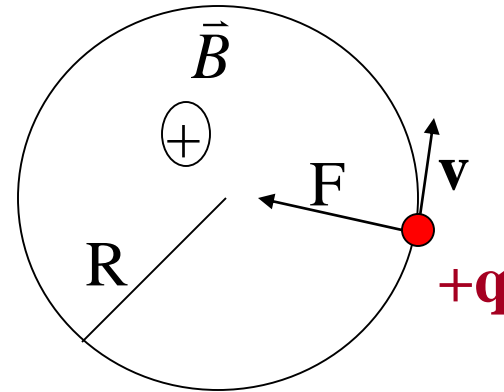


$$F = qv_{\perp} \quad B = qv_{\perp} B_{\perp}$$



$$\mathbf{F} = I \ell \mathbf{B}_{\perp} = I \ell B \sin(\theta)$$

$$\mu_0 I = \sum_{\text{edge}} B_{\parallel} \Delta l \quad \text{Ampere's Law}$$

$$\mu_0 I = B 2\pi R \quad \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

$$B = I n \mu_0$$

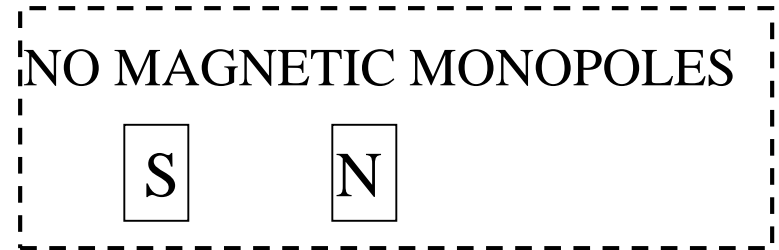
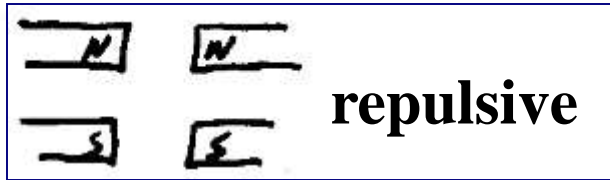
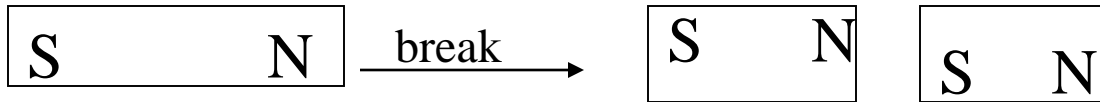
Magnetic Interactions

1.) Force at a distance

2.) Repulsive and attractive

} Like electrostatics!

Permanent magnets

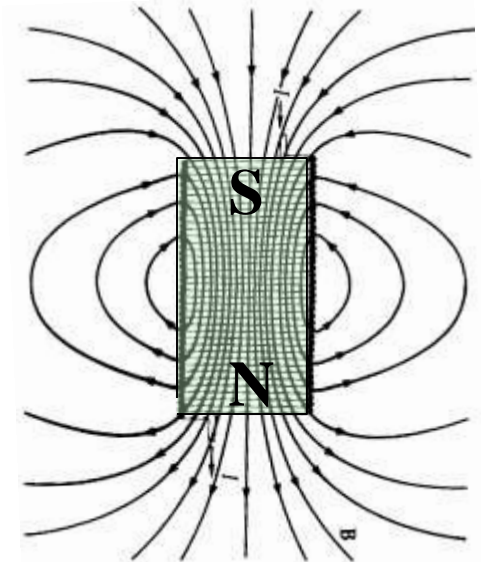


Magnets and Magnetic fields : Dipole Fields

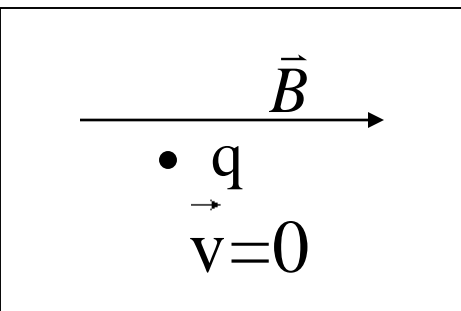
Magnetic field \vec{B} {Like E}

Magnetic field lines

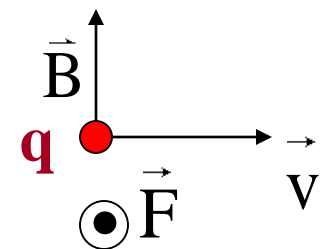
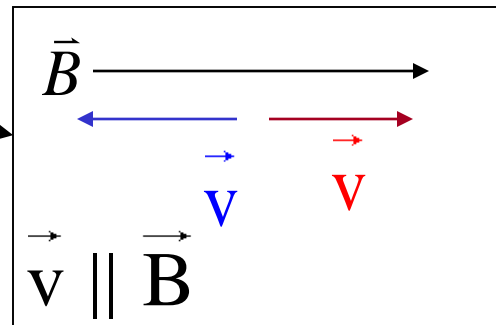
$$B - \text{units Tesla} = \frac{\text{kg}}{\text{CS}}$$



Forces on charge due to \vec{B} (magnetic field)

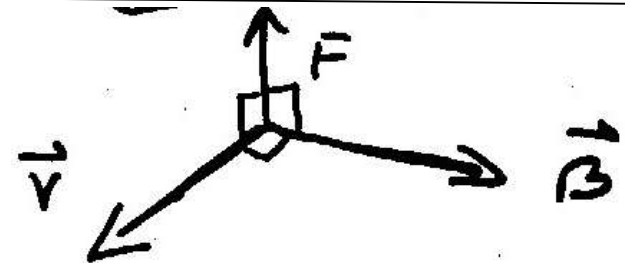


No force



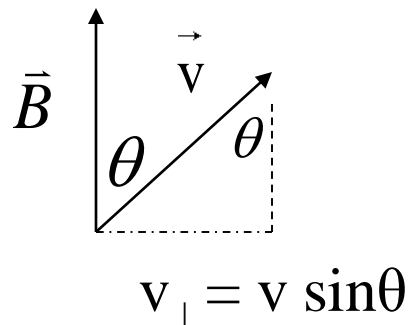
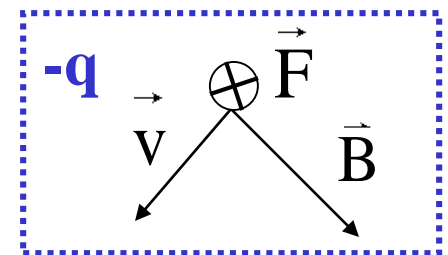
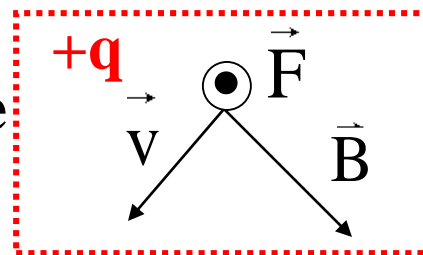
$\vec{v} \perp \vec{B} \Rightarrow F$ force

direction by **right hand rule!**



note: \odot = vector at you
 \otimes = vector into page

**+q & -q
 opposite**



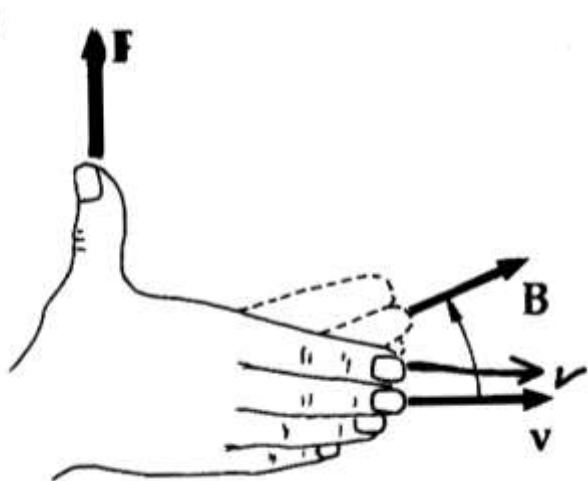
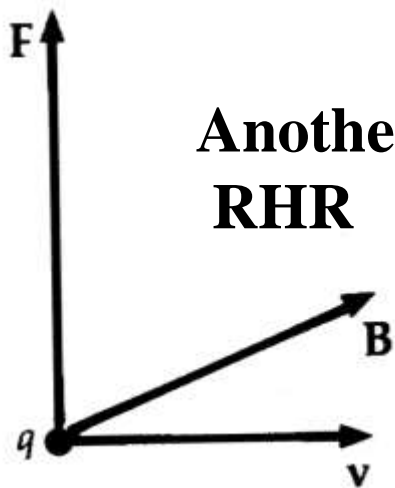
$$F = qv_{\perp} B = qv B_{\perp}$$

