360^\circ$$

$$a^2 + b^2 = c^2$$



$$\sin(\theta) = \frac{a}{c}$$

$$\cos(\theta) = \frac{b}{c}$$

$$\tan(\theta) = \frac{a}{b}$$

$$\theta = \sin^{-1}\left(\frac{a}{c}\right)$$

$$\theta = \cos^{-1}\left(\frac{b}{c}\right)$$

$$\theta = \tan^{-1}\left(\frac{a}{b}\right)$$

See link below for demo on Pythagorean theorem

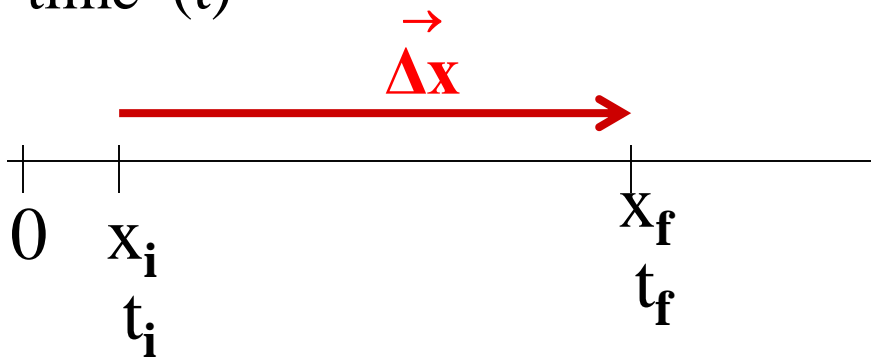
<http://i.imgur.com/W8VJp.gif>

vector in 1D - magnitude (length) - direction

one dimension motion

position coordinate (x) – measure of position

time (t)



→ define + direction

initial position time

final position time

displacement $\Delta \mathbf{x} = \mathbf{x}_f - \mathbf{x}_i$

- how far
- what direction + or -

magnitude direction

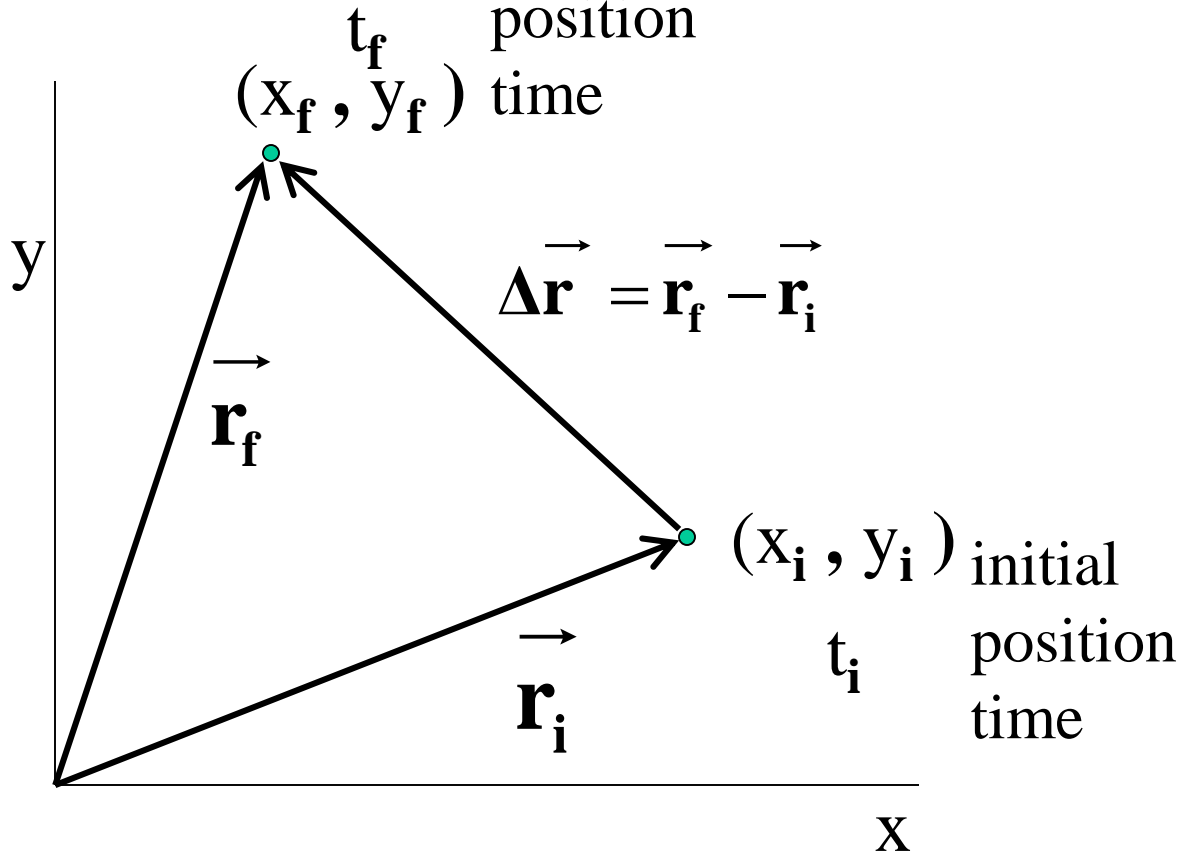
average velocity $\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathbf{x}_f - \mathbf{x}_i}{t_f - t_i}$

- how fast
- what direction + or -

2 dimensions, 3 dimensions, ...

vector (position and displacement)

