Order Fractionalization

www.physics.rutgers.edu/~coleman/talks/iip_natal.pdf

Piers Coleman

Center for Materials Theory, Rutgers U, USA
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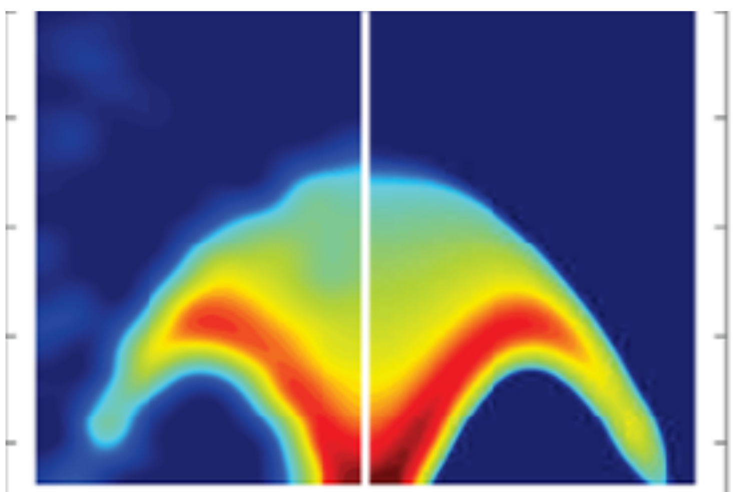

