The Challenge of Heavy Fermions

Piers Coleman\textsuperscript{(1,2)}

(1) CMT, Rutgers U, NJ, USA
(2) Royal Holloway, U. London, UK.
The Challenge of Heavy Fermions

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Quantum Criticality & Strange Metals
The Challenge of Heavy Fermions

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Quantum Criticality & Strange Metals

Heavy Fermion SC Composite Pairs

\[ \Psi^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+ \]

NpPd\(_3\)Al\(_2\), \( T_c = 4.5K \) (Aoki et al, 2009)
The Challenge of Heavy Fermions

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Quantum Criticality & Strange Metals

Heavy Fermion SC Composite Pairs

URu$_2$Si$_2$

Hidden Order

Altarawneh et al., (2012)
# Collaborators.

## QCP:

<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
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<tbody>
<tr>
<td>Q. Si</td>
<td>Rice</td>
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<td>R. Ramazashvili</td>
<td>Toulouse</td>
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<td>C. Pepin</td>
<td>CEA, Saclay</td>
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<td>Aline Ramires</td>
<td>Rutgers</td>
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## Composite Order

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<tr>
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<tr>
<td>Rebecca Flint</td>
<td>Iowa State</td>
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<td>Maxim Dzero</td>
<td>Kent State</td>
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<tr>
<td>Andriy Nevidomskyy</td>
<td>Rice</td>
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<td>Alexei Tsvelik</td>
<td>Brookhaven NL</td>
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<td>Hai Young Kee</td>
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<td>Natan Andrei</td>
<td>Rutgers</td>
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<td>Onur Erten</td>
<td>Rutgers</td>
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## Experiment

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<td>H. von Lohneysen</td>
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<td>Kent State</td>
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<td>S. Nakatsuji</td>
<td>ISSP</td>
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<tr>
<td>G. Lonzarich</td>
<td>Cambridge</td>
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<td>F. Steglich</td>
<td>Dresden/Zhejiang</td>
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## Hidden Order

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<td>R. Flint</td>
<td>Iowa State</td>
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<td>Premi Chandra</td>
<td>Rutgers</td>
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Notes:


General reading:


OUTLINE: CHALLENGE OF HEAVY FERMIONS.
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• Heavy Fermions: intro.
OUTLINE: CHALLENGE OF HEAVY FERMIONS.

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• Quantum Criticality
OUTLINE: CHALLENGE OF HEAVY FERMIONS.

- Heavy Fermions: intro.
- Quantum Criticality
- Heavy Fermion Superconductivity
OUTLINE: CHALLENGE OF HEAVY FERMIONS.

• Heavy Fermions: intro.
• Quantum Criticality
• Heavy Fermion Superconductivity
• Hidden Order
Heavy Fermions: Introduction
Fruit-Fly: 20th C.
Fruit-Fly: 20th C.
Fruit-Fly of the 20th C.

Fruit-Fly of the 21st C.
Fruit-Fly: 20th C.

QUANTUM EMERGENCE

PuCoGa$_5$: 20 K Superconductor

Heavy Electron Physics

Fruit-Fly of the 21st C
Fruit-Fly : 20th C.

QUANTUM EMERGENCE

nm

μm

Ψ

Heavy Electron Physics

PuCoGa$_5$ : 20 K Superconductor

Fruit-Fly of the 21st C.
Fruit-Fly : 20th C.

Heavy Electron Physics
PuCoGa$_5$ : 20 K Superconductor

Fruit-Fly of the 21st C
Increasing localization 4f 5f 3d

Rare Earth
Actinide
Transition metal

Represented by a single, neutral spin operator $S = \bar{\hbar}^2 \sigma$, where $\sigma$ denotes the Pauli matrices of the localized electron. Localized moments develop within highly localized atomic wavefunctions. The most severely localized wavefunctions in nature occur inside the partially filled 4f shell of rare earth compounds (Fig. 1) such as cerium ($Ce$) or ytterbium ($Yb$). Local moment formation also occurs in the localized 5f levels of actinide atoms as uranium and the slightly more delocalized 3d levels of first row transition metals (Fig. 1). Localized moments are the origin of magnetism in insulators, and in metals their interaction with the mobile charge carriers profoundly changes the nature of the metallic state via a mechanism known as the "Kondo effect".

In the past decade, the physics of local moment formation has also reappeared in connection with quantum dots, where it gives rise to the Coulomb blockade phenomenon and the non-equilibrium Kondo effect.
Increasing localization

FIGURE 1. Depicting localized 4\textit{f}, 5\textit{f}, and 3\textit{d} atomic wavefunctions.

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Smith and Kmetko (1983)
Increasing localization

**FIGURE 1.** Depicting localized 4f, 5f, and 3d atomic wavefunctions. Represented by a single, neutral spin operator $S = \frac{\hbar}{2} \sigma$ where $\sigma$ denotes the Pauli matrices of the localized electron. Localized moments develop within highly localized atomic wavefunctions. The most severely localized wavefunctions in nature occur inside the partially filled 4f shell of rare earth compounds (Fig. 1) such as cerium (Ce) or ytterbium (Yb). Local moment formation also occurs in the localized 5f levels of actinide atoms as uranium and the slightly more delocalized 3d levels of first row transition metals (Fig. 1). Localized moments are the origin of magnetism in insulators, and in metals their interaction with the mobile charge carriers profoundly changes the nature of the metallic state via a mechanism known as the "Kondo effect".

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Smith and Kmetko (1983)
Many things are possible at the brink of magnetism.
Heavy Fermions + Kondo

Spin (4f,5f): basic fabric of heavy electron physics.
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Scales to Strong Coupling

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \vec{S} \cdot \vec{\sigma}(0) \]

J. Kondo, 1962
Heavy Fermions + Kondo

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J. Kondo, 1962

\[ \chi \sim \frac{1}{T} \]

Curie

\[ \chi \]

\[ T \]

Resistance/Resistance(T=0 Celsius) x 10000

(from W.J. de Haas and G.J. van den Berg, Physica vol. 3, page 440, 1936)
Heavy Fermions + Kondo

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Curie

\[ X_k \]

\[ J \]

\[ H = \]

\[ \text{Curie} \]

\[ \text{Scales to} \]

\[ \text{Strong Coupling} \]

\[ \text{Weak coupling} \]

\[ \text{J. Kondo, 1962} \]
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“Kondo temperature”

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\[ T_K = W \sqrt{J \rho e^{-\frac{1}{2J\rho}}} \]

Resistance/Resistance (T=0 Celsius) x 10000

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Low temperature resistivity of Au
Heavy Fermions + Kondo

Spin screened by conduction electrons: **entangled**

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J. Kondo, 1962
Heavy Fermions + Kondo

