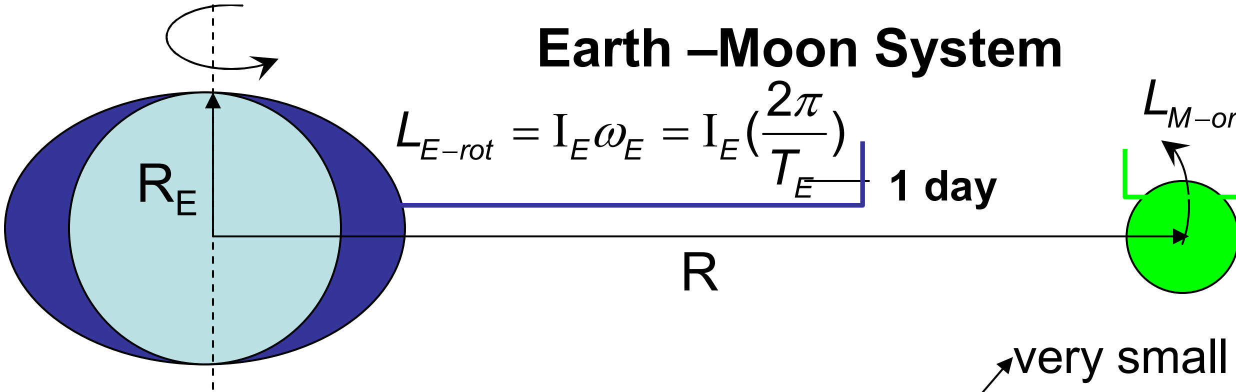
Earth –Moon System



$$L_{E-rot} = \mathrm{I}_E \omega_E = \mathrm{I}_E(\frac{2\pi}{T_E})$$

1 day

$$L_{M-orb} = \mathrm{I}_{M-orb}\omega_{M-orb} = \mathrm{m}_M R^2(\frac{2\pi}{T_{M-orb}})$$

27 days

$$\mathrm{I_E} = \frac{2}{5}\mathrm{M_E R_E^2}$$

very small (neglect)

$$\mathrm{L_{tot}=L_{E\text{-}rot}+L_{M\text{-}orbit}+L_{M\text{-}rot}}$$

constant

$$\boxed{L_{tot} = \mathrm{I}_E(\frac{2\pi}{T_E}) + \mathrm{m}_M R^2(\frac{2\pi}{T_{M-orb}})}$$

$$K_{E-rot} = \frac{1}{2}\mathrm{I}_E\omega_E^{\ 2} = \frac{1}{2}\mathrm{I}_E(\frac{2\pi}{T_E})^2$$

solid Earth – rotates under ocean tidal budge

- frictional energy loss

$$\mathbf{\Delta E_E = \Delta K_{E\text{-}rot} = \text{-}W_{tidal\text{-}friction}}$$

$$-W_{tidal} = \Delta K_{E-rot} = [\frac{(2\pi)^2}{2}\mathrm{I}_E](\frac{1}{T_E(later)^2} - \frac{1}{T_E(earlier)^2})$$

Earth's rotation slows with time

$$T_E(later) > T_E(earlier)$$

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constant

$$L_{tot} = \boxed{\mathrm{I}_E(\frac{2\pi}{T_E})} + \boxed{\mathrm{m}_M R^2(\frac{2\pi}{T_{M-orb}})}$$

