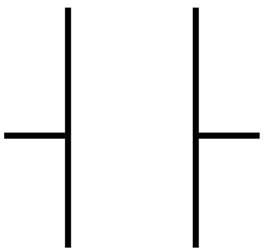
$V = \frac{U}{q}$ electric potential

potential energy

$V = k\frac{q}{r}$ point charge potential

Energy conservation

electric potential lines $\perp$ electric field



capacitor

$\sigma = \frac{Q}{A}$ $E = \frac{\sigma}{\varepsilon_o}$ $V = Ed$

$Q = VC$ $C = \kappa\frac{A\varepsilon_o}{d}$

$W = \frac{1}{2}qV$

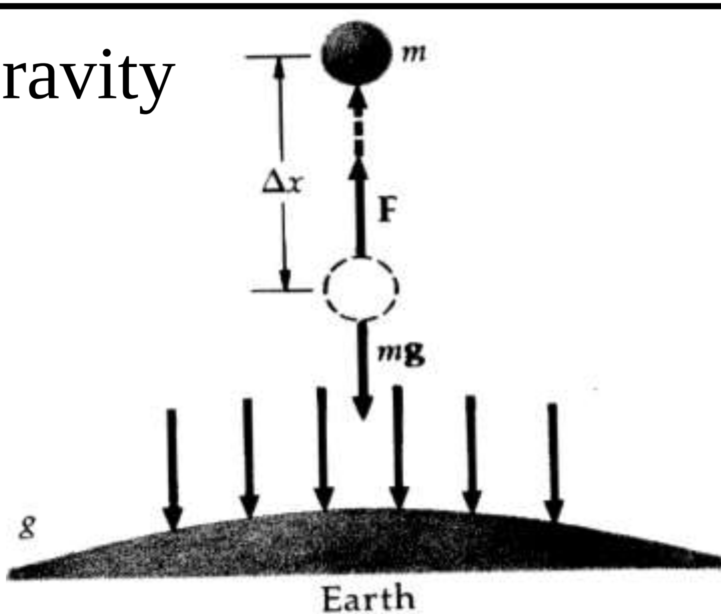
Recall for gravity

force = $\vec{F}$ → W= work = $F_{\parallel}$ d → U= potential energy = -W

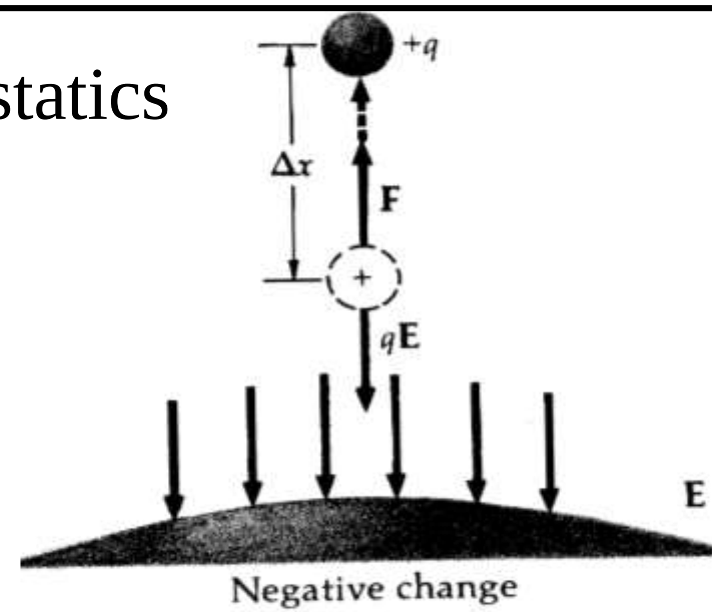
N   N m = J   J

examples	force		potential energy	
gravity near earth's surf.	mg	→	mgh	Set U=0 anywhere
gravity in general.	$\frac{Gm_1m_2}{r^2}$	→	$-\frac{Gm_1m_2}{r}$	Set U=0 at $r=\infty$

Gravity



Electrostatics

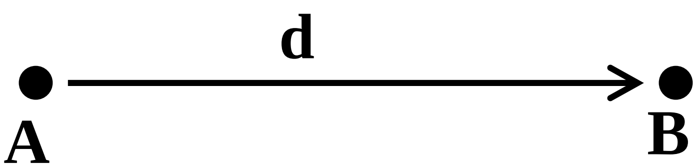


Electric Potential V

$$\vec{E} = \frac{\vec{F}}{q} \longrightarrow \frac{W}{q} = \frac{F_{\parallel}\, d}{q} = E_{\parallel}\, d = V \longrightarrow$$ **V = electric potential**

$$\frac{N}{C} \qquad \frac{J}{C} = V = \text{Volt} = \frac{\text{kg m}^2}{\text{s}^2\ \text{C}}$$

electric potential <u>difference</u> (allows V=0 to be set locally)



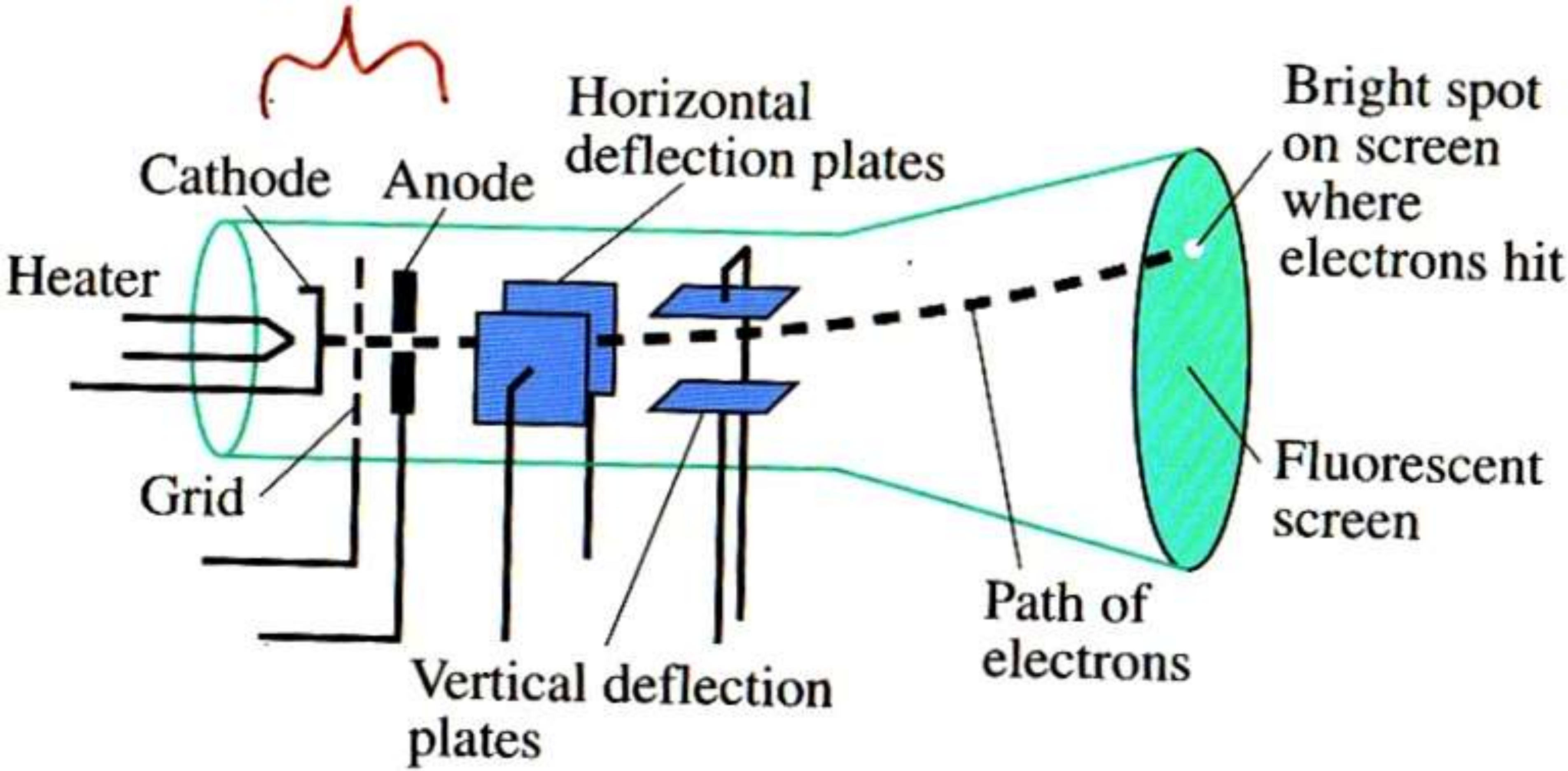
$$V_{AB} = \frac{W_{AB}}{q} = E_{\parallel}\, d$$