Spin screened by conduction electrons: **entangled**

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Heavy Fermions + Kondo

Spin screened by conduction electrons: *entangled*

\[ S(T') = \int_{0}^{T} \frac{C_V}{T'} dT' \]

Spin entanglement entropy

\[ T_K = W \sqrt{J \rho e^{-\frac{1}{2J\rho}}} \]

"Kondo temperature"
Heavy Fermions + Kondo

Electron sea

Spin screened by conduction electrons: entangled

$S(T') = \int_0^T \frac{C_V}{T'} dT'$

Spin entanglement entropy

$TK = W \sqrt{J \rho} e^{-\frac{1}{2J\rho}}$

$\chi \sim \frac{1}{T}$

Curie

Pauli

“Kondo temperature”

$\gamma \sim \frac{C_V}{T} \frac{R \ln 2}{TK}$

Linear S. Heat
Heavy Fermions + Kondo

Spin screened by conduction electrons: entangled

\[ S(T) = \int_{0}^{T} \frac{C_{V}}{T'} dT' \]

Spin entanglement entropy

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Curie

Linear S. Heat

S=0.5 R ln2
DONIACH’S Hypothesis.
Doniach (1977)

\[ H = \sum \varepsilon_k c_k^{\dagger} c_k + J \sum_j (\psi_j^{\dagger} \vec{\sigma} \psi_j) \cdot \vec{S}_j \]

Kondo Lattice Model
(Kasuya, 1951)
DONIACH’S Hypothesis.

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\[ T_{RKKY} > T_K \]

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Large Fermi surface of composite Fermions

Kondo Lattice Model (Kasuya, 1951)

\[ T_K = D \exp[-1/2J\rho] \]

\[ T_N \sim J^2 \rho \]

AFM

QCP
The main result ... is that there should be a second-order transition at zero temperature, as the exchange is varied, between an antiferromagnetic ground state for weak $J$ and a Kondo-like state in which the local moments are quenched.

$$T_{RKKY} > T_K$$

$$T_K \sim D \exp \left[-\frac{1}{2J\rho}\right]$$

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Doniach (1977)

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Large Fermi surface of composite Fermions
Heavy Fermion Primer
\[ H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + J \sum_j \vec{S}_j \cdot \vec{\sigma}(j) \]
$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_{j} \mathbf{S}_j \cdot \mathbf{\sigma}(j)$

“Kondo Lattice”

Entangled spins and electrons

→ Heavy Fermion Metals
Incoherent scattering

Ott et al (1976)

Entangled spins and electrons

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\[ H = \sum_{k\sigma} \epsilon_{k} c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_{j} \vec{S}_{j} \cdot \vec{\sigma}(j) \]
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Coherent Heavy Fermions

Ott et al (1976)

Entangled spins and electrons $\rightarrow$ Heavy Fermion Metals

"Kondo Lattice"

\( H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_j \vec{S}_j \cdot \vec{\sigma}(j) \)

\[ \rho(T) = \rho_0 + AT^2 \]

Coherent Heavy Fermions
Incoherent scattering

Ott, Fisk, Smith 1983

Coherent Heavy Fermions

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Entangled spins and electrons → New kinds of superconductor

"Kondo Lattice"

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_j \vec{S}_j \cdot \vec{\sigma}(j) \]
Digression:
Landau Fermi Liquid Theory
Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.
Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

"Quasiparticle"

\[ H = H_0 + \lambda V \]

\[ |e^-\rangle \quad \xrightarrow{\text{Interactions adiabatically}} \quad |qp^-\rangle \]
Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

\[
\left| e^- \right> \quad \xrightarrow{\text{Interactions adiabatically}} \quad \left| qp^- \right>
\]
Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

\[
\begin{align*}
|e^-\rangle & \quad \text{Interactions} \\ & \quad \text{adiabatically} \\ & \quad |qp^-\rangle
\end{align*}
\]

\[
\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}
\]

Landau, JETP 3, 920 (1957)
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Landau, JETP 3, 920 (1957)
Heavy Fermions: magnetically polarizable Landau Fermi liquids.

\[ E_p = \frac{p^2}{2m^*}, \quad N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3} \]
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"Wilson" or "Sommerfeld" ratio.
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"Wilson" or "Sommerfeld" ratio.

eg Cu vs CeCu₆ (copper, spin doped)
 \( \gamma \text{Cu} \sim 1 \text{ mJ/mol/K}^2 \),
Heavy Fermions: magnetically polarizable Landau Fermi liquids.

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eg Cu vs CeCu\(_6\) (copper, spin doped)
\[ \gamma_{\text{Cu}} \sim 1 \text{ mJ/mol/K}^2, \]
\[ \gamma[\text{CeCu}_6] \sim 1000 \text{ mJ/mol/K}^2, \]
\[ m^*/m_e \sim 1000 \]
What happens when the interaction becomes too large?
What happens when the interaction becomes too large?
What happens when the interaction becomes too large?

Fermi Liquid

$X_c$

Long range order

$X \sim U/t$

Wigner/ Landau 1934/36

“Electrons order”
What happens when the interaction becomes too large?

Wigner/Landau 1934/36
Peierls/Mott 1939

“Electrons order”
“Electrons localize”
What happens when the interaction becomes too large?

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“Electrons localize”

Anderson 1961

“Moments form”

Fermi Liquid

$X_c$

$X \sim U/t$

Long range order
What happens when the interaction becomes too large?

Fermi Liquid

Strange Metal

0

T

$X \sim U/t$

“QCP”

Long range order

Wigner/Landau 1934/36

Peierls/Mott 1939

Anderson 1961

“Electrons order”

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What happens when the interaction becomes too large?

- **Fermi Liquid**
- **Strange Metal**

“Moments form” Anderson 1961

“Electrons order” Wigner/Landau 1934/36

“Electrons localize” Peierls/Mott 1939

“Moments form” Anderson 1961

Long range order

“QCP” Hertz, 1976, Doniach, 1977

One of theFixed Points Kenneth Wilson 1936-2013

X \sim U/t
The development of a parallel formalism and approach to strongly correlated electron systems is still in its infancy, as our understanding of metallic behavior in most conventional materials is not yet complete. 

The 'standard model' for metals is built on the expansion of the Fermi energy, one arrives at an 'exhaustion paradox', which suggests that magnetic impurities exhaust the conduction electrons available to screen each local moment. This issue was later resolved by Nozières and Luttinger (1962), providing the foundation for the use of gauge theories. When written as a field theory, constraints on the low-energy electronic dynamics are removed by a cloud of low-energy electrons within an energy of the order of the Debye cutoff. From this perspective, the Kondo length is analogous to the coherence length of the Fermi liquid. 