- magnitude (length)
 - direction
- final position
 t_f time



Note: unit vector notation discussed at end of lecture

$$\vec{r} = x \hat{x} + y \hat{y}$$

two dimensions

vector position label $\vec{\mathbf{r}}$

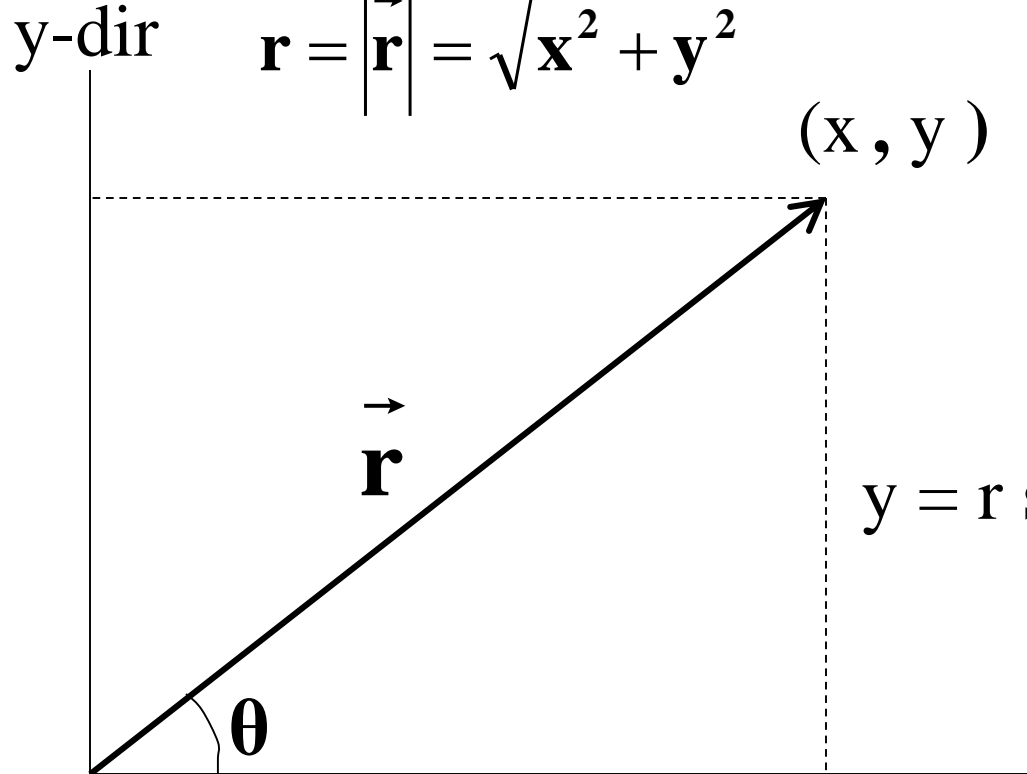
2 ways to represent position

\mathbf{x} component
\mathbf{y} component

\mathbf{r} magnitude
θ direction

$$r = |\vec{\mathbf{r}}| = \sqrt{\mathbf{x}^2 + \mathbf{y}^2}$$

(x, y)



$$y = r \sin(\theta)$$

$\sin(\theta) = \frac{y}{r}$	$\theta = \sin^{-1}\left(\frac{y}{r}\right)$
$\cos(\theta) = \frac{x}{r}$	$\theta = \cos^{-1}\left(\frac{x}{r}\right)$
$\tan(\theta) = \frac{y}{x}$	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$x = r \cos(\theta)$$

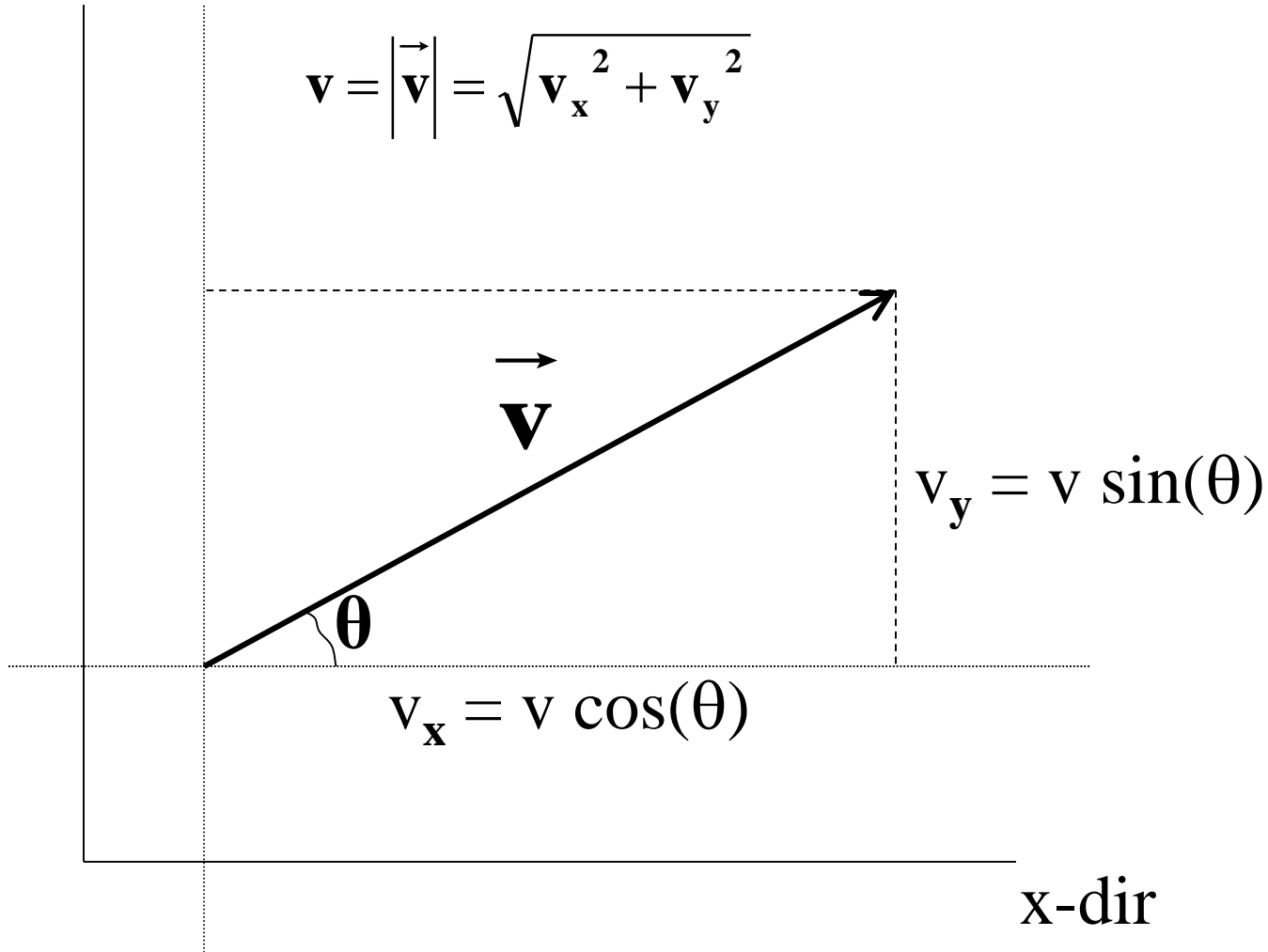
x-dir position vector

- magnitude (length)
- direction

general vector not tied to particular origin

y-dir

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$



$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\theta = \cos^{-1}\left(\frac{v_x}{v}\right)$$