Potential energy change $$U = qV$$

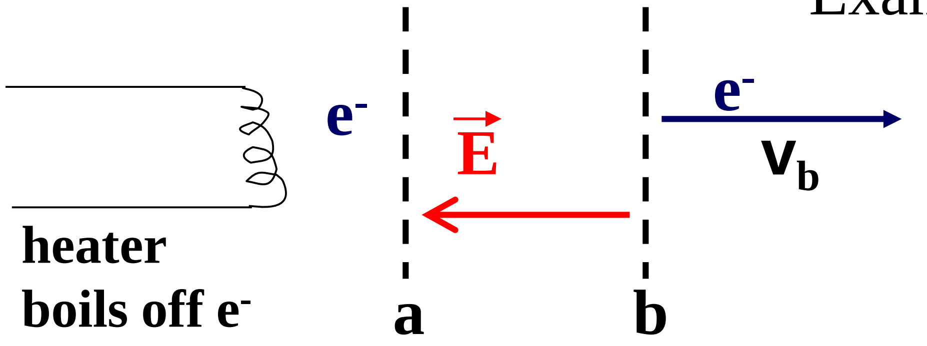
V = work/unit charge to bring charge <u>from</u> ∞

(in cases when V=0 set at ∞) – will treat 2nd

focus of problem



Example accelerator grids in old TV



$$E = \frac{1}{2}mv^2 + qV$$

total energy (E), KE ($\frac{1}{2}mv^2$), U (qV)

$V_a = 0$ **$V_b = +5000$ V**

tot. energy at "a"

$$E_a = \frac{1}{2}mv_a^2 + 0$$

$v_a = 0$ so $E_a = 0$

tot. energy at "b"

$$E_b = \frac{1}{2}mv_b^2 + (-e)V_b$$

$E_a = E_b$ (energy conservation)

$$0 = \frac{1}{2}mv_b^2 + (-|e|)\,V_b$$

$$-\frac{1}{2}mv_b^2 = -|e|\,V_b$$

$$v_b = \sqrt{\frac{2\,e\,V_b}{m}} = \sqrt{\frac{2\,(1.6)(10)^{-19}\ (5000)}{9.1\,(10)^{-31}}} \qquad \sqrt{\frac{C\,V}{kg}} = 4.2\,(10)^7\ \frac{m}{s}$$

$$\sqrt{\frac{C\,\frac{J}{C}}{kg}} = \sqrt{\frac{C\,\frac{1}{C}\,\frac{kg\,m^2}{s^2}}{kg}} = \frac{m}{s}$$

For point charge q

Electric field for charge q

$$E = k\frac{q}{r^2}$$

like gravity

$\rightarrow$

Electric Potential for q

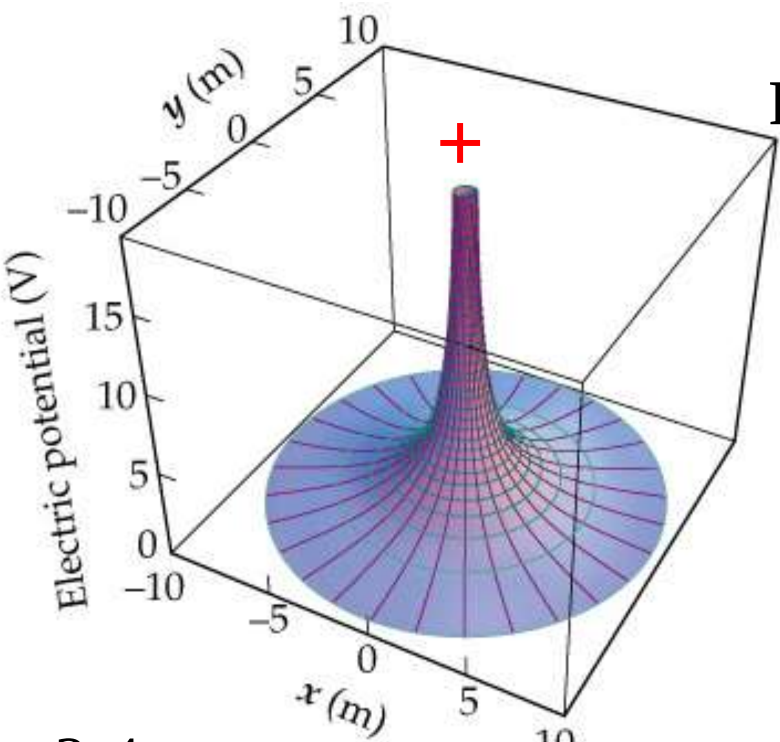
$$V = k\frac{q}{r}$$

like gravity

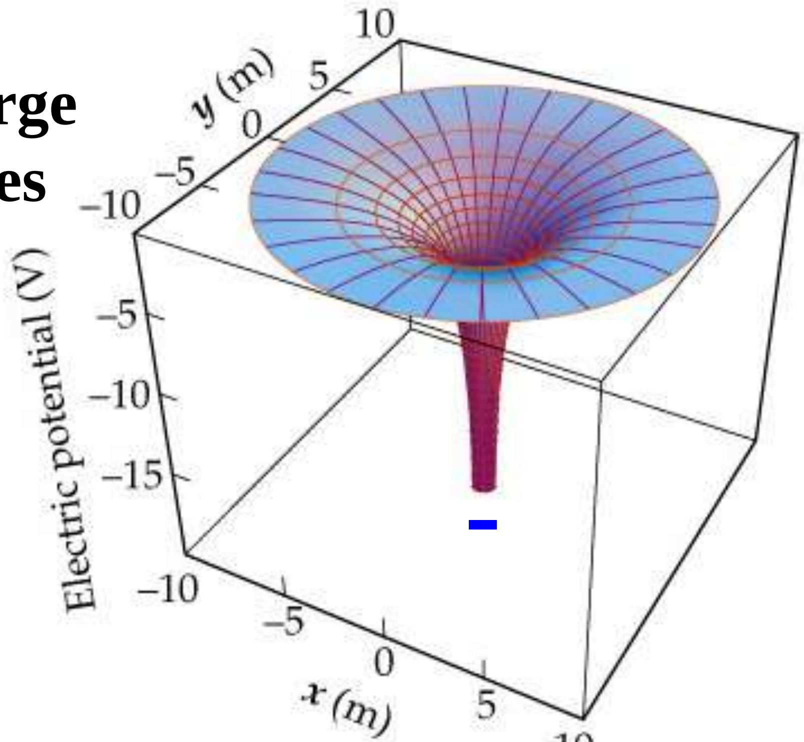
$$r \rightarrow \infty \quad V = 0$$

choice of V=0

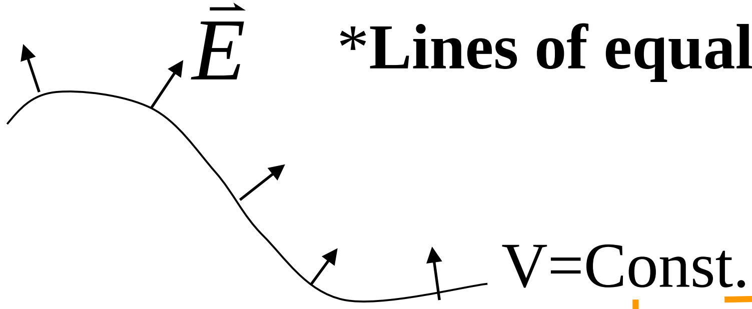
more complicated cases ----   superimpose many charges.