The resolution to the exhaustion paradox lies in the more modern perception that spin screening of local moments is primarily by a cloud of low-energy electrons within an energy of the Fermi energy, one arrives at an ‘exhaustion paradox’. 

Since the scaling enhancement effect stretches out across several decades of energy, it is largely robust against crystal field effects. The heavy fermion behavior persists to much higher densities. 

The Kondo effect is a consequence of the screening of localized magnetic moments by conduction electrons. The screening process is characterized by a temperature scale, the Kondo temperature, which is typically much lower than the Fermi temperature.

The diagram illustrates the relationship between the Kondo temperature and the exchange coupling constant. The Kondo temperature is given by:

\[ T_K \sim D_{\text{exp}}[-1/J\rho] \]

where \( D_{\text{exp}} \) is the exchange splitting and \( J\rho \) is the exchange coupling constant.

The diagram also shows the behavior of the magnetic ordering temperature, \( T_N \), as a function of the exchange coupling constant, with \( T_N \sim J^2\rho \). The diagram highlights the competition between the Kondo effect and the antiferromagnetic ordering, with a phase transition occurring at a critical exchange coupling constant \( J_{\rho_c} \).

The diagram is labeled with the following references:

- Mott, 1973
- Doniach 1976
- Wilson 1975
- New Fixed Points
There are, however, aspects to the Doniach argument that do not provide a detailed mechanism connecting the heavy-fermion phase to the local moment AFM. It is purely a comparison of energy scales and does not address the local moment nature of AFM. The resolution to the exhaustion paradox lies in the more modern perception that spin screening of local moments is the origin of Cooper pair formation, which involves electron spin correlations.

Magnetism

$T_K \sim D_{\text{exp}}[-1/J_\rho]$

$T_N \sim J^2 \rho$

AFM

QCP

Landau Fermi Liquid

New

Multipolar

SC

New Fixed Points

Wilson 1975

Mott, 1973

Doniach 1976
New kinds of superconductor
New kinds of insulator

Kondo Insulators

→ New kinds of superconductor

Ott, Fisk, Smith 1983

UBe_{13}

Specific heat

$\int_0^T \frac{C_V}{T'}dT' = \text{Spin Entropy (T)}$
New kinds of insulator

Topological Kondo Insulators

[Graph and diagram here]

New kinds of superconductor

[Graph and diagram here]
New kinds of insulator

Topological Kondo Insulators

\[ R(T) \]

\[ \rho(T) \]

SmB\(_6\)

Sm\(^{2.7+}\)

New kinds of superconductor

UBe\(_{13}\)

specific heat

\[ \frac{\int_0^T C_V \, dT}{T} = \text{Spin Entropy (T)} \]

Surface Fermi Surface

URu\(_2\)Si\(_2\)

New kinds of Electron Order
New kinds of insulator
Topological Kondo Insulators

\[ R(T) \]

\[ 10^3 \gamma \] vs. \[ T \] in [mK] for SmB$_6$ and Sm$^{2.7+}$

New kinds of Electron Order

New kinds of superconductor

UBe$_{13}$

Specific heat

\[ \int_0^T T \frac{dC}{dT} = \text{Spin Entropy} (T) \]

Surface Fermi Surface

Energy (eV)

(a)

(b)

Altarawneh et al., (2012)

URu$_2$Si$_2$

Ising Electrons: Hastype order?

\[ \Psi = \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix} \]
New kinds of insulator

Topological Kondo Insulators

\[ \text{SmB}_6 \]

\[ \text{Sm}^{2.7+} \]

New kinds of Electron Order

NpPd\textsubscript{5}Al\textsubscript{2} \(T_C = 4.5\text{K}\) (Aoki et al, 2009)

\[ \Psi = \left( \langle \Psi_\uparrow \rangle \right) \]

Ising Electrons: Hstatic order?

New kinds of superconductor

Ott, Fisk, Smith 1983

\[ \text{UBe}_{13} \]

specific heat

\[ \int_0^T \frac{C_V}{T^2} dT = \text{Spin Entropy (T)} \]

Composite Pairing
New kinds of insulator

Topological Kondo Insulators

$\text{SmB}_6$

$\text{Sm}^{2.7+}$

$\Psi = \left( \begin{array}{c} \langle \Psi \uparrow \rangle \\ \langle \Psi \downarrow \rangle \end{array} \right)$

Ising Electrons: Hstatic order

New kinds of Phase Transition

$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\uparrow} S^+_z$

Composite Pairing

$\text{NpPd}_5\text{Al}_2, T_C = 4.5K$ (Aoki et al., 2009)

$\text{Altarawneh et al., 2012}$

$\text{URu}_2\text{Si}_2$
New kinds of insulator

Topological Kondo Insulators

New kinds of phase transition

Quantum Criticality

SmB$_6$ $\rightarrow$ Sm$^{2.7+}$

$10^3\gamma_T$ vs $T$ (mK$^{-1}$)

$R$ (Ω) vs $T$ (K)

$\Psi = \begin{pmatrix} \langle \Psi \uparrow \rangle \\ \langle \Psi \downarrow \rangle \end{pmatrix}$

Ising Electrons: Hstatic order

New kinds of electron order

YbRh$_2$Si$_2$, $B \parallel c$

$\chi$ (×10$^{-3}$ cm$^3$/mol)

$\Psi^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$

$NpPd_5Al_2$, $T_C = 4.5K$ (Aoki et al, 2009)

$H \parallel [100]$

$\Psi \rightarrow$ Composite Pairing

Altarawneh et al., (2012)

$URu_2Si_2$
→ New kinds of insulator

Topological Kondo Insulators

![SmB$_6$ and Sm$^{2.7+}$ structure](image)

Ψ = \( \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix} \)

Ising Electrons: Hstatic order

→ New kinds of Phase Transition

![YbRh$_2$Si$_2$](image) B || c

\( T^2 T^2 \)

→ Strange Metals

→ Quantum Criticality

![NpPd$_5$Al$_2$, T$_C$ = 4.5K (Aoki et al, 2009)](image)

Composite Pairing

Altarawneh et al., (2012) URu$_2$Si$_2$
Quantum Criticality
Quantum Criticality and Superconductivity
Quantum Criticality and Superconductivity

3d Cu

Superconductivity

Pseudo-gap phase

Strange metal

Fermi liquid

Doping

QCP

3d Fe

BaFe$_2$(As$_{1-x}$P$_x$)$_2$:

Superconductivity (SC)

Non-Fermi liquid (Non-FL)

Quantum Critical Point (QCP)

4f Ce

CeRhIn$_5$
Wanted: a unified conceptual description of magnetism, quantum criticality and superconductivity.
Quantum Criticality: divergent specific heat capacity

Heavy Fermion Materials

Quantum Critical Point
Quantum Criticality: divergent specific heat capacity

Heavy Fermion Materials

Quantum Critical Point

H. Von Lohneyson (1996)
Quantum Criticality: divergent specific heat capacity

Heavy Fermion Materials

\[ \frac{C_V}{T} \sim \frac{Q}{T_0} \log \frac{T_0}{T} \]

Quantum Critical Point

H. Von Lohneyson (1996)
Quantum Criticality:
divergent specific heat capacity

Heavy Fermion Materials

\[ \frac{C_V}{T} \sim \frac{Q}{T_0} \log \frac{T_0}{T} \]

AFM metal

Quantum Critical Point

H. Von Lohneyson (1996)
Quantum Criticality: divergent specific heat capacity breakdown of Landau Fermi Liquid

Quantum Criticality:
divergent specific heat capacity
breakdown of Landau Fermi Liquid

(a) YbRh$_2$Si$_2$: Field tuned quantum criticality.