$$= q v B \sin \theta$$

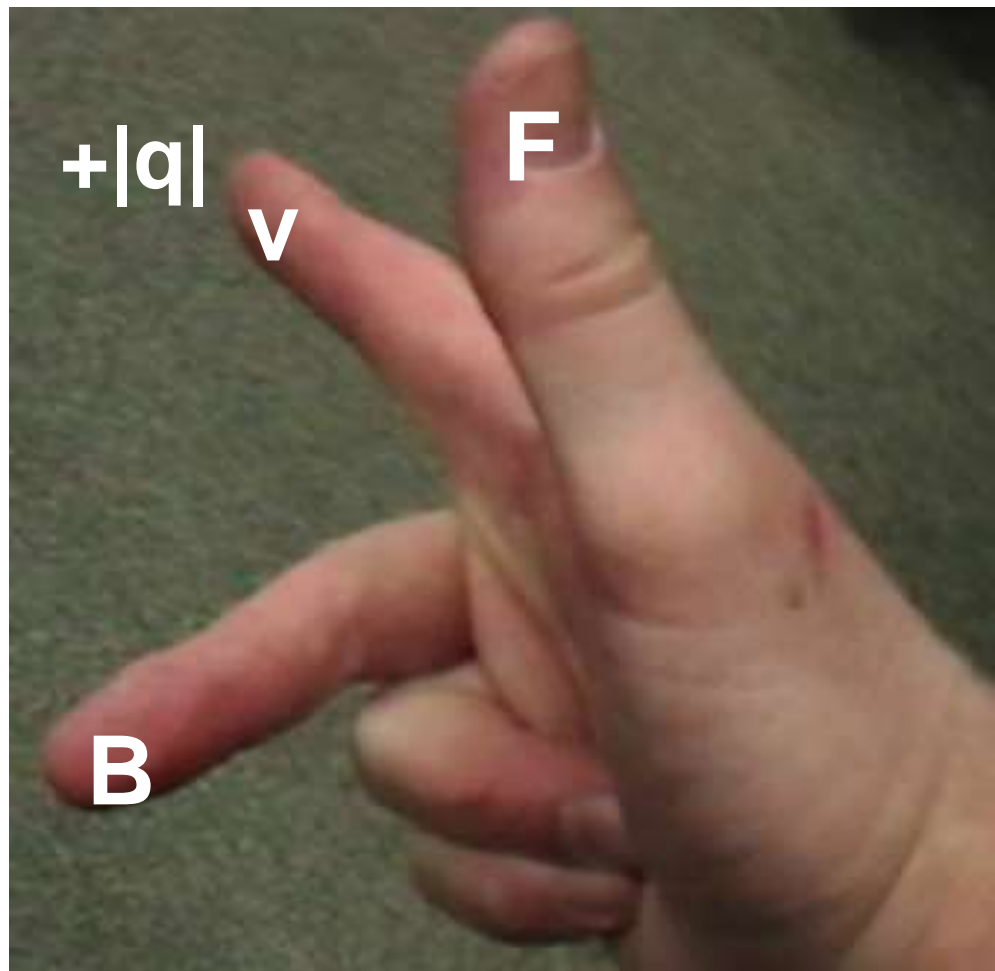
Croft preferred RHR

direction by **right hand rule!**

Another equivalent
RHR

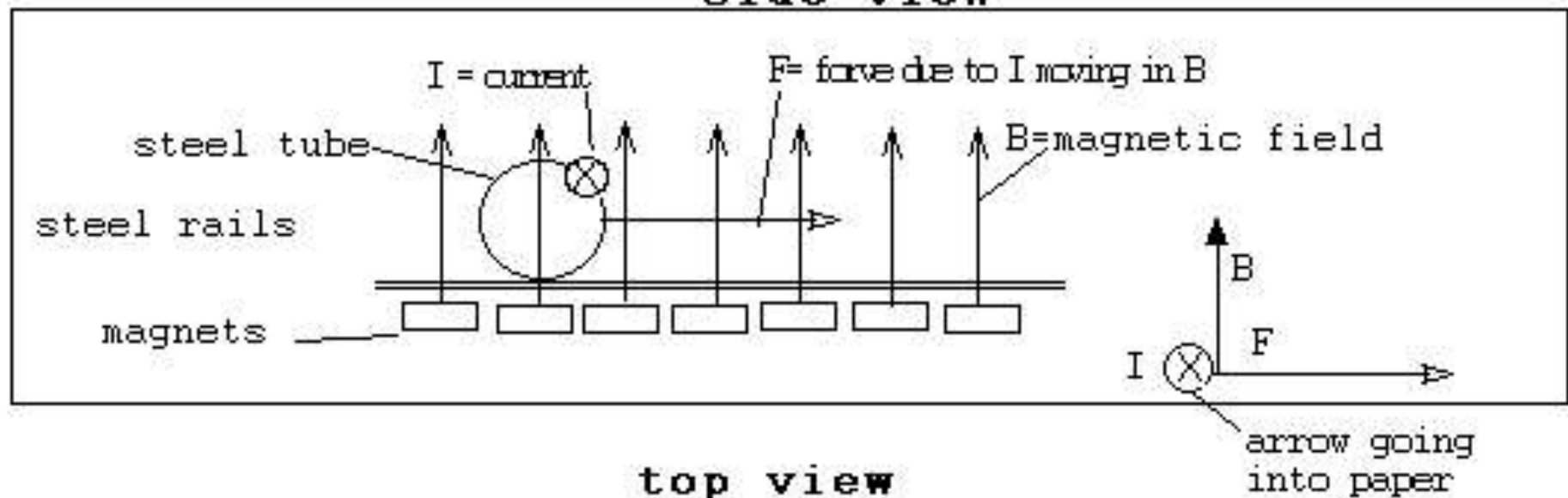


$-|q|$ flip thumb

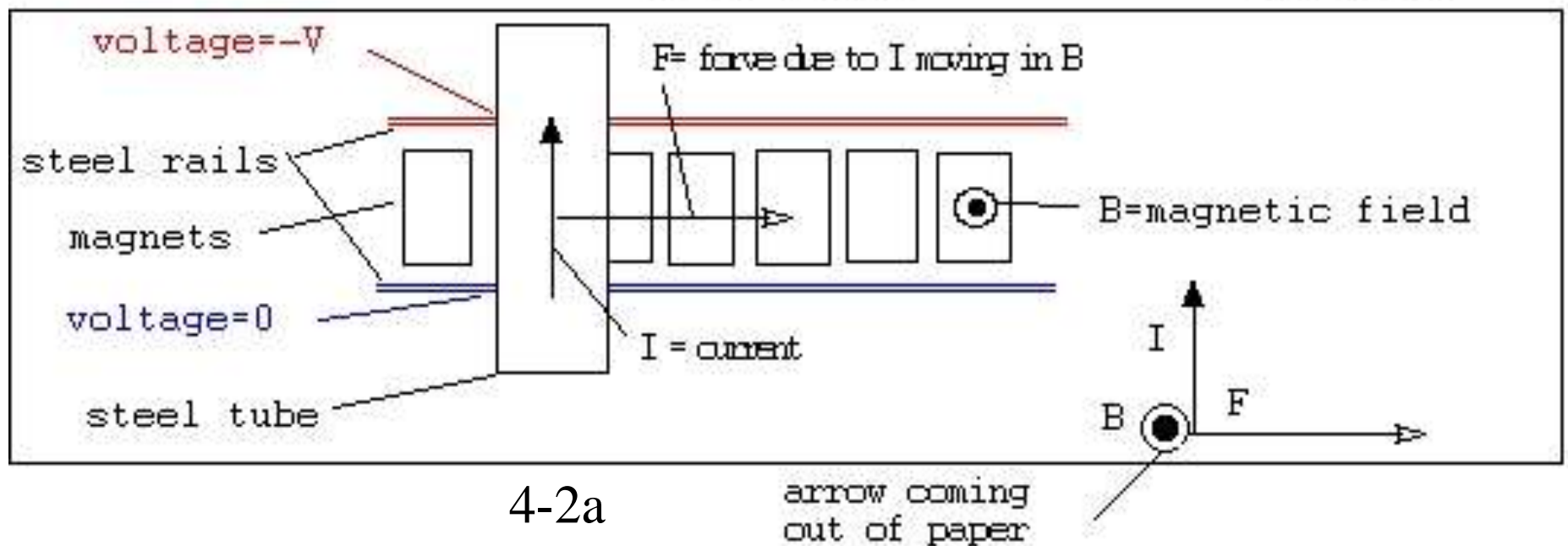


Faraday Motor

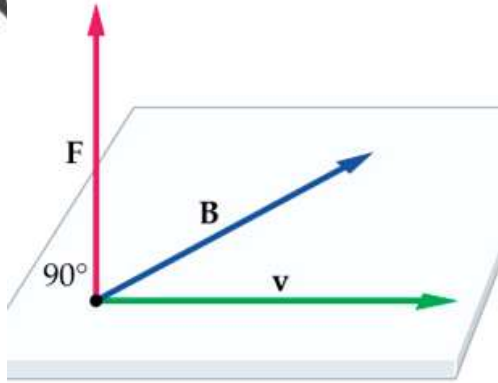
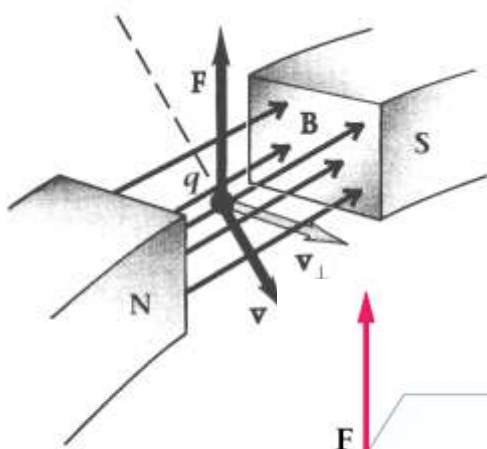
side view



