Einstein's Special Theory of Relativity

Postulate 1: The laws of physics have the same form in all inertial reference frames.

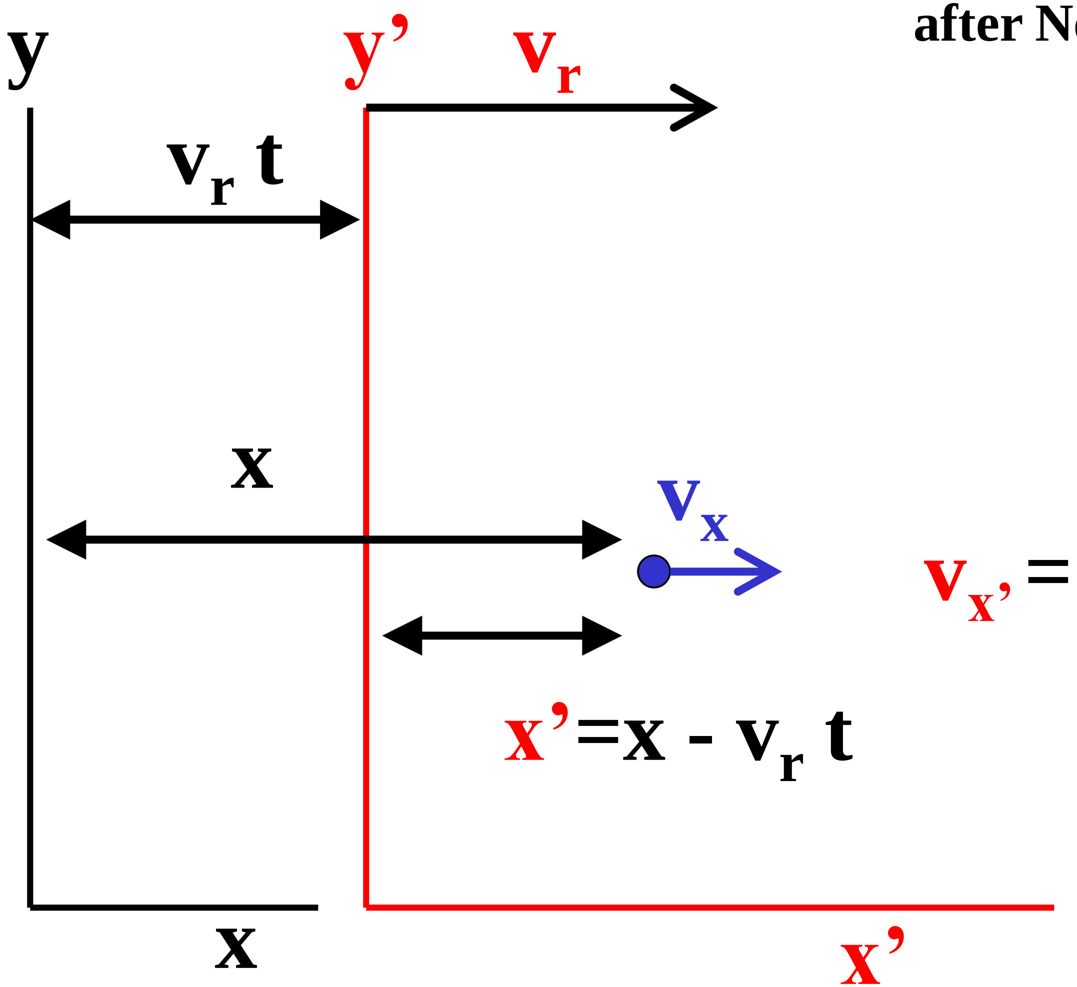
Postulate 2: Light propagates through empty space with a definite speed (c) independent of the speed of the source or of the observer.

Modifications to Newtonian physics as speed approaches c.

Newton	Einstein	
$E = \frac{1}{2}mv^2$	$E = m^* c^2$ $KE = m^* c^2 - m_0 c^2$	$m^* = \frac{m_0}{\sqrt{1 - v^2/c^2}}$
$p = mv$	$p = m^* v$	
$L = L_0$	$L = L_0\sqrt{1 - v^2/c^2}$	$E = \frac{p^2}{2m}$ $E^2 = p^2c^2 + m_0^2c^4$
$\Delta t = \Delta t_0$	$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$	

Inertial reference frame (constant v, a=0, F=ma is ok)

after Newton & Galileo



$y' = y$

$v_{y'} = v_y$

$v_{x'} = v_x - v_r$

$x' = x - v_r\, t$

Viewed from moving reference frame

Initial and final "conditions" (x, v) change

Problem transforms but physics the same (F=ma same !!!)