YbRh$_2$Si$_2$: Field tuned quantum criticality.
Reconstruction of the Fermi Surface and mass divergence


CeRhIn$_5$ material
Reconstruction of the Fermi Surface and mass divergence

Reconstruction of the Fermi Surface and mass divergence

CeRhIn$_5$

115 material


Reconstruction of the Fermi Surface and mass divergence

Reconstruction of the Fermi Surface and mass divergence


CeRhIn$_5$

$T_c = 68 \text{mK}$

$B_{c2} = 28 \text{mT}$

$\rho \sim T^\frac{3}{2}$

Max($-dM/dT$)

Onset $\rho(T) \sim T^2$

Strange Metal

SC

FL

$\frac{C}{T}, \chi \sim \frac{1}{\sqrt{B}}$

$\rho(T) \sim T^2$
E/T Scaling:

E/T Scaling:

\[ \chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right) \]
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CeCu_{6-x}Au_x (x=0.1)
E/T Scaling:

\[ \chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right) \]

Physics Below the upper Critical Dimension.


CeCu_{6-x}Au_x \quad (x=0.1)

Standard Model: Quantum SDW?

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

\[ d_{\text{eff}} = d + z \]

\[ \chi_0^{-1} = -J(Q) \]

F.S. instability
Standard Model: Quantum SDW?

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

\[ d_{\text{eff}} = d + z \]

\[ \chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega / \Gamma) \]

\[ \chi_0^{-1} = -J(Q) \]

F.S. instability
Standard Model: Quantum SDW?

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

\[ d_{\text{eff}} = d + z \]

\[ \chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega / \Gamma) \]

\[ \tau^{-1} \propto \xi^{-2} \]
Standard Model: Quantum SDW?

- Moriya, Doniach, Schrieffer (60s)
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- Millis (93)

\[ d_{\text{eff}} = d + z \]

\[ \chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega / \Gamma) \]

\[ \tau^{-1} \propto \xi^{-2} \]

Time counts as \( z = 2 \) scaling dimensions

F.S. instability

Fermi Surface

vertex non-singular

\[ \chi_0^{-1} = -J(Q) \]
Standard Model: Quantum SDW?

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

\[ d_{\text{eff}} = d + z \]

If \( d + z = d + 2 > 4 \):
\( \phi^4 \) terms “irrelevant”
Critical modes are Gaussian.
T is not the only energy scale.

\[ \chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega / \Gamma) \]

\[ \tau^{-1} \propto \xi^{-2} \]

Time counts as \( z = 2 \) scaling dimensions
New Ideas
New Ideas

- Local quantum criticality
  (Si, Ingersent, Smith, Rabello, Nature 2001):
  Spin is the critical mode,
  Fluctuations critical in time.

Requires a two dimensional spin fluid
New Ideas

• Local quantum criticality
  (Si, Ingersent, Smith, Rabello, Nature 2001):
  Spin is the critical mode,
  Fluctuations critical in time.

Requires a two dimensional spin fluid

• Two fluid scenario.
  
D. Pines Z. Fisk S. Nakatsuji Y. Yang

New Ideas

- **Local quantum criticality**
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Requires a two dimensional spin fluid

- **Two fluid scenario.**

D. Pines Z. Fisk S. Nakatsuji Y. Yang

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D. Pines Z. Fisk S. Nakatsuji Y. Yang

Description of unconventional QCP requires
new formalism.
New Ideas

- **Local quantum criticality**
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Requires a two dimensional spin fluid

- **Two fluid scenario.**

![Diagram showing phase transitions and critical points](image)

D. Pines Z. Fisk S. Nakatsuji Y. Yang

- **Supersymmetry?**
  Coleman, Pepin, Tsvelik (1999)
  Ramires Coleman (2014)

Description of unconventional QCP requires new formalism.
New Ideas

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D. Pines Z. Fisk S. Nakatsuji Y. Yang

Magnetism | LM + Kondo | Pure Kondo
Spin = B | Spin = B,F | Spin = F

Supersymmetry?
Coleman, Pepin, Tsvelik (1999)
Ramires Coleman (2014)

Description of unconventional QCP requires new formalism.
New Ideas

- **Local quantum criticality**
  (Si, Ingersent, Smith, Rabello, Nature 2001):
  Spin is the critical mode,
  Fluctuations critical in time.

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- **Two fluid scenario.**

D. Pines Z. Fisk S. Nakatsuji Y. Yang


- **Supersymmetry?**
  Coleman, Pepin, Tsvelik (1999)
  Ramires Coleman (2014)

Description of unconventional QCP requires
new formalism.

**Strange Metal = Unbroken Susy?**
Heavy Fermion Systems: Why Supersymmetric Spins?

C/T ~ log T
ρ ~ T^{α<2}
Heavy Fermion Systems: Why Supersymmetric Spins?

\[
C/T \sim \log T
\]
\[
\rho \sim T^{(\alpha<2)}
\]

Bosonic

\[
S_B = b^\dagger_\alpha \Gamma_{\alpha\beta} b_\beta
\]
\[
|\Psi\rangle = P_G |\Psi_B\rangle
\]

Fermionic

\[
S_F = f^\dagger_\alpha \Gamma_{\alpha\beta} f_\beta
\]
\[
|\Psi\rangle = P_G |\Psi_F\rangle
\]
Heavy Fermion Systems: \textbf{Why Supersymmetric Spins?}

How to describe the generic HF phase diagram in its entirety?