top view



4-2a

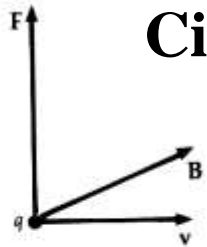


$$F = q v_{\perp} B \quad \text{Units B field} \quad \underline{\text{tesla}}$$

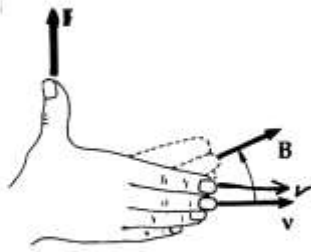
$$\therefore N = C \frac{m}{s} T$$

$$\Rightarrow T = \frac{NS}{mC} = \text{kg} \frac{m}{s^2} \frac{s}{mC} = \boxed{\frac{\text{kg}}{Cs} = T}$$

Circular motion of charge moving \perp to magnetic field



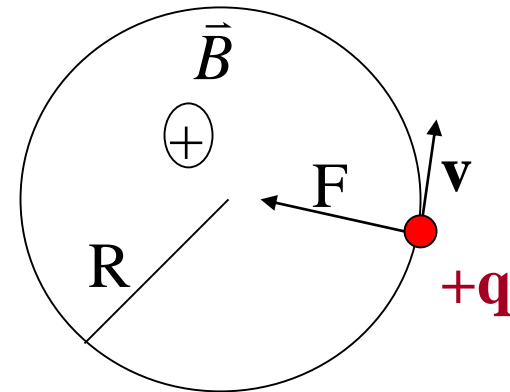
(a)



(b)

$$F = m a = m \frac{v^2}{R}$$

$$qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB}$$

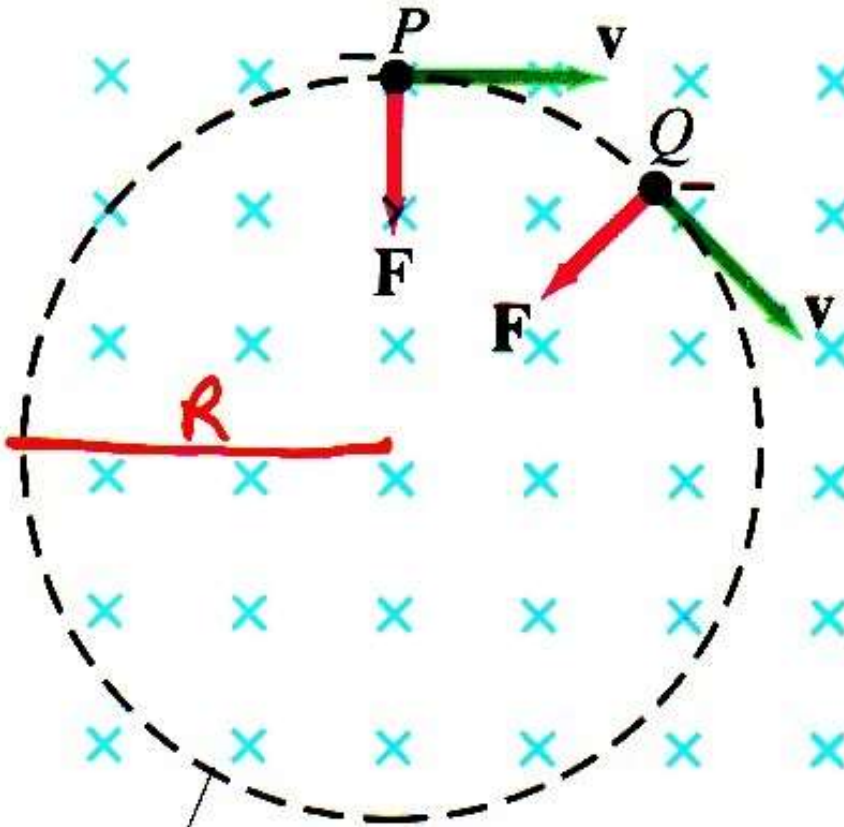


uniform B field $\rightarrow \vec{v} \perp \text{to } \vec{B} \Rightarrow$ circular motion.

17 Circular motion of a charge in a uniform magnetic field

Recall

$$F = ma = \frac{mv^2}{R} = \overset{\substack{\text{electron} \\ \downarrow}}{e} v B$$



$$\frac{e}{m} = \frac{v}{RB}$$

or

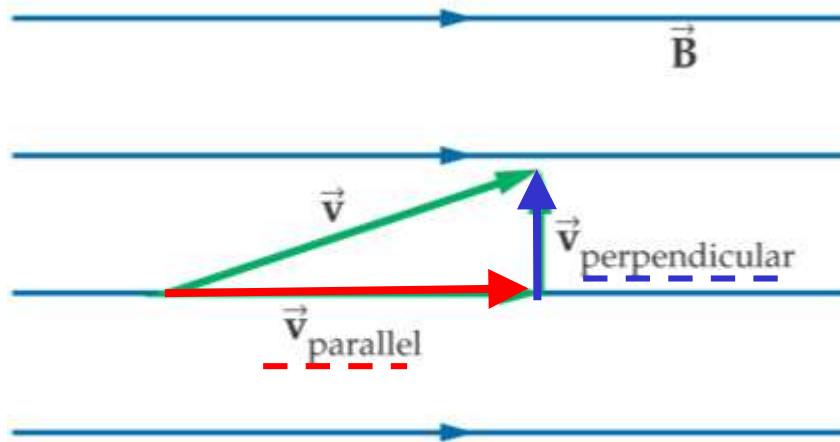
$$v = \left(\frac{e}{m} \right) RB$$

save for later

Path of electron

B is into the page

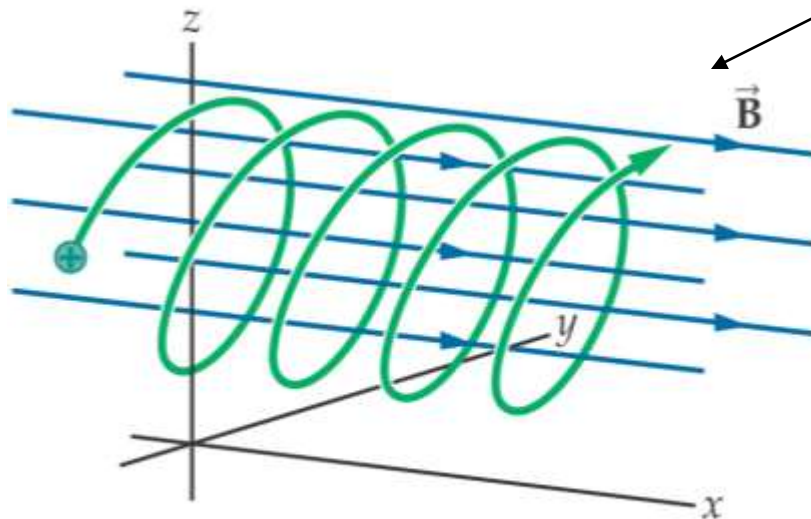
General motion of charged particle in a magnetic field



(a)

