$\phi = B_{\perp}A$ = magnetic flux $\qquad \Phi = \sum B_{\perp}\Delta A$

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} \qquad \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

Lentz's Law Direction of current **opposes** $\Delta\varphi$ which created it.

$$(e^{-\frac{t}{L/R}})$$

$$V_{max} = I_{max}\,Z \qquad Z_R = R \qquad Z_L = X_L = \omega L \qquad Z_C = X_C = \frac{1}{\omega C}$$

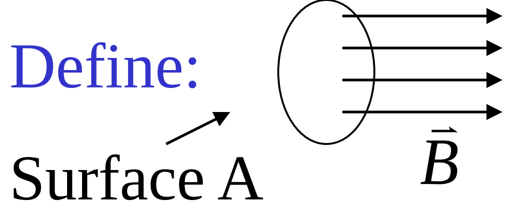
$$Z_{RLC} = \sqrt{R^2 + (X_L - X_C)^2}$$

<u>Recall:</u> moving charges (I) in $\vec{B}$ $\longrightarrow$ force

moving charges (I) $\longrightarrow$ created. $\vec{B}$

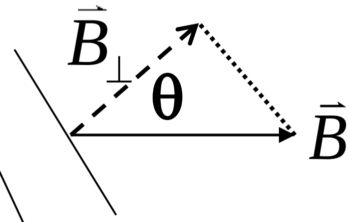
Now we consider (B) $\longrightarrow$ I connection.

Define:



$\phi = B_{\perp} A =$ magnetic flux

$\phi = AB_{\perp} = A\,B\cos\theta$



$B \perp$ to surface.

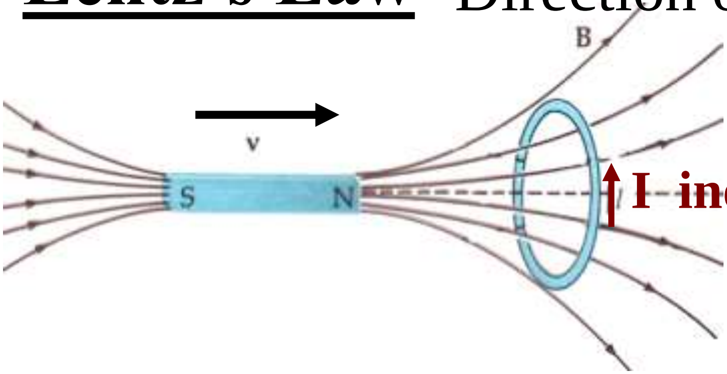
Faraday's Law

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t}$$

EMF created by $\dfrac{\Delta\phi}{\Delta t}$

Lentz's Law

Direction of current opposes $\Delta\varphi$ which created it.



I induced

$\vec{B}$ $\qquad \Delta\vec{B} = \vec{B}_{final} - \vec{B}_{initial}$

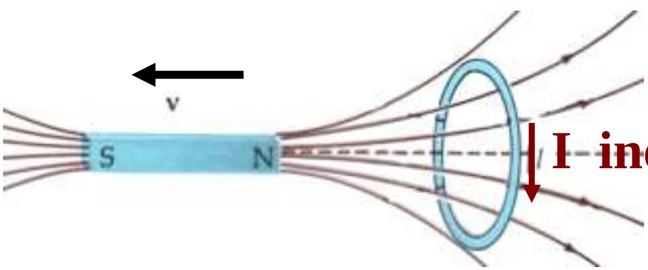
B induced

$\vec{B}$ later

$$\Delta\varphi = \Delta(AB_{\perp})$$

In these examples A=constant

$$\therefore \quad \Delta\varphi = A\Delta(B_{\perp})$$



I induced

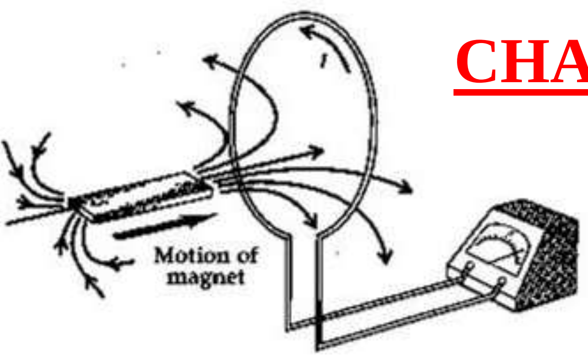
$\vec{B}$

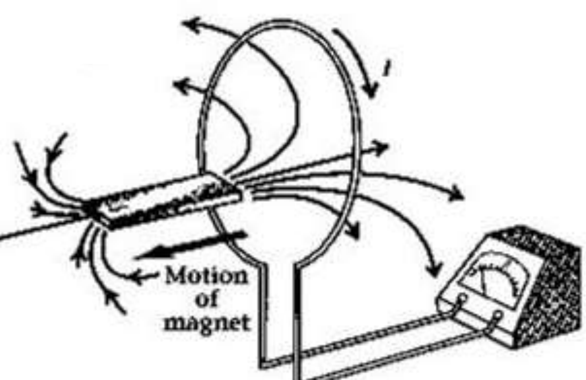
$\vec{B}$ induced

$\vec{B}$ later

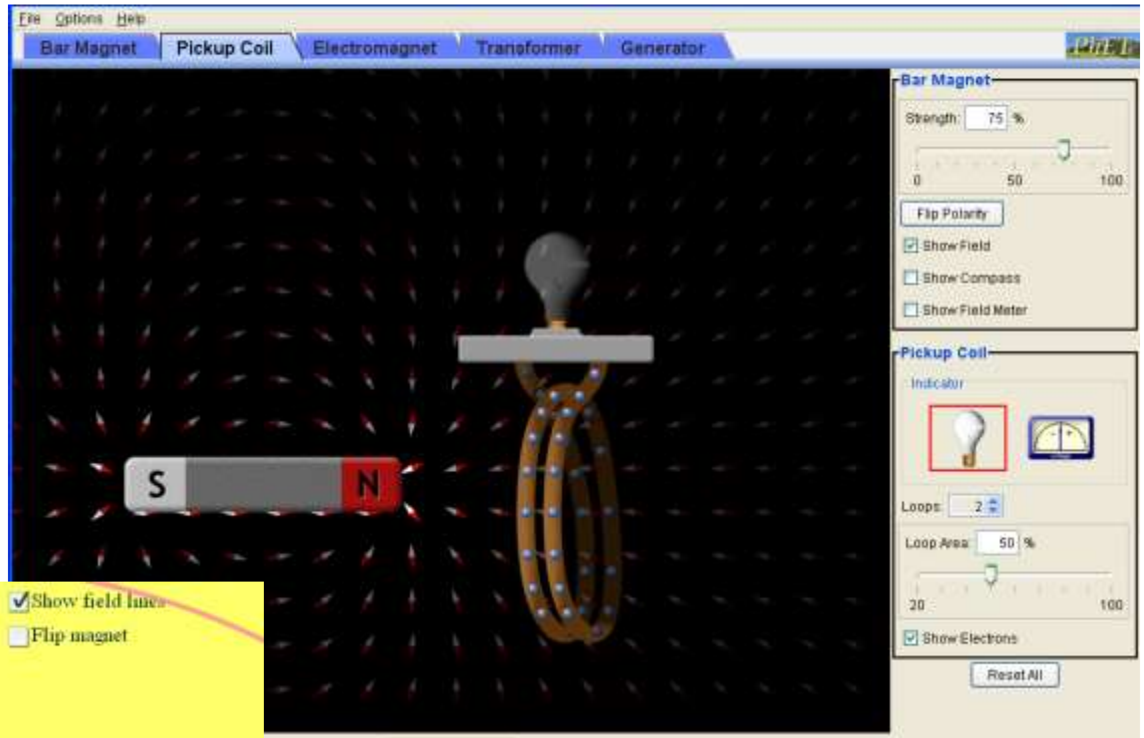
$$\Delta\vec{B} = \vec{B}_{final} - \vec{B}_{initial}$$