Supersymmetric Spin

\[ S = f_{\alpha}^{\dagger} \Gamma_{\alpha \beta} f_{\beta} + b_{\alpha}^{\dagger} \Gamma_{\alpha \beta} b_{\beta} \]

\[ |\Psi\rangle = P_{G} |\Psi_{B}\rangle \otimes |\Psi_{F}\rangle \]

Gan, Coleman and Andrei, 1992
Coleman, Pepin, Tsvelik, 2000
Symmetries of the SUSY-SP(N) Spin

\[ S = f_\alpha^\dagger \Gamma_{\alpha\beta} f_\beta + b_\alpha^\dagger \Gamma_{\alpha\beta} b_\beta, \]

SP(N) generators

Spin commutes with the following operator bilinears:

- \( \Psi_0 = n_b + N/2 \), \( n_b = b_\alpha^\dagger b_\alpha \)
- \( \Psi_1 = \frac{\psi^\dagger + \psi}{2} \), \( \psi = \bar{\alpha} f_\alpha f_{-\alpha} \)
- \( \Psi_2 = \frac{\psi^\dagger - \psi}{2i} \), \( \psi^\dagger = \bar{\alpha} f_{-\alpha}^\dagger f_\alpha^\dagger \)
- \( \Psi_3 = n_f - N/2 \), \( n_f = f_\alpha^\dagger f_\alpha \)
- \( X_1 = \theta + \eta \), \( \theta = b_\alpha^\dagger f_\alpha \)
- \( X_2 = \theta - \eta \), \( \eta = \bar{\alpha} f_\alpha b_{-\alpha} \)

“Super-Algebra”: SU(2\|1)

- \( [\Psi_i, \Psi_j] = 2i \epsilon_{ijk} \Psi_k \) \( i, j, k = \{1, 2, 3\} \)
- \( \{X_i, X_j^\dagger\} = 2(\Psi_0 \delta_{ij} + \Psi_3 a_{ij}) \), \( i, j = \{1, 2\} \)
- \( \{X_i, X_j\} = 0 \), \( i, j = \{1, 2\} \)
Results

Within a static mean field solution the free energy have the following closed form:

\[
F = -2 \sin(\pi n_f) - \frac{\pi J_H}{T_K} (q_0 - n_f) (q_0 - n_f + 1)
\]

\[n_f + n_b = q_0\]

The energy will be minimized by different representations in different areas of the phase diagram:

- **F+B Phase → Coexistence**;
- **2nd order transition F → F+B**;
- **Fermionic modes go soft**;
- **Unusual critical behavior**;
Open Challenges.

- QCPs: Explanation of universality of $C/T \sim \log(T_0/T)$, $\rho \sim T^{1+\alpha}$?

- Co-existence heavy fermions & LM AFM = Two fluid behavior? [Supersymmetry? B/F]
Heavy Fermion Superconductivity
The Nature of Magnetic Pairing.
The remarkable case of NpPd$_5$Al$_2$
The remarkable case of NpPd₅Al₂

![Graph showing the resistivity of NpPd₅Al₂](image)

**Graph Description:**
- The graph shows the resistivity ($\rho$) of NpPd₅Al₂ as a function of temperature ($T$) with the magnetic field parallel to the [100] direction.
- The inset highlights the superconducting transition, where the resistivity drops sharply and becomes zero.
- The temperature-dependent specific heat is also shown, indicating the upper critical field ($H_{c2}$).

**Diagram Description:**
- The tetragonal crystal structure of NpPd₅Al₂ is depicted, showing the arrangement of Np, Pd, and Al atoms.
- The magnetic field direction is indicated by arrows, showing the anisotropy in the superconducting transition.

**Additional Information:**
- The specific heat jump at the upper critical field ($H_{c2}$) is indicated.
- The electronic state is considered to be quasi-two-dimensional, indicating an ellipsoidal Fermi surface.
- The topology of the Fermi surface is discussed, noting the anisotropic behavior in the superconducting transition.
- The crystallographic axes [001], [100], and [010] are marked.

**Equations:**
- $c_2 = \frac{m_{[001]}}{C_{138}}$:
The mass anisotropy ratio for [001].
- $S = \frac{m_{[100]}}{C_{18}}$:
The mass anisotropy ratio for [100].
- $\Delta T = \frac{m_{[001]}}{C_{3}}$:
The mass anisotropy ratio for [010].
- $D = \frac{c_2}{C_{138}}$:
The slope of the upper critical field ($H_{c2}$).
- $T_{c} = \frac{1}{4} \left( \frac{c_2}{C_{138}} \right)^{1/2}$:
The superconducting transition temperature.
- $T_{m} = \frac{1}{5} \left( \frac{c_2}{C_{18}} \right)^{1/2}$:
The magnetic transition temperature.

**References:**
- Aoki et al. 2007.
The remarkable case of NpPd$_5$Al$_2$

NpPd$_5$Al$_2$ $T_C = 4.5K$

4.5K Heavy Fermion S.C
NpAl$_2$Pd$_5$
Aoki et al 2007
The remarkable case of NpPd$_5$Al$_2$

NpPd$_5$Al$_2$ $T_C = 4.5K$

4.5K Heavy Fermion S.C

Aoki et al 2007
The remarkable case of NpPd$_5$Al$_2$

NpPd$_5$Al$_2$ $T_c = 4.5K$

Magnetic moments

$\chi (x10^{-3}\text{ emu/mol})$

$H // [100]$

$[001]$

$[010]$

$[100]$

4.5K Heavy Fermion S.C

NpAl$_2$Pd$_5$

Aoki et al 2007
The remarkable case of $\text{NpPd}_5\text{Al}_2$

$\text{NpPd}_5\text{Al}_2$ $T_C = 4.5\text{K}$

4.5K Heavy Fermion S.C

$\text{NpAl}_2\text{Pd}_5$

Aoki et al 2007
The remarkable case of NpPd$_5$Al$_2$

NpPd$_5$Al$_2$ $T_C = 4.5K$

$\sim 1/3 \, R \, \ln(2)$
The remarkable case of $\text{NpPd}_5\text{Al}_2$

$\text{NpPd}_5\text{Al}_2$ $T_C = 4.5\text{K}$

$\chi(x) = 10^{-3} \text{emu/mol}$

$H \parallel [100]$

$\sim 1/3 R \ln(2)$

How does the spin form the condensate?

4.5K Heavy Fermion S.C
$\text{NpAl}_2\text{Pd}_5$
Aoki et al 2007
The remarkable case of NpPd$_5$Al$_2$

NpPd$_5$Al$_2$ $T_C = 4.5$K

How does the spin form the condensate?