\parallel direction
 $v_{\parallel} = \text{constant}$
magnitude & direction

\perp direction
circular motion
speed = $|v_{\perp}|$



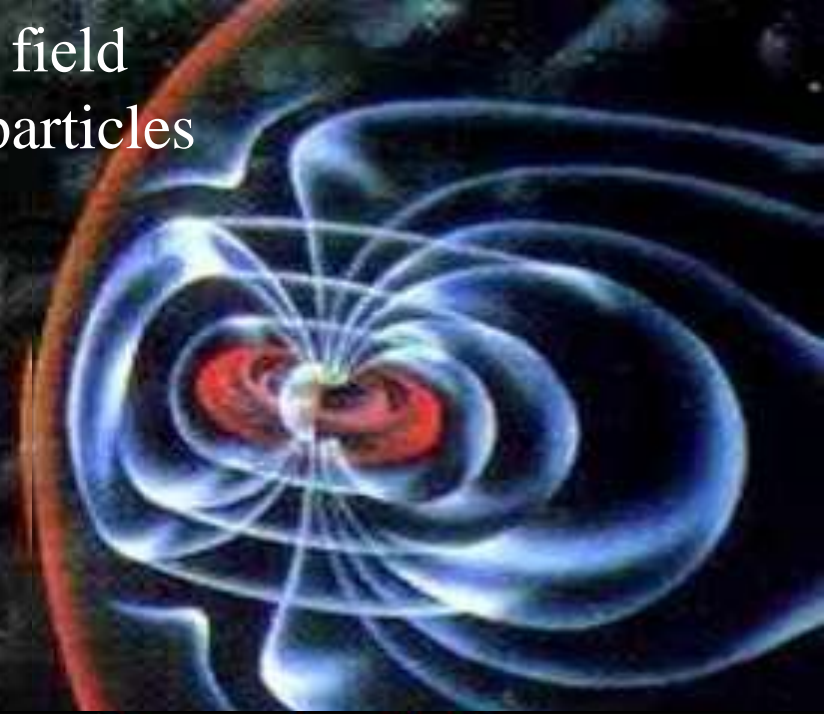
(b)

helical-motion

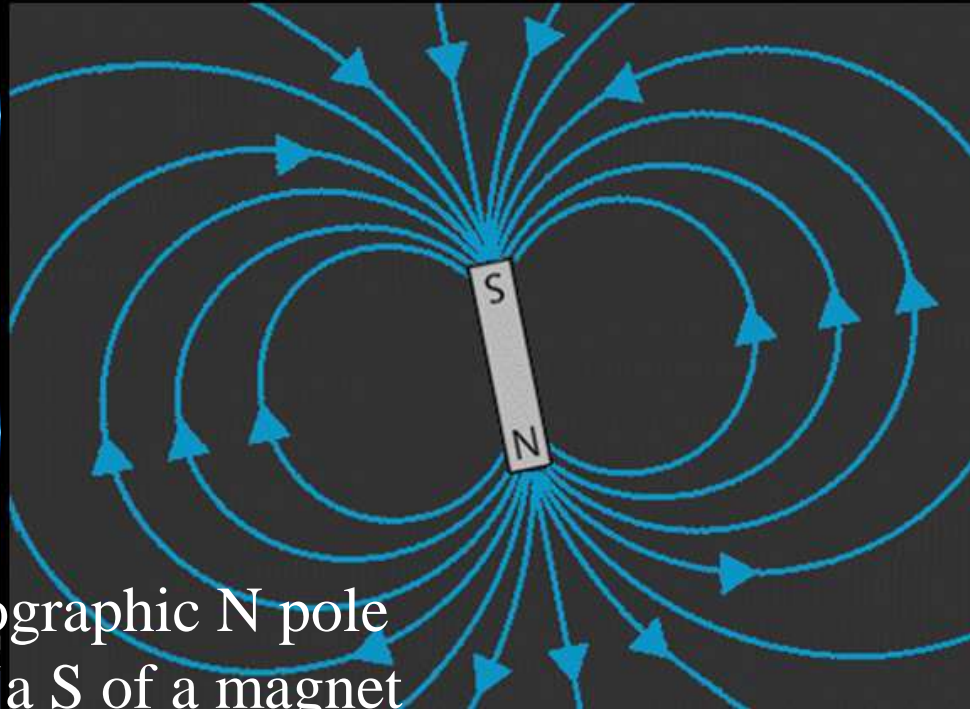
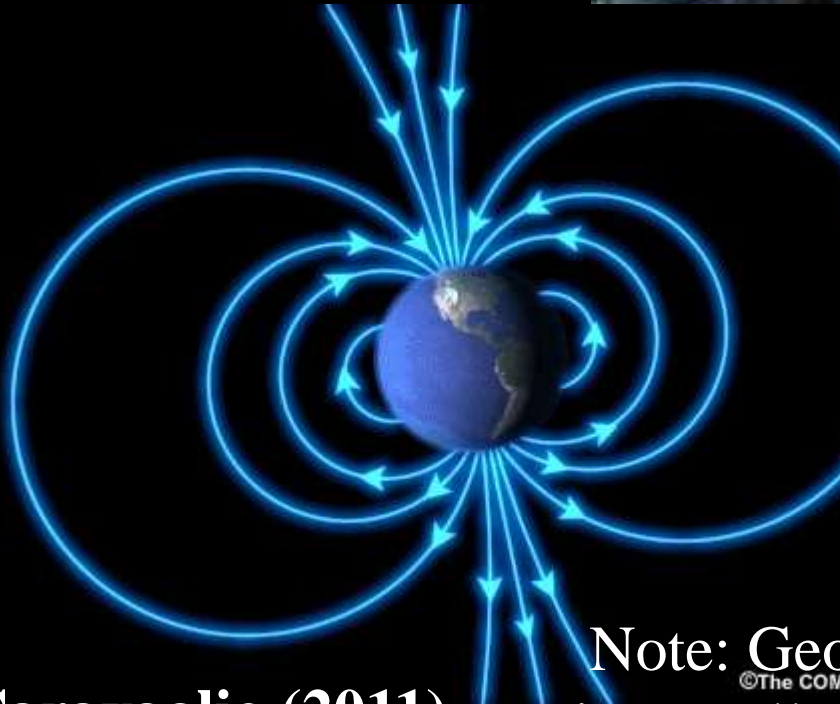


Earth's magnetic field
deflects charge particles
from Sun

solar wind →



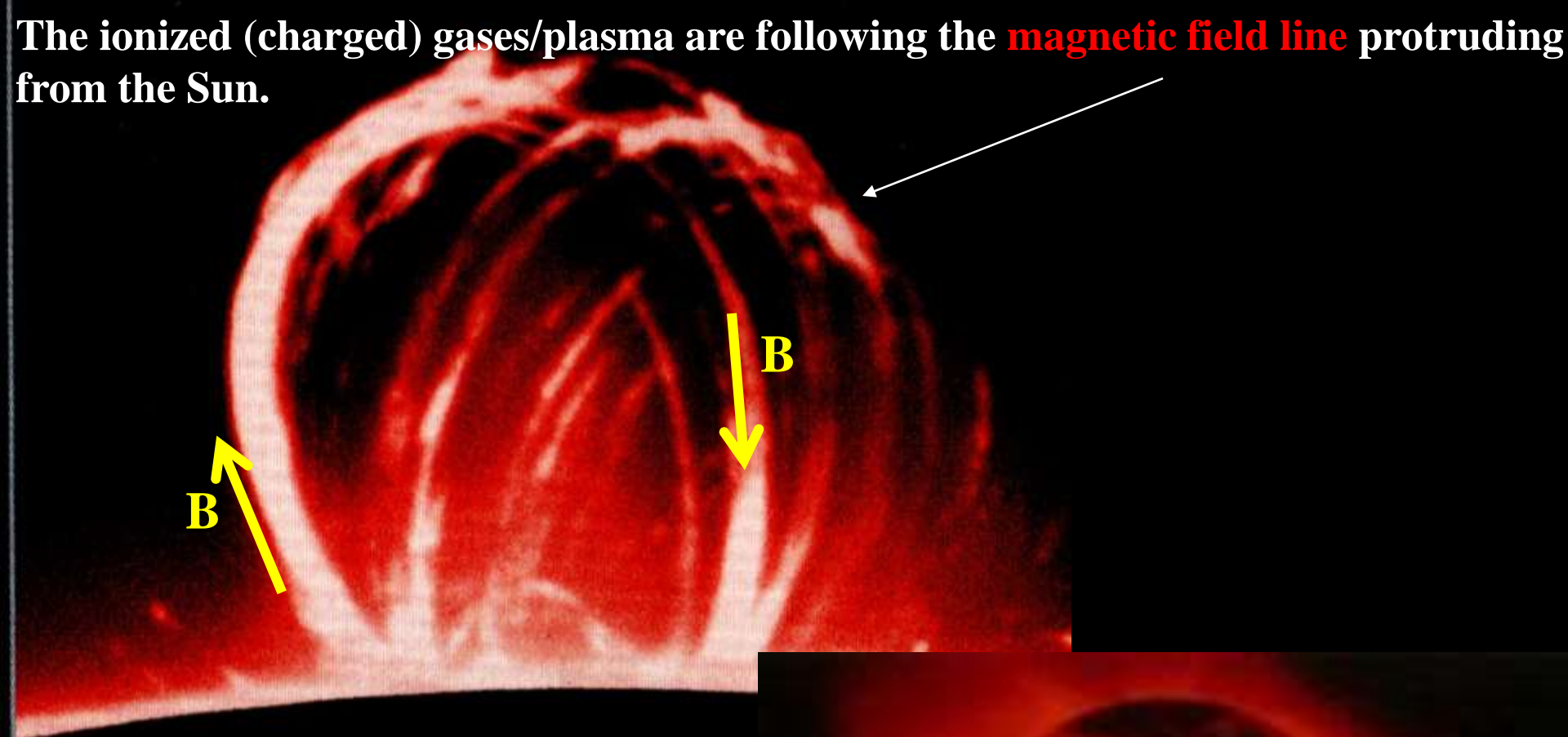
charged particles
spiral down
magnetic field lines
near magnetic poles create aurora