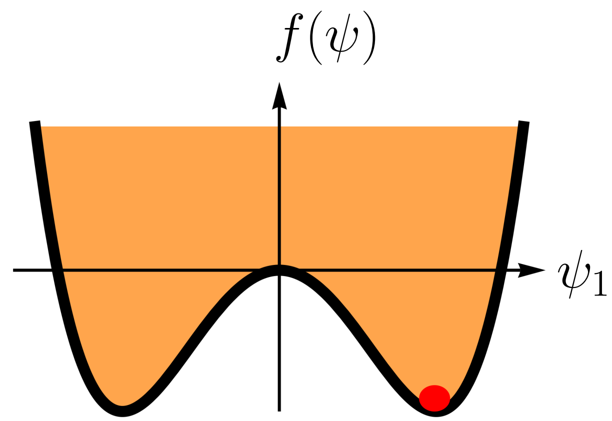

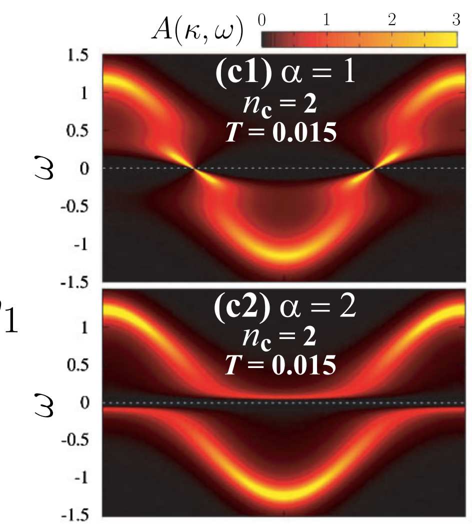

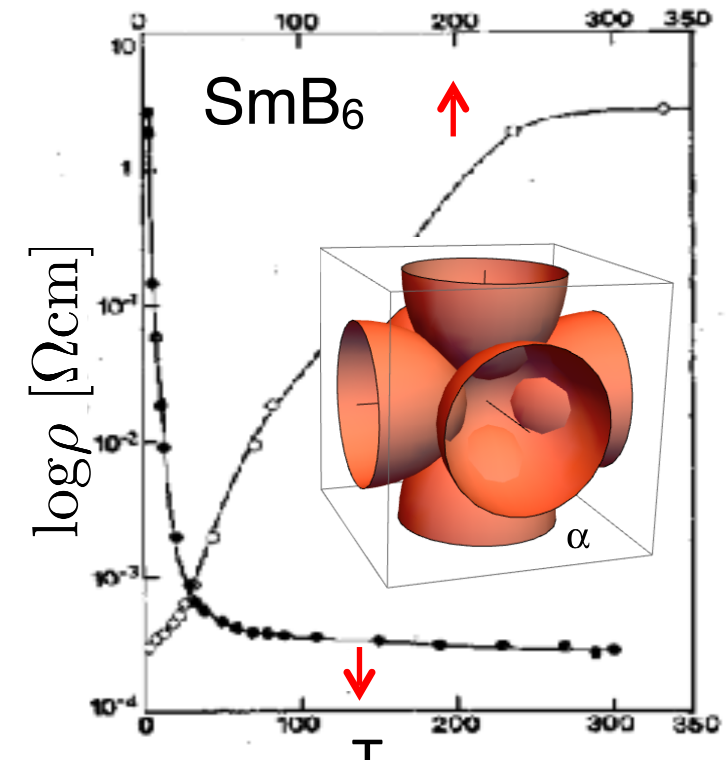

Yashar Komijani (Rutgers),
Anna Toth (TU, Budapest),
Premi Chandra (Rutgers)
Ari Wugalter (Rutgers)

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IIP, Natal
29th Aug 2018
Emergent Phenonon in Strongly Correlated Quantum Matter.

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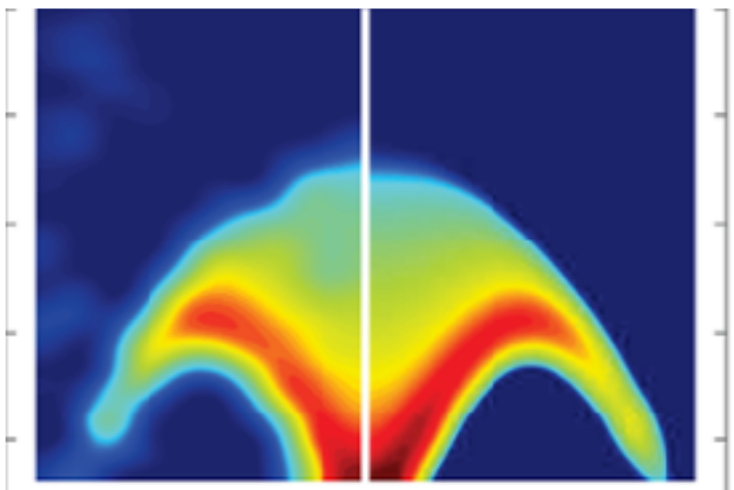