"COMPOSITE PAIR"

Abrahams, Balatsky, Schrieffer and Scalapino (1994)

Flint, Dzero, Coleman (2008)
Composite pairing

$\text{NpPd}_5\text{Al}_2 \ T_C = 4.5\text{K}$
Composite pairing

$\text{NpPd}_5\text{Al}_2 \quad T_C = 4.5\text{K}$
Composite pairing

NpPd$_5$Al$_2$ \( T_C = 4.5\text{K} \)

Heavy Cooper pair = (pair x spinflip)
Composite pairing

\[ \text{NpPd}_5\text{Al}_2 \ T_C = 4.5\text{K} \]

Heavy Cooper pair = (pair x spinflip)

\[ \Psi^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+ \]
Composite pairing

\[ \Psi^\dagger = c_1^{\dagger \downarrow} c_2^{\dagger \downarrow} S_+ \]

\[ \text{NpPd}_5\text{Al}_2 \quad T_C = 4.5\text{K} \]

\[ \text{Abrahams, Balatsky, Scalapino, Schrieffer 1995} \]
A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei     PRB   60, 3605 (1999).
Flint, PC,                        PRL, 105, 246404 (2010).
Flint, Nevidomskyy, PC, PRB     84, 064514 (2011).
\[ H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger(j) \psi_{1b}(j) + J_2 \psi_{2a}^\dagger(j) \psi_{2b}(j) \right) S^{ba}(j) \]

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Single FS, two channels.

\[ \psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_\Gamma k c_k e^{ik \cdot x_j} \]
\[ Z = \int \text{Fields} \ e^{-S[\psi]} \]

\[ H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger(j) \psi_{1b}(j) + J_2 \psi_{2a}^\dagger(j) \psi_{2b}(j) \right) S^{ba}(j) \]

Single FS, two channels.

\[ \psi_T(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{1k} c_k e^{i k \cdot x_j} \]

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)
$$Z = \int_{\text{Fields}} e^{-S[\psi]}$$

$$H = \sum_k \epsilon_k c_k^\dagger c_k + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger(j) \psi_{1b}(j) + J_2 \psi_{2a}^\dagger(j) \psi_{2b}(j) \right) S^{ba}(j)$$

Wild quantum fluctuations!

Single FS, two channels.

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot x_j}$$
How can we tame the wild Quantum fluctuations?

\[ Z = \int \text{Fields} e^{-S[\psi]} \]

\[ H = \sum_k \epsilon_k c_k^{\dagger} c_k + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^{\dagger} (j) \psi_{1b} (j) + J_2 \psi_{2a}^{\dagger} (j) \psi_{2b} (j) \right) S^{ba} (j) \]

Single FS, two channels.

\[ \psi (x, \tau) = \frac{1}{\sqrt{V}} \sum_k \gamma_{1k} c_k e^{i k \cdot x} \]

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)
Large $N$ expansion.

\[
Z = \int \operatorname{Fields} e^{-S[\psi]}
\]

\[
H = \sum_k \epsilon_k c_k^{\dagger} c_k + \frac{1}{N} \sum_{k,k'} \left(J_1 \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_2 \psi_{2a}^{\dagger}(j) \psi_{2b}(j)\right) S^{ba}(j)
\]

Single FS, two channels.

\[
\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot x_j}
\]

cf Cox, Pang, Jarell (96)
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Large $N$ expansion.

\[ Z = \int e^{-S[\psi]} \]

\[ \sigma \in \left( -\frac{1}{2}, \frac{1}{2} \right) \rightarrow \left( -\frac{N}{2}, \frac{N}{2} \right) \]

\[ H = \sum_k \epsilon_k c_k^{\dagger} c_k + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_2 \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j) \]

Single FS, two channels.

\[ \psi_T(j) = \frac{1}{\sqrt{V}} \sum_k \gamma^{\Gamma k} c_k e^{ik \cdot x_j} \]

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)
Large $N$ expansion.

$$Z = \int \text{Fields} e^{-N S[\psi]}$$

$$\sigma \in \left( -\frac{1}{2}, \frac{1}{2} \right) \rightarrow \left( -\frac{N}{2}, \frac{N}{2} \right)$$

$$H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_1^{\dagger a}(j) \psi_1^b(j) + J_2 \psi_2^{\dagger a}(j) \psi_2^b(j) \right) S^{ba}(j)$$

Single FS, two channels.

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{ik\cdot x_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)
Large $N$ expansion.

$$Z = \int \text{Fields} \; e^{-\frac{S[\psi]}{1/N}}$$

$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \rightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

$$H = \sum_k \epsilon_k c_k^{\dagger} c_k + \frac{1}{N} \sum_{k,k'} \left(J_1 \psi_{1a}^\dagger (j) \psi_{1b} (j) + J_2 \psi_{2a}^\dagger (j) \psi_{2b} (j) \right) S^{ba} (j)$$

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\]

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Single FS, two channels.

\[
\psi_{\Gamma} (j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot x_j}
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\[ H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_1^{\dagger}(j) \psi_1^b(j) + J_2 \psi_2^{\dagger}(j) \psi_2^b(j) \right) S^{ba}(j) \]

Single FS, two channels.

\[ \psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot x_j} \]
Scott Thomas,
Rutgers NHETC.

PC: why don’t you ever use the group $SP(N)$?

$$H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger (j) \psi_{1b} (j) + J_2 \psi_{2a}^\dagger (j) \psi_{2b} (j) \right) S^{ba} (j)$$

Single FS, two channels.

$$\psi_{1\gamma}(j) = \frac{1}{\sqrt{V}} \sum_k \gamma \tau_k c_k e^{i k \cdot x_j}$$
Scott Thomas,
Rutgers NHETC.

**PC:** why don’t you ever use the group $\text{SP}(N)$?

**Scott:** “Simple, no Baryons.”

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**SU(N):**

<table>
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<th>Mesons</th>
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<td>$\bar{q}q$</td>
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$\text{SU}(N)$: \begin{align*}
\text{Mesons} & \quad \bar{q}q \\
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\end{align*}

$\text{SP}(N)$:
\begin{align*}
\bar{q}q & \quad \text{Cooper pairs} \\
q_a q_{-a}
\end{align*}

\[ H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger (j) \psi_{1b} (j) + J_2 \psi_{2a}^\dagger (j) \psi_{2b} (j) \right) S^{ba} (j) \]

Single FS, two channels.