decreases **increases**

L transferred to moon

moon moves away with time

$$\boxed{L_{tot} = \mathrm{I}_E(2\pi/T_E) + m_M\sqrt{GM_E}R^{1/2}}$$

$$\frac{m^3}{s^2 kg} = G$$

$$\mathrm{T^2 = \frac{(2\pi)^2}{GM_E}R^3}$$

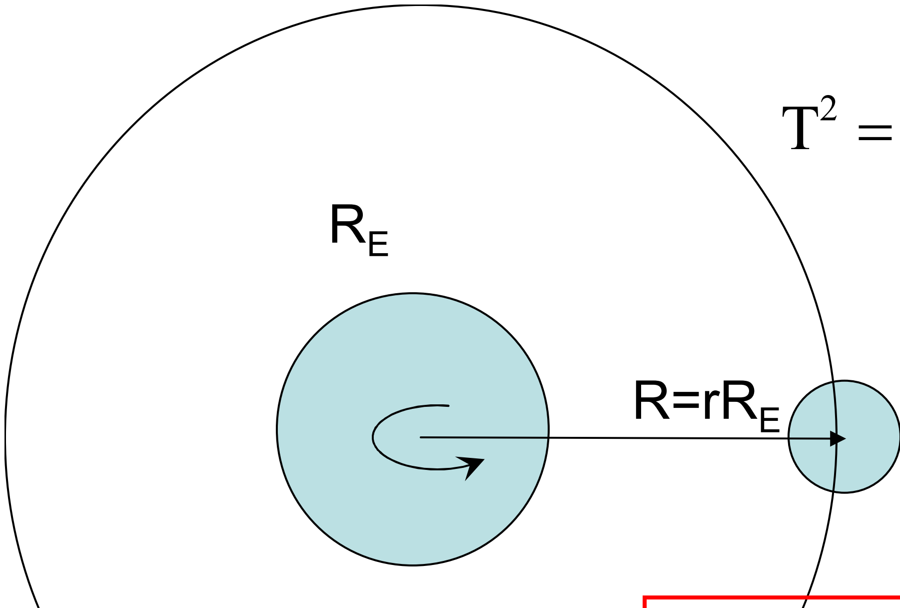
$$\mathrm{T^{-2} = \frac{GM_E}{(2\pi)^2 R^3}}$$

$$\mathrm{\frac{1}{T} = \frac{\sqrt{GM_E}}{(2\pi)}\frac{1}{R^{3/2}}}$$

$$T^2 = \frac{(2\pi)^2}{GM_E} R^3$$

$$T_{\text{M-orbit}} = 27 \text{ days} = T_{\text{E-rot27}}(now)$$

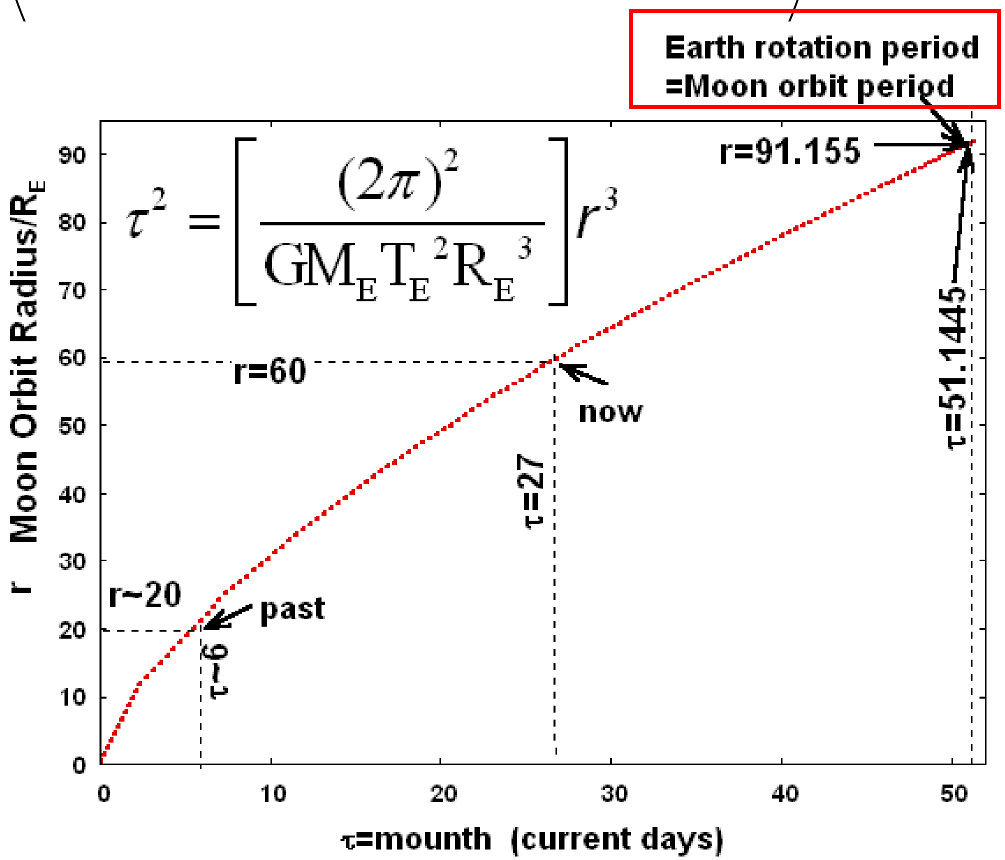
$$T_{\text{M-orbit}} = \tau T_{\text{E-rot}}(now)$$