original reference frame

object dropped

$v_i = 0$

accelerated downward motion

$$y = y_i - gt^2/2$$

$$v_y = -gt$$

moving reference frame

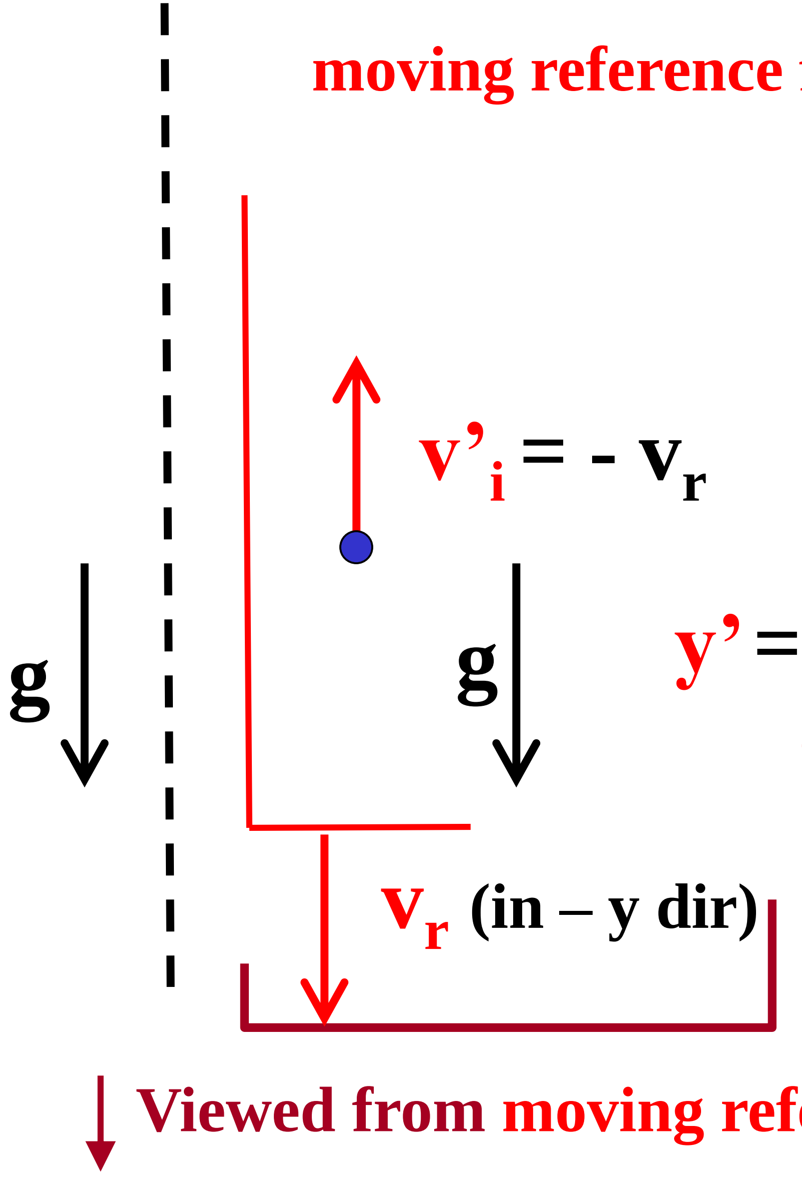
object projected upward

$v'_i = -v_r$

$$y' = y_i' + v_r t - gt^2/2$$

$$v_y' = v_r - gt$$

upward motion
stop
downward



Viewed from moving reference frame

Problem transforms but physics the same (F=ma same !!!)

original reference frame

object dropped

$v_i = 0$

g

accelerated downward motion

$$y = y_i - gt^2/2$$

$$v_y = -gt$$

moving reference frame

$v'_{xi} = v_r$

object projected horizontally

g

parabolic trajectory

v_r (in -x dir)

$$y' = y_i - gt^2/2$$

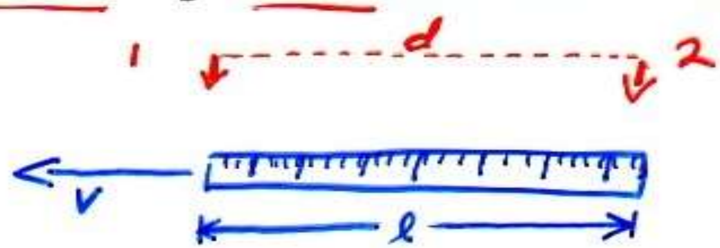
$$x' = x_i + v_r t$$

Viewed from moving reference frame

Problem transforms but physics the same (F=ma same !!!)

Illustration- measure length of moving ruler

- 1-flashes light to 2 when the end of the ruler is opposite to him

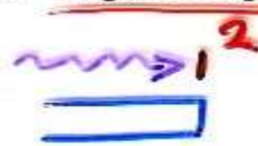


- adjust positions until 2 sees other end opposite to him at the same time he gets light signal from 1