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“Symplectic Large N” \( \rightarrow \) R. Flint and PC ‘08

\[ S^{b\alpha} = f_b^\dagger f_{a} - \text{sgn}(a)\text{sgn}(b)f_{-b}^\dagger f_{-a} \]

**SU(N):**

- **Mesons**
  - \( \bar{q}q \)

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  - \( q_{\alpha} q_{-\alpha} \)

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Nozieres and Blandin 1980

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Singular composite pair fluctuations

Emery and Kivelson 1992

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Real-space structure of pair

Ce
Real-space structure of pair

Magnetic pair: intercell

\[ \Psi_M^\dagger = \Delta_d (1 - 2) f_1^\dagger (1) f_2^\dagger (2) \]
Real-space structure of pair

Magnetic pair: intercell

\[ \Psi_M^{\dagger} = \Delta_d (1 - 2) f_{\uparrow}^{\dagger} (1) f_{\downarrow}^{\dagger} (2) \]
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Real-space structure of pair

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\[ \Psi_{M}^{\dagger} = \Delta_d (1 - 2) f_{1}^{\dagger} f_{1}^{\dagger} f_{2}^{\dagger} (2) \]

Composite pair

\[ \Psi_{C}^{\dagger} = c_{1\uparrow}^{\dagger} c_{2\downarrow}^{\dagger} S_{+} \]

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998)

Flint, Dzero, Coleman, Nat. Phys, (2008)
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Real-space structure of pair

Magnetic pair: intercell

$$\Psi_M^\dagger = \Delta_d (1 - 2) f_1^\uparrow (1) f_2^\downarrow (2)$$

Composite pair: intra-cell boson

$$\Psi_C^\dagger = c_{1\downarrow}^\dagger c_2^\dagger S_+$$

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Andrei, Coleman, Kee & Tsvelik PRB (1998)

Flint, Dzero, Coleman, Nat. Phys, (2008)

Extreme Resilience to doping on Ce site.

Table I. Superconducting parameters for samples of the system exhibits heavy FL behavior at low temperatures. If we consider the specific heat data, after which it slowly recovers towards the electrical resistivity, assuming that transients are clearly observed in strong electronic correlations persist up to electronic specific heat and the coefficient of specific heat gives the relationship between the coefficient of specific heat data.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T_c$ (K)</th>
<th>$\Delta_c^2$ (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce</td>
<td>3K</td>
<td>$1.75 \times 10^{-3}$</td>
</tr>
<tr>
<td>Yb</td>
<td>3K</td>
<td>$2.16 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

FIG. 3. (a) Coherence temperature reduces to 0.5 for CoIn at high temperature. (b) Circles: $T_c$ vs temperature for Ce-In and Yb-In. The ratio of magnetic susceptibility along the ab axis, after which it slowly recovers towards Curie-Weiss behavior is observed for intermediate between what is expected for Ce- and Yb-based heavy fermion compounds. The solid line shows the suppression of magnetic susceptibility along the ab axis, after which it slowly recovers towards Curie-Weiss behavior is observed for intermediate between what is expected for Ce- and Yb-based heavy fermion compounds.
Real-space structure of pair

Magnetic pair: intercell

$$\Psi^+_M = \Delta_d (1 - 2) f^+_1 (1) f^+_{\downarrow} (2)$$

Composite pair: intra-cell boson

$$\Psi^+_{C} = c^+_{1\downarrow} c^+_{2\downarrow} S^+$$

Abrahams, Balatsky, Scalapino, Schrieffer 1995
Andrei, Coleman, Kee & Tsvelik PRB (1998)
Flint, Dzero, Coleman, Nat. Phys, (2008)

Lei Shu et al, PRL, (2011)
M. Tanatar et al (unpublished)
Erten and PC arXiv1402.7361
\[ \Psi^+ = c_1^{\uparrow} c_2^{\downarrow} S_+ \]
The transition is first order for temperature as the amounts of magnetic, relative strengths of the Kondo and RKKY couplings, ing dome as the Néel temperature vanishes[1], further, where moderate pressure reveals a superconduct- canonical example of a magnetically paired superconduc- a light Fermi liquid with free local moments when all pa- finite, below

In gauge fluctuations, however, it is an interesting

In, a ss h o in F i g . 2 . T h e r ei sn ol o gr a n g e

and

are finite, with symmetry

λ

2

k
−
∆
th e ir e will lead to a sharp positive shift in the NQR
counting for the linear increase in

115 doping phase diagram[3], described by the orange ellips

and the f-ion will develop equal hole densities in

3 ) .

y
−
∆
C

(17)

is obtained from a Schrie,

λ

−
δ

(14)

and

from its coupling to the changes in the chemical poten-

T

terials will be antiferromagnetic for

115 superconductors is known to increase linearly with

the sensitivity of the Kondo

∆

1

µ
−
∆
C

(13 ) =

E
→
E

(7)

are the bare hybridiza-

J

tial,

from

H

K

c

and the f-ion will develop equal hole densities in

always nonzero in the heavy Fermi liquid because the hy-

∆

2

k
−
∆

δ

(3)

k
p
−
−
∆

| 2

−
−
δ

(12)

| 2

−
−
δ

(10)

| 2

−
−
δ

(9)

(11)

(8)

(7)

(6)

(5)

(4)

(3)

(2)

(1)

\[ \Psi^\dagger = c_1^\dagger c_2^\dagger S_+ \]

\[ Q_{zz} \propto \Psi_C^2 \]


[26] A nonzero

would indicate a composite nematic phase,


The transition is first order for materials could tune the relative coupling strengths (see Fig).

We obtain four equations for the relative strengths of the Kondo and RKKY couplings, producing a new element to this relationship. Changes in the superconductivity is stable with respect to the magnetic fluctuations, however, it is an interesting rattle in the phase diagram in Fig 2, chosen for its similarity to the Ce formation of a two-channel Anderson model, which gives a natural interpretation: as the condensate quadrupole moment grows abruptly below $T_c$.

The sensitivity of the Kondo effect between $f$-electron valence and the Kondo temperature vanishes.$^1$

$\Psi^\dagger = c_1^\dagger c_2^\dagger S_+$

$Q_{zz} \propto \Psi_C^2$

Flint et al, PRB 84, 064054, (2011)

$\Delta \nu \propto |\Psi|^2 \sim (T_c - T)$
\[ \Psi^+ = c_1^+ c_2^+ S_+ \]

\[ Q_{zz} \propto \Psi^2 \]

\[ \Delta \nu \propto |\Psi|^2 \sim (T_c - T) \]

Flint et al, PRB 84, 064054, (2011)

Bauer, G. Koutroulakis Yasuoko, (2014)
Open Challenges.

- HFSC: how is the spin incorporated into the condensate?
- Composite pairs?
- Possibility of molecular pairing. (see Onur Erten and Coleman, arXiv1402.7361)
URu$_2$Si$_2$:
The Hidden Order Mystery
Hidden Order in URu$_2$Si$_2$
Hidden Order in URu$_2$Si$_2$
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\[ \Delta S = \int_{0}^{T_0} \frac{C_V}{T} dT \]
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= 0.14 \times 17.5 \text{ K}  \\
= 2.45 \text{ J/mol/K}  \\
= 0.42 \text{ R ln 2}  \\

(Palstra et al., 85)
Hidden Order in URu$_2$Si$_2$

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Large entropy of condensation.
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= 0.14 x 17.5 K
= 2.45 J/mol/K
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Large entropy of condensation.

