Summary Page week 1

charges + - opposite attract like repel

Cancelation/screening

\[ F = \frac{k Q_1 Q_2}{r^2} \]  
Coulomb's Law

vector sum of forces

Superposition Principle – solution (A+B)= solution (A)+ solution (B)

\[ \vec{E} = \frac{\vec{F}}{q_T} \]  
Electric field q_T (+ test charge)

E Field Lines

E Field behavior in/at-surface-of metals

\[ \sum_{\text{surf}} E_\perp \Delta A = \frac{q_{\text{inside}}}{\varepsilon_0} \]  
Gausses' Law

\[ k = \frac{1}{4\pi \varepsilon_0} \]
Electrostatics (stationary charges)

Back drop Newton's famous work on gravity
- attractive force at a distance \( F = G\frac{m_1 m_2}{R^2} \)

Electrostatic forces

- sometimes attractive !
- sometimes repulsive !

du Fay 1733 2 kinds of electricity
- rubbed amber
- rubbed glass

Ben. Franklin assigned opposite signs to 2 types of electricity !!

- + & - charges leads to attractive & repulsive forces

Unlike charges attract (attractive force)

\[ \text{\begin{tikzpicture}[scale=0.5] \begin{scope}[very thick,deeptomagenta] \draw[->] (0,0) -- (1,0); \end{scope} \end{tikzpicture}} \]

Like charges repel (repulsive force)

\[ \text{\begin{tikzpicture}[scale=0.5] \begin{scope}[very thick,deepred] \draw[->] (0,0) -- (1,0); \end{scope} \end{tikzpicture}} \]
• + & - charges leads to notion of "cancellation"

{!!! charge does not disappear !! }

charge close together "screen" or hide each others presence

• quantity of charge \( Q \) [measured in Coulomb = C]

Millikan Oil drop experiment (1906)
added small charges to oil drop & found smallest units of \( Q \)

\[
Q(\text{electron, } e^-) = -1.6 \times 10^{-19} \text{ C}
\]

\[
Q(\text{proton, } p^+) = +1.6 \times 10^{-19} \text{ C}
\]

Actually \( 1 \text{ C} \) is huge

usually use \( \overbrace{1 \times 10^{-6} \text{ C}}^{\text{micro coulomb}} = 1 \mu \text{C} \)
Examples

\[ F = \text{force on } q_1 \text{ (due to } q_2) \]
\[ q_1 \quad -6 \text{ C} \]
\[ F = \text{force on } q_2 \text{ (due to } q_1) \]
\[ q_2 \quad +4 \text{ C} \]

\[ F = F \text{ by Newton’s 3’rd Law (and later, Coulomb’s Law)} \]

\[ F = \text{force on } q_1 \text{ (due to } q_2) \]
\[ q_1 \quad +6 \text{ C} \]
\[ F = \text{force on } q_2 \text{ (due to } q_1) \]
\[ q_2 \quad +4 \text{ C} \]

Note: forces along straight line between charges
All matter contains + & - charges: organized in Atoms

Atoms
Example Li

- electrons (light) e<sup>-</sup>
- charge in orbits

Nucleus (very small)
3 protons (heavy) p<sup>+</sup> + charge

**Net charge on an object ("cancellation")**

neutral object
Q<sub>net</sub> = 0

+ charged object
more protons

- charged object
more electrons

when + & - charges become separated & charge on object "created"

Conservation of charge
The total charge of an isolated system can not change.

charge can be moved around but sum is constant.
2 point charges

\[ F = k \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \]

Coulomb's Law

- Force depends on magnitude of both charges \((q_1 q_2)\) like \(m_1 m_2\) in Newton's Gravity
- Force decreases with distance like \(1/r^2\)
- Force acts along straight line between point charges (again like Newton's Gravity)
- Force very big \(10^{39}\) bigger than Gravity

\[ k = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \]
Consider the He nucleus:

\[ \begin{array}{c}
p^+ & n \\ 
p^+ & n \\ \hline 
\end{array} \]

\( r \) About \( 10^{-15} \) m

What is the repulsion force between protons?

\[
F = \frac{k q^2}{r^2} = \frac{8.99 \times 10^9 \times [1.6 \times 10^{-19}]^2}{(10^{-15})^2} [\frac{Nm^2}{C^2}][\frac{C^2}{m^2}]
\]