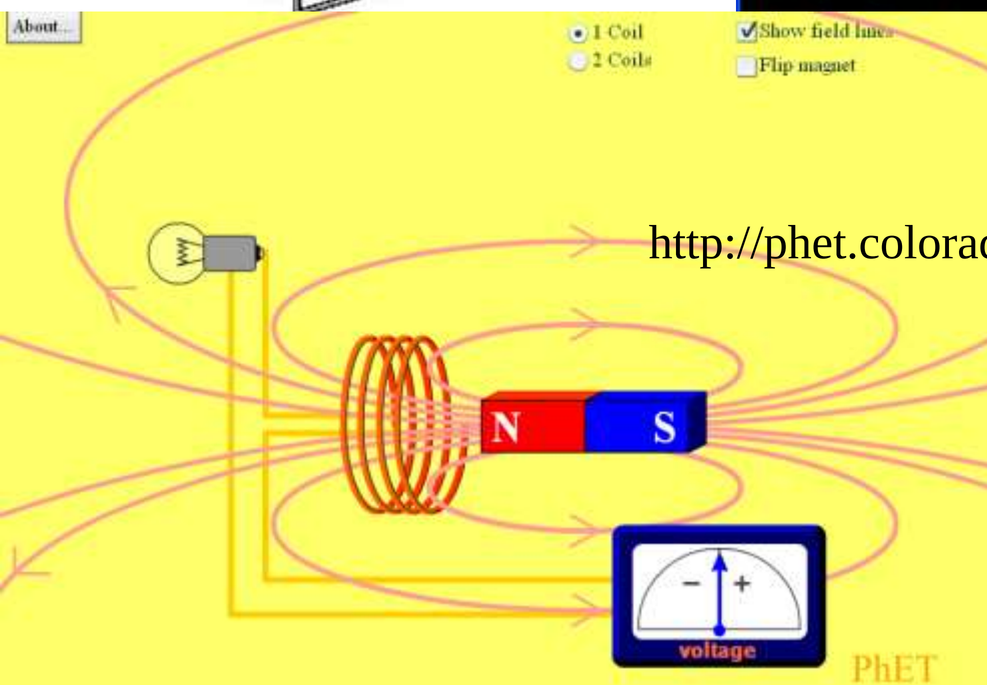
CHANGE IN FLUX CAUSES CURRENT !!!





http://phet.colorado.edu/en/simulation/faraday





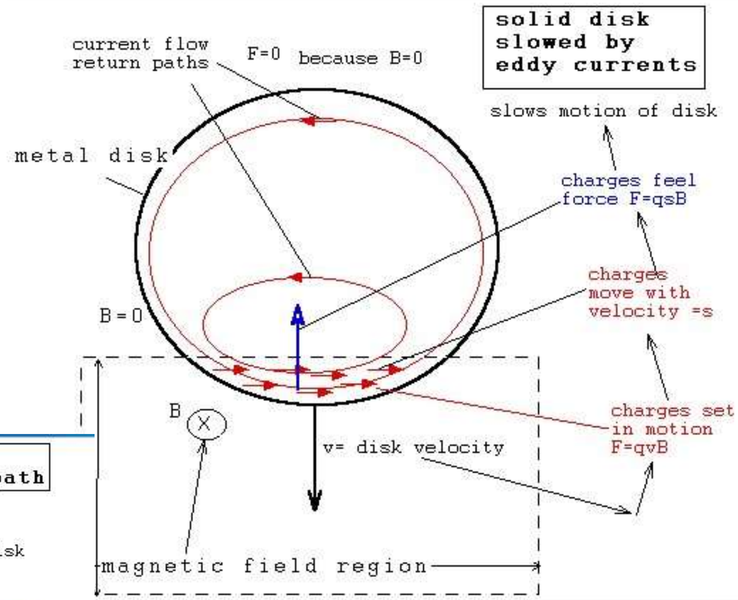
http://phet.colorado.edu/sims/faradays-law/faradays-law_en.html