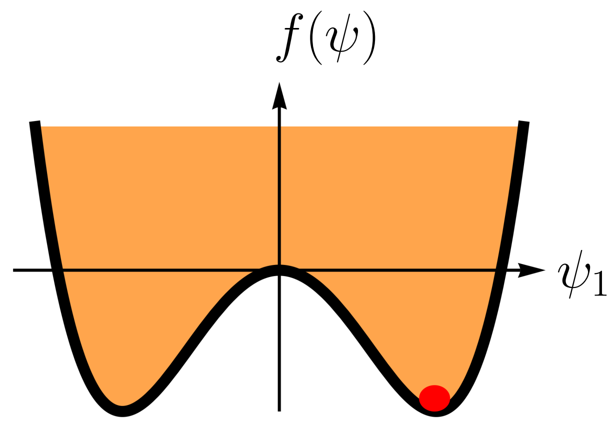

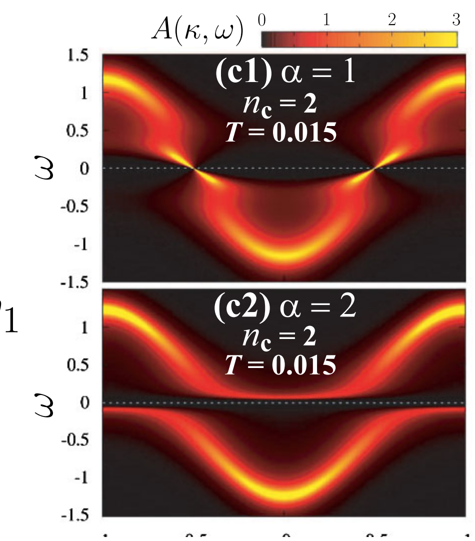

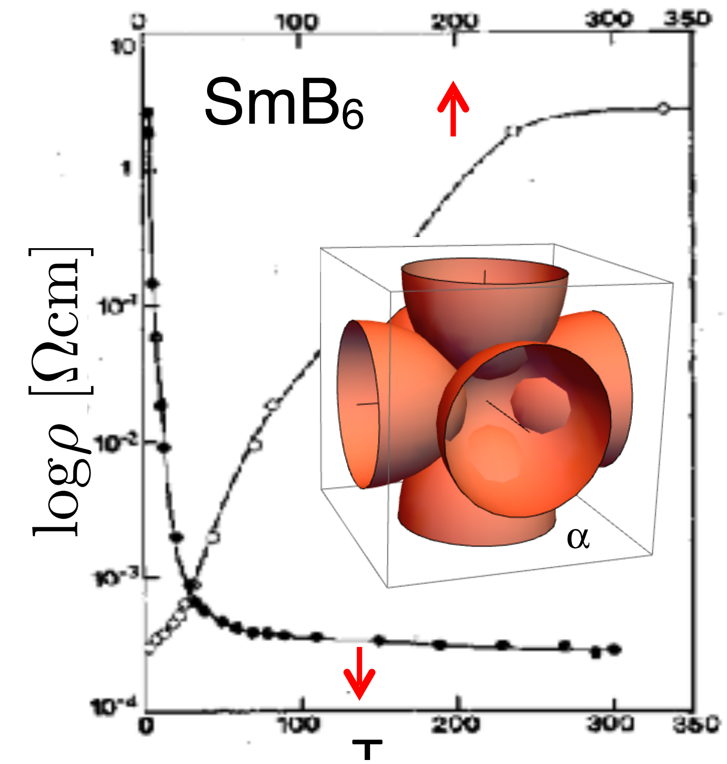

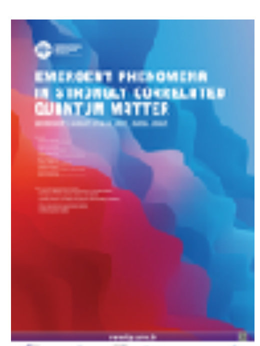

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NSF

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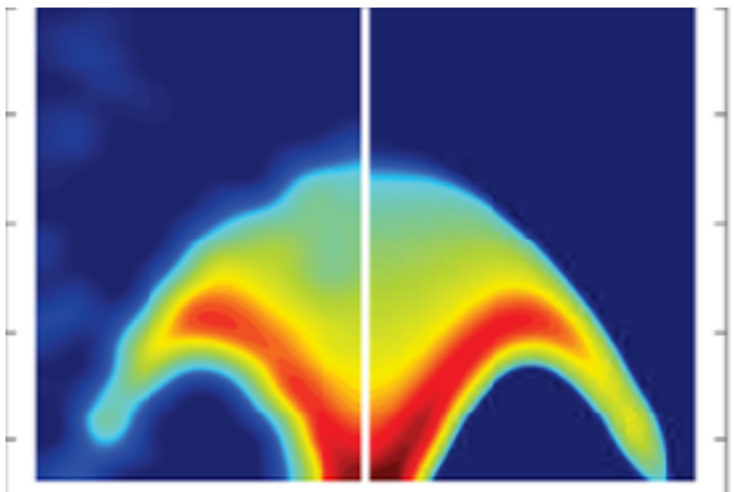