\( F \sim 230 \) N repulsive

Strong nuclear force (see end of sem.) balances this repulsion.
Example

Forces and directions on $Q_1$ & $Q_2$

\[ r = 1 \text{m} \]

$F_1$ [left] $Q_1 = -6(10)^{-6} \text{C}$ -6 $\mu$C

$F_2$ [right] $Q_2 = +4(10)^{-6} \text{C}$ +4 $\mu$C

\[ F = \frac{kQ_1Q_2}{r^2} = \frac{9(10)^9 \left[ 6(10)^{-6} \right] \left[ 4(10)^{-6} \right]}{1^2} \]

\[ \text{Nm}^2 \frac{\text{C}^2}{\text{m}^2} \]

= 216(10)^{-3} \text{N} in magnitude

signs come from + definition

\[ F_1 = F_{\text{on}1 \text{ due to } 2} = +0.216 \text{N (attractive = toward other charge)} \]

\[ F_2 = F_{\text{on}2 \text{ due to } 1} = -0.216 \text{N (attractive = toward other [1] charge)} \]

Force reaction force pair NEWTON’s 3rd LAW
If two one-second collections of 1 Coulomb each were concentrated at points one meter apart, the force between them could be calculated from Coulomb's Law. For this particular case, that calculation becomes

\[ F = \frac{(9 \times 10^9 N \cdot m^2 / C^2)(1C)(1C)}{1m^2} = 9 \times 10^9 N \]

\[ F = (9 \times 10^9 N)(1lb / 4.45N)(1\text{ ton} / 2000 \text{ lb}) = 1.01 \text{ Million tons!} \]

If two such charges could indeed be concentrated at two points a meter apart, they would move away from each other under the influence of this enormous force, even if they had to rip themselves out of solid steel to do so!

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefor.html#c1
Net force $\rightarrow$ superposition of forces due to individual charges

Vector addition

\[
\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2
\]
Example: 3 co-linear charges
Consider force on $Q_2$

$$F_{23} = k \frac{Q_3 Q_2}{r_{23}^2} = \frac{9 \times 10^9 \left[ \frac{4 \times 10^{-6}}{(2)^2} \right]}{N}$$

Consider total force $F$ on $Q_2$

$$F = (4.5 - 1.5) N = +3 N$$

$\rightarrow$ direction def. to right
Electric Field = Force per unit charge

divide universe in 2 parts [we only care about \( q_0 \)]

1) collection of charges \( \{Q_1, Q_2, \ldots\} \) exert net force \( F \) on charge \( q_0 \)

Define \( \vec{E} = \frac{\vec{F}}{q_0} \) or \( \vec{F} = q_0 \vec{E} \)

collection of charges create \( \vec{E} \) & \( q_0 \) comes along and feels \( F \)

Even if \( q_0 \) had not come along the electric field would be sitting there in space waiting for any charge that eventually happened by !!!
Electric field of one point charge

**Magnitude:**

\[ E = \frac{kq}{r^2} \]

**Direction:**

Direction = direction of force on + test charge!

on straight line between points

**Group of point charges:**

add electric fields of point charges like vectors!!
Example: E at point, P, between 2 point charges

\[ E = k \frac{q}{r^2} \]

\[ |E_1| = 8.99 \times 10^9 \frac{3}{(3000)^2} \]

\[ = 8.99 \times \frac{3}{3^2} \times (10)^3 \]

\[ |E_1| = 3(10)^3 \text{ N/C} \]

\[ |E_2| = 8.99 \times 10^9 \frac{4}{(2000)^2} \]

\[ = 8.99 \times \frac{4}{2^2} \times (10)^3 \]

\[ |E_2| = 8.99(10)^3 \text{ N/C} \]

\[ \vec{E} = \vec{E}_1 + \vec{E}_2 = [-|E_1| + |E_2|] \hat{x} \]

\[ \vec{E} = \vec{E}_1 + \vec{E}_2 = [-3 + 8.99] (10)^3 \hat{x} \text{ N/C} \]

\[ \vec{E} = 5.99 (10)^3 \hat{x} \text{ N/C} \]
Vector addition of E fields: use symmetry to simplify when possible