Note: Geographic N pole
is actually a S of a magnet

* Caravaglio (2011)

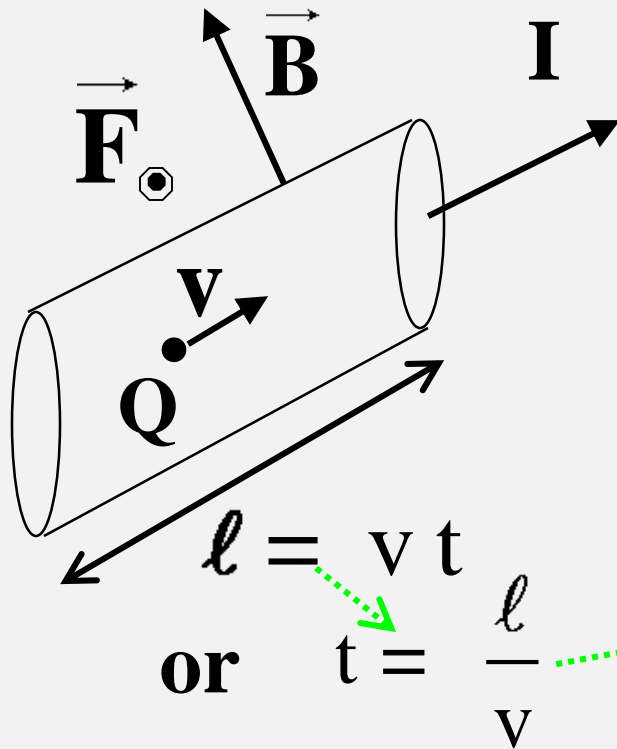
The ionized (charged) gases/plasma are following the **magnetic field line** protruding from the Sun.



PROMINENCES: Surging clouds of ionized gas are best visible along the limb; loop prominences can tower tens of thousands of miles high.



Force on a straight wire with I due to B



Note: here $v \perp B \perp F$

$$\mathbf{F} = Q \mathbf{v} \mathbf{B}$$

but $I = \frac{Q}{t}$

so $I = \frac{Q}{\ell/v} = \frac{Q v}{\ell}$

and so $I \ell = Q v$

$$\Rightarrow \mathbf{F} = (Q \mathbf{v}) \mathbf{B} = (I \ell) \mathbf{B}$$

$$\mathbf{F} = (I \ell) \mathbf{B}$$

units

$$\mathbf{N} = \mathbf{A m T}$$

$$\mathbf{T} = \frac{\mathbf{N}}{\mathbf{A m}}$$

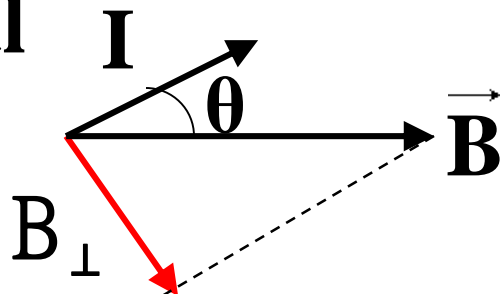
$$\frac{\mathbf{kg m}}{\mathbf{s}}$$

$$\mathbf{T} = \frac{\mathbf{s^2}}{\mathbf{C m}}$$

$$\mathbf{T} = \frac{\mathbf{kg}}{\mathbf{Cs}}$$

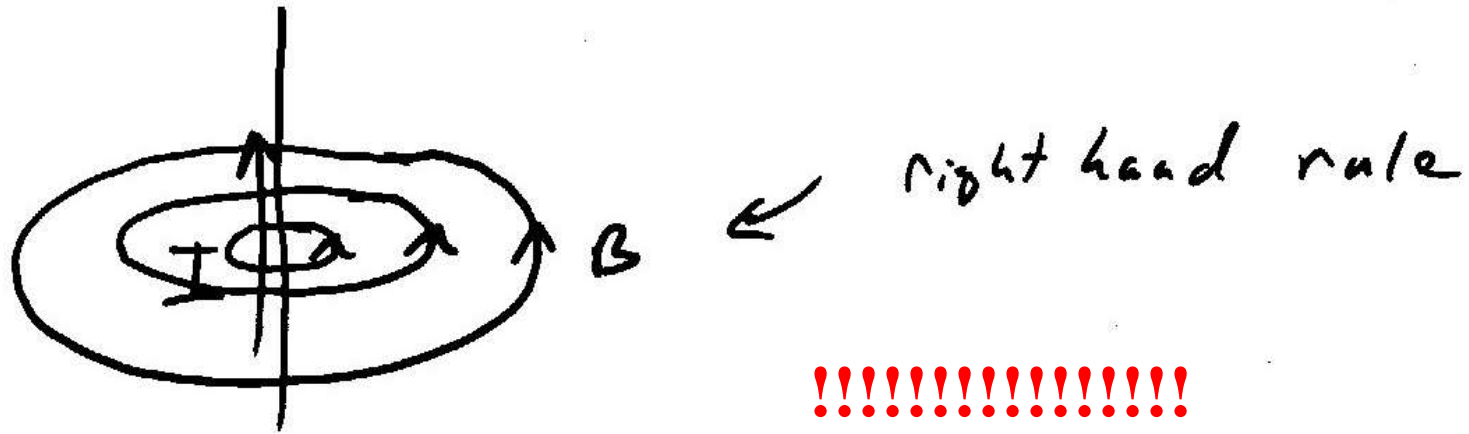
ok

In general



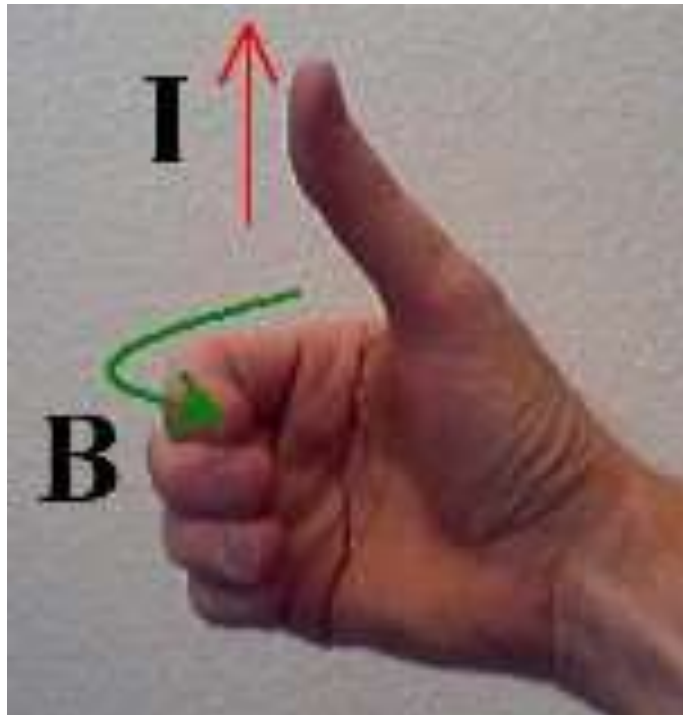
$$\mathbf{F} = I \ell \mathbf{B}_{\perp} = I \ell \mathbf{B} \sin(\theta)$$

Flowing charge (current) creates magnetic field.



!!!!!!!!!!!!!!!!!!!!