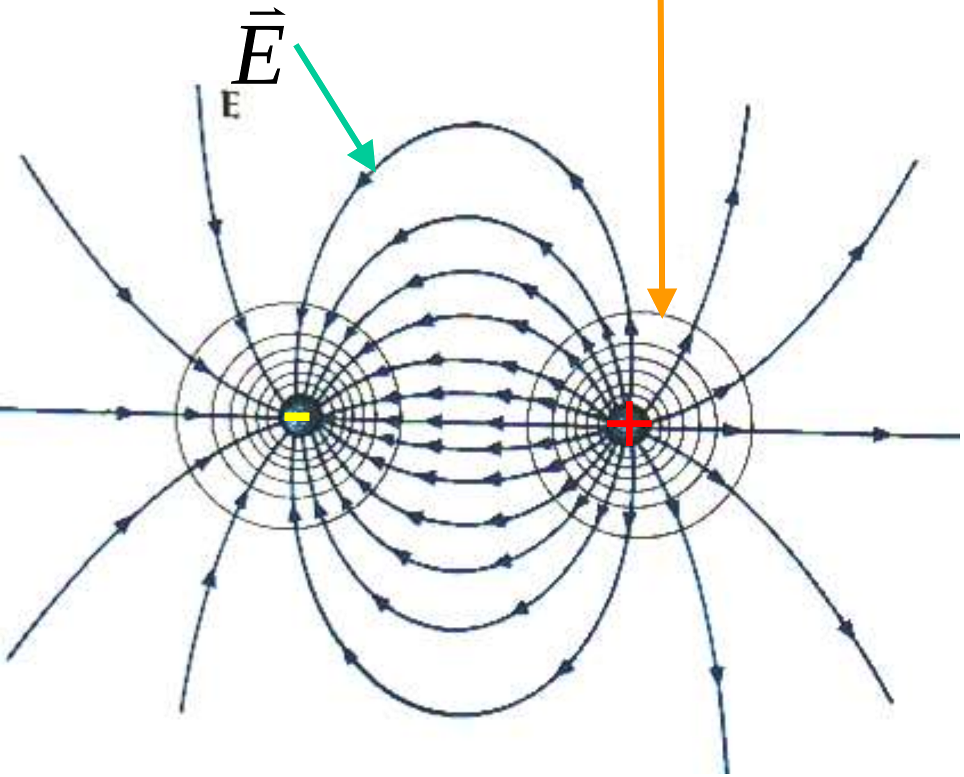


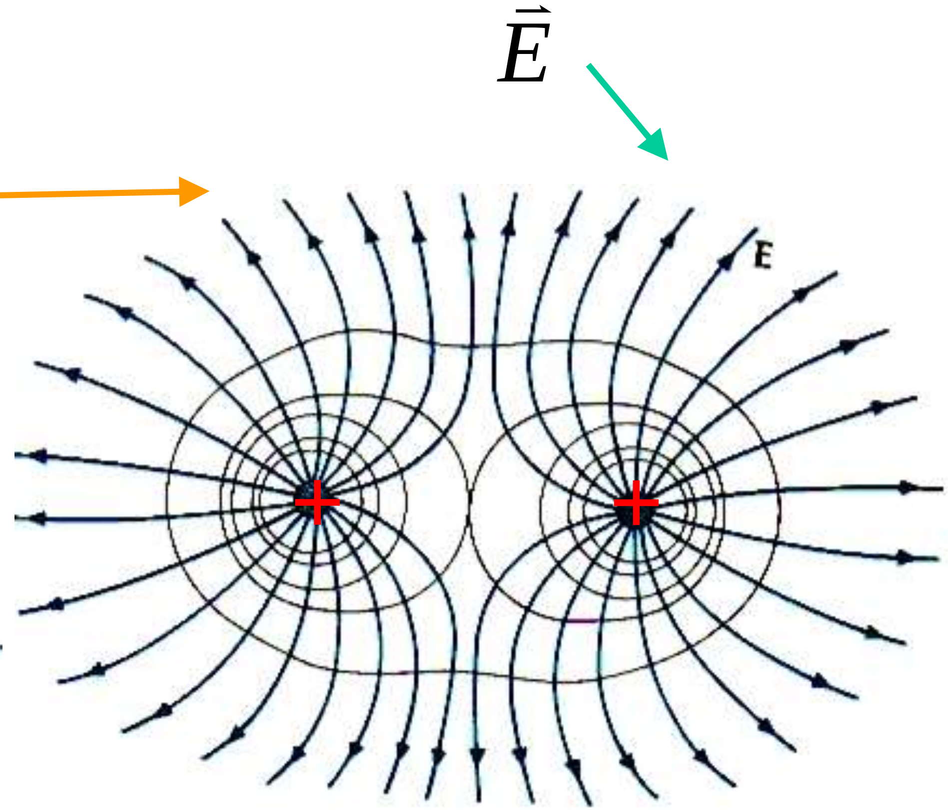
Point charge
V surfaces



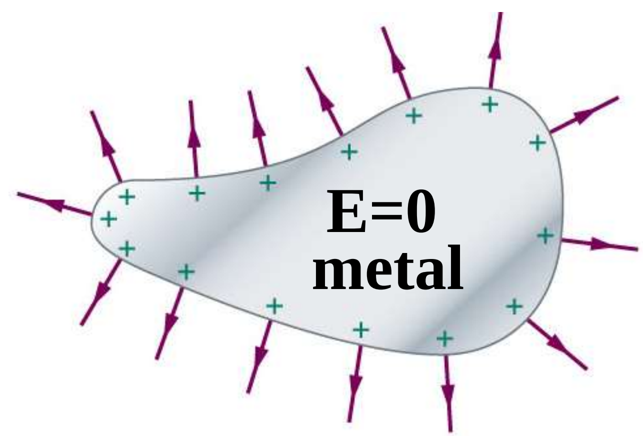
***Lines of equal potential $\perp$ to $\vec{E}$ field lines.**







Recall E $\perp$ surface of metal



V=constant throughout interior and surface of metal

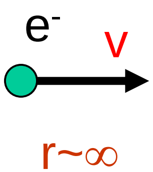
2-4a

An e^- is shot (from a great distance away) with a velocity v= 1000 m/s straight at a **fixed** charge of e^-. How close (d=?) do the two charges get to each other?

$$K=\frac{1}{2}mv^2$$

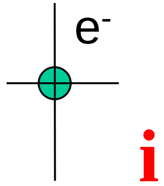
$$U=k\frac{(q_1)(q_2)}{r}$$

recall



$$E_i=\frac{1}{2}mv^2+k\frac{(-e)(-e)}{r}$$

0 because r~∞



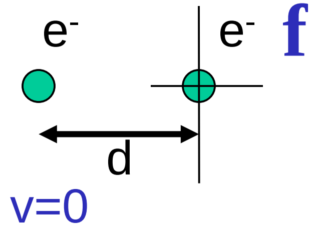
Conserve energy

$$E_i = E_f$$

$$\frac{1}{2}mv^2=k\frac{e^2}{d}$$

$$E_f=\frac{1}{2}mv^2+k\frac{(-e)(-e)}{d}$$

0 because v= 0 at closest point



$$\frac{1}{2}v^2=\frac{ke^2}{m}\frac{1}{d}$$

$$\frac{1}{2}v^2=[\frac{ke^2}{m}]\frac{1}{d}=[252]\frac{1}{d}\{\frac{m^2}{s^2}m\}$$