Q. Electric field at center (place + test charge at center)

E fields cancel so $E_{\text{tot}} = 0$

$$E = k \frac{q}{r^2}$$

$q$’s the same $r$’s the same $\Rightarrow$ E’s the same

$$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$
Vector addition of E fields: symmetry continued

Q. Electric field at center

\[ E = k \frac{q}{r^2} \]

\[ r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 \]

- q's the same r's the same \(\Rightarrow\) E's the same

\[ E_y = k \frac{q}{r^2} \sin(45^\circ) = k \frac{q}{r^2} \frac{1}{\sqrt{2}} \]

\[ E_{tot} = 4E_y \]

\(-\hat{y}\) direction

\[ \hat{k} \sin(45^\circ) = \hat{r} \]

\[ 2E_y \]

\[ 2E_y \]

- q

- q

cancel

superimpose

homework problem help
E Field Lines
- direction = force direction on + test charge (away from +) (toward -)

- line density $\alpha$ field magnitude

repulsive            attractive
Static (no charge motion) condition

For Metal

1. $e^-$ (charge) free to flow
2. all charge on outside surface (static conditions)
3. $E$ inside metal $\equiv 0$ (static conditions)
4. $E$ field at surface of metal is $\perp$ to the surface

$E$ = later
$E$ = no place to go
as long as charge not flowing

\[ E_{\text{total}} = E_1 + E_2 \]

\[ E_{\text{total}} = 0 \]
For Metal

1. $e^-$ (charge) free to flow
2. all charge on outside surface (static conditions)
3. $E_{\text{inside metal}} = 0$ (static conditions)
4. $E$ field at surface of metal is $\perp$ to the surface
5. $E$ concentrate at sharp points (low radius of curvature)

minimize force (energy)

Electric field $\perp$ surface!!!
E concentrate at sharp points (low radius of curvature)

can move apart
-decrease density

can’t move apart
(surf. contains)

enough E
can actually rip charges from surface
Faraday Bucket

can accumulate charge !!
Van de Graff generator

continuous charge accumulation !!

sharp points – big electric field

+ charge left on rubber belt

- charge on rubber belt drained

- static charge induced on inside of rubber belt

sharp points – big electric field

charge transferred

Metal dome

+ pulled off belt

metal pulley

plastic pulley

Belt

Ground

Insulator
Electric Flux

- Electric Flux, $\Phi_E$, through a surface of area, $A$

\[ E_\perp = \text{component of } \vec{E} \perp \text{ to surface} \]

Surface (side view) (surface area $A$)

\[ \Phi_E = A E_\perp \]

(units $\text{m}^2 \frac{\text{N}}{\text{C}}$)

Gauss's Law

- Larger field, larger flux
- Area not perpendicular to field, smaller flux
- Area parallel to field, zero flux

\[ E_\perp = 0 ! \to \Phi_E = 0 \]
Gauss's Law

The net electric flux ($\Phi_E$) through any closed surface is directly proportional to the net electric charge (Q) enclosed by that surface.

$$\Phi_E = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

$$\sum_{\text{surf}} E_\perp \Delta A_i = \Phi_{E} = \oint_{\text{surf}} E_\perp \, dA$$

Gauss's Law

any Gaussian Surface

Special Surface Sphere

$$E_\perp = \frac{kQ}{R^2}$$
Point Charge (Gauss's Law Applications cont.)

\[ E = E_{\perp} \]

\[ E_{\perp} A = \frac{q}{\varepsilon_0} \]

\[ E \frac{4\pi r^2}{4\pi r^2} = \frac{q}{\varepsilon_0} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

Note: add \( q_0 \) at \( r \) and find force on \( q_0 \)

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q \varepsilon_0}{r^2} \]

Coulomb's Law

Gauss's Law
Surface charge density & E field for metal

\[ \sigma = \frac{Q}{A} \]

\( \sigma \) units: C/m\(^2\)