What really happens

t=0 1-sends signal

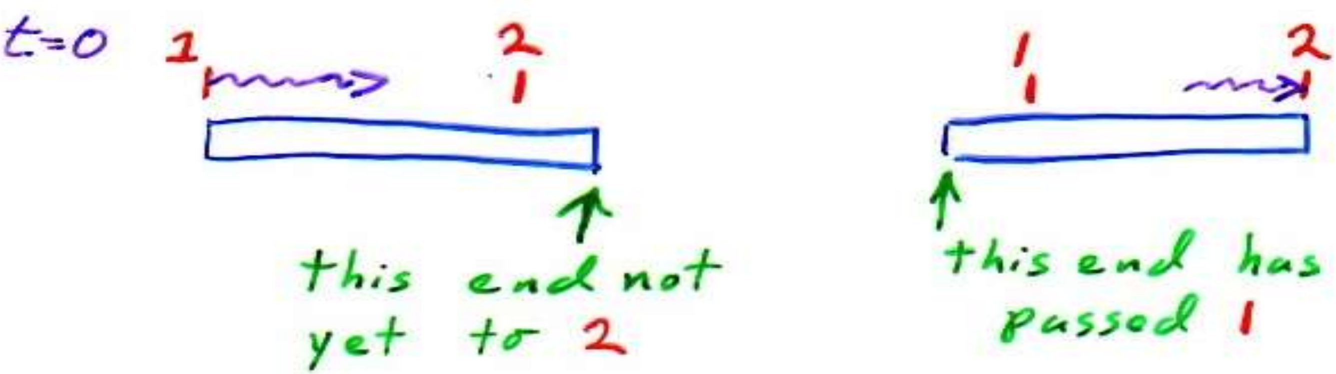
time-t later 2 gets signal & sees end



It took light time-t to travel distance d

d= ct distance = velocity X time

∴ ruler must have moved in time t



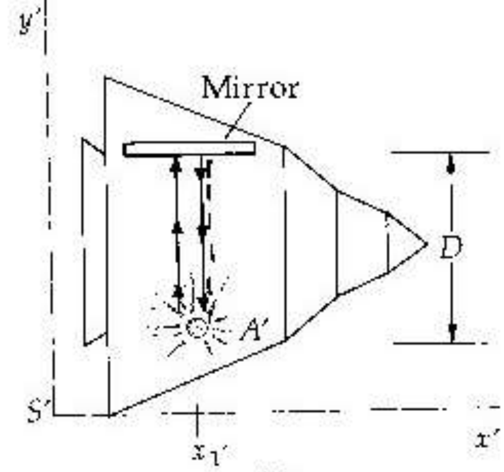
Time Dilation

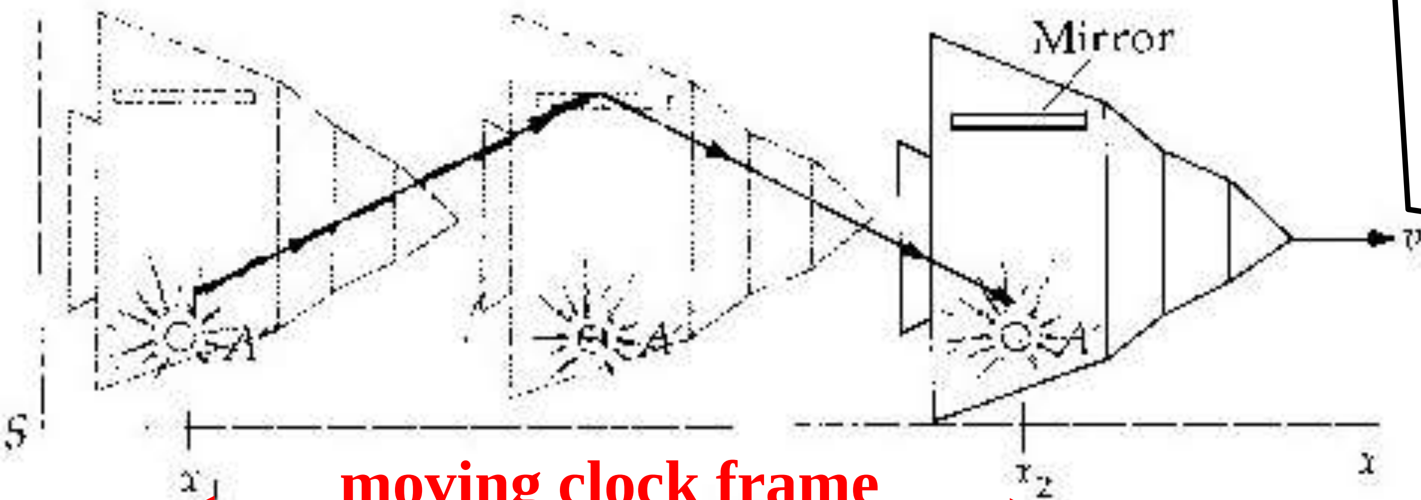
rest frame for clock

$$D = \frac{c\Delta t_0}{2}$$

save for later

1 clock at one place !!!





moving clock frame

clock 1
record time

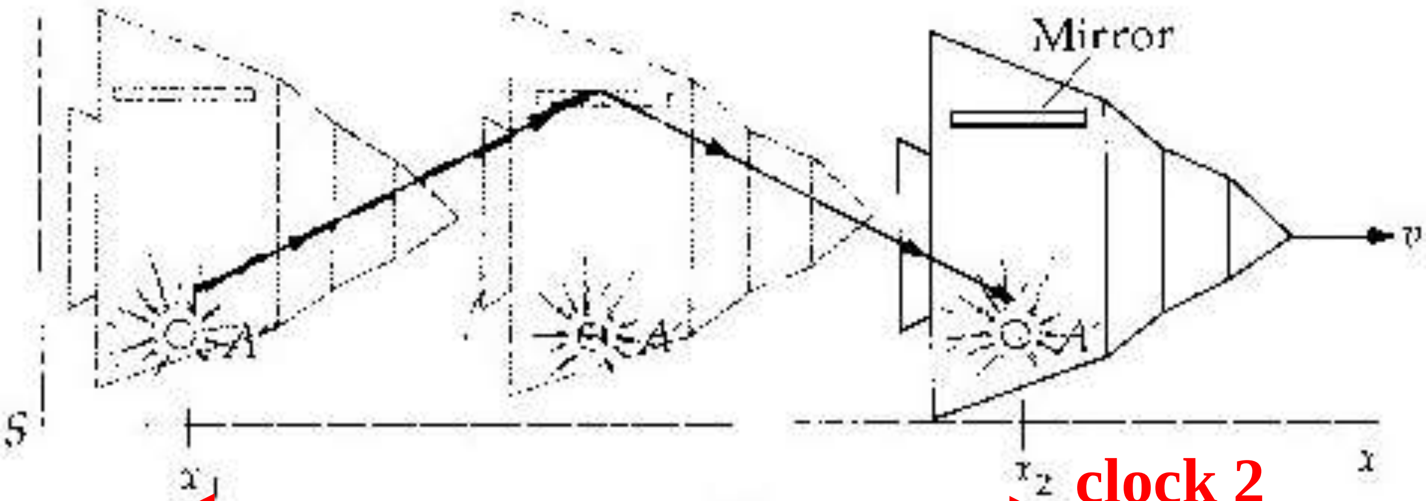
clock 2
record time

clock 1 & 2

a) had to be synchronized before experiment (at same place)

b) have to be brought together and times compared after

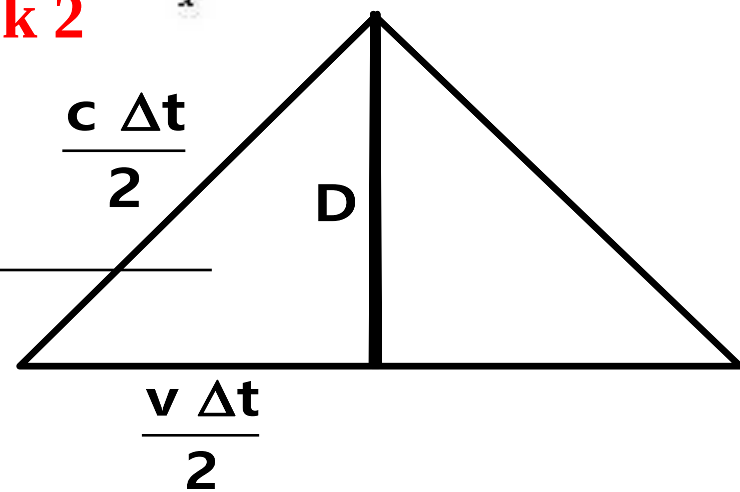
Time Dilation



clock 1 — clock 2

moving clock frame (2 clocks)

9-6a



$$\left(\frac{c\Delta t}{2}\right)^2 = D^2 + \left(\frac{v\Delta t}{2}\right)^2$$

recall

$$D = \frac{c\Delta t_0}{2}$$

$$D^2 = \frac{\Delta t^2\ c^2}{2^2}\left(1 - \frac{v^2}{c^2}\right)$$

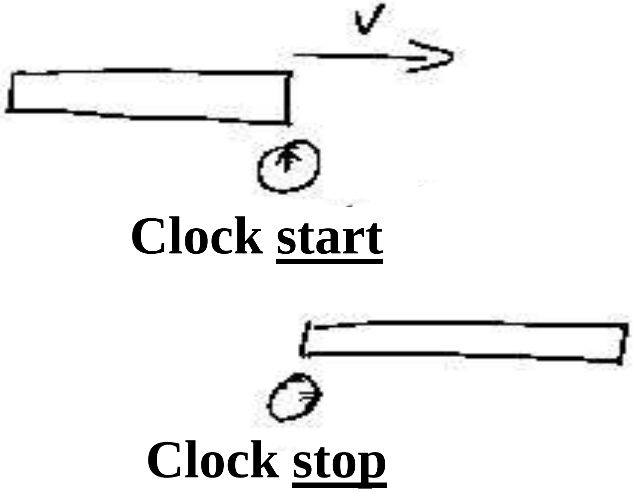
$$\therefore \quad \frac{\Delta t_0{}^2\ c^2}{2^2} = \frac{c^2 \Delta t^2}{2^2}\left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

time dilated in moving ref. frame !!!