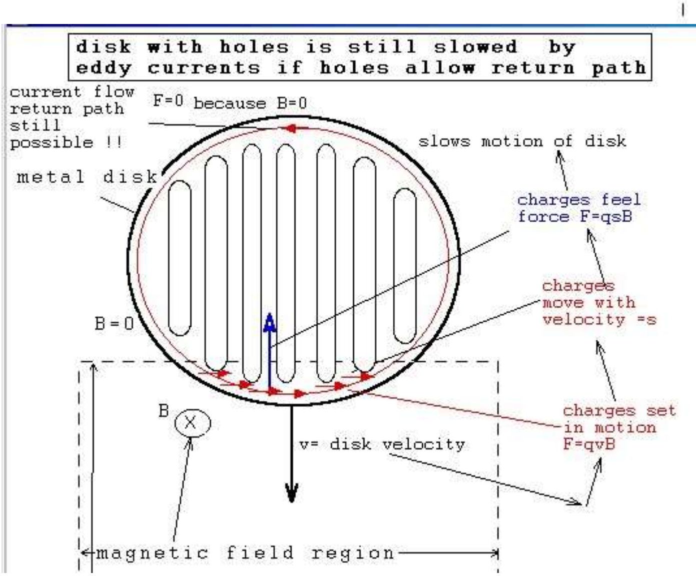
5-2a

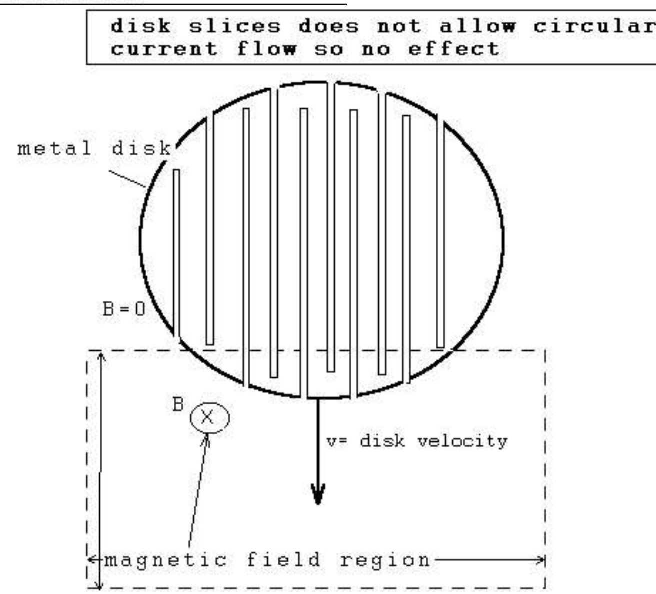
Eddy currents

Falling Magnet Applet
http://web.mit.edu/jbelcher/www/java/falling/falling.html

Magnet in Cu tube http://www.physics.rutgers.edu/~croft/magnetintube.wmv



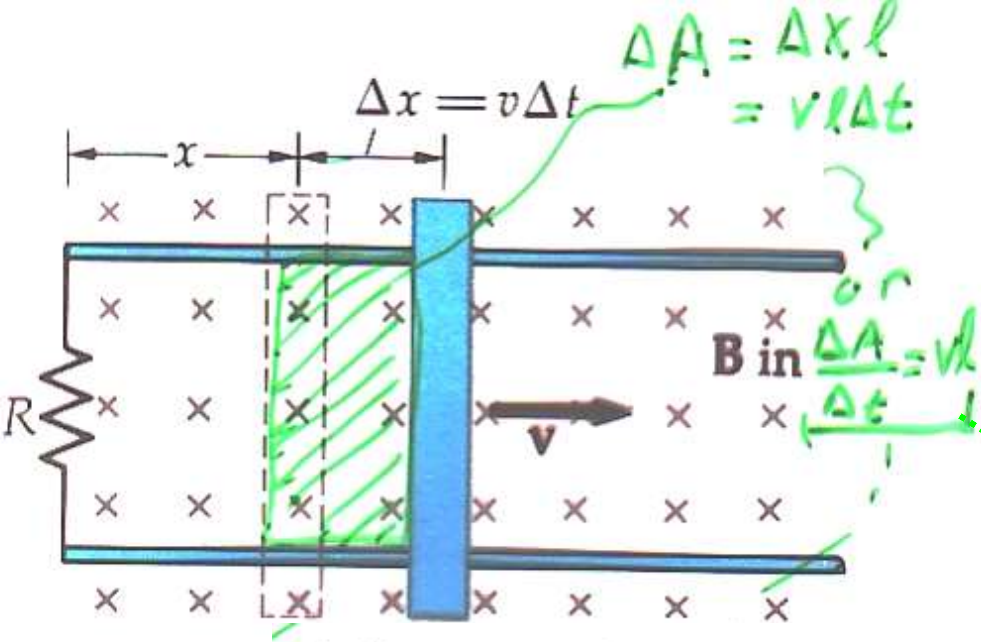


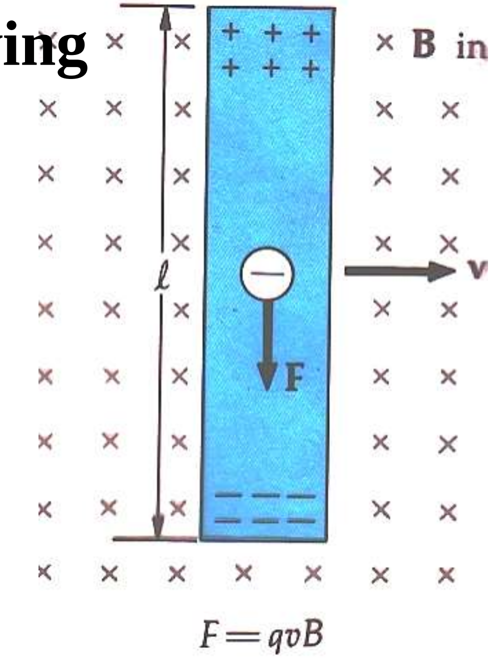


Magnet- on Cu Inclined plane

http://www.physics.rutgers.edu/~croft/2007magnetonplane.wmv

metal rod moving on metal rails in B field





Voltage or EMF across rod

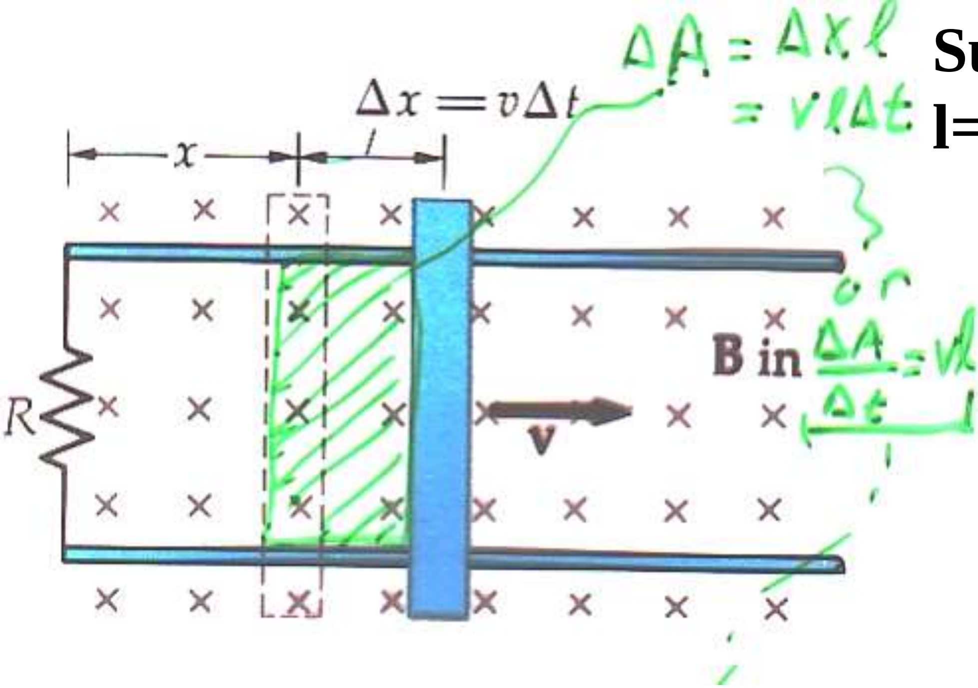
$\therefore \quad |\mathcal{E}| = V = \frac{W}{q}$ = work done / charge moved length of Rod.

$\therefore \quad |\mathcal{E}| = \frac{W}{q} = \frac{F\ell}{q} = \frac{q\ell v B}{q} = \ell v B \quad$ ie $\quad |\mathcal{E}| = \ell v B$

$$\ell v = \frac{\Delta A}{\Delta t}$$

$\therefore \quad |\mathcal{E}| = \frac{\Delta A}{\Delta t} B \quad$ or $\quad \phi = BA$

$$|\mathcal{E}| = \frac{\Delta \phi}{\Delta t}$$



Suppose

l=.2m, B=0.5T, v = .1 m/s & R = 3 Ω

What is the I in the circuit ?

$$|\varepsilon| = \frac{\Delta \phi}{\Delta t}$$

$$\frac{\Delta\phi}{\Delta t} = \frac{B\Delta A}{\Delta t} = B\ell\frac{\Delta x}{\Delta t} = B\ell v$$

$$\frac{\Delta\phi}{\Delta t} = B\ell v$$

$$V = \frac{\Delta\phi}{\Delta t} = (0.5\ \mathrm{T})(0.2\ \mathrm{m})(0.1\ \mathrm{m/s}) = 0.01\ \mathrm{Tm^2/s} = 0.01\ \mathrm{V}$$

Units next page

$$V = IR \quad \Rightarrow \quad I = \frac{V}{R} = \frac{0.01\ \mathrm{V}}{3\ \Omega} = .0033\mathrm{A}$$

recall $F = qvB$

units $N = C\frac{m}{s}T$

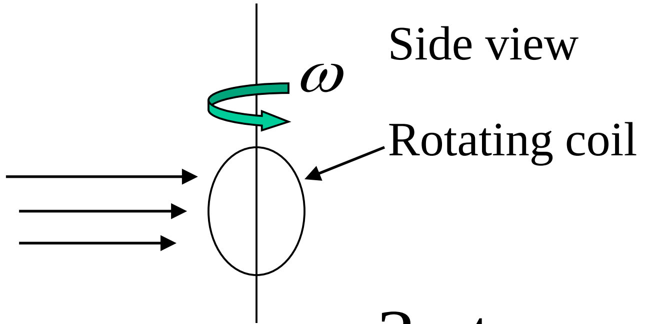
$$\frac{N}{C}\frac{s}{m} = T$$

units

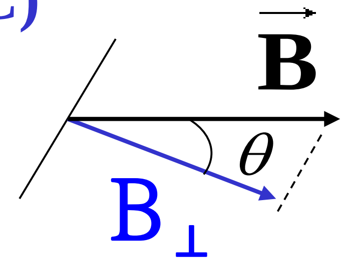
$$\frac{[T]m^2}{s} = [\frac{N}{C}\frac{s}{m}]\frac{m^2}{s} = \frac{N}{C}m = \frac{J}{C} = V$$

$$V = \frac{\Delta\phi}{\Delta t}$$

Generator: creates alternating current (AC)



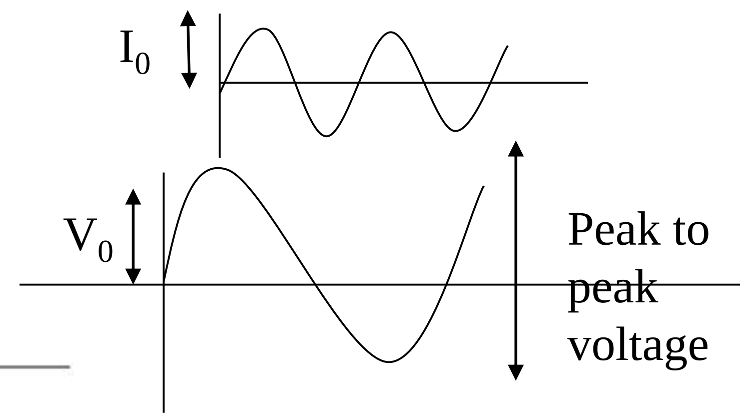
Top view



$$\phi = B_\perp A = BA\cos\theta$$

$$\theta = \omega t = 2\pi\, ft = \frac{2\pi t}{T}$$

AC current (will discuss later)

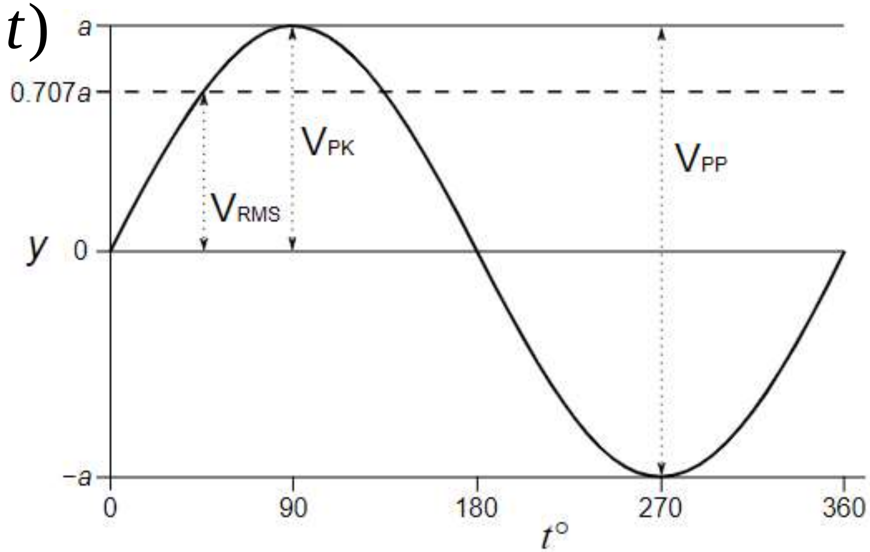


$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} = -BA\,\frac{\Delta\cos(\omega t)}{\Delta t}$$

$$\mathcal{E} = BA\omega\,\sin(\omega t)$$

$$V = V_0\sin(\omega\, t)$$

$$\omega = \frac{2\pi}{T}$$



$$V_{\text{Peak to peak}} = 2V_0$$

Alternating Current (AC)