See next page

$$d=\frac{ke^2}{m}\frac{2}{v^2}=252\frac{2}{(1000)^2}\{\frac{m^2}{s^2}m\}\frac{1}{\frac{m^2}{s^2}}=0.000504\text{ m}$$

$k = 8.99\,(10)^{9}\ Nm^2/C^2$ **$m = 9.11\,(10)^{-31}\ kg$** **$e = 1.6\,(10)^{-19}\ C$**

$$\left[\frac{ke^2}{m}\right] = \frac{8.99\,(10)^9\,[1.6\,(10)^{-19}]^2}{9.11(10)^{-31}}\left\{\frac{Nm^2}{\cancel{C^2}}\ \frac{\cancel{[C]^2}}{kg}\right\}$$

$$\frac{ke^2}{m} = \frac{8.99\ [1.6]^2}{9.11}\ \frac{(10)^9(10)^{-38}}{(10)^{-31}}\left\{\frac{\cancel{kg}\,m}{s^2}\,m^2\,\frac{1}{\cancel{kg}}\right\}$$

$$\frac{ke^2}{m} = 2.52(10)^2\left\{\frac{m^3}{s^2}\right\}$$

Two e⁻'s are shot (from a great distance away) **each** with a velocity v= 1000 m/s straight at each other. How close (d=?) do the two charges get to each other?

$$K=\frac{1}{2}mv^2$$

$$U=k\frac{(q_1)(q_2)}{r}$$

recall

e⁻ v → r~∞ ← v r~∞ **i**

$$E_i=\frac{1}{2}mv^2+\frac{1}{2}mv^2+k\frac{(-e)(-e)}{r}$$

0 because r~∞

Conserve energy

$$E_i=E_f$$

$$2\frac{1}{2}mv^2=k\frac{e^2}{d}$$

$$v^2=\frac{ke^2}{m}\frac{1}{d}$$

f

0 because v= 0 at closest point

$$E_f=0+k\frac{(-e)(-e)}{d}$$

e⁻ e⁻ ↔ d v=0 v=0

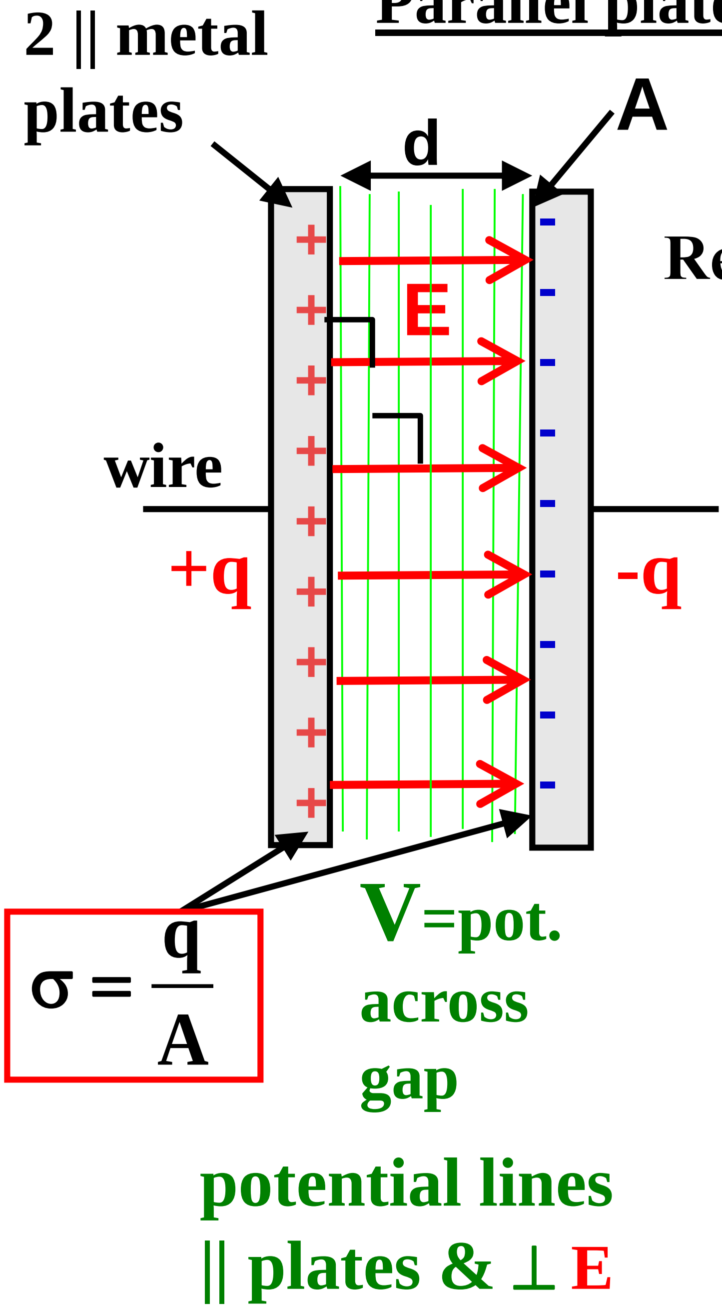
See previous page

$$v^2=[\frac{ke^2}{m}]\frac{1}{d}=[252]\frac{1}{d}\{\frac{m^2}{s^2}m\}$$

$$d=\frac{ke^2}{m}\frac{1}{v^2}=252\frac{1}{(1000)^2}\{\frac{m^2}{s^2}m\}\frac{1}{\frac{m^2}{s^2}}=0.000252\ m$$

Parallel plate capacitor

- $\vec{E}$ straight across gap
- $\vec{E}$ constant & ⊥ to plate



$$\sigma = \frac{q}{A}$$

Recall Gauss's Law for plate gave

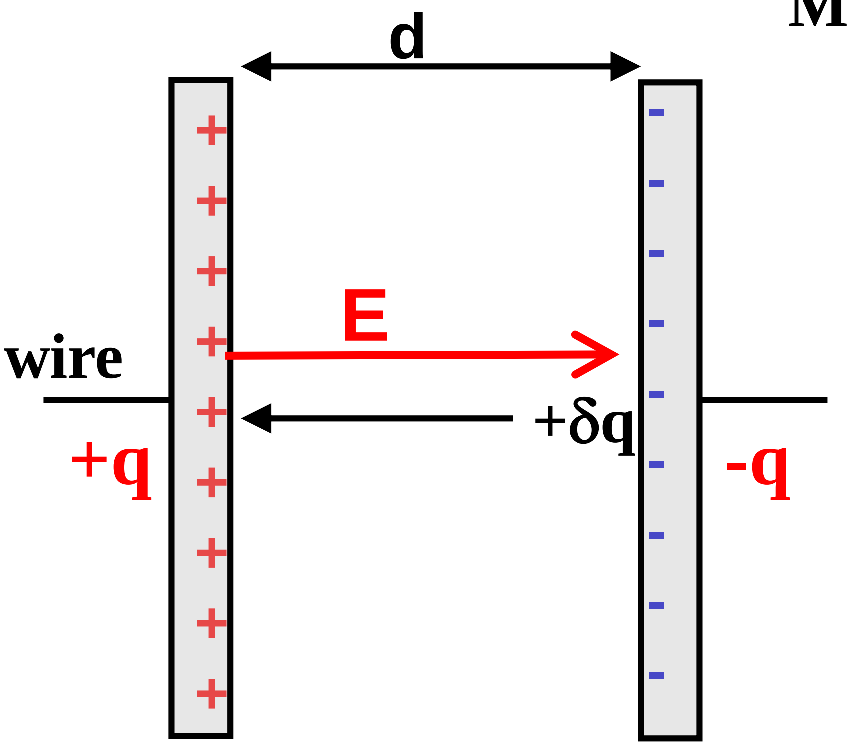
$$E = \frac{\sigma}{\varepsilon_o}$$

Since E constant

$$V = Ed$$

(Proof next page)

Parallel plate capacitor (cont.)