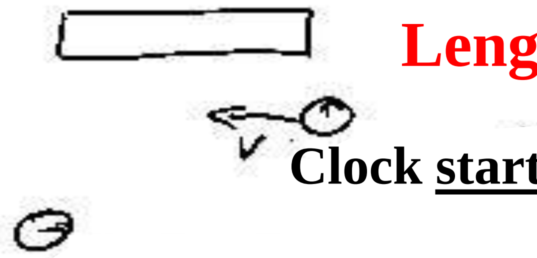
Experiment: measure the length of a ruler with a moving clock

Length contraction



rest frame for clock:
moving frame for ruler

$$L = \{ v\,\Delta t_0 \}$$



Clock stop
(when it reaches other end

rest frame for ruler:
moving frame for clock (actually 2 clocks)

$$L_0 = v\,\Delta t$$

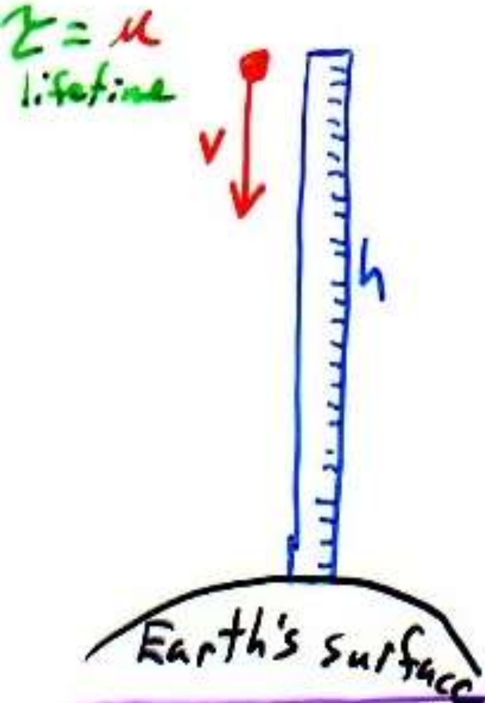
but $\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$

$$\therefore \quad L_0 = \frac{\{ v\,\Delta t_0 \}}{\sqrt{1 - v^2/c^2}}$$

so $$L_0 = \frac{L}{\sqrt{1 - v^2/c^2}}$$

$\Rightarrow \quad L = L_0 \sqrt{1 - v^2/c^2}$ less than 1 (length contracts)

Classic example of problem
The case of the μ-mesons that live too long



μ-mesons created in upper atmosphere

v/c close to 1 so relativity important

h= height at which μ-meson created

Problem
- know h at which μ-meson created
- know speed v
- know how long μ-mesons live (half life) ~τ

--> ≈ **None of the μ-mesons should reach the earth's surface** (they should die first)

But lots of μ-mesons reach the surface !!!!!

μ-meson problem (time dilation viewpoint)

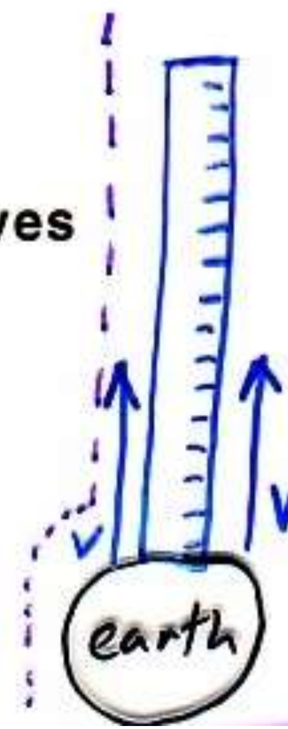
clock at rest on the μ-meson - lives life at one spot (from its view point)

"proper or rest frame of reference" for the clock

- all reference frames moving with respect to the proper frame will measure a dilated or lengthened time

clock rest frame

μ-meson "thinks" he lives just the right lifetime



clock moving frame

time measured is dilated
longer time

observer "thinks" μ-meson lives too long

μ-meson problem (length contraction viewpoint)

ruler at rest with respect to the earth

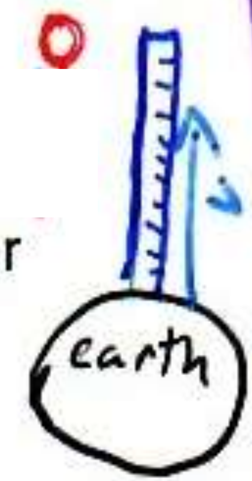
earth is rest frame for ruler

all moving frames will measure a contracted (shorter) length

<u>ruler moving frame</u>

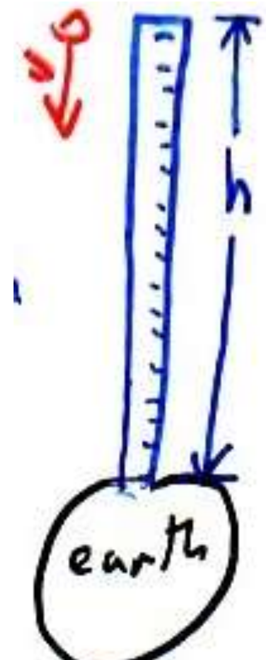
μ-meson sees contracted ruler running past

μ-meson "says" it is easy to cover this shortened distance in his lifetime!!!



<u>ruler rest frame</u>

ruler - full length = h



μ meson: time dilation view

μ - meson

$\tau_0 \simeq 2.2(10)^{-6}$ s

v = .998 c = (.998) 3 (10)^8 m/s

Guess (wrong) how far it should travel

$\tau_0 \cdot v = (.998)\, 3(10)^8\, 2.2(10)^{-6} = 660$ m

Actually we see μ born at 1 place and die at another

$\therefore$ we are in the moving reference frame for the clock.

Time dilation

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} = \frac{\tau_0}{\sqrt{1-(.998)^2}} = \tau_0 \, 16$$

$\therefore$ we see it travel

$\tau v = \tau_0 \cdot 16 \cdot v \simeq 11$ km !!