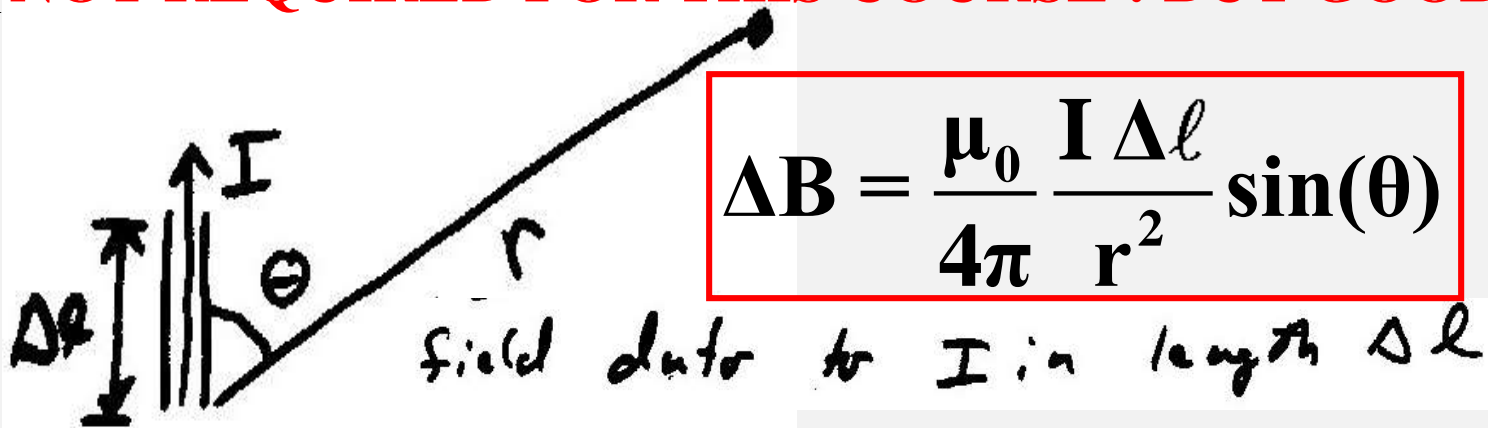
2'nd right hand rule: RHR for B direction created by current



Magnitude of field (Biot & Savart Law)

{Like point charge electric field law but for current}

{ NOT REQUIRED FOR THIS COURSE : BUT GOOD TO KNOW }



To calculate tot field add (*as vectors) up all ΔB from each Δl

[B&S Law- magnetic field equivalent of electric field point charge]

$$\Delta B = k' \frac{I \Delta l}{r^2} \sin(\theta)$$

4-6a

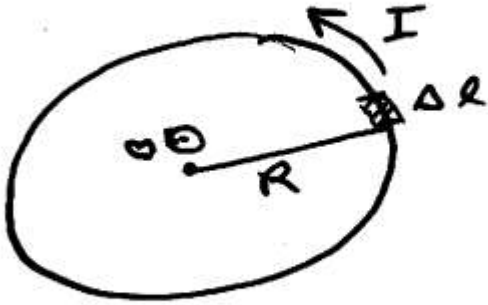
$$k' = (10)^{-7} \frac{N}{A^2}$$

also $k' = \frac{\mu_0}{4\pi}$

B field - simple example

(using Biot & Savart Law)

circular loop - field at center



$$\Delta B = \frac{\mu_0 I}{4\pi R^2} \Delta l$$

divide circle into N parts
of Δl each

$$\Delta l = \frac{2\pi R}{N} \Rightarrow N \Delta l = 2\pi R$$

note every ΔB same \therefore

$$B = N \Delta B = N \frac{\mu_0 I}{4\pi R^2} \Delta l$$

$$= \frac{\mu_0 I}{4\pi R^2} (N \Delta l)$$

$$= \frac{\mu_0 I}{4\pi R^2} 2\pi R$$

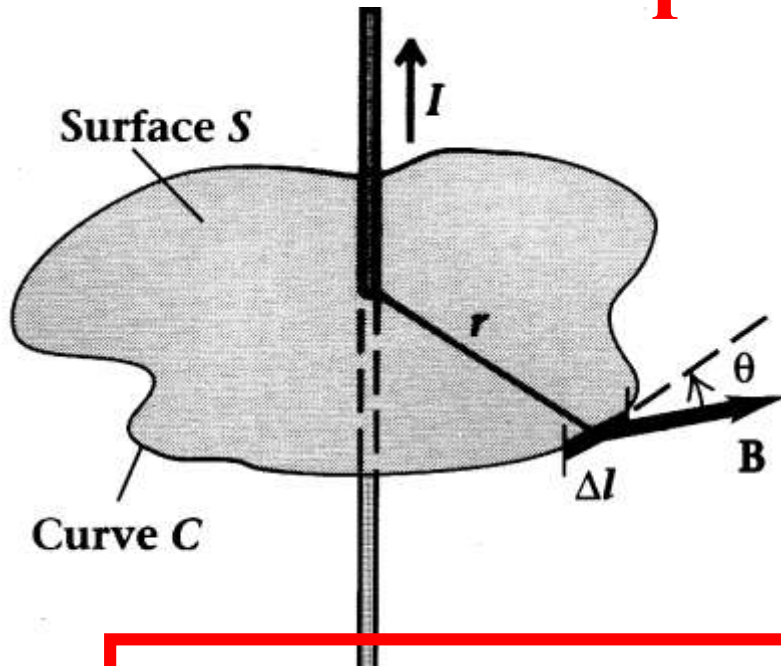
**{ NOT REQUIRED FOR THIS COURSE :
BUT GOOD TO KNOW }**

$$B = \frac{2\pi \mu_0}{4\pi} \frac{I}{R}$$

$$B = \frac{2\pi \mu_0 I}{4\pi R}$$

or $B = \frac{\mu_0 I}{2R}$

Ampere's Law



$I = \text{curr. through surface}$

$$\mu_0 I = \underbrace{[B_{\parallel} \Delta l]}_{\substack{\text{B along } \Delta l \\ \text{add up around surface edge}}}$$

$$B_{\parallel} = B \cos(\theta)$$

here

closed edge- line integral form.

$$\mu_0 I = \sum_{\text{edge}} B_{\parallel} \Delta l$$

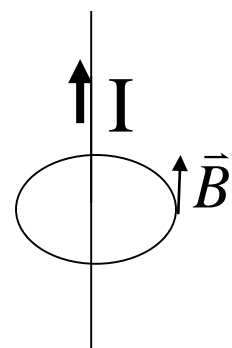
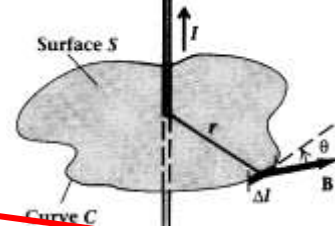
$I = \text{current through surface}$

$B \text{ along } \Delta l$

$$\mu_0 I = \oint_{\text{edge}} B_{\parallel} dl$$

Add up around surface edge.

recall Ampere's Law $\mu_0 I = \sum_{\text{edge}} [B_{\parallel} \Delta l]$



B field around an ∞ wire with I

\vec{B} along circle & B always constant on circle.
[circumference]

$$\mu_0 I = B [2\pi R]$$

From Ampere's Law

$$k' = \frac{\mu_0}{4\pi}$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

← For a wire

or

$$B = \frac{2k'I}{R}$$

example

Suppose $I = 1\text{A}$: $R = 1\text{m}$: $B = ?$

$$B = \frac{[4\pi(10)^{-7} \frac{\text{N}}{\text{A}^2}] 1\text{A}}{2\pi \quad 1\text{m}}$$

$$= 2(10)^{-7} \frac{\text{NA}}{\text{A}^2 \text{m}} \quad B = 2(10)^{-7} \frac{\text{N}}{\text{A m}}$$

$$B = 2(10)^{-7} \text{ T very small!}$$

Units check

Recall: $F = qVB$ $F = I\ell B$

$$B = \frac{F}{I\ell} \Rightarrow \text{T} = \frac{\text{N}}{\text{Am}}$$

- Two long magnetic wires carry currents of 2.0 A and 6.0 A . The wires are 1.0 m apart and the currents flow in opposite directions. Determine the magnetic field strength and its direction at a point (P) 2.0 m away from the right long straight wire.