Move a small charge δq between plates

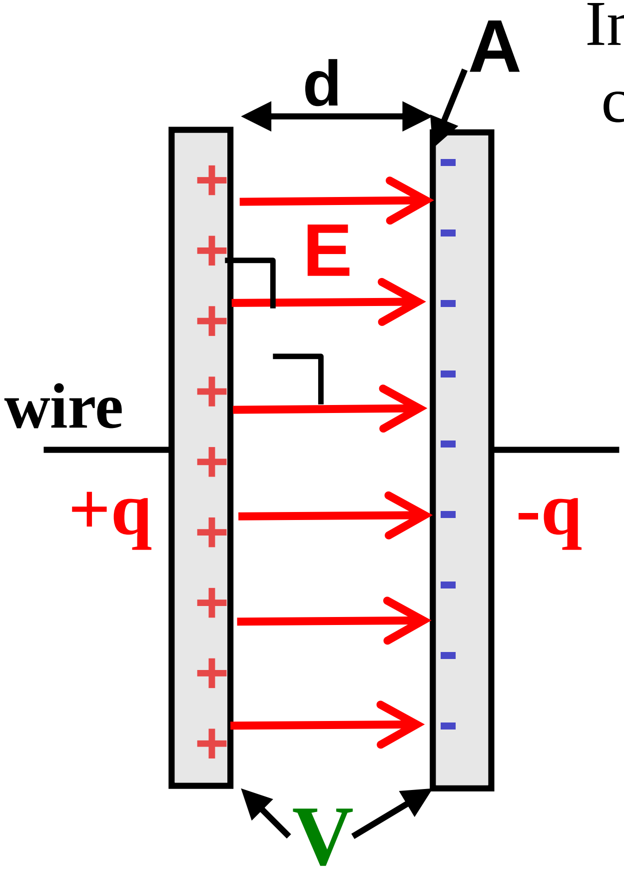
W=work done

$$W = [F]\, d = [\delta q\, E]\, d$$

$$V = \frac{W}{\delta q} = \frac{[\delta q\, E]\, d}{\delta q}$$

$$\boxed{V = E\, d}$$

Parallel plate capacitor (cont.)



In applications often only need to know how much charge a capacitor can store at a given voltage

$$q = VC \quad \leftarrow \text{capacitance}$$

$$C = ?$$

$$\sigma = \frac{q}{A} \qquad V = Ed \qquad E = \frac{\sigma}{\varepsilon_o}$$

$$\sigma A = Ed\,C$$

$$\sigma A = \frac{\sigma}{\varepsilon_o} d\,C$$

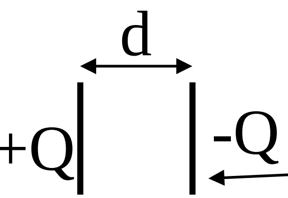
$$C = \frac{A\varepsilon_o}{d}$$

Capacitance units **farads**

$$C = \frac{q}{V} \longrightarrow \text{farads} = \frac{\text{Colombs}}{\text{Volts}} = \frac{C}{V} = \frac{C}{J/C} = \frac{C^2}{J}$$

Capacitor : given d, A, & Q **find E and V**

$Q = 0.5\ \mu C = 0.5(10)^{-6}\ C$



$d = 1mm = (10)^{-3}\ m$

$A = 250cm^2 = 250\ [(10^{-2})m]^2 = .025m^2$

$dE = V$

$C = \frac{\varepsilon_0 A}{d}$

$Q = VC$

$Q = (dE)\left(\frac{\varepsilon_0 A}{d}\right) = E\ \varepsilon_0 A$

Or realize !!

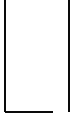
$$\mathbf{E = \frac{\sigma}{\varepsilon_o} = \frac{Q/A}{\varepsilon_o}}$$

$$E = \frac{Q}{A\varepsilon_0} = \frac{.5\ (10)^{-6}}{(.025)[8.85\ (10)^{-12}]}\ \frac{C}{m^2[\frac{C^2}{Nm^2}]} \Rightarrow E = 2.3(10)^6\ \frac{N}{C}\ \text{or}\ \frac{V}{m}$$

Note $V = \frac{J}{C} \rightarrow$ and $\frac{N}{C} = \frac{(mN)}{mC} = \frac{J}{mC} = \frac{V}{m}$

Question what is voltage?

$V = Ed = 2.3(10)^6\ \frac{V}{m}\ \ 10^{-3}m = 2.3(10)^3 V$



d

Example capacitor with variable d:

d_2=2mm

d_1=1mm

What is the change in V?

V_2=?

V_1=50 V

$$VC=Q \Rightarrow V=\frac{Q}{C} \leftarrow C=\frac{\varepsilon_0 A}{d}$$

$$V=\frac{Qd}{\varepsilon_0 A}$$

$$\frac{V_2}{V_1}=\frac{\left[\frac{Qd_2}{\varepsilon_0 A}\right]}{\left[\frac{Qd_1}{\varepsilon_0 A}\right]}=\frac{d_2}{d_1} \Rightarrow \frac{V_2}{V_1}=\frac{2}{1}$$

$$V_2=2V_1$$

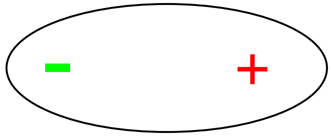
$$V_2=2(50\text{ V})=100\text{ V}$$

last slide 9/13/17 lec

Dielectric Materials

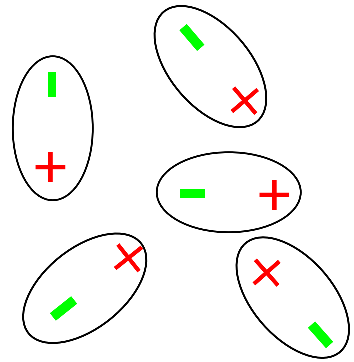
$\vec{E}=0$

polar molecule



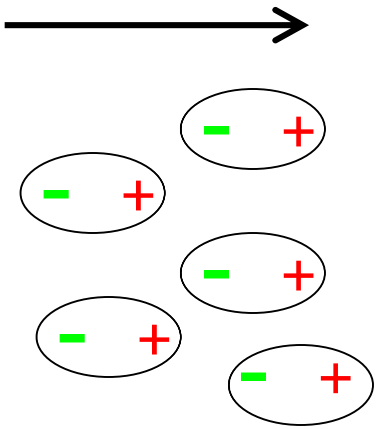
natural + and - ends

$\vec{E}=0$



E =0 random

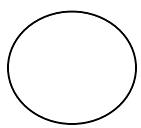
$\vec{E}$



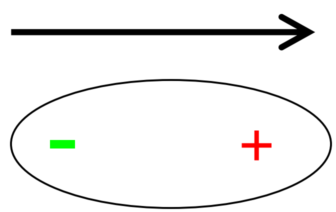
E field polarizes medium (orients dipoles)