μ meson: length contraction view

$\tau_0 = 2.2(10)^{-6}$ s is lifetime

How long does the 11 km distance to Earth look to the μ-meson who views it in a moving reference frame?

$$L = L_0\sqrt{1 - v^2/c^2}$$

$$v = .998\,c$$

$$\Rightarrow L = L_0 \frac{1}{16}$$

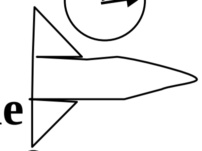
$$\therefore\ L = \frac{11\text{ km}}{16} = \underline{660\text{ m}}$$

$$\tau_0\,(.998c) = 660\text{ m}$$

everything is ok for μ

Space ship travel to 100 light year (ly) distant star

clock at rest
⇒ ship
proper ref. frame



v=0.999 c

earth

star

100 ly [= {3(10)^8 m/s} {365 d} {24 hr/d} {3600 s/hr}= 9.5 10^{15} m]

clock at 2 places ⇒ earth-star in moving ref. frame

$$\Delta t_{\text{earth-star}} = \frac{\Delta t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

time dilation view

$$1-\frac{v^2}{c^2} = 1-0.999^2 = 1-0.998001 = 0.001999$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 22.4$$

$$\sqrt{1-\frac{v^2}{c^2}} = .0447$$

100 years = 22.4 Δt_0

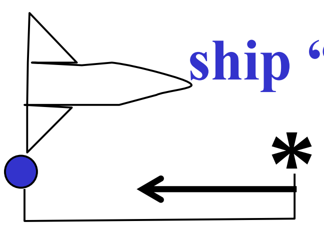
Δt_0 = 100/22.4 yr= 4.47 yr

Length contraction view

earth

star

ruler at rest ⇒ 100 ly proper length



ship "sees"

v=0.999 c

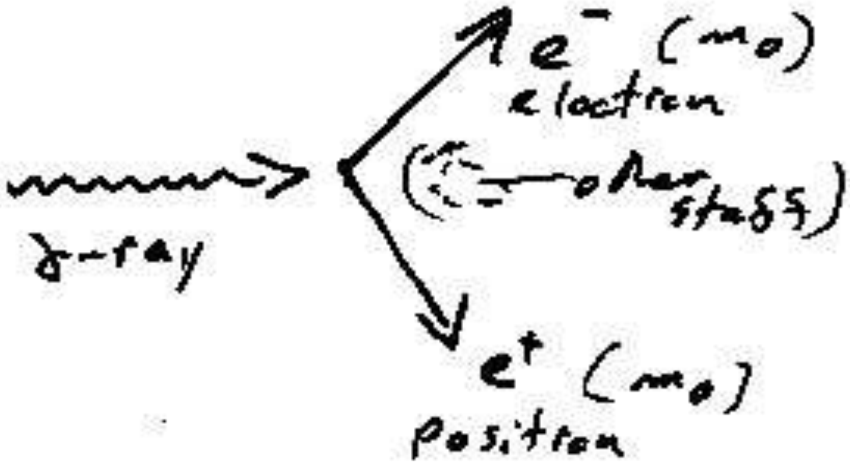
moving ruler ⇒ contracted

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$L=100\ (.0447)=4.47\ \text{ly}$$

Simple Example of $E = mc^2$

Pair creation

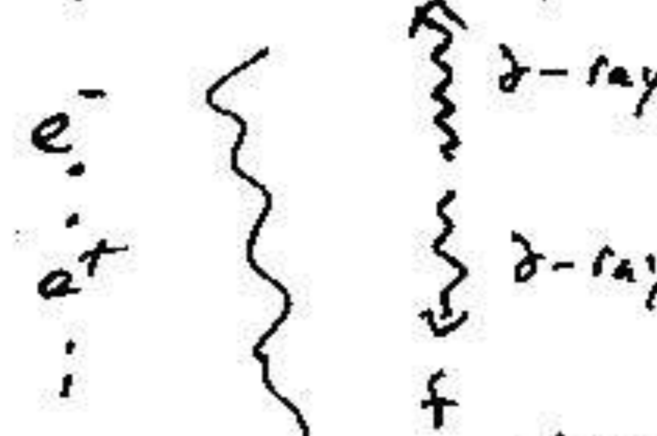


Min. energy of γ-ray needed

$E = m_0 c^2 + m_0 c^2$

$E_{min} = 2 m_0 c^2$

Pair annihilation



$E_i = 2 m_0 c^2$

Plancks const ↓

$E_f = 2hf$

$E_i = E_f$ (in next example)

$2 m_0 c^2 = 2hf$

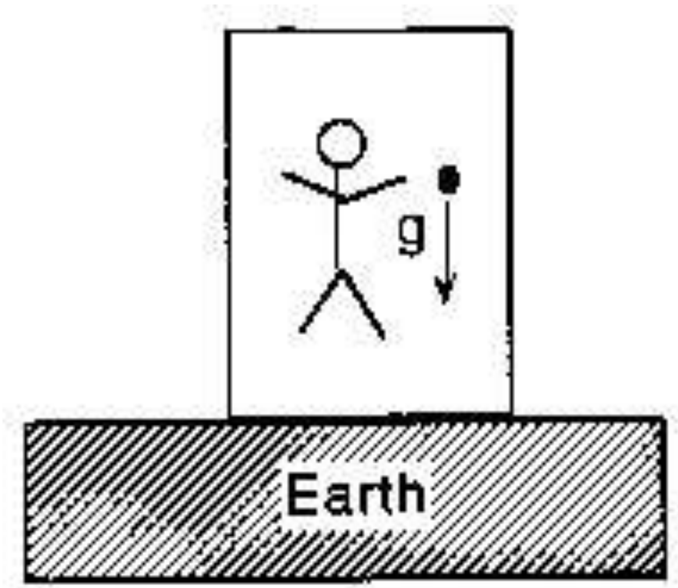
Q what is the freq.

$$f = \frac{m_0 c^2}{h} = \frac{9(10)^{-31}\text{kg}[3(10)^8\text{m/s}]^2}{6.6(10)^{-34}\text{Js}} = \frac{81(10)^{-31}(10)^{16}\text{J}}{6.6(10)^{-34}\text{Js}} = 1.2(10)^{20}\text{Hz}$$

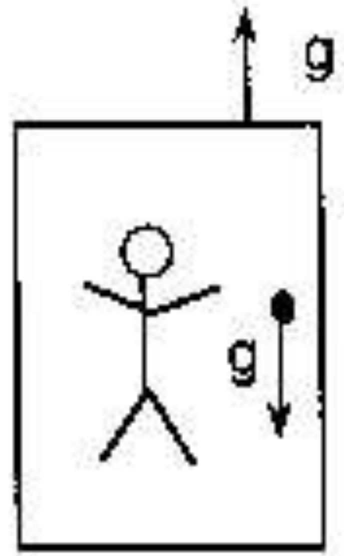
Einstein's General Theory of Relativity (1915)