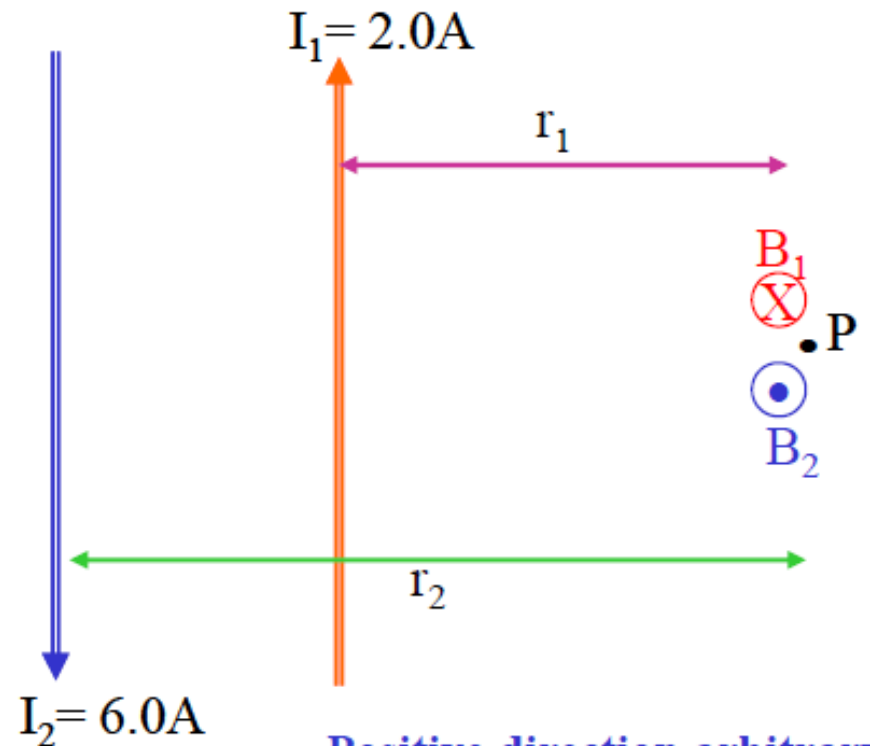
$$\mathbf{B}_1 = \frac{\mu_0 \mathbf{I}_1}{2\pi r_1} = 2.0 (10^{-7})\text{ T}$$

$$\mathbf{B}_2 = \frac{\mu_0 \mathbf{I}_2}{2\pi r_2} = 4.0 (10^{-7})\text{ T}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

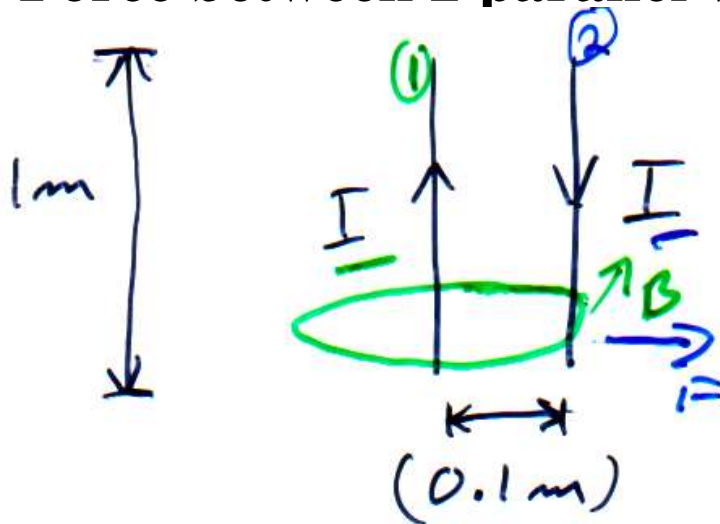
Out of the page

$$\mathbf{B} = +\mathbf{B}_1 - \mathbf{B}_2 = -2.0 (10^{-7})\text{ T}$$



Positive direction arbitrary
chosen to be into the page(+)

Force between 2 parallel wires $L=1\text{ m}$, at 0.1 m , $I = \pm 50\text{ A}$



B due to 1

$$B = \frac{\mu_0 I}{2\pi R}$$

$$F_{on 2} = l I B$$

$$= l \frac{I \mu_0 I}{2\pi R}$$

$$F = \frac{\mu_0}{2\pi} \frac{l I^2}{R}$$

$$F = \frac{1\text{ m}}{(0.1\text{ m})} (50)^2 \text{ A}^2 \cdot 2 \cdot 10^{-7} \frac{\text{N}}{\text{A}^2}$$

$$= (10) \cdot 2500 \cdot 2 (10)^{-7} \text{ N}$$

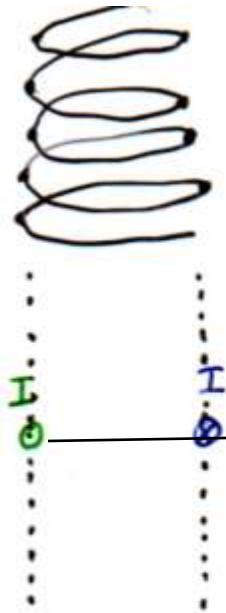
$$= 50,000 (10)^{-7} \text{ N}$$

$$F = .005 \text{ N}$$

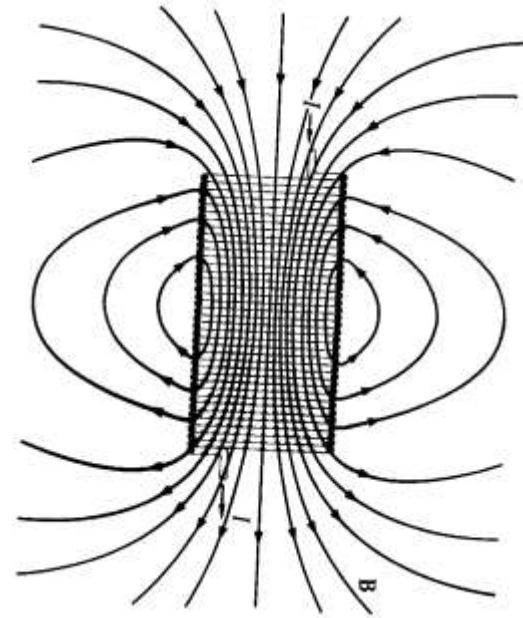
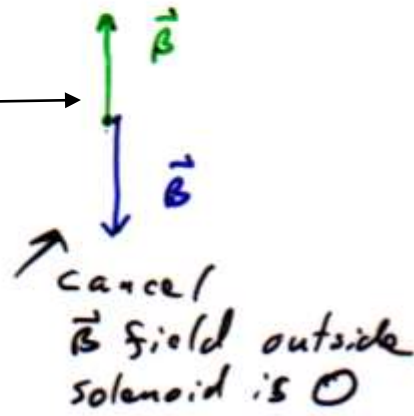
$$.01 \text{ m} \Rightarrow .05 \text{ N}$$

$$.001 \text{ m} \Rightarrow .5 \text{ N}$$

Magnetic field of long solenoid



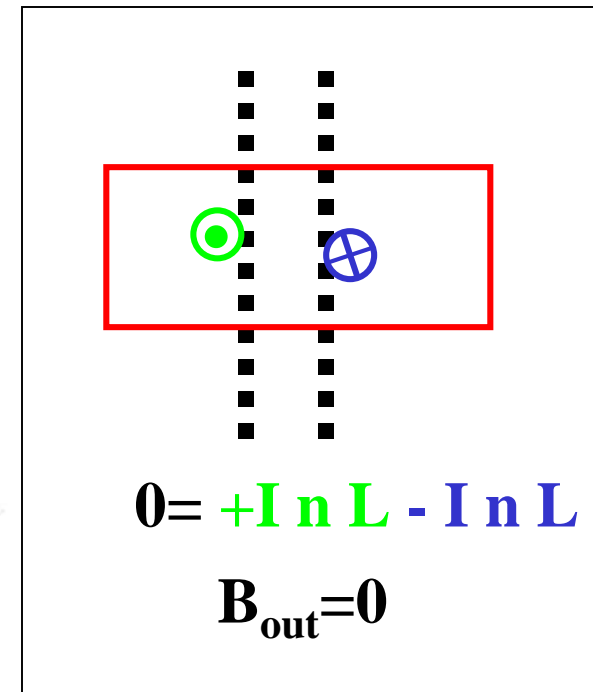
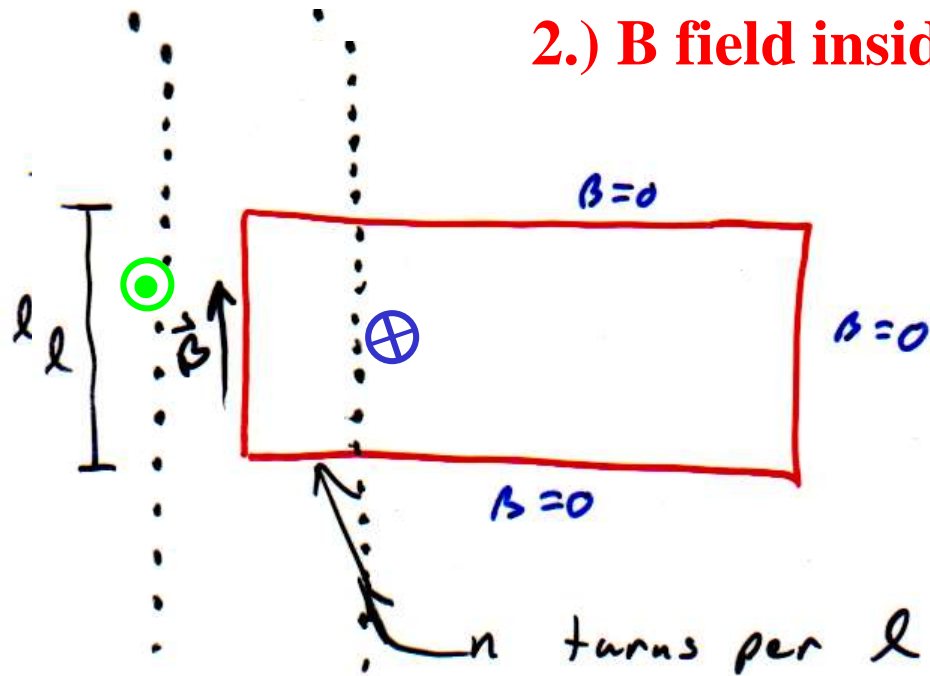
1.) B field outside goes to 0