polarizable molecule

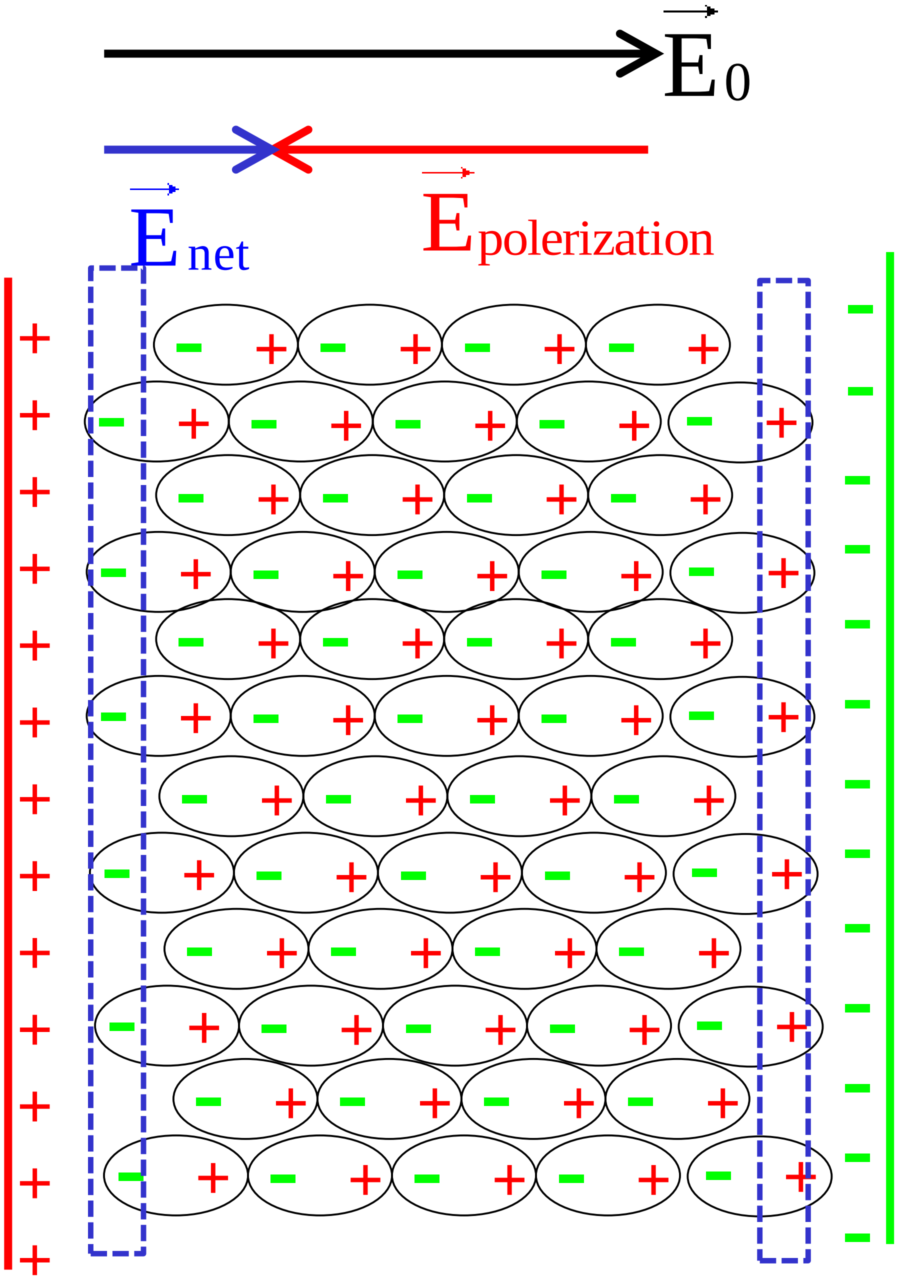
$\vec{E}=0$



$\vec{E}$



E field polarizes molecule (distorts)
(i.e. + &- charges displace oppositely)



$$\vec{E}_{net} = \vec{E}_0 + \vec{E}_{polerization}$$

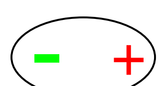
reduces

$$\vec{E}_{net} = \frac{1}{\kappa}\vec{E}_0 \qquad \kappa > 1$$

$$C = \kappa\,\varepsilon_o \frac{A}{d} = \kappa C_{\kappa=1}$$

$$\frac{V_\kappa}{V_{\kappa=1}} = \frac{q/C_\kappa}{q/C_{\kappa=1}} = \frac{C_\kappa}{C_{\kappa=1}} = \kappa$$

$$V_\kappa = \frac{1}{\kappa} V_{\kappa=1}$$



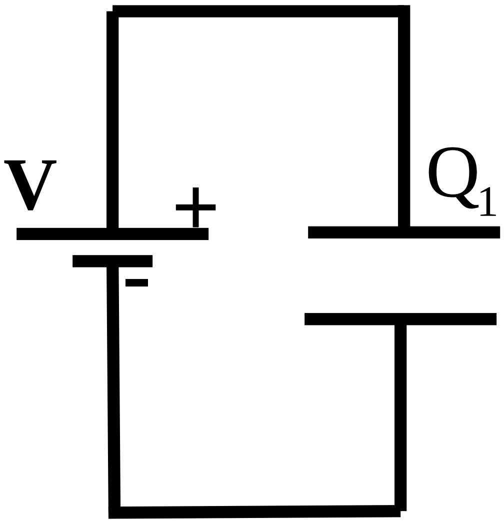
q same voltage reduced !!

2-9a

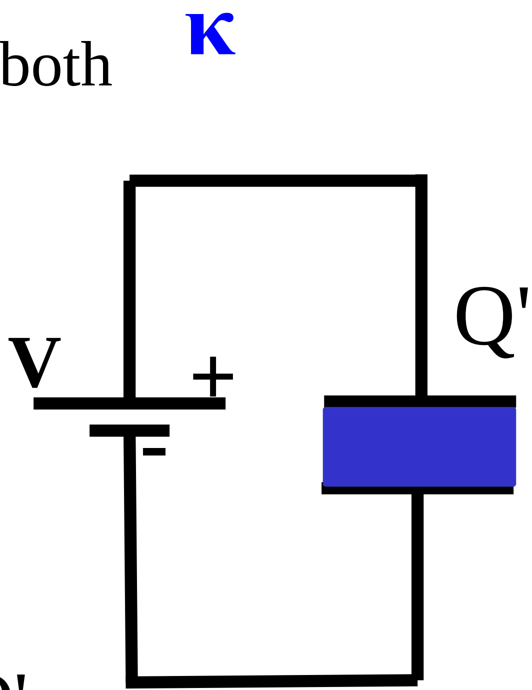
Capacitor without/with dielectric material: Battery connected