- **considers gravity and accelerating reference frames on same footing**

in General Theory of Relativity
Gravitational forces ⇒ curvature of space



On Earth: person sees object accelerate down with g



in deep space

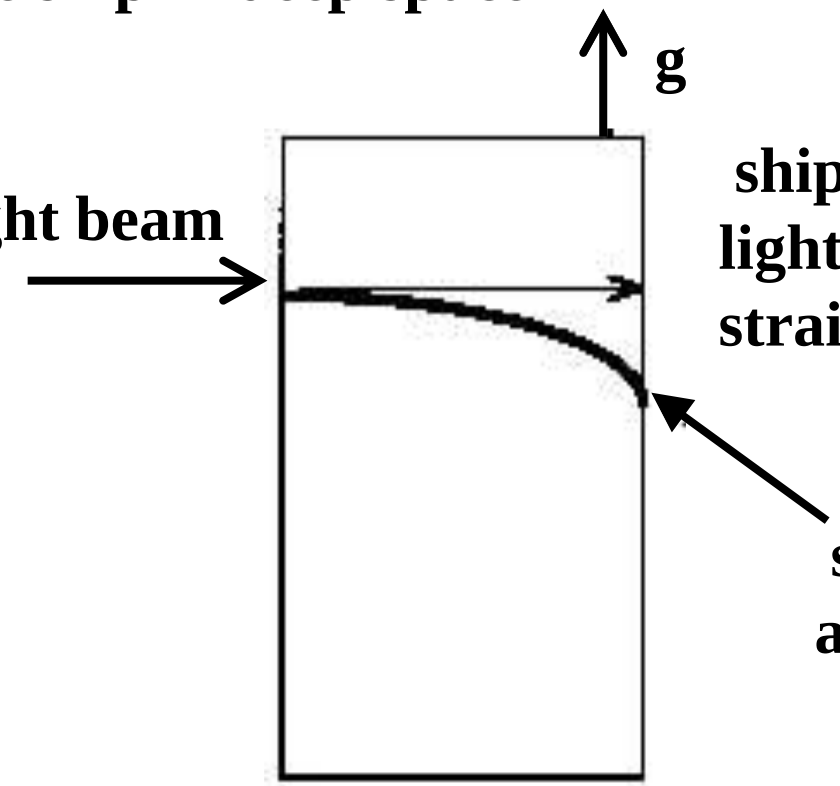
In space ship accelerating at a person sees object accelerate down with g

- **no way to tell whether gravity or acceleration of box/ship (reference frame) is causing the object's acceleration**

Principal of Equivalence !!!

Now what about light.

space ship in deep space

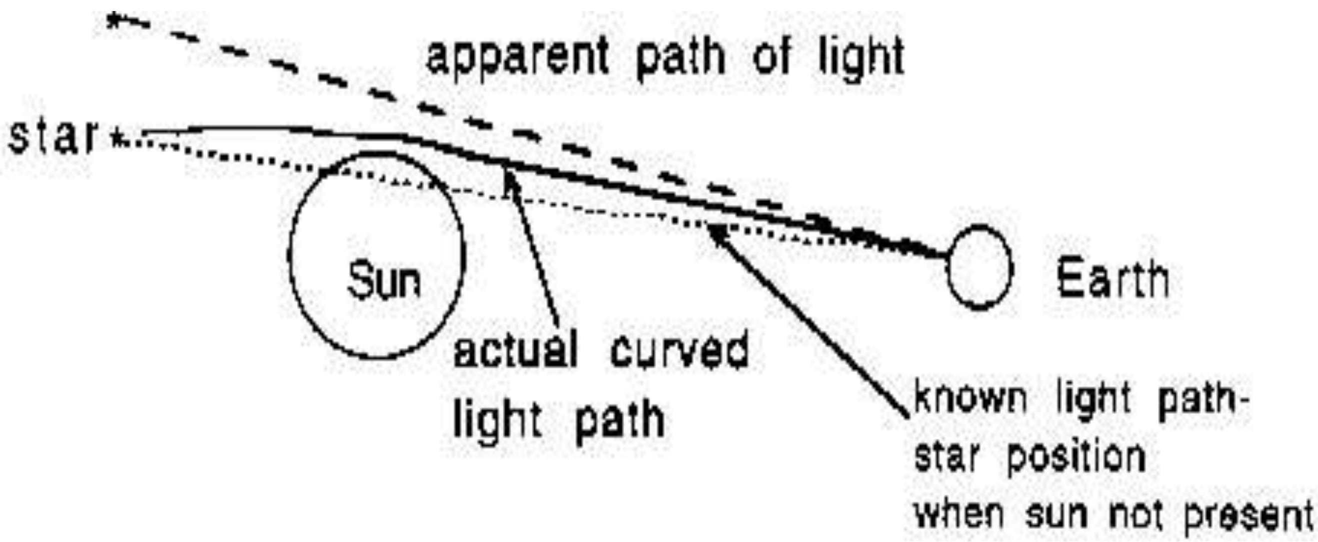


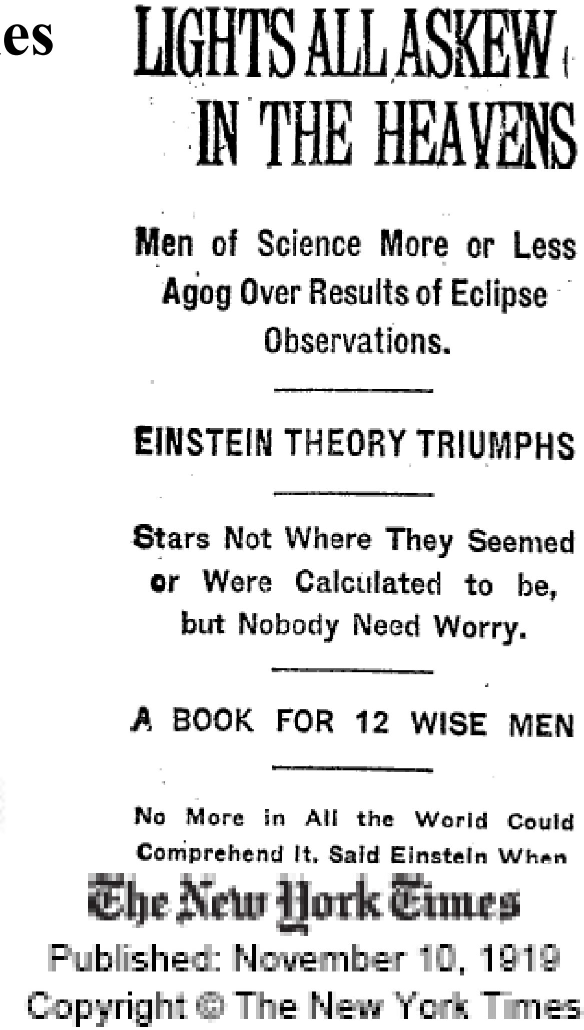
- **light follows curved path in accelerating reference frame !!!**