$R=rR_E$

$$\tau^2 = \left[\frac{(2\pi)^2 R_E{}^3}{GM_E T_E{}^2}\right] r^3$$

$$\tau^2 = [0.003584]\, r^3$$



distant future

moon-orbit at 91 R_E

$\tau^2 = [0.0033375]\, r^3$

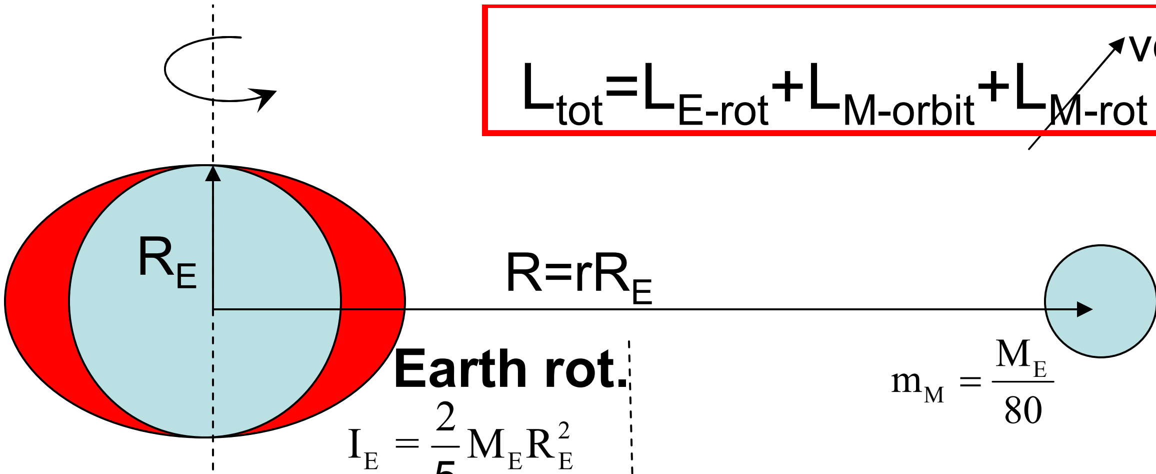
day'=month'=51 days (current)

distant past

moon- much closer
day shorter

$$L_{tot}=L_{E\text{-}rot}+L_{M\text{-}orbit}+L_{M\text{-}rot}$$

very small (neglect)



$$m_M = \frac{M_E}{80}$$

Earth rot.

$$I_E = \frac{2}{5}M_E R_E^2$$

$$L_{E\text{-}rot} = \{I\}\frac{2\pi}{T_E}$$

$$L_{E\text{-}rot} = \frac{2}{5}\left[\frac{M_E R_E^2 2\pi}{T_E}\right]$$

$$T_{E\text{-rot}}(\text{at other time}) = t\,T_{E\text{-rot}}(now)$$

$$L_{E\text{-}rot} = \frac{2}{5t}\left[\frac{M_E R_E^2 2\pi}{T_E}\right]$$

Moon orbit

$$T_{M\text{-orbit}} = 27\text{ days} = T_{E\text{-rot}27}(now)$$

$$T_{M\text{-orbit}} = \tau T_{E\text{-rot}}(now)$$

$$\tau = 27\ (now)$$

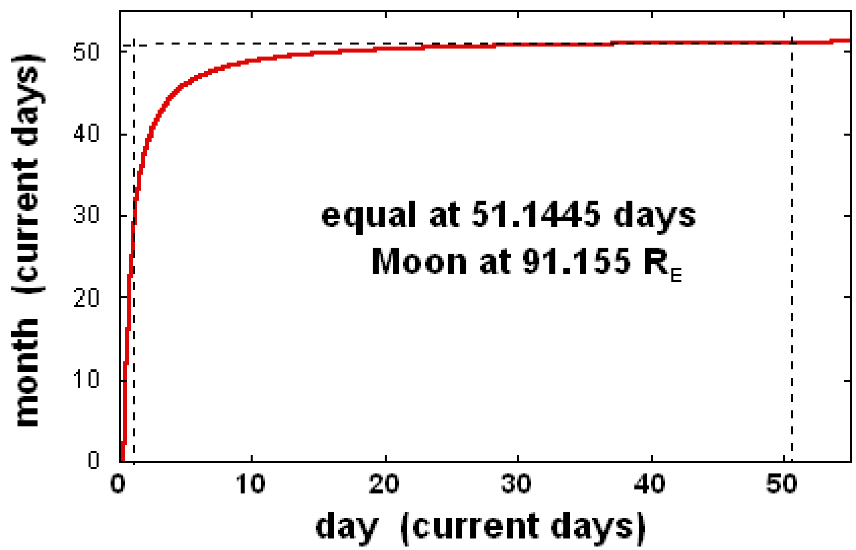
$$L_{M\text{-orbit}} = m_M R^2\left(\frac{2\pi}{T_{M\text{-orbit}}}\right)$$