V, voltage same for both

$$V = \frac{Q}{C}$$



$$C_1 = \varepsilon_o \frac{A}{d}$$



$$C' = \kappa\, \varepsilon_o \frac{A}{d}$$

$$C' = \kappa\, C_1$$

$$\frac{Q_1}{C_1} = V = \frac{Q'}{C'}$$

$$\frac{Q_1}{C_1} = \frac{Q'}{\kappa C_1}$$

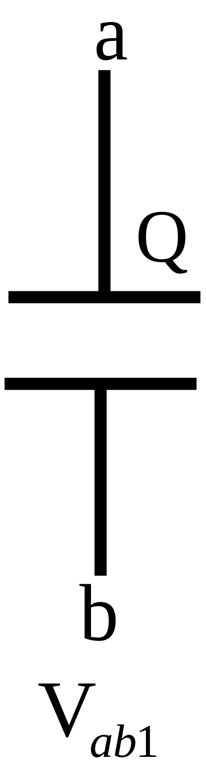
$$\boxed{\kappa Q_1 = Q'}$$

battery supplies more charge

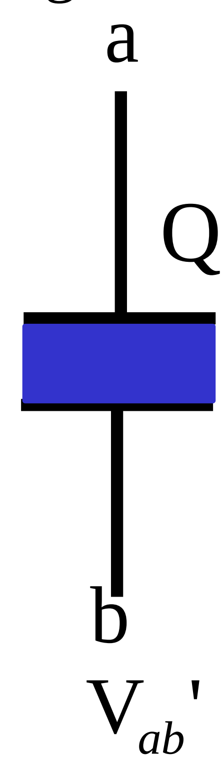
same V larger Q

Capacitor w/wo dielectric material: charged, battery-disconnected

Q, stays same for both – no battery to supply charge $\quad Q = VC$



$$C_1 = \varepsilon_o \frac{A}{d}$$



$$C' = \kappa\, \varepsilon_o \frac{A}{d}$$

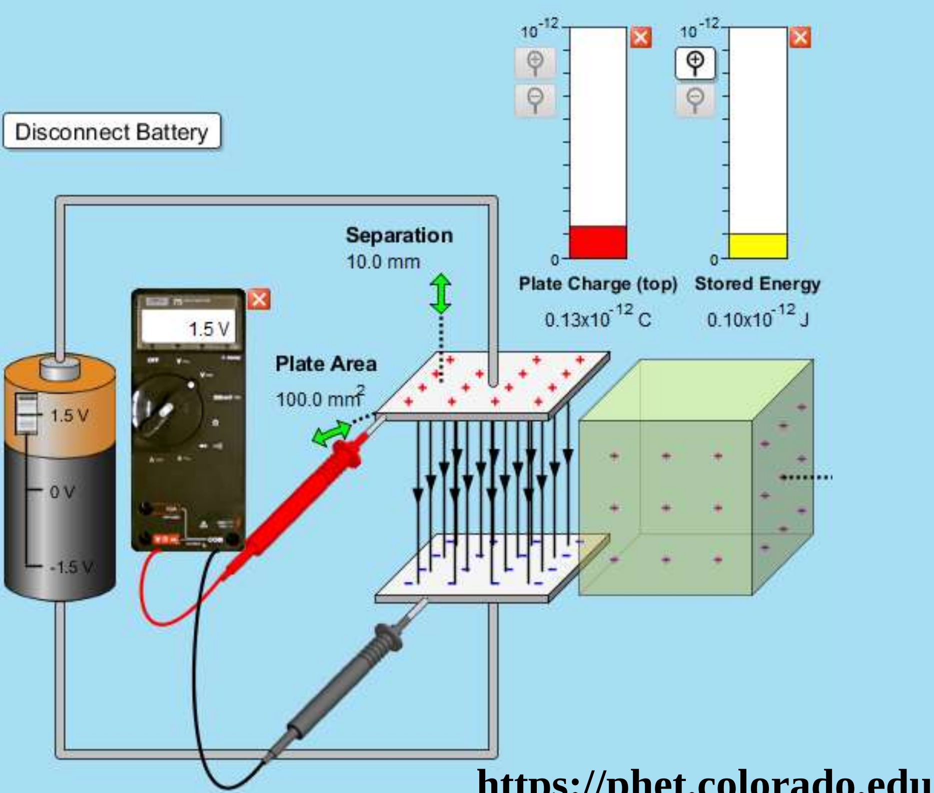
$$C' = \kappa\, C_1$$

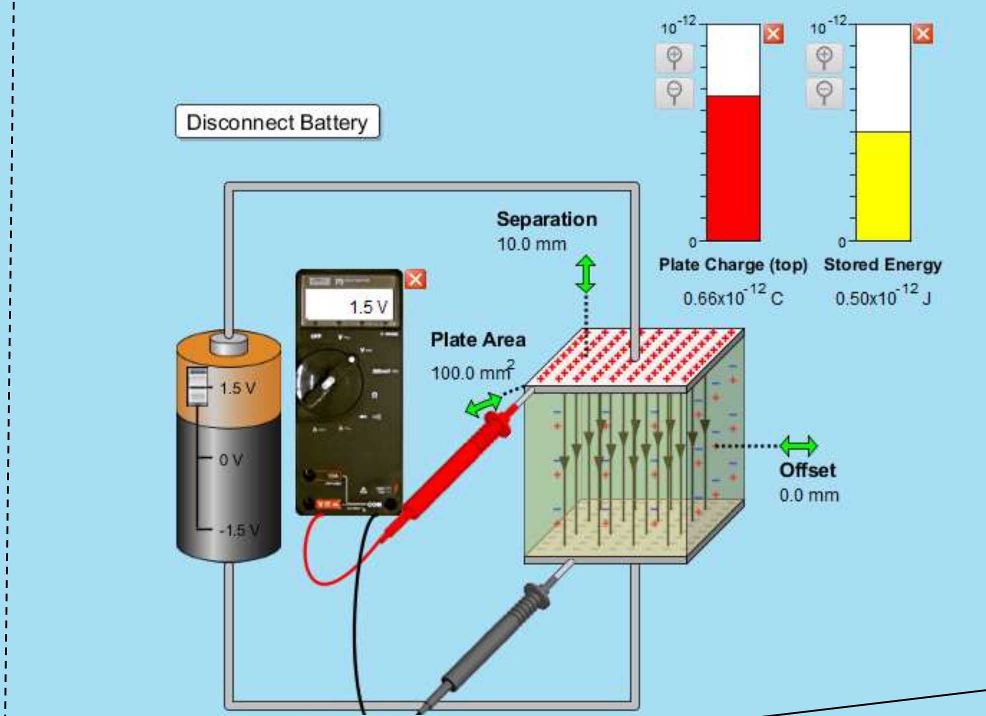
$$V_{ab1} C_1 = Q = V_{ab}'\, C'$$

$$V_{ab1} C_1 = V_{ab}'\, \kappa\, C_1$$

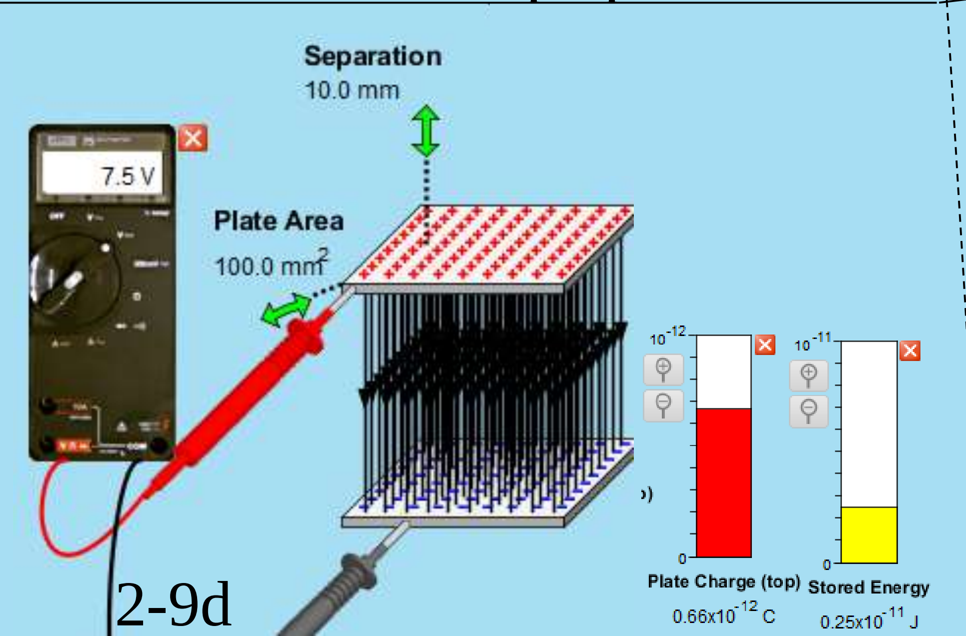
$$\frac{V_{ab1}}{\kappa} = V_{ab}'$$

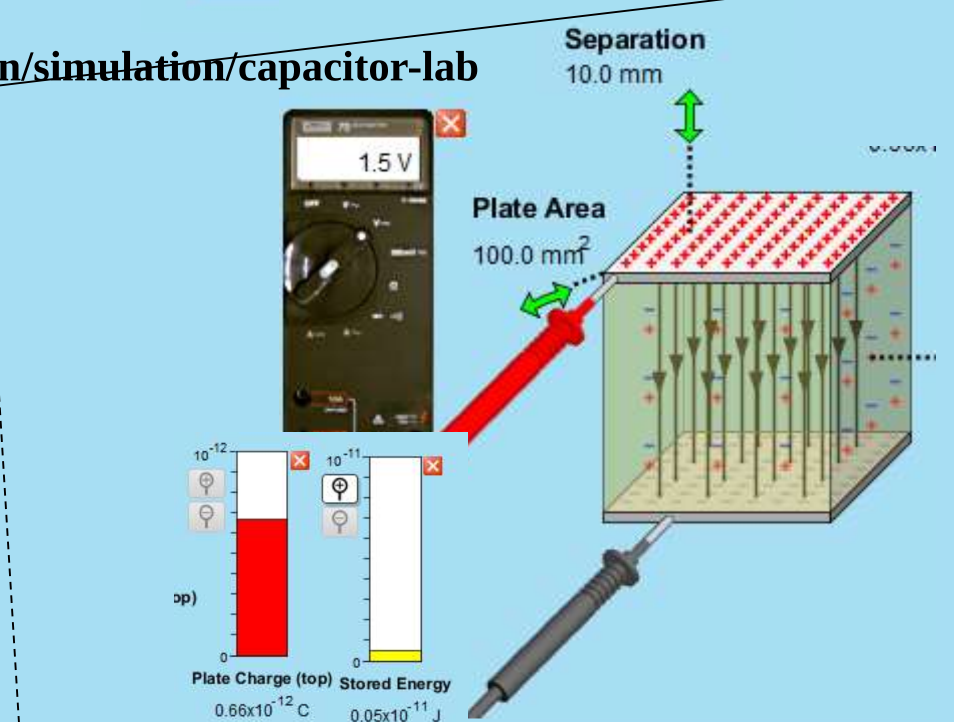
same Q smaller V





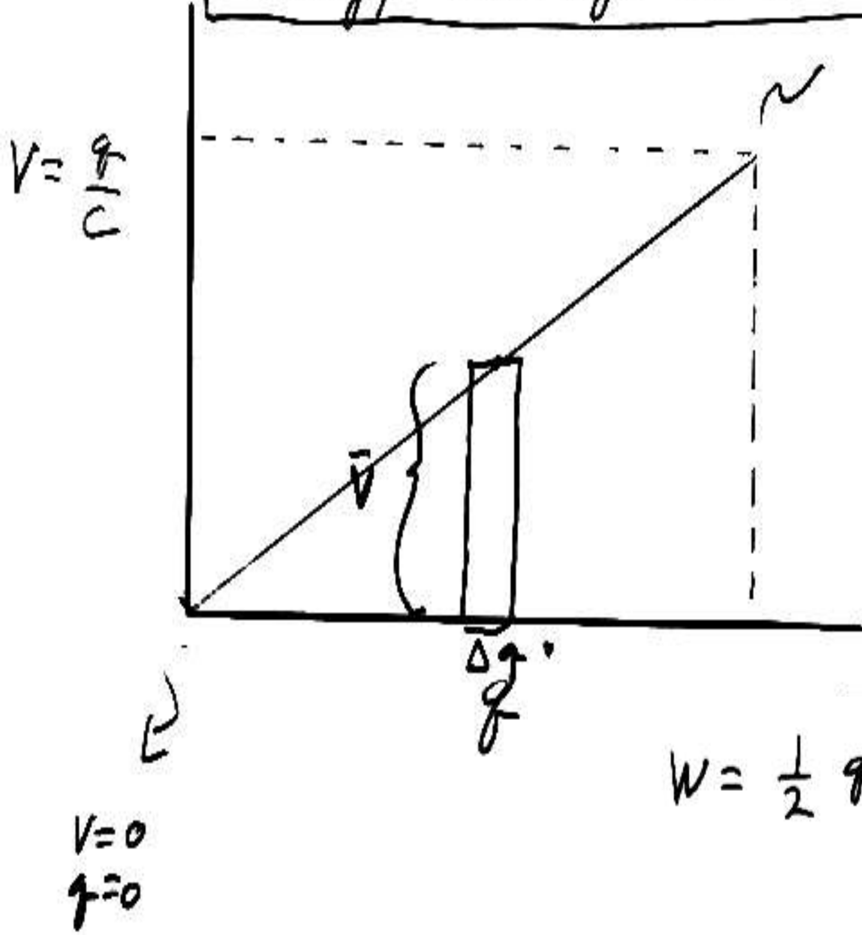