$$\theta = \sin^{-1}\left(\frac{v_y}{v}\right)$$

x-dir

vector components can be found using different angles

or - #

$$A_x = A \cos(\theta)$$

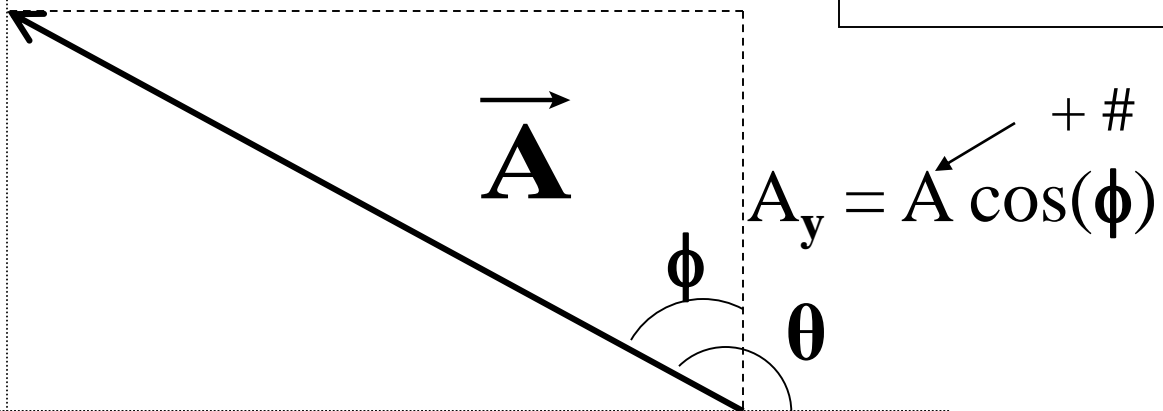
$$A_y = A \sin(\theta)$$

+ #

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

y-dir

$$A = \left| \vec{A} \right| = \sqrt{A_x^2 + A_y^2}$$



+ #

$$A_y = A \cos(\phi)$$

- #

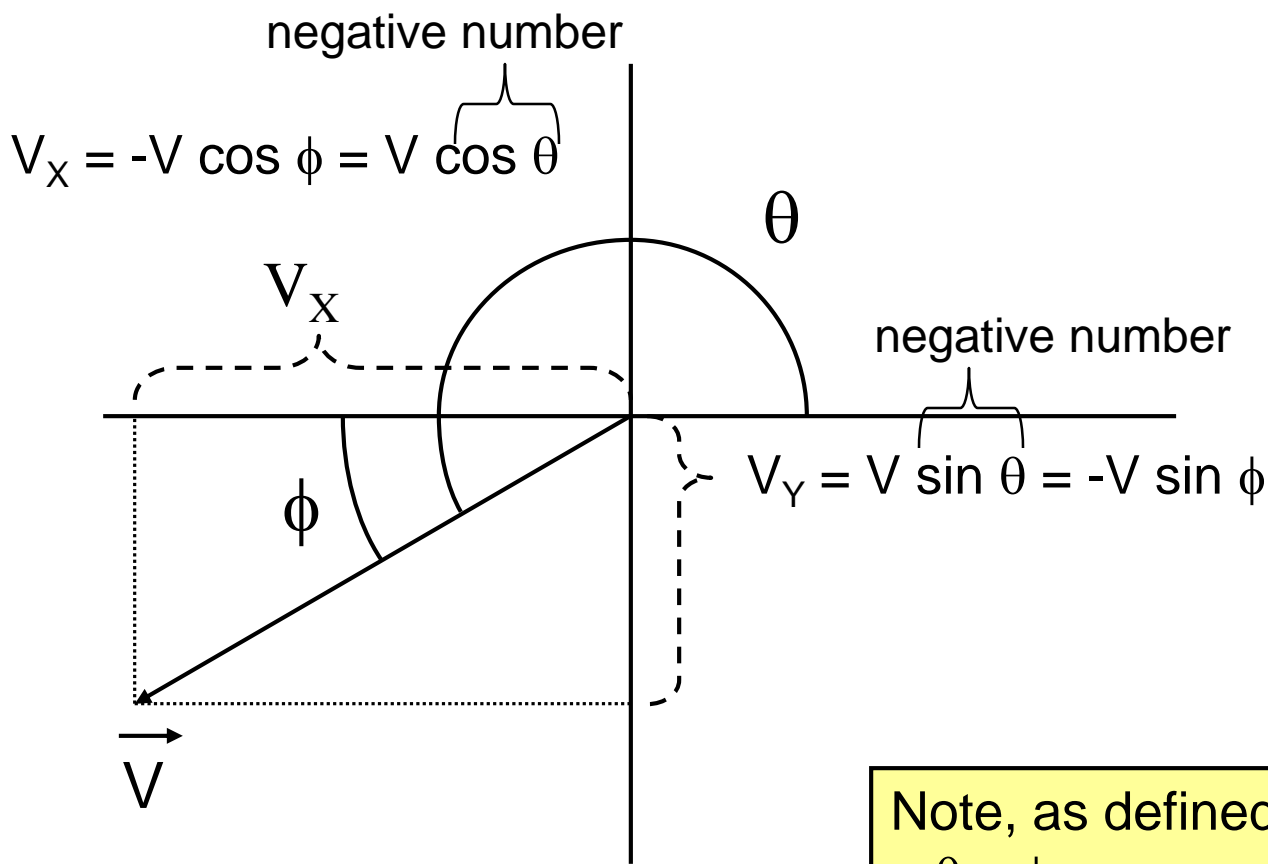
$$A_x = -A \sin(\phi)$$

$$\phi = \tan^{-1}\left(\frac{|A_x|}{|A_y|}\right)$$

x-dir

vector components can be found using different angles

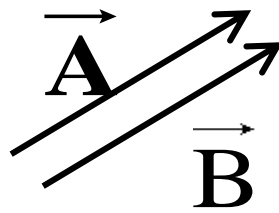
(angle bigger than 90° example)



Note, as defined above:
 $\theta = \phi + \pi$
 $\theta = \phi + 180^\circ$
 $\sin \theta = \sin (\theta+2\pi)$