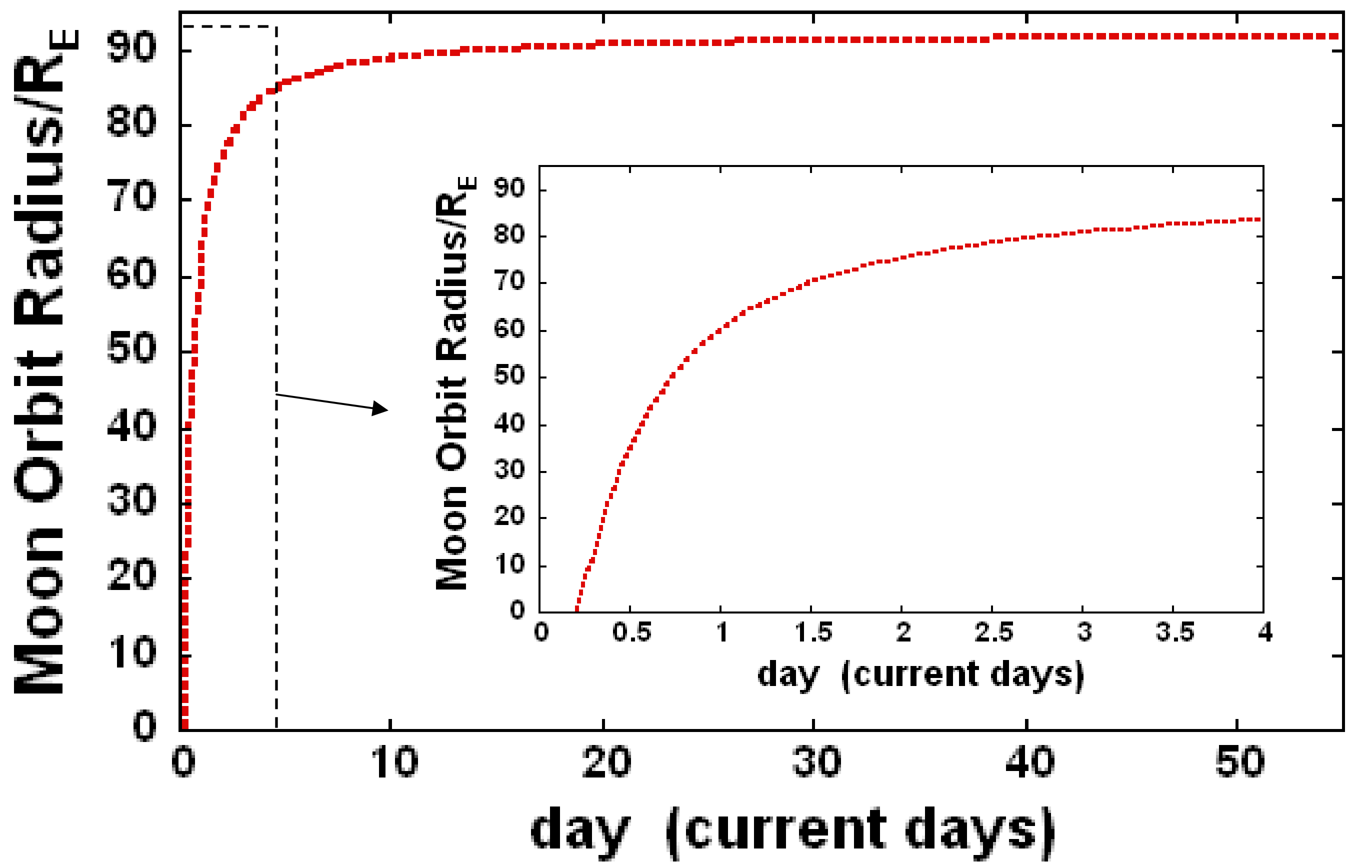
$$L_{M\text{-orbit}} = \frac{M_E}{80}(rR_E)^2\left(\frac{2\pi}{\tau T_{E\text{-rot}}}\right)$$