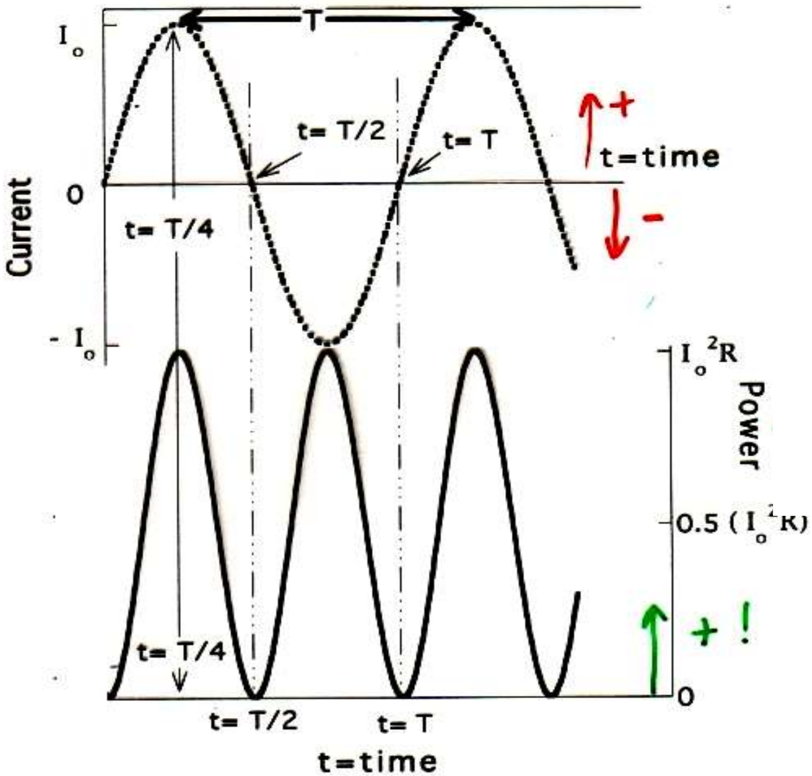
$I = I_0 \sin(2\pi f t) = I_0 \sin(\omega t)$

$V = V_0 \sin(2\pi f t) = V_0 \sin(\omega t)$

T= period (sec.)

$\frac{1}{T} = f$ = freq. (cyc. / sec)

$\omega = 2\pi f$ = ang. freq. (rad/sec.)



$$V = I\,R$$

$$P = I^2 R = I\,V$$ power

$$P = I_0^2 R \sin^2(\omega t) = I_0 V_0 \sin^2(\omega t)$$

* Average power over time:

$$\bar{P} = \frac{I_0 V_0}{2} \qquad \overline{\sin^2 2\pi f t} = \frac{1}{2}$$

$$\bar{P} = I_{RMS} V_{RMS}$$

$$\frac{I_0}{\sqrt{2}} = I_{RMS} = .707 I_0$$

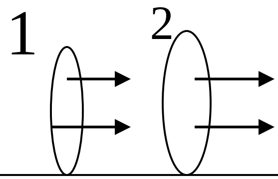
$$\frac{V_0}{\sqrt{2}} = V_{RMS} = .707 V_0$$

Example line voltage

V_{RMS} = 120 Volts

$V_0 = V_{RMS} (\sqrt{2}) =$ 170 Volts

*Ni



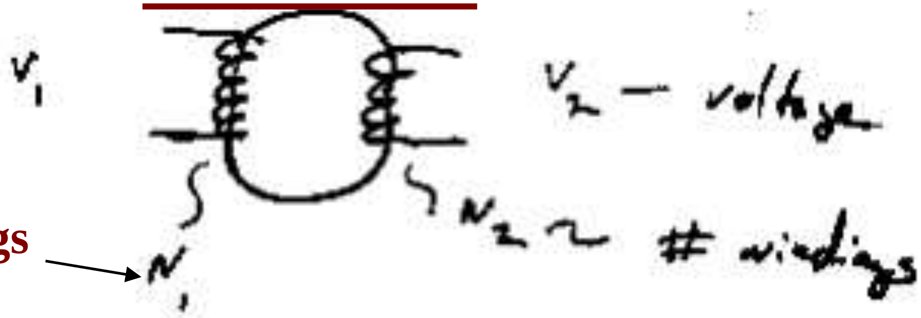
Magnetic field coupling between coils

ΔI_1 creates ΔB_1 $\rightarrow$ induces I_2

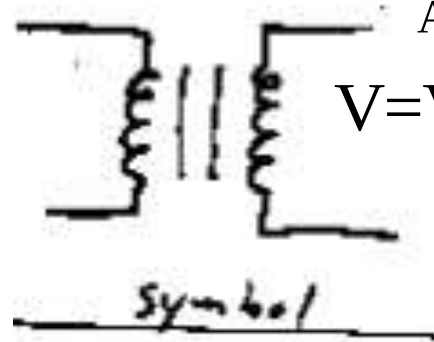
Principle of transformer

voltage

windings



AC current



$V = V_o \sin \omega t$

Same for both 1 and 2

$$V = -N\frac{\Delta\phi}{\Delta t} \rightarrow \frac{V}{N} = -\frac{\Delta\phi}{\Delta t} \rightarrow \boxed{\frac{V_1}{N_1} = \frac{V_2}{N_2}}$$

More coils more V

Energy conservation $\rightarrow I_1V_1 = I_2V_2 \rightarrow \boxed{\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}}$

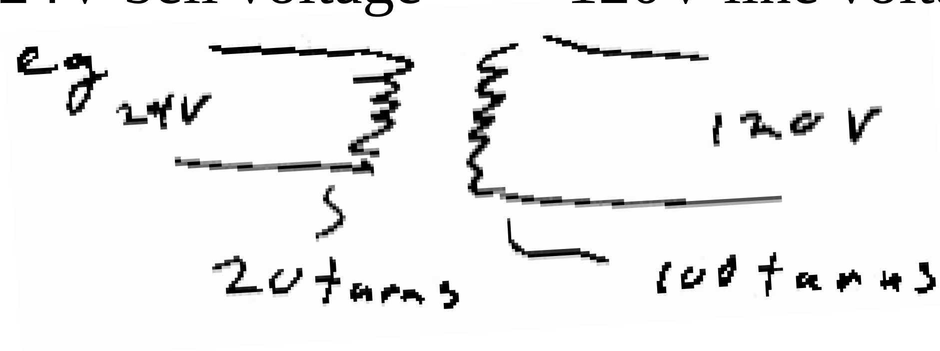
More coils more V but less I

Energy conservation

$$I_1 V_1 = I_2 V_2 \rightarrow \boxed{\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}}$$

Step down transformer ex. Door bell

24V bell voltage 120V line voltage



$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{120}{24} = 5$$

$$I_1 V_1 = I_2 V_2$$

Long distance power transmission
high V modest current

---

step down transformer to 120 V & high current

Inductance (L) {Self field coupling}

$$\phi_{\text{solenoid (total)}} = N\,(AB) = N\,(A\,[\mu_0 n I])$$

N: # loops; (AB): ϕ/loop; $[\mu_0 n I]$: B_z

$$n = \frac{\#\ \text{loops}}{\text{length}} = \frac{N}{\ell}$$

$$\phi = N A \mu_0 \frac{N}{\ell} I = \left[\mu_0 \frac{N^2 A}{\ell}\right] I$$

$\left[\mu_0 \frac{N^2 A}{\ell}\right]$ = [L]

so $\phi = L I$

$$L = \left[\mu_0 \frac{N^2 A}{\ell}\right]$$

L contains

geometry (A, ℓ) + μ_0 + N + material

$$\mu_r = \mu / \mu_0$$

Material	μ/(H m^{-1})	μ_r	Application
Ferrite U 60	1.00E-05	8	UHF chokes
Ferrite M33	9.42E-04	750	Resonant circuit RM cores
Nickel (99% pure)	7.54E-04	600	-
Ferrite N41	3.77E-03	3000	Power circuits
Iron (99.8% pure)	6.28E-03	5000	-
Ferrite T38	1.26E-02	10000	Broadband transformers
Silicon GO steel	5.03E-02	40000	Dynamos, mains transformers
supermalloy	1.26	1000000	Recording heads