Vector equality

$$\vec{\mathbf{A}} = \vec{\mathbf{B}}$$



$$A_x = B_x$$

$$A_y = B_y$$

$$|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$$

$$\theta_A = \theta_B$$

Scalar Multiplication

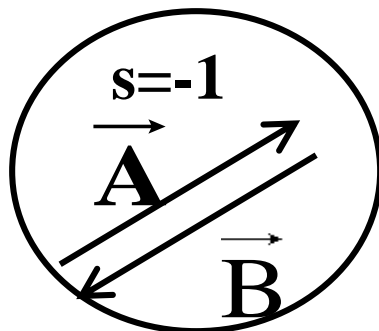
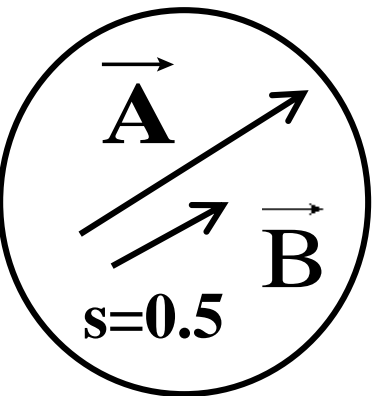
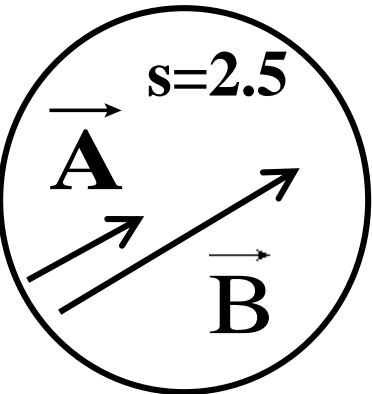
$$s = \#$$

$$\vec{\mathbf{B}} = s \vec{\mathbf{A}} \Rightarrow \begin{cases} B_x = s A_x \\ B_y = s A_y \end{cases}$$

or

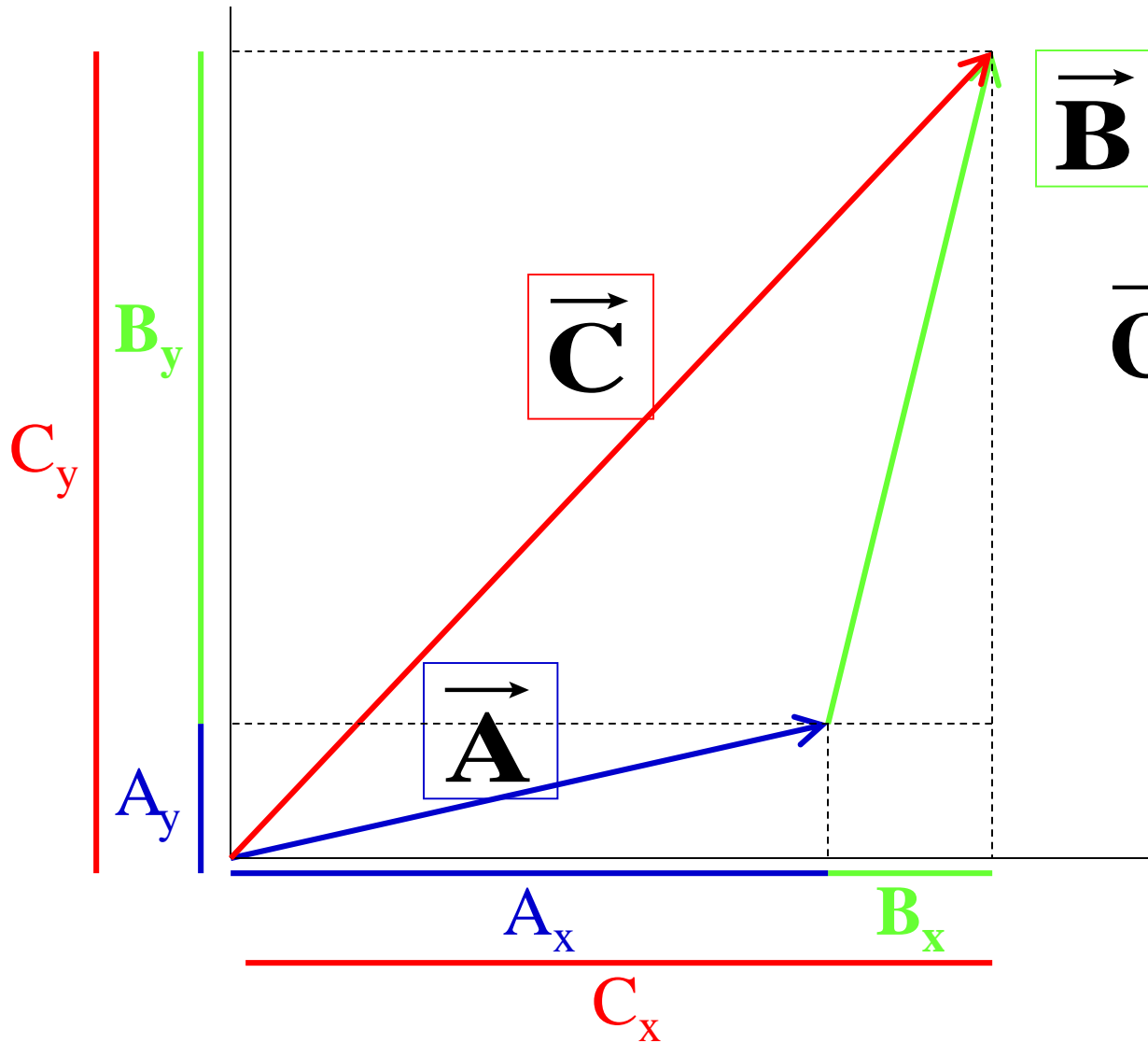
$$\text{same direction} \begin{cases} \theta_A = \theta_B \\ |\vec{\mathbf{B}}| = s |\vec{\mathbf{A}}| \end{cases}$$