“Principle of Equivalence” therefore implies

- **Light follows curved path in a gravitational field !!!**

"Principle of Equivalence" therefore implies




LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

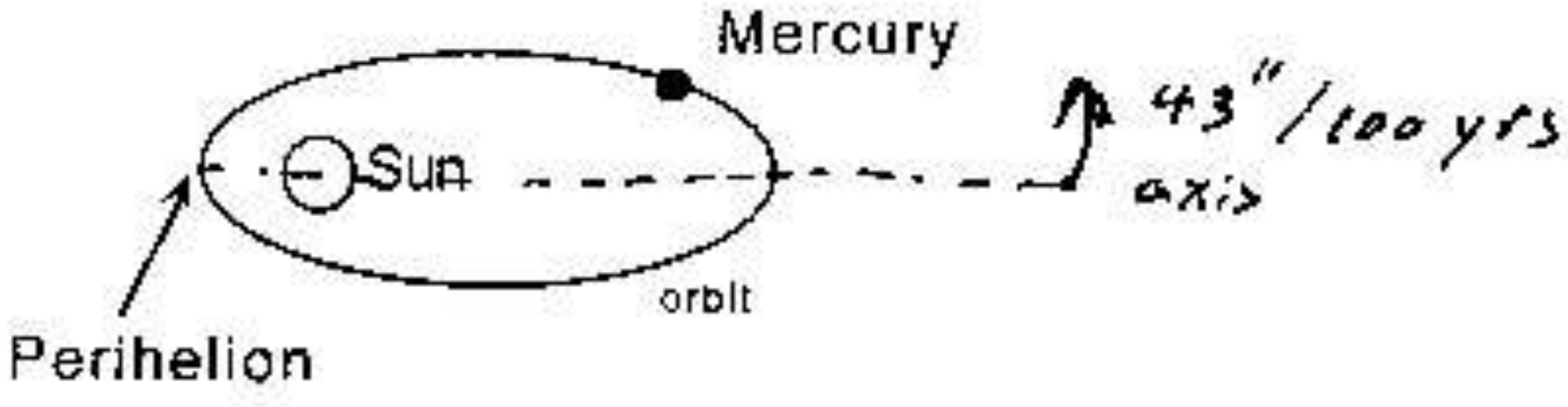
No More in All the World Could Comprehend It, Said Einstein When

The New York Times

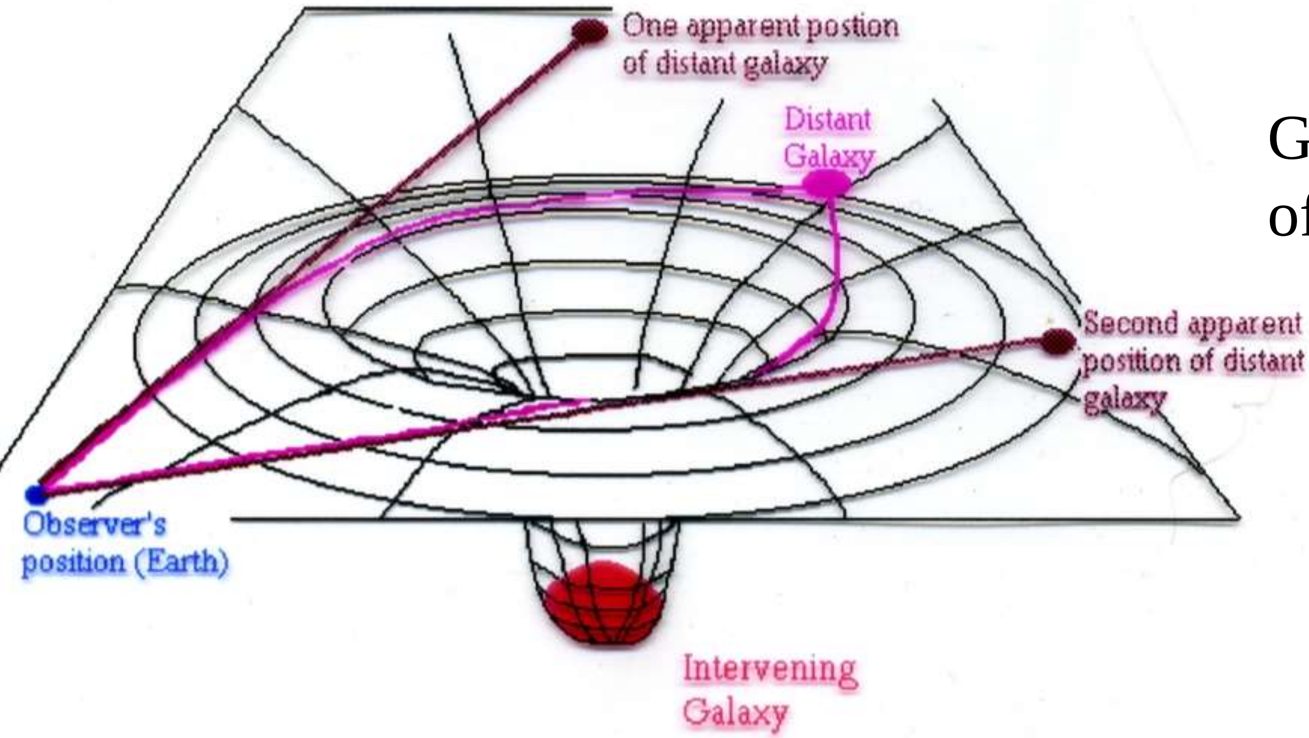
Published: November 10, 1919

- **observation of deflection of star light path at solar eclipse confirmed Einstein's ideas in the General Theory of Relativity**
- **Einstein's General Theory of Relativity**
  - gravitational forces replaced by the curvature of space
  - light just follows the curvature of space

Advance of Mercury's orbit also proves quantitative confirmation of Einstein's GR Theory