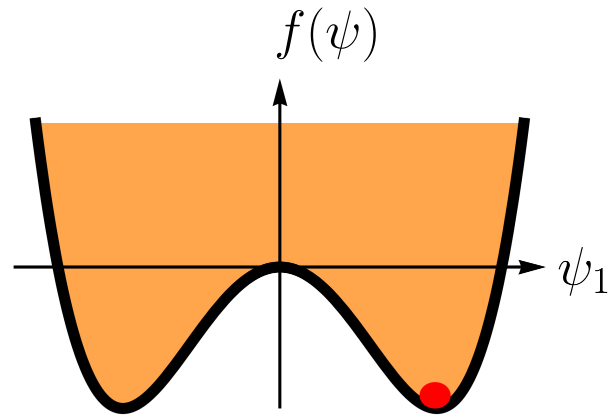

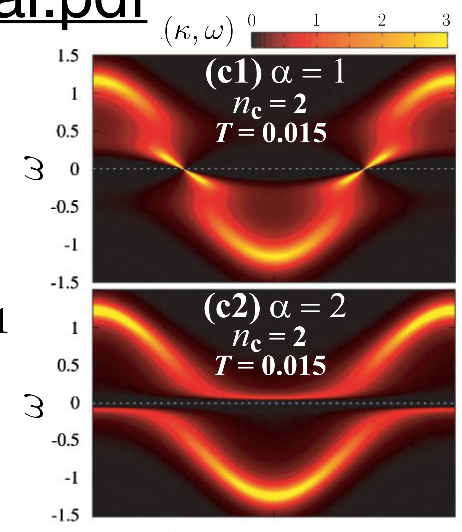

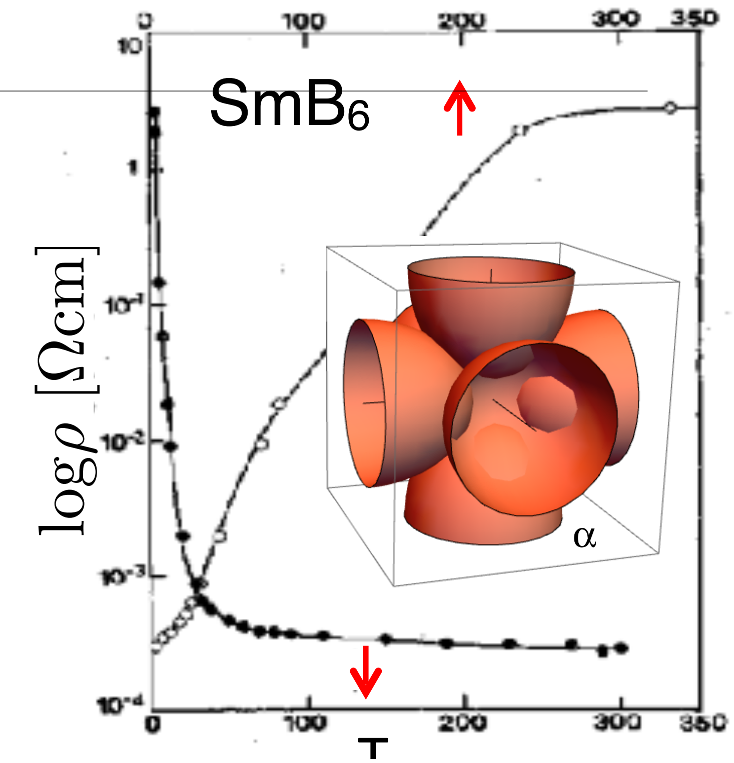

- Order
- Fractionalization
- Order + Fractionalization
- Motivation from Heavy Fermions
- Induced Order Fractionalization
- First attempts at a classification

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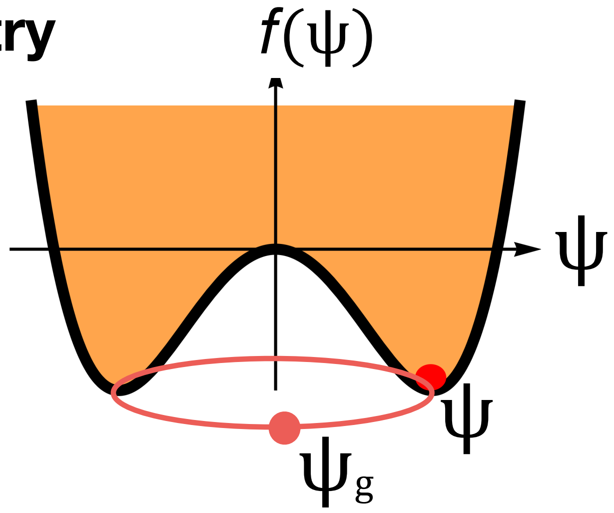
Order