Q. What is relation between E and \( \sigma \) ?
Electric field above charged metal surface

(pill box shape)
Gassian surface

\[ Q = \sigma A \]

\[ \sigma = \frac{Q}{A} \]

\[ \text{Q units C/m}^2 \]

\[ \text{E is } \perp \text{ top} \]

\[ \text{E is } \parallel \text{ side in air} \]

\[ \text{E=0 in metal} \]

\[ \text{E=0 in metal} \]

\[ \text{E=0 in metal} \]

\[ \text{E is } \perp \text{ top} \]

\[ \phi_{\text{top}} = EA_{\text{top}} \]

\[ \phi_{\text{bottom}} = 0 \]

\[ \phi_{\text{sides}} = 0 \]

\[ \phi_{\text{total}} = \phi_{\text{top}} + \phi_{\text{bottom}} + \phi_{\text{sides}} \]

\[ \phi_{\text{total}} = \phi_{\text{top}} + \phi_{\text{bottom}} + \phi_{\text{sides}} \]

\[ \phi_{\text{total}} = \frac{Q_{\text{in}}}{\varepsilon_0} \]

\[ \phi_{\text{total}} = \frac{\sigma A_{\text{top}}}{\varepsilon_0} \]

\[ \frac{\sigma A_{\text{top}}}{\varepsilon_0} = EA_{\text{top}} \]

\[ \Rightarrow \]

\[ E = \frac{\sigma}{\varepsilon_0} \]

1-13a
Consider 2 parallel plates connected to charge reservoirs.

1. The E field, by symmetry, must run straight across the gap.

2. Apply Gauss's Law

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]

\[ \Rightarrow \mathbf{E} = \text{constant in gap} \]

3. Metal Plates

\[ \Rightarrow \text{we know} \quad E = \frac{\sigma}{\varepsilon_0} = \frac{\varepsilon_{\text{Tot}}}{A_{\text{Tot}}} \frac{1}{\varepsilon_0} \]

\[ E_1 = E_2 \Rightarrow E_{\text{constant}} \]
hollow spherical shells: $R_i$ (charge $+Q$) and $R_o$ (charge $-Q$)

- Region 1 $r < R_i$
  $Q_{\text{in}} = 0 \Rightarrow E_1 = 0$

- Region 2 $R_i < r < R_o$
  
  $E_2 = \frac{+Q}{4\pi \varepsilon_0 r^2}$

- Region 3 $r > R_o$
  $E_3 = 0 \Rightarrow 4\pi r^2 \frac{(+Q - Q)}{\varepsilon_0} = 0$

3 Gaussian surfaces with $r$ in region 1, 2 & 3
hollow spherical shells: $R_i$ (charge $+Q$) and $R_o$ (charge $-Q$).

Region 1 $r < R_i$

$$Q_{in} = 0 \implies E_1 = 0$$

Region 2 $R_i < r < R_o$

$$E_2 \ 4\pi r^2 = \frac{+Q}{\varepsilon_o}$$

Region 3 $r < R_o$

$$E_3 \ 4\pi r^2 = \frac{(+Q - Q)}{\varepsilon_o} = 0 \implies E_3 = 0$$
Gauss's Law application

Line of charge of density $\lambda$ ($\frac{C}{m}$)

$E_1 = 0$ top
$E_1 = E$ side of cylinder
$A_{side of cylinder} = 2\pi rl$
$E_1 = 0$ bottom

(top view)

$E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r}$

$EA = \frac{\Phi_{enc.}}{\varepsilon_0}$
Brute force electric force vector addition

\[ \mathbf{F}_{32} = \frac{9 \times (10)^9 \left[ 65 \times (10)^{-6} \right] \left\lfloor \mathbf{50} \times (10)^{-6} \right\rfloor}{0.3^2} = \frac{29250 \times (10)^{-3}}{0.09} = 325 \text{ N} = \mathbf{F}_{32} \text{ away from 2} \]

\[ \mathbf{F}_{31} = \frac{9 \times (10)^9 \left[ 65 \times (10)^{-6} \right] \left\lfloor \mathbf{86} \times (10)^{-6} \right\rfloor}{(0.6)^2} = \frac{50310 \times (10)^{-3}}{0.36} = -50.31 \text{ N} = \mathbf{F}_{31} \text{ toward 1} \]

will assign direction absolute
\[ F_{x} = F_{31x} = 121 \, N \]
\[ F_{y} = F_{32y} + F_{34y} = 325 - 70 = 255 \, N \]