Approximate maximum permeabilities

How L is used → AC circuits or just time change in I

$\mathcal{E}$ across $\mathcal{E} = -\frac{\Delta\phi}{\Delta t} = -L\frac{\Delta I}{\Delta t}$

$$\mathcal{E} = -L\frac{dI}{dt}$$

L in Henrys (H)

Units

L in Henrys

$$\phi_{T_{sol}} = L_{sol} I \qquad L_{sol} = \frac{\mu_0 N^2 A}{\ell}$$

$$T \cdot m^2 = H\, A$$

S

Henry's

$$\Rightarrow \quad H = \frac{T \cdot m^2}{A}$$

most basic units

$$\frac{T \cdot m^2}{A} = \left(\frac{N}{A m}\right) \cdot \frac{m^2}{A} = \frac{N m}{A^2} = \frac{kg\, m}{s^2} \frac{m}{C^2/s^2} \quad \to \quad H = \frac{kg\, m^2}{C^2}$$

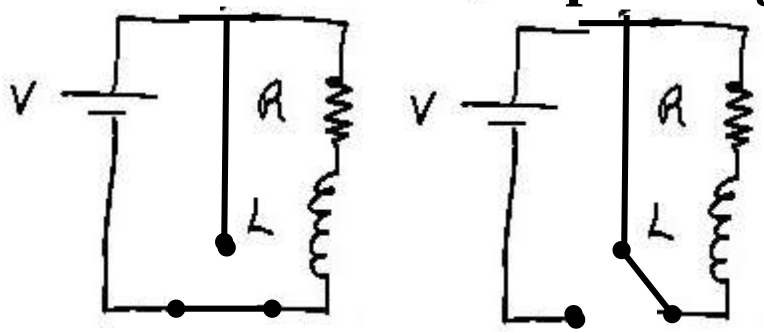
$$\mathcal{E} \equiv -\frac{\Delta \phi}{\Delta t} = -L \frac{\Delta I}{\Delta t}$$

$$V = H \frac{A}{s} \Rightarrow H = \frac{V \cdot s}{A}$$

Check units

$$\frac{V \cdot s}{A} = \frac{N \cdot m \cdot s}{C/s} = \frac{kg\, m}{s^2} \frac{m}{C^2} = \frac{kg\, m^2}{C^2}$$

Collapse magnetic field in L



Boundary conditions

$$t = 0 \quad V = I_0 R \qquad I_0 = \frac{V}{R}$$

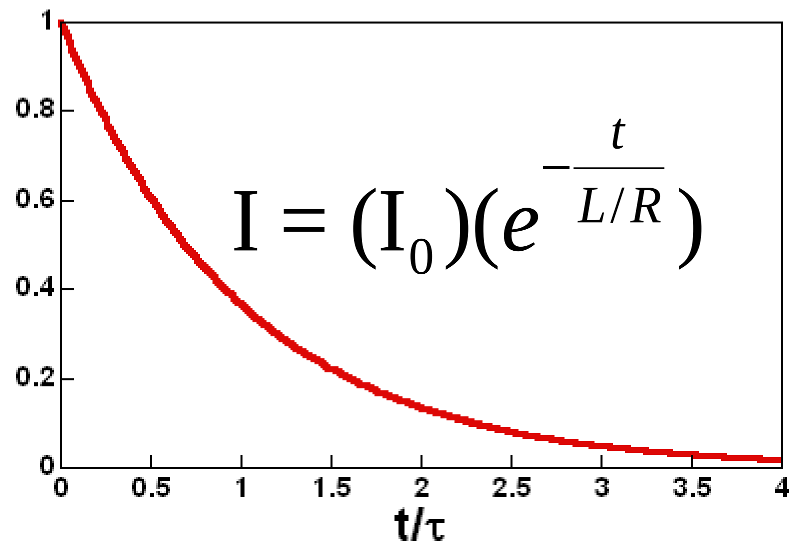
$$t = \infty \quad I = 0 \qquad \frac{\Delta I}{\Delta t} = 0$$

t=0 move switch.

$$L\frac{\Delta I}{\Delta t} + I\,R = 0$$

$$\therefore \text{ here} \qquad \frac{\Delta I}{\Delta t} + \frac{R}{L}\,I = 0$$

$$I = (I_0)(e^{-\frac{t}{L/R}})$$



$$\tau = \frac{L}{R}$$

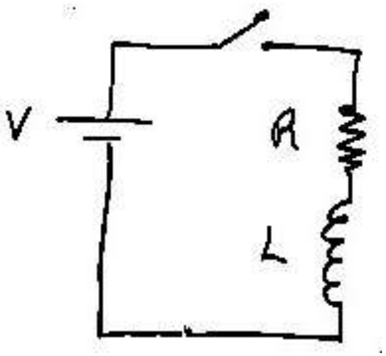
Recall:

$$\frac{\Delta Q}{\Delta t} + \frac{1}{RC}Q = 0 \quad \Rightarrow Q = Q_0 e^{-\frac{t}{RC}}$$

5-8

I at t=∞

Establishing magnetic field in L



t=0 close switch.

$$V = IR + L\frac{\Delta I}{\Delta t}$$

$$\frac{\Delta I}{\Delta t} + \frac{R}{L} I = \frac{V}{L}$$

Boundary conditions

$t = 0 \quad I = 0$

$t = \infty$

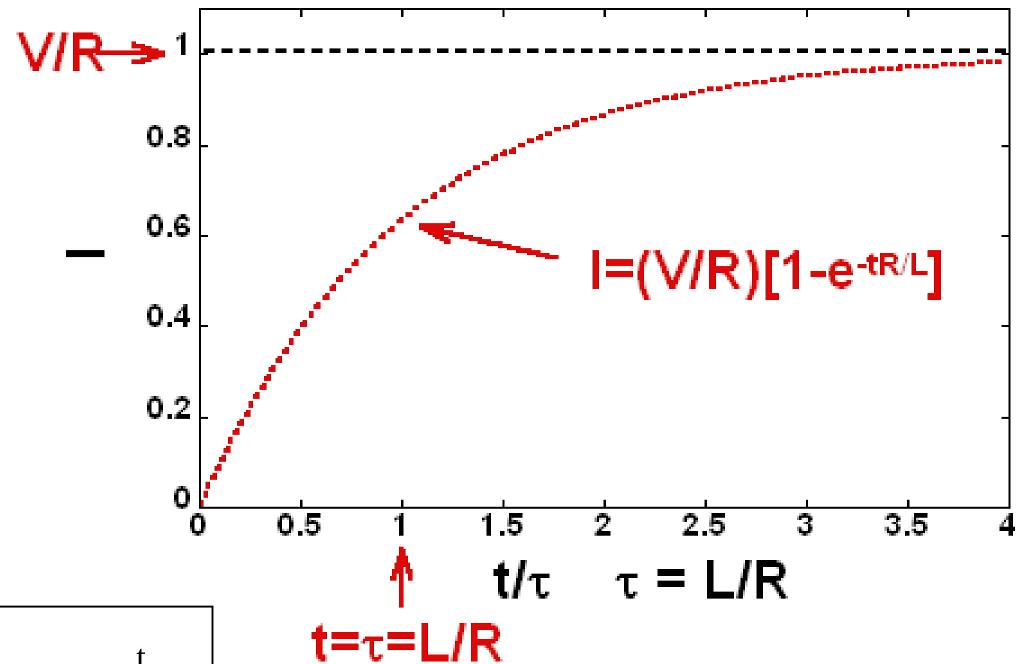
I=constant V=IR

$$\frac{\Delta I}{\Delta t} = 0$$

$\therefore$ here

$$I = \left(\frac{V}{R}\right)\left(1 - e^{-\frac{tR}{L}}\right)$$

I at t=∞



Recall:

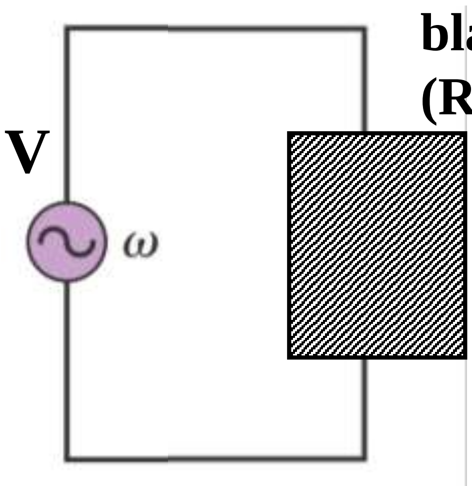
$$\frac{\Delta Q}{\Delta t} + \frac{1}{RC} Q = \frac{V}{R} \Rightarrow$$

$$Q = VC\left(1 - e^{-\frac{t}{RC}}\right)$$

$$I = \frac{V}{R} e^{-\frac{t}{RC}}$$

5-8a

General AC circuit



black box
(R, C, L combinations)