s times longer



**note: if $s < 0$ then
reverse vector direction**

vector addition



$$\vec{\mathbf{B}}$$

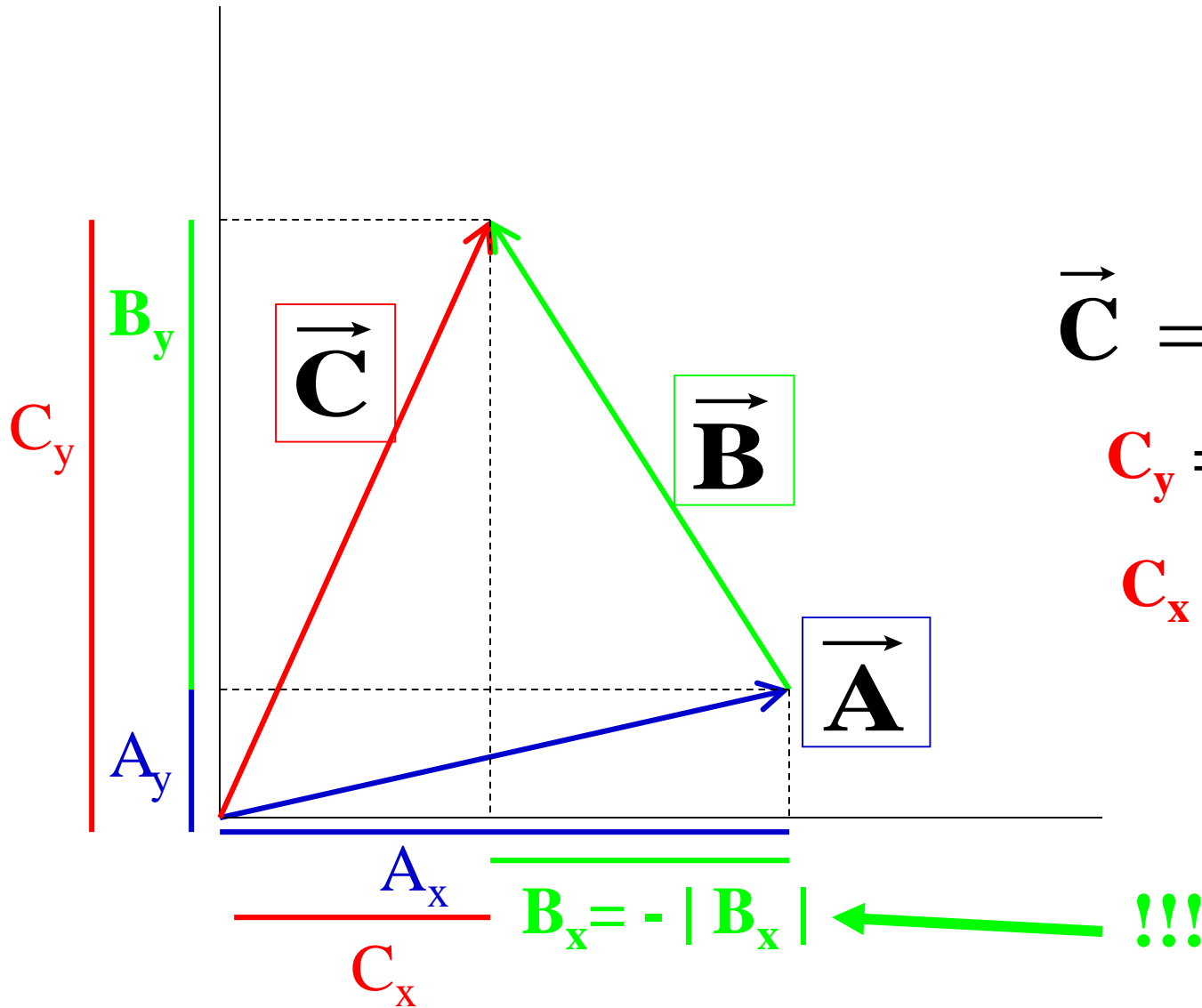
$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