Order

Broken Symmetry

Landau (1937)
Order Parameter



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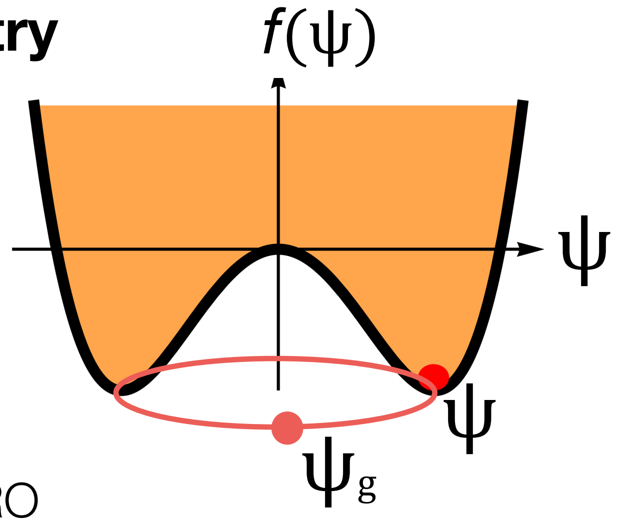

CN Yang 1962 ODLRO

$$\langle \Psi(x)\Psi^{\dagger}(y)\rangle \xrightarrow{|x-y|\to\infty} \langle \Psi(x)\rangle\langle \Psi(y)\rangle^{*}$$

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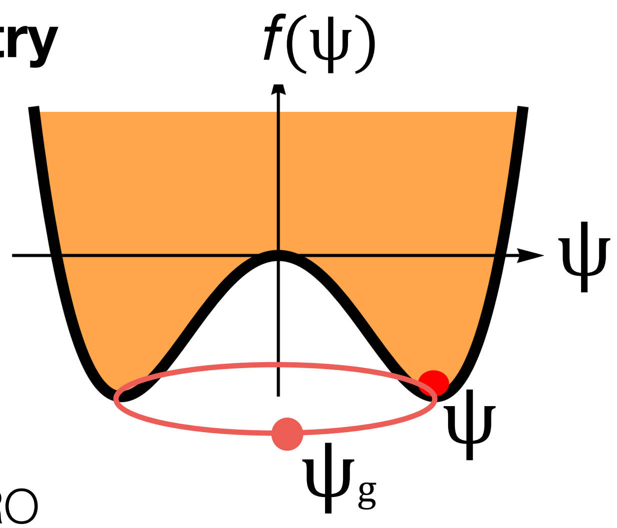

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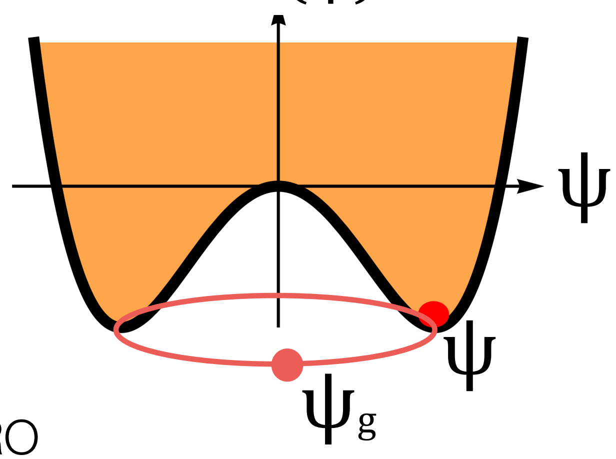

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