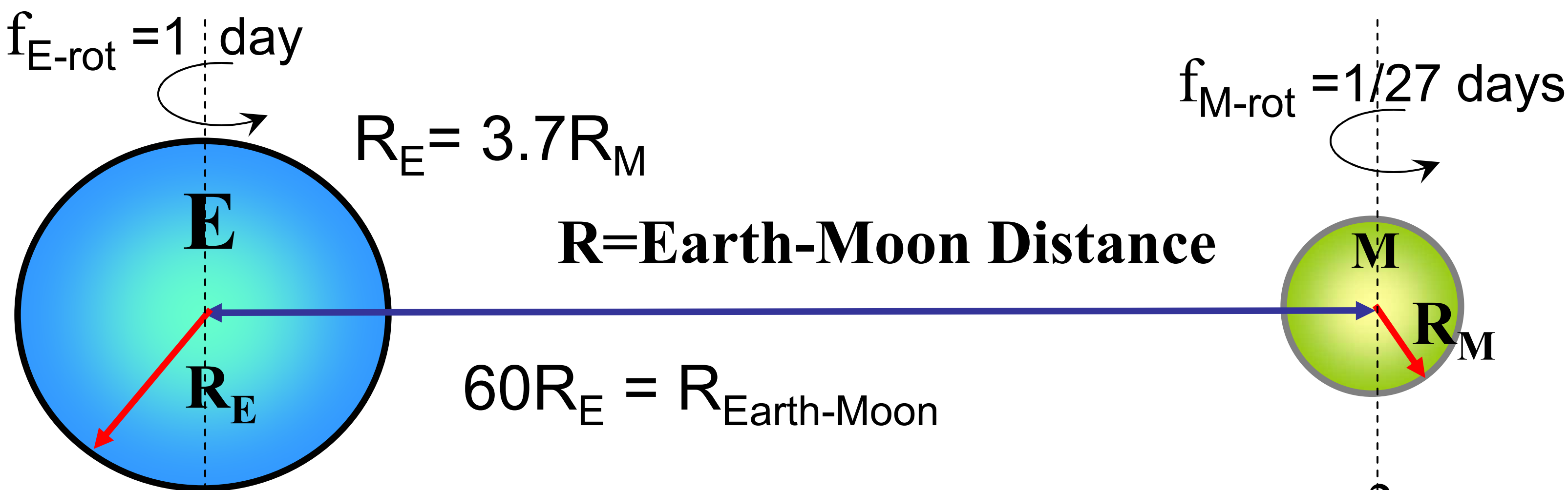
$$L_{M\text{-orbit}} = \frac{r^2}{80\tau}\left[\frac{M_E R_E{}^2 2\pi}{T_E}\right]$$

$$L_{totM\text{-orbit}} = L_{E\text{-rot}} + L_{M\text{-orbit}}$$

$$L_{totM\text{-orbit}} = \left\{\frac{2}{5t}+\frac{r^2}{80\tau}\right\}\left[\frac{M_E R_E{}^2 2\pi}{T_E}\right]$$


month (current days)
day (current days)
equal at 51.1445 days
Moon at 91.155 R_E


Moon Orbit Radius/R_E
day (current days)
Moon Orbit Radius/R_E
day (current days)