http://phet.colorado.edu/sims/vector-addition/vector-addition_en.html

vector addition



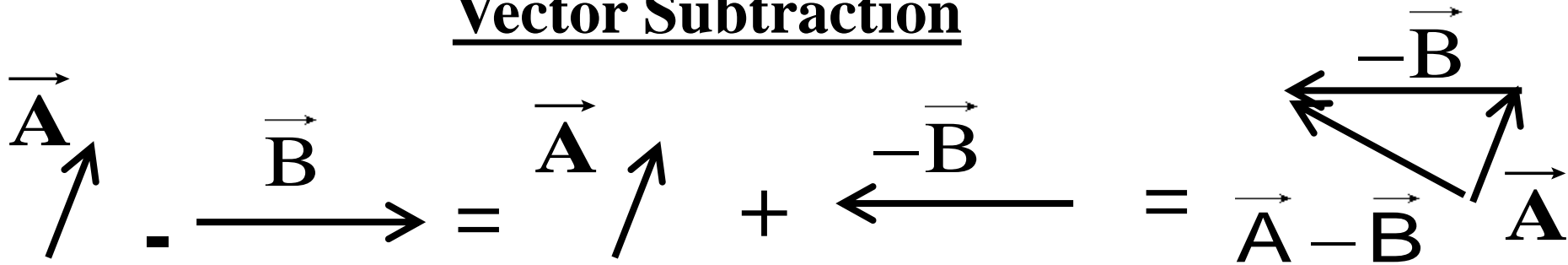
$$\vec{C} = \vec{A} + \vec{B}$$

$$C_y = A_y + B_y$$

$$C_x = A_x + B_x$$

$$= A_x - |B_x|$$

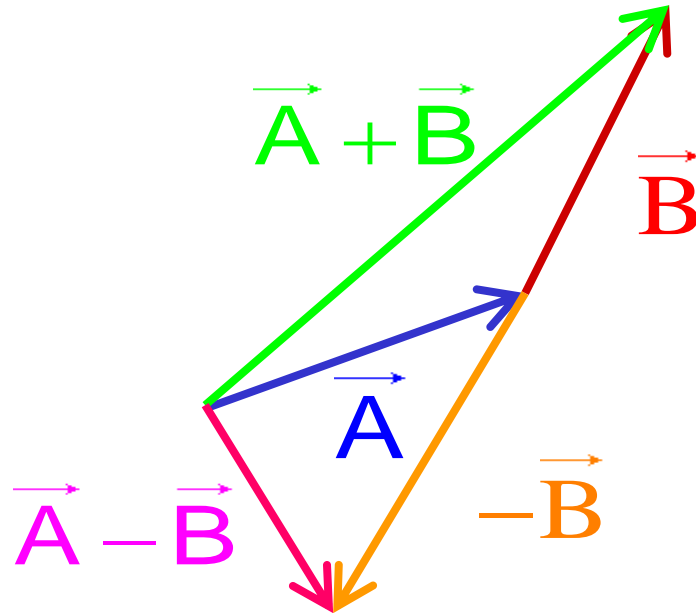
Vector Subtraction

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$


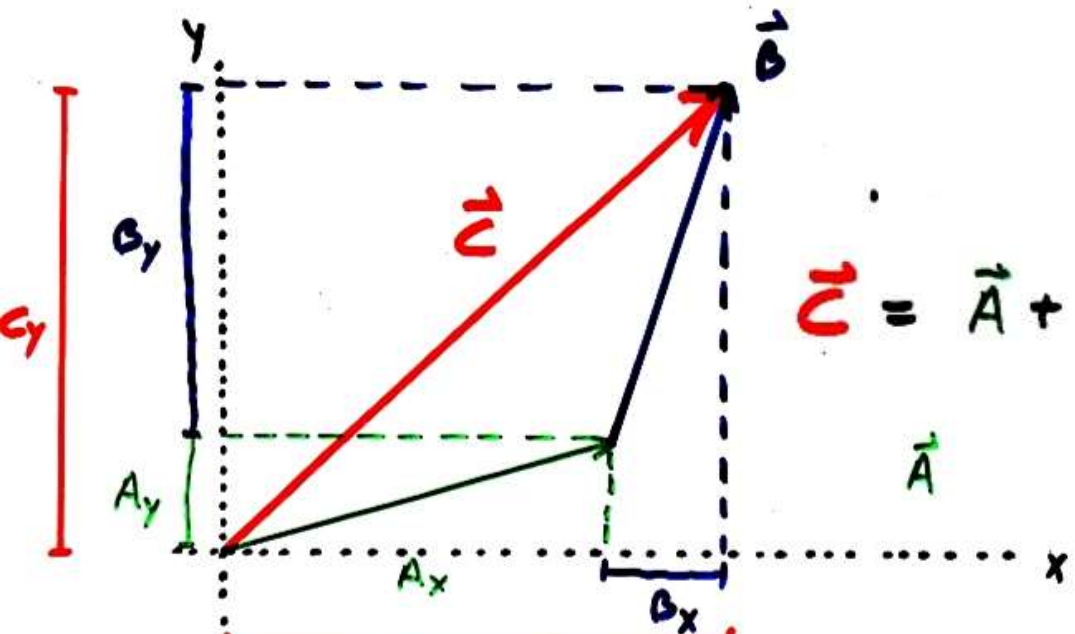
$$\vec{A} - \vec{B} = \vec{A} + \{-\vec{B}\}$$

{ follows from vector addition + -1 vector reversal }

Comparison
vector
addition
subtraction



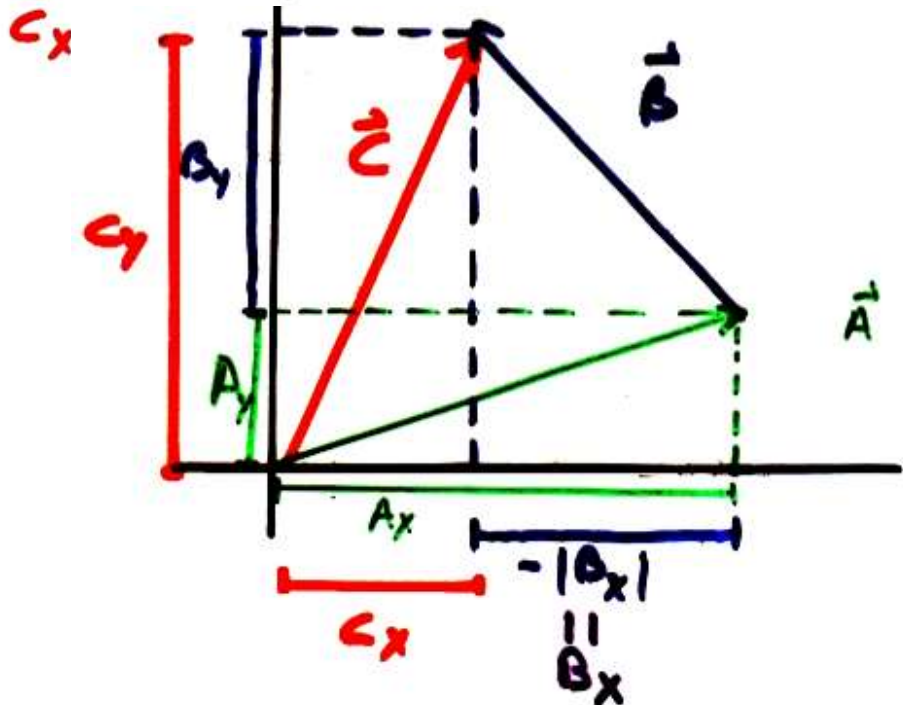
Vector Addition



$$\vec{C} = \vec{A} + \vec{B}$$

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$



$$C_y = A_y + B_y$$

$$C_x = A_x + B_x = A_x + (-|B_x|)$$

Unit vectors their uses (book sometimes uses)

$$\begin{array}{c} \uparrow y \\ \rightarrow x \end{array} \quad |y|^2 = |x|^2 = 1$$

\hat{x} Unit vectors along x and y directions
On length “1”