https://phet.colorado.edu/en/simulation/capacitor-lab



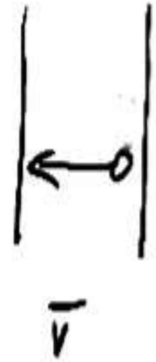


2-9d

Energy Storage in a Capacitor



$V = \frac{q}{C}$



$\Delta W = \bar{V} \Delta q$

add up all pieces

= area under V vs q curve

$W = \frac{1}{2} q V$ (area of triangle)

$W = \frac{1}{2} q \frac{q}{C} = \frac{1}{2} \frac{q^2}{C} = W$

or $W = \frac{1}{2} C V^2$

energy stored in capacitor

$J = \frac{C^2}{\text{farads}}$

$J = (\text{farads})(\text{Volts})^2$

Example

Consider a capacitor bank of 40 capacitors with 440 μf each

$C_{bank} = (40)\,440\,\mu f = 17{,}600\,(10)^{-6} f = 0.0176 f$!! big

① Charge to $V = 300$ Volts

a.) what is Q?

$Q = VC = 300\,(.0176)\ V f$

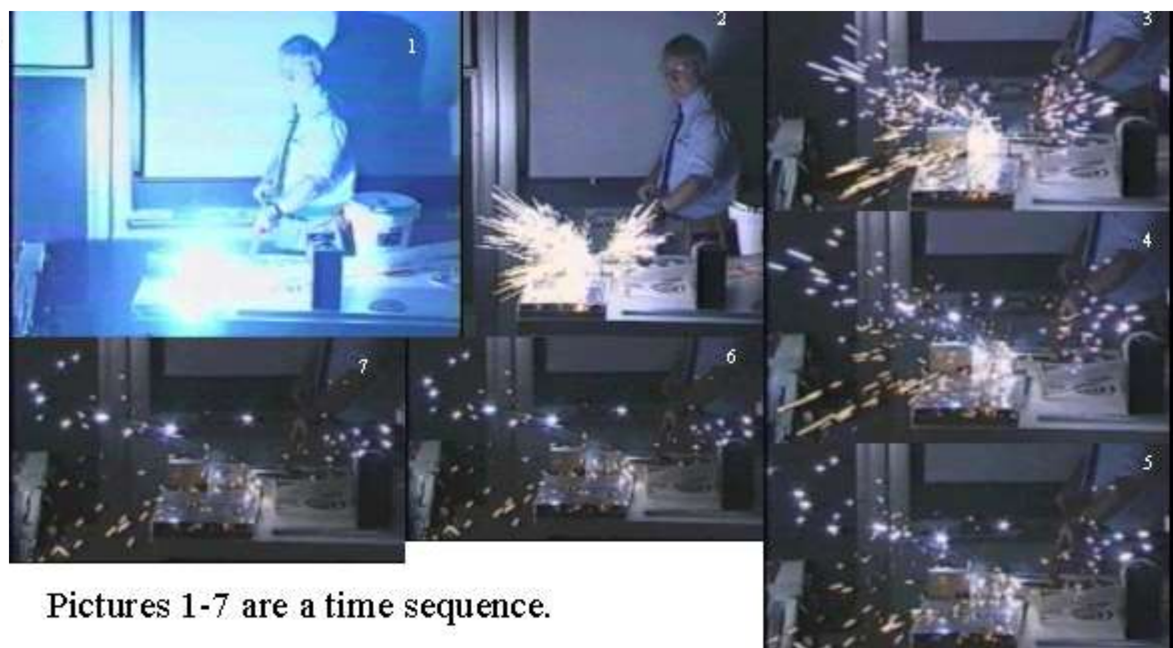
$Q = 5.28\ V\,\frac{C}{V}$!!

b) What is the energy stored in this C bank?

$W = \frac{1}{2}\,Q\,V = \frac{1}{2}\,(5.28)(300)\ C\,V$

$W = 792\ C\,\frac{J}{C}$!!!

Pro baseball pitcher throws with ~ 100 J of energy (Check!)



Pictures 1-7 are a time sequence.