M_E = 80m_M

$$I_M = \frac{2}{5} m_M R_M^2$$

$$I_E = \frac{2}{5} M_E R_E^2$$

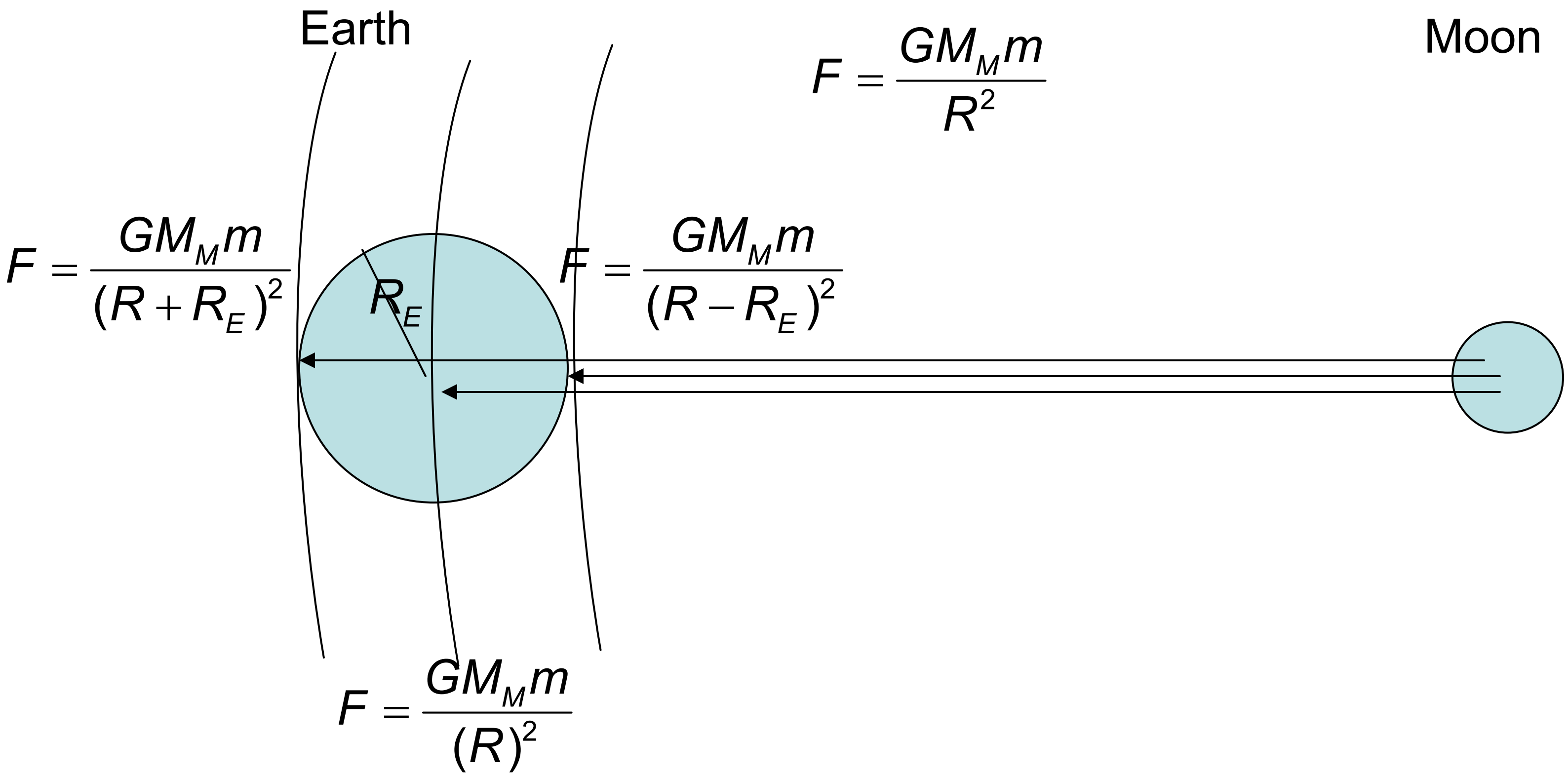
$$I_M = \frac{2}{5} m_M R_M^2 = \frac{2}{5}\frac{M_E}{80}\left(\frac{R_E}{3.7}\right)^2 = (0.000913) I_E$$

$$L_{E\text{-rot}} = I_E\,(2\pi f_{E\text{-rot}})$$

$$L_{M\text{-rot}} = I_M\,(2\pi f_{M\text{-rot}})$$

$$L_{M\text{-rot}} = (0.000913) I_E\,(2\pi f_{E\text{-rot}})\,\frac{1}{27}$$

$$L_{M\text{-rot}} = (0.0000338) L_{E\text{-rot}}$$


Earth
Moon
$F = \frac{GM_M m}{R^2}$
$F = \frac{GM_M m}{(R + R_E)^2}$
R_E
$F = \frac{GM_M m}{(R - R_E)^2}$
$F = \frac{GM_M m}{(R)^2}$