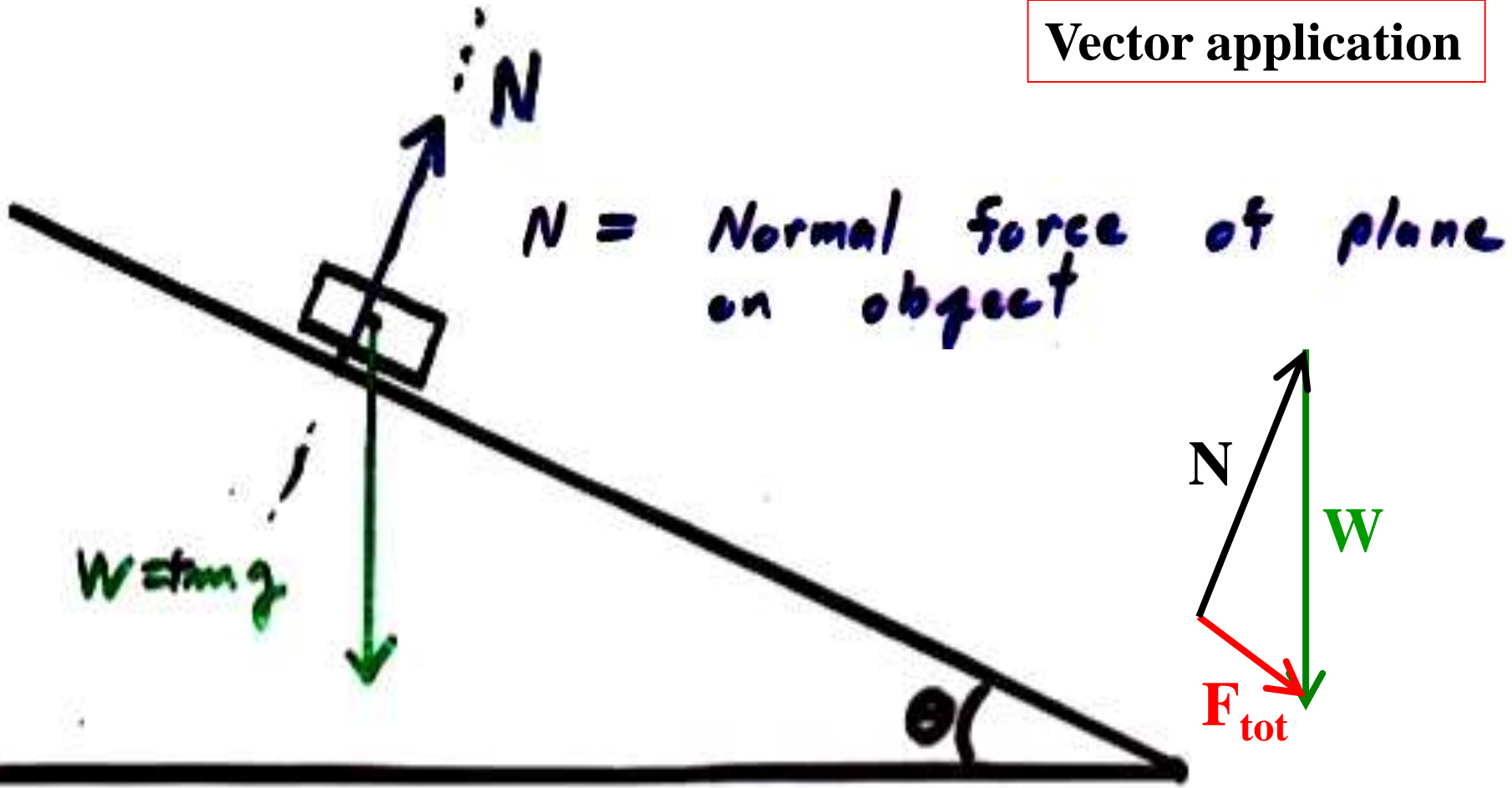
$$\vec{A} = A_x \hat{x} + A_y \hat{y}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y}$$

$$\vec{A} + \vec{B} = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y}$$

Notation allows keeping track of general vectors with unit vectors to keep track of direction and scalar multiplication to give magnitude.

Vector application

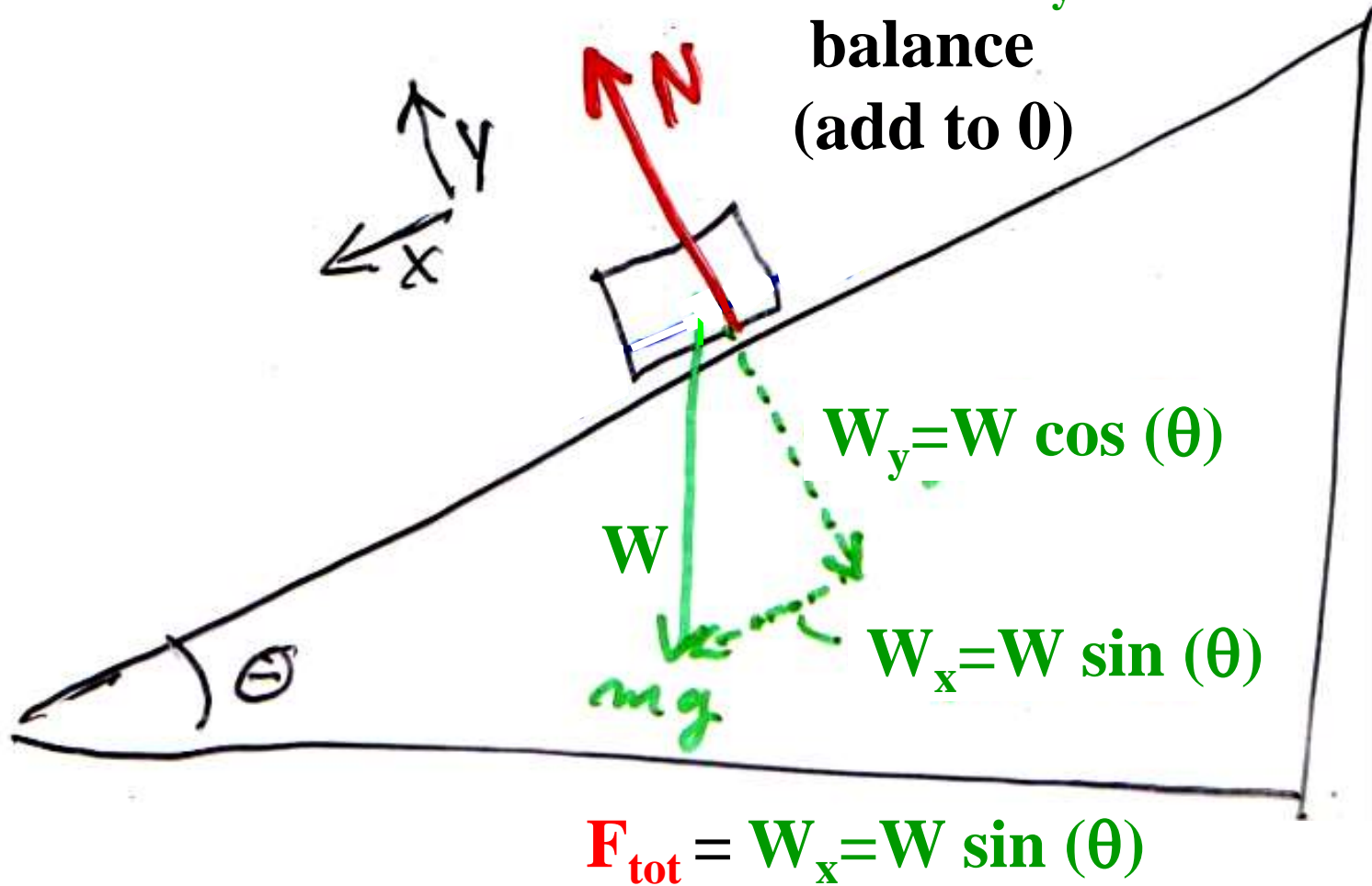


$W = +mg =$ Force of gravity on object
(Pull of earth on object)