What is the nature of the hidden order?
High pressures, high fields

Villaume et al. (08)

Kim et al. (03)
Ising order, present in LMAF, vanishes in the hidden order state. (NMR,MuSR).
## 25 Years of Theoretical Proposals

### Local

- Barzykin & Gorkov, '93 (three-spin correlation)
- Santini & Amoretti, '94, Santini ('98) (Quadrupole order)
- Amitsuka & Sakihabara ($\Gamma_5$, Quadrupolar doublet, ‘94)
- Kasuya, ’97 (U dimerization)
- Kiss and Fazekas ’04, (octupolar order)
- Haule and Kotliar ’09 (hexa-decapolar)

### Landau Theory

- Shah et al. (’00) “Hidden Order”,
- Ramirez et al, ’92 (quadrupolar SDW)
- Ikeda and Ohashi ’98 (d-density wave)

### Itinerant

- Okuno and Miyake ’98 (composite)
- Tripathi, Chandra, PC and Mydosh, ’02 (orbital afm)
- Dori and Maki, ’03 (unconventional SDW)
- Mineev and Zhitomirsky, ’04 (SDW)
- Varma and Zhu, ’05 (spin-nematic)
- Ezgar et al ’06 (Dynamic symmetry breaking)
- Pepin et al ’10 (Spin liquid/Kondo Lattice)
- Dubi and Balatsky, ’10 (Hybridization density wave)
- Fujimoto, 2011 (spin-nematic)
- Rau and Kee 2012 (Rank 5 pseudo-spin vector)
Cause Célèbre: state of the art spectroscopies

Scanning Tunneling Microscopy
Pegor Aynajian et al, PNAS (2010)

dependence of the hidden-order DOS from the shown as open squares. Again, by subtracting the fitted Fano spectrum in Sr range –7.75 mV to 6.75 mV while the measured Fig. 4a–f we show simultaneous images of (the simultaneous topograph is shown in Supplementary Fig. 4). In resolution and atomic spatial resolution on these U-terminated surfaces rapidly below the Si-terminated surfaces upon cooling below atoms produced intense scattering interference and allowed success-purpose because the Fourier transform of its (the predominant effects occurring between –4 meV and 3 eV). For U-terminated surfaces (see Fig. 3a), the spatially averaged density Evolution of density of states at Si- and U-termination surfaces are already detectable in tunnelling within 1 K below the bulk transition NATURE | Vol 465 | 2010. For QPI studies of the hidden-order transition we therefore measure alterations is consistent with broken symmetry at the surface is likely to be visualized. 

$\Delta T = 15 K, 13 K, 11.7 K, 10.2 K, 8.4 K, 6.6 K$

Hybridization of U 5f states develops at $T_0$
The Giant Ising Anisotropy.
Strange electron spin of URu$_2$Si$_2$
Strange electron spin of URu$_2$Si$_2$

electron spin

$magnetic\ field\ B$

\[ g\mu_B B \]
Strange electron spin of URu$_2$Si$_2$
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$\mathbf{B} = g \mathbf{\mu}_B B$

$magnetic \ field \ B$

electron \ spin

$M = g(\theta) \mu_B = 2\mu_B$

Isotropic moment
Strange electron spin of URu$_2$Si$_2$

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Isotropic moment

$S=1/2$
Strange electron spin of URu$_2$Si$_2$

$g\mu_B B$

URu$_2$Si$_2$
Strange electron spin of URu$_2$Si$_2$
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electron spin

$g\mu_B B$

No splitting in transverse direction
Strange electron spin of URu$_2$Si$_2$

No splitting in transverse direction

\[ M = g\mu_B \cos \theta = M_z \]

Magnetic moment only along z-axis
Strange electron spin of URu$_2$Si$_2$

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Strange electron spin of URu$_2$Si$_2$

- Electron spin
- URu$_2$Si$_2$

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Magnetic moment only along z-axis

"Ising moment"

S~integer?
Quantum Oscillations: Giant Ising Anisotropy
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Quantum Oscillations: Giant Ising Anisotropy

\[ M \propto \cos \left[ 2\pi \frac{\text{Zeeman}}{\text{cyclotron}} \right] \]

\[ \frac{m^*}{m_e} g(\theta) = 2n + 1 \]

Spin Zero condition
Quantum Oscillations: Giant Ising Anisotropy

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17 spin zeros!
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Ising quasiparticle with giant Ising anisotropy > 30.
Pauli susceptibility anisotropy > 900
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Pauli susceptibility anisotropy > 900
Confirmed from upper critical field measurements

Quantum Oscillations: Giant Ising Anisotropy

Electrons hybridize with Ising 5f state to form Heavy Ising quasiparticles.

$M \propto \cos \left[ \frac{2\pi}{\text{Zeeman/cyclotron}} \right]$  

$m^* g(\theta) = 2n + 1$

Spin Zero condition

Ising quasiparticle with giant Ising anisotropy $> 30$. Pauli susceptibility anisotropy $> 900$

Confirmed from upper critical field measurements

17 spin zeros!

URu$_2$Si$_2$: Electronic Polaroid
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URu$_2$Si$_2$: Electronic Polaroid

Light
URu$_2$Si$_2$: Electronic Polaroid
URu$_2$Si$_2$: Electronic Polaroid
URu$_2$Si$_2$: Electronic Polaroid

Light

S=1/2

S=1

Electron
URu$_2$Si$_2$: Electronic Polaroid

Order parameter carries half-integer spin

“Spinor”
Hastatic order in the heavy-fermion compound URu$_2$Si$_2$

Premala Chandra$^1$, Piers Coleman$^{1,2}$ & Rebecca Flint$^3$

Order parameter carries half-integer spin

“Spinor”
URu$_2$Si$_2$: Electronic Polaroid

Hastatic order in the heavy-fermion compound URu$_2$Si$_2$
Premala Chandra$^1$, Piers Coleman$^{1,2}$ & Rebecca Flint$^3$

\textit{Hasta: Spear (Latin)}

Order parameter carries half-integer spin

“Spinor”
Open Challenges.

- QCPs: Origin of \( C/T \sim \log(T_0/T) \)? \( \rho \sim T \)?
- Co-existence heavy fermions & LM AFM = Two fluid behavior? [Supersymmetry? B/F]
- HFSC: how is the spin incorporated into the condensate? [Composite pairs?]
- Hidden order (HO). Origin of ISING qps? [1/2 integer spinor OP]
- HO: complex MDW (multipolar density wave) vs Fractional spinor order.
Thank you!