$V=V_{max}\sin(\omega t)$

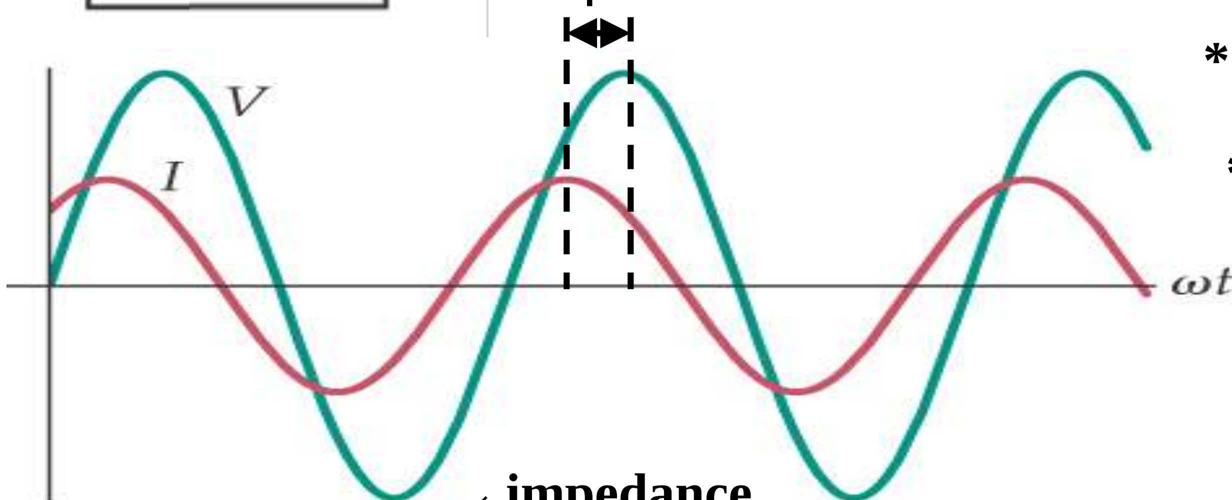
$I=I_{max}\sin(\omega t+\varphi)$

Note: You can use current as reference for phase (book)

$I=I_{max}\sin(\omega t)$

$V=V_{max}\sin(\omega t-\varphi)$

φ phase difference between I and V



* if R only then φ=0

* if L or C present φ≠0

impedance

$V_{max}=I_{max}\,Z$

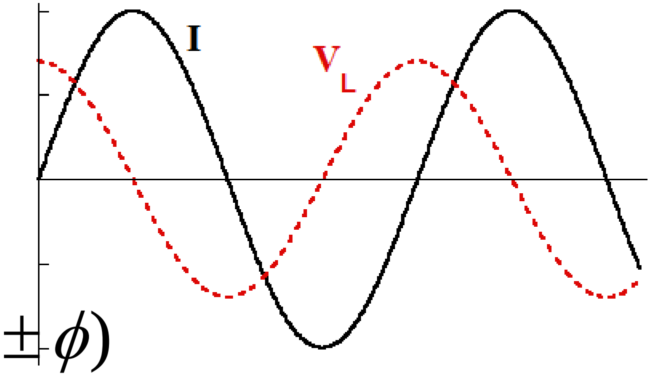
(Ohm's Law generalization)

Z → 2 parts

resistance R

reactance X_C X_L

X_L



$$I = I_0 \sin(\omega\, t) \Rightarrow \quad V = V_0 \sin(\omega\, t \pm \phi)$$

$$\frac{dI}{dt} = I_0\ \omega \cos(\omega\, t)$$

$$V = L\frac{dI}{dt} = L\ I_0\ \omega \cos(\omega\, t) = \omega L\ I_0\ \sin(\omega\, t + \pi/2)$$

$$V = [\omega L]\ \ I_0 \sin(\omega\, t + \pi/2)$$

$[V_0]$

$$V_0 = (\omega L)\ I_0$$

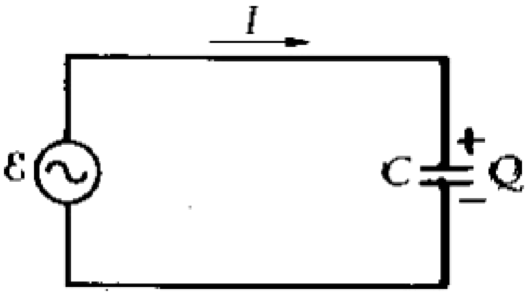
X_L

$X_L = \omega L$

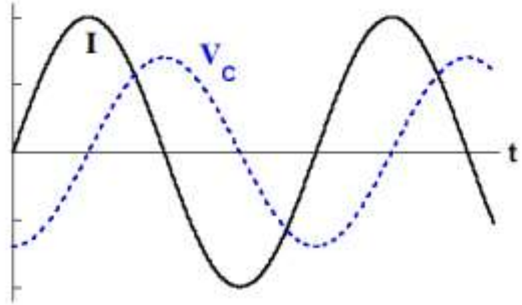
$$I_0 = \frac{V_0}{\omega L}$$

$\omega \to 0\ \ X_L \to 0$ inductive load $\to 0$

$\omega \to \infty\ X_L \to \infty$ inductive load $\to \infty$



X_C



$X_C = \frac{1}{\omega C}$ $\varphi = 90°$

$$I = I_0 \sin(\omega t) \Rightarrow V = V_0 \sin(\omega t \pm \phi)$$

$$I = \frac{dQ}{dt}$$

$$Q = \int I \, dt = \int I_0 \sin(\omega t) \, dt = I_0 \frac{-\cos(\omega t)}{\omega}$$

$$V = \frac{Q}{C}$$

$$V = \frac{1}{\omega C} I_0[-\cos(\omega t)] = [\frac{1}{\omega C}] I_0 \sin(\omega t - \pi/2)$$