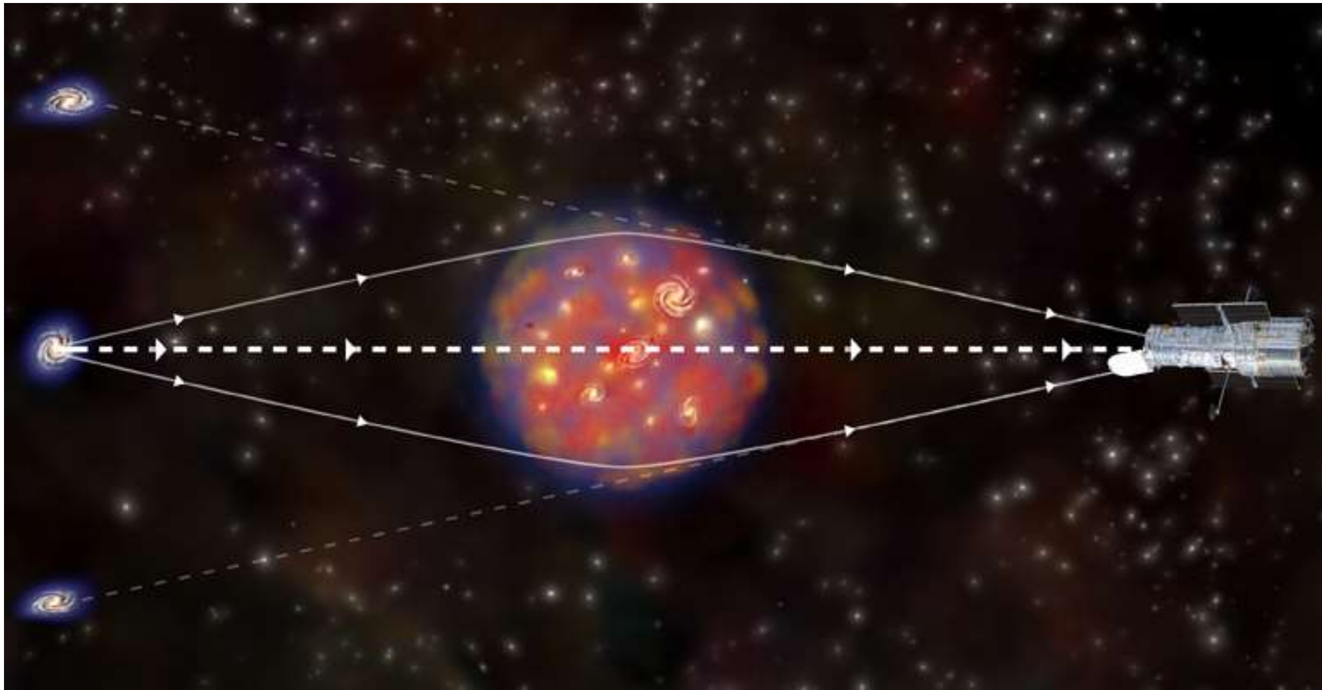


Advance of the perihelion of Mercury had been long known deviation from Newton's Laws.



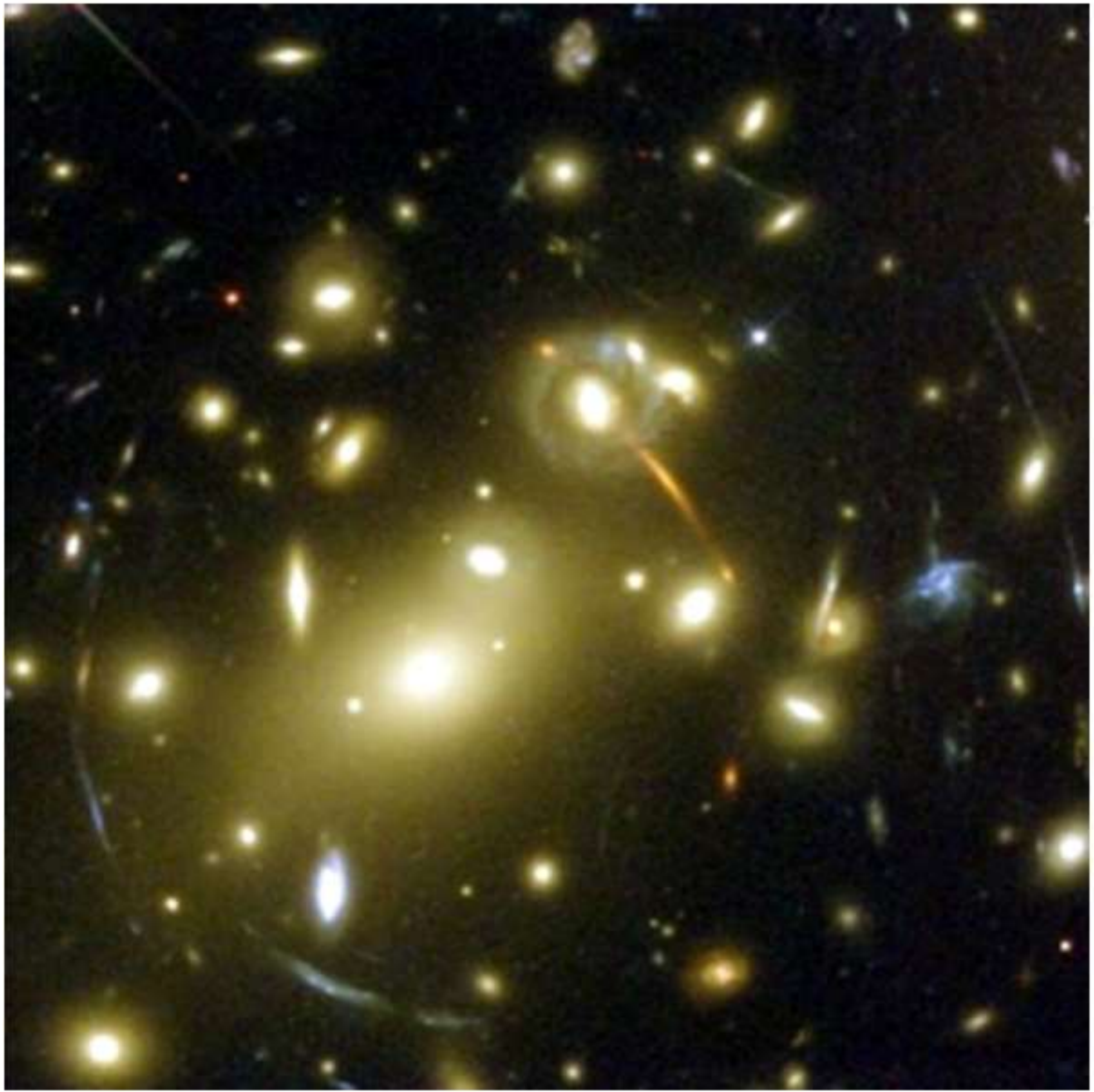
Gravitational Lensing
of distant objects

9-19



C. Keaton Rutgers

4x splitting of background galaxy image by foreground galaxy



Arc distortions of background galaxies images by foreground galaxy (at center)