(= 0 ; ∞)

Magnetic field of long solenoid

2.) B field inside



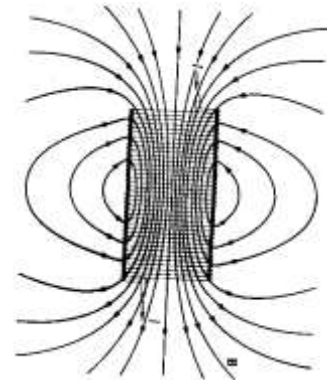
$I \cdot n l = \text{tot curr through surface}$

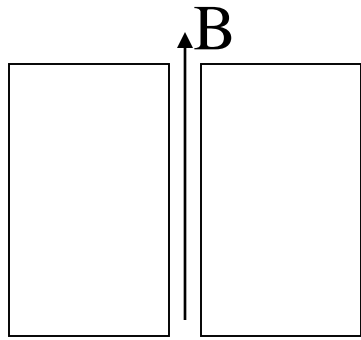
Baround loop

$$B \cdot l + 0 = I n l \mu_0$$

$$\therefore \boxed{B = I n \mu_0} \propto \text{solenoid}$$

4-11a





↔ R=0.025m

Superconducting solenoid

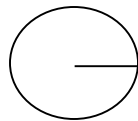
$$B = I n \mu_0$$

$$= 100\text{A} [100,000 \frac{\text{turns}}{\text{m}}] [4\pi(10)^{-7}] \frac{\text{N}}{\text{A}^2}$$

$$=(10)^2 10^5 4\pi (10)^{-7} \frac{\text{NA}}{\text{mA}^2} = 4 [3.14] \text{ T}$$

$$B = 12\text{T}$$

Q: What is outward force on inside of such a solenoid?



R=0.025m

$$l = 2\pi R = 2\pi 0.025\text{m}$$

$$F = I l B = (100\text{A}) (.025\text{m} 2\pi) 12\text{T}$$

$$= (100) (.025) (6.28) 12 \frac{\cancel{\text{ANm}}}{\cancel{\text{Am}}}$$

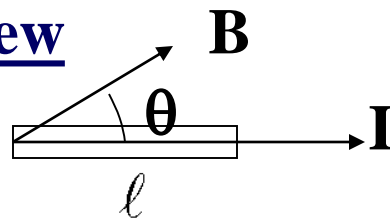
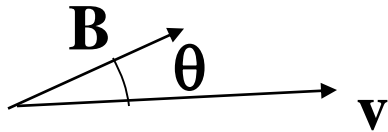
$$=(2.5)(6.28) 12\text{N}$$

$$=150\text{N}$$

$$\sim 33.7 \text{ lbs.}$$

Magnetic Field Review

$$F = qv_{\perp}B \quad \text{or} \quad F = qvB \sin \theta$$



Right Hand
Rule

$$F = I \ell B_{\perp} \quad \text{or} \quad F = I \ell B \sin \theta$$

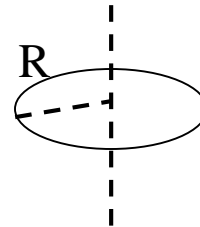
$$\Delta B = k' \frac{I \Delta l}{r^2} \sin \theta$$

Not required:

$$k' = \frac{\mu_0}{4\pi}$$

Long wire:

$$B = k' \frac{2I}{R} \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

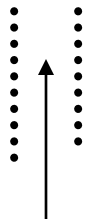


Center of Loop (circular):



$$B_{\text{center}} = \frac{2\pi k' I}{R} = \frac{\mu_0 I}{2R}$$

Solenoid:

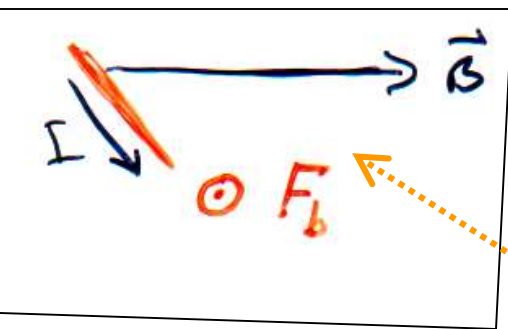
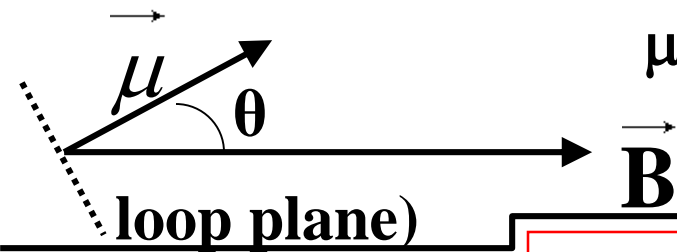


$$B = \mu_0 n I = 4\pi k' n I$$

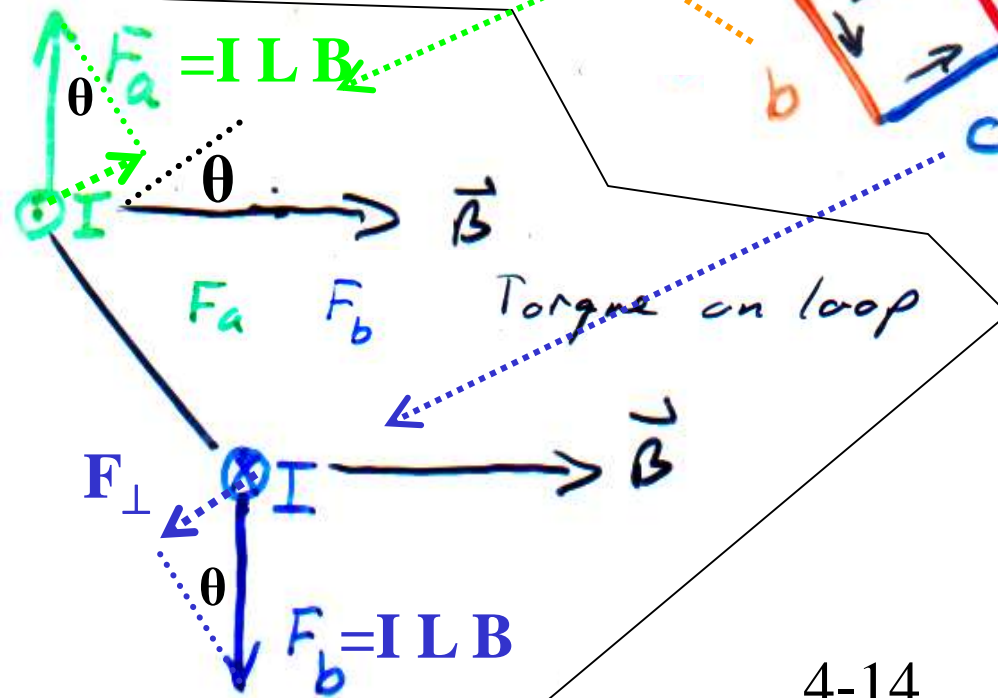
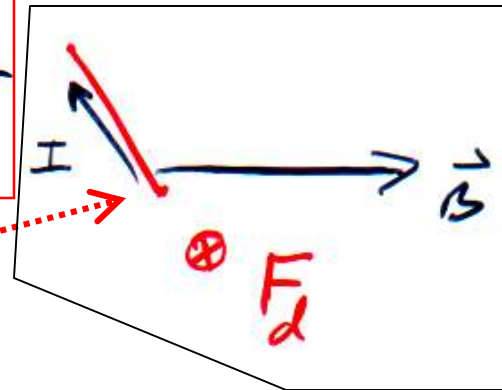
Torque, τ , on current loop (assume square) in B field

μ = magnetic moment (like electric dipole)

$$\tau = \mu B \sin(\theta)$$

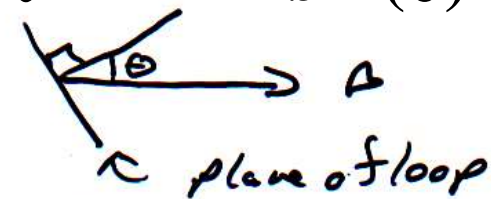


F_d F_b try to pull loop out but no Torque



$$\tau = I L^2 B F_{\perp}$$

$$\tau = I A B \sin(\theta)$$



$$\mu_{\text{loop}} = I A$$

$$\tau = \mu B \sin \theta$$