$\frac{1}{\omega C} I_0 \to V_0$

$$V_0 = [\frac{1}{\omega C}] I_0$$

Note! ωt usually measured in radius use degrees for simplicity

$$X_C = \frac{1}{\omega C}$$

$$V_0 = X_C I_0$$

Note: $\omega \to \infty$ $X_c \to 0$

High frequency looks like short circuit

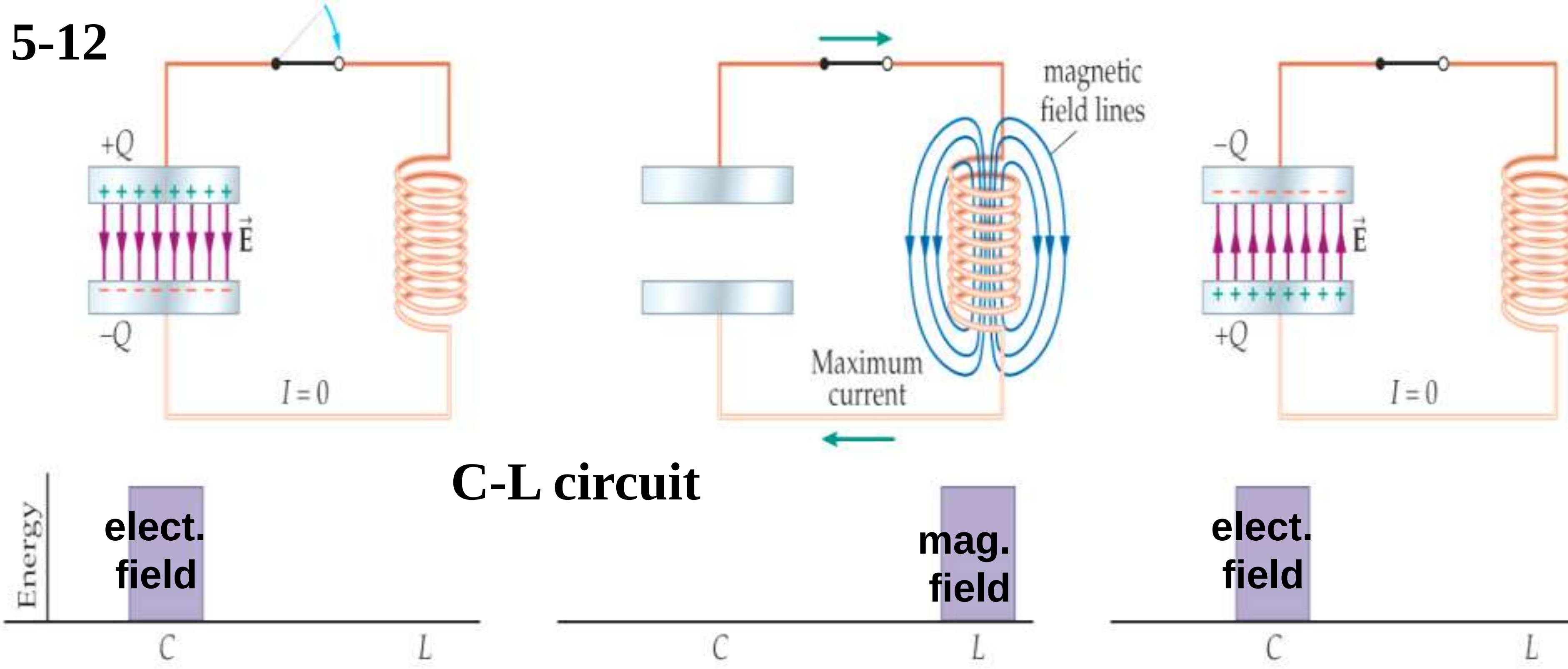
$\omega \to 0$ $X_c \to \infty$

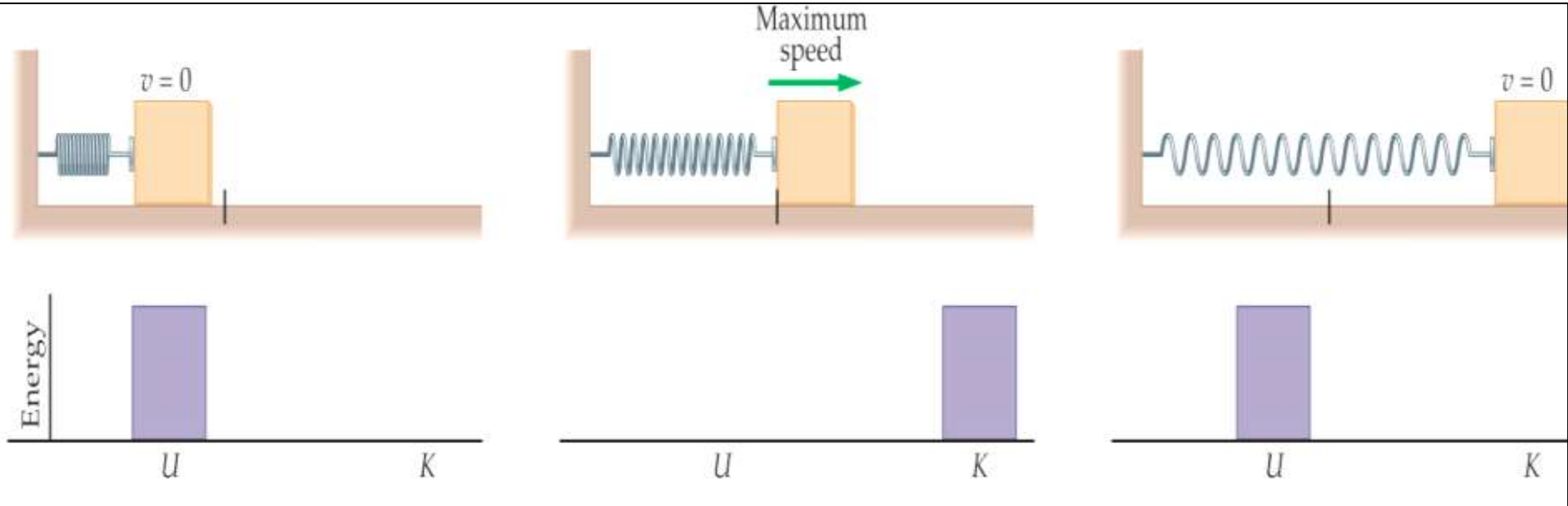
0 frequency no AC current

5-11cap

5-12

C-L circuit

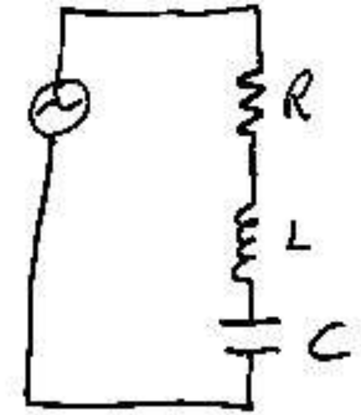




Analogies Between a Mass on a Spring and an *LC* Circuit

Mass-spring system		*LC* circuit	
position	x	charge	q
velocity	$v = \Delta x/\Delta t$	current	$I = \Delta q/\Delta t$
mass	m	inductance	L
force constant	k	inverse capacitance	$1/C$
natural frequency	$\omega = \sqrt{(k/m)}$	natural frequency	$\omega = \sqrt{(1/LC)}$

RLC series circuit –Current resonance



$$V = V_R + V_L + V_C$$

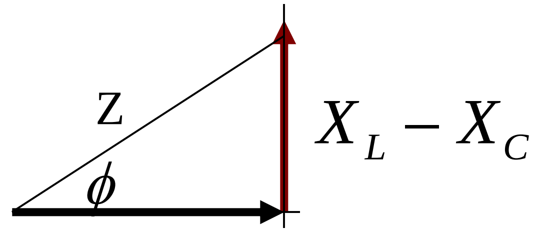
$$\frac{AC}{V = V_0 \sin(wt)}$$

V=IR worked well so look for relation

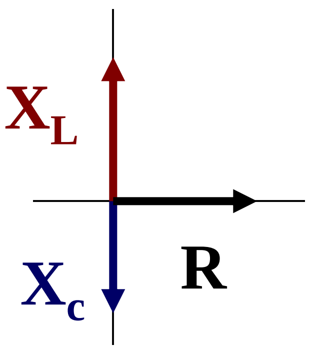
$$V_0 = I_0 Z$$

phase

$$I = I_0 \sin(\omega t + \phi)$$



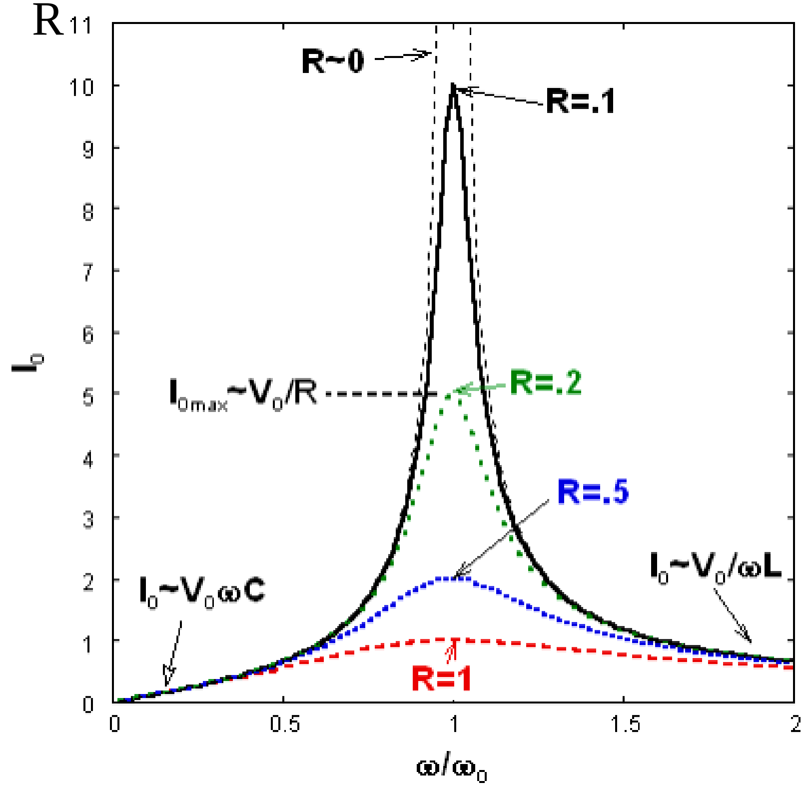
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