Forces add like vectors

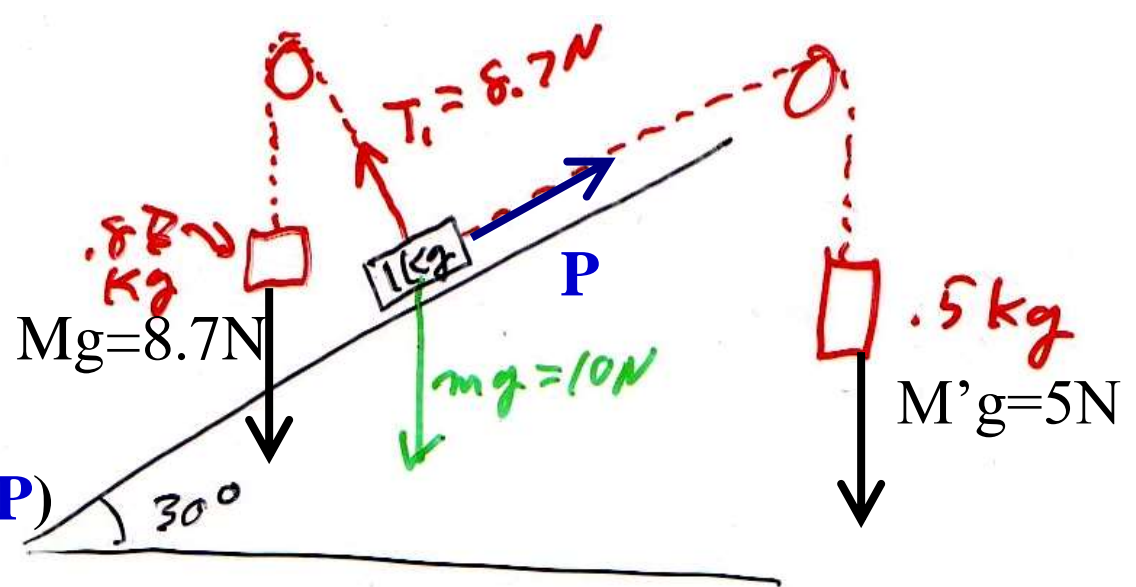
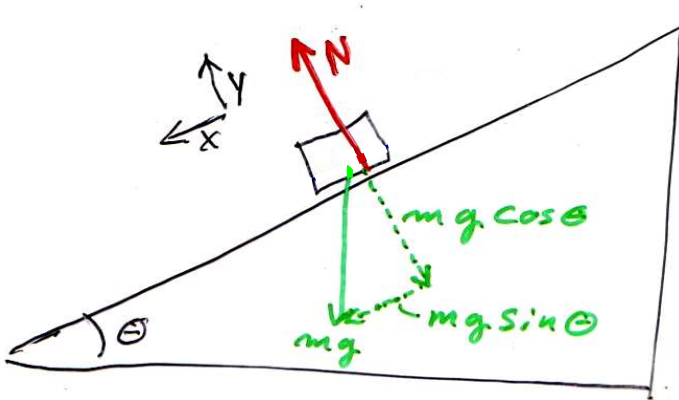
N and **W_y**
balance
(add to 0)

1-18b



As we will see later:

unbalanced force causes acceleration down plane

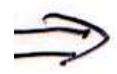


Tension in ropes (T_1 and P)
replace N and W .

$$\sum F_x = -P + mg \sin \theta = \bar{F}_x = 0$$

$$\sum F_y = +T_1 - mg \cos \theta = \bar{F}_y = 0$$

1-18c



$$P = mg \sin \theta$$

$$T_1 = mg \cos \theta$$

Demo
Forces add like vectors
 $F_{tot} = 0 \Leftrightarrow a = 0$

$$m = 1 \text{ kg} \quad \theta = 30^\circ \quad \left\{ \begin{array}{l} \sin \theta = \frac{1}{2} \\ \cos \theta = .866 \end{array} \right.$$

$$P = 1 \text{ (kg)} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot \frac{1}{2}$$

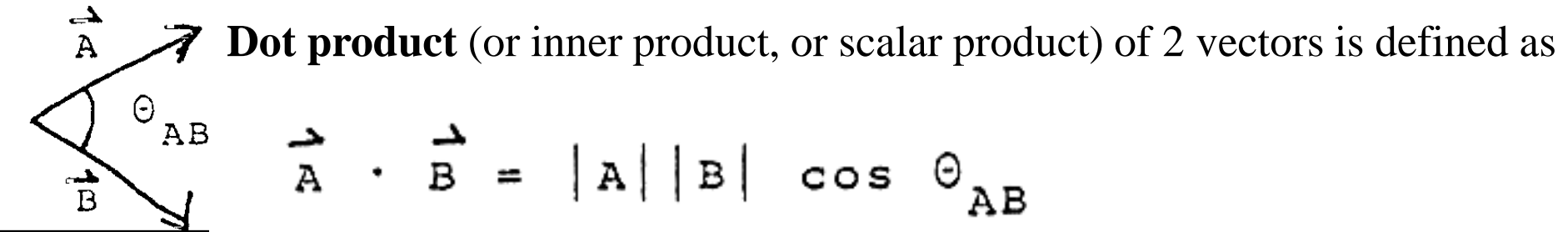
$$T_1 = 1 (9.8) \cdot .866$$

Vector application

$$P \approx 5 \text{ N}$$

$$T_1 \approx 8.7 \text{ N}$$

Physics/engineering and advanced students should be aware of the following.



Here A_x (A_y), is the x (y) component of \vec{A} along the x (y) axis and \hat{x} (\hat{y}) is the unit vector along the x (y) direction. (Note A_x (A_y) are just scalar numbers.).

$$\vec{A} = A_x \hat{x} + A_y \hat{y}$$

For unit vectors one has $\hat{x} \cdot \hat{y} = 0$ $\hat{x} \cdot \hat{x} = |\hat{x}|^2 = 1$ $\hat{y} \cdot \hat{y} = |\hat{y}|^2 = 1$

Therefore $\hat{x} \cdot \vec{A} = A_x$ $\hat{y} \cdot \vec{A} = A_y$

Thus the dot product with a unit vector can be obtain (“project out”) the component of the vector in the direction of the unit vector.

Advanced (not required for this course) topic. For physics, engineering, math, ... majors “An introduction to generalized vector spaces and Fourier analysis” see below