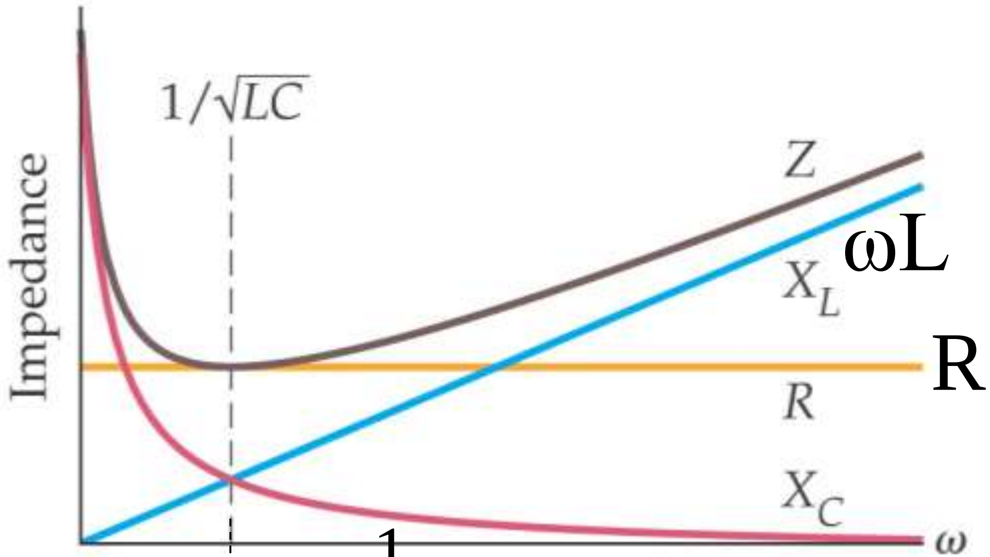
$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Resonance where $X_L - X_C \Rightarrow 0$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

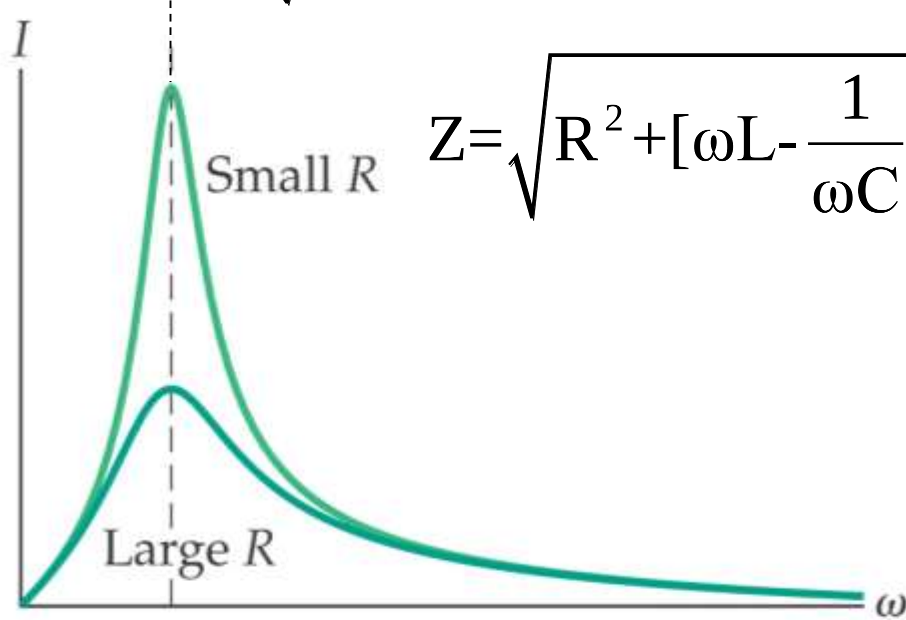


$$|Z| = \sqrt{R^2 + [\omega L - \frac{1}{\omega C}]^2}$$



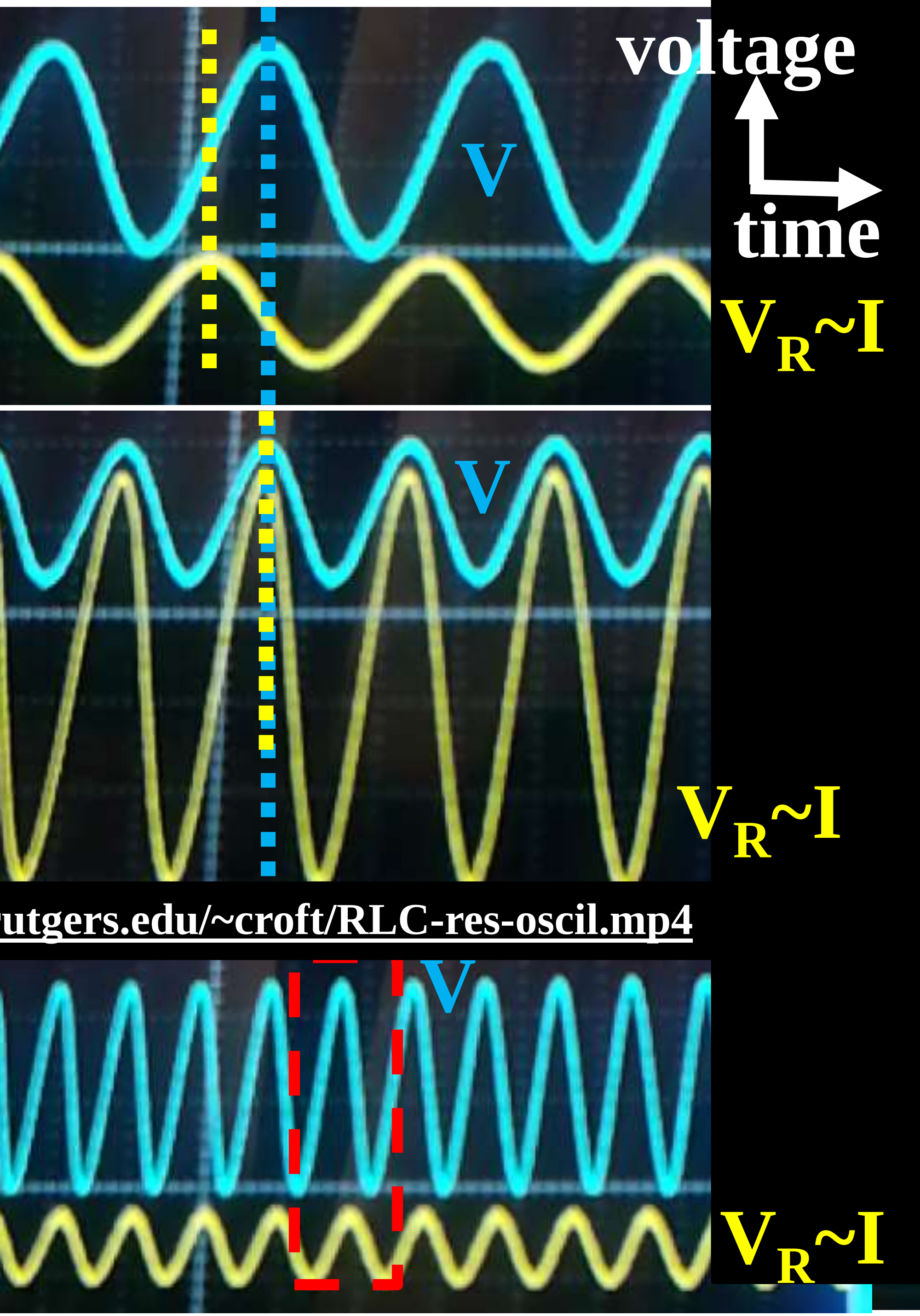
$$\frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$Z = \sqrt{R^2 + [\omega L - \frac{1}{\omega C}]^2}$$

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 + [\omega L - \frac{1}{\omega C}]^2}}$$



$\omega << \omega_0$

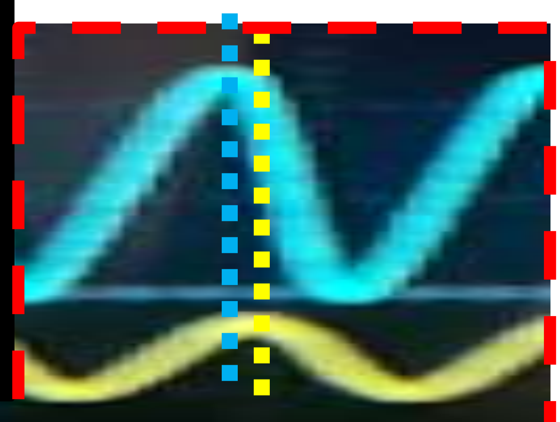
I small $Z \sim \sqrt{\left[-\frac{1}{\omega C}\right]^2}$

phase C -like

$\omega = \omega_0$ **phase R –like (0)**

$Z \sim \sqrt{R^2}$

$\omega >> \omega_0$



$Z \sim \sqrt{[\omega L]^2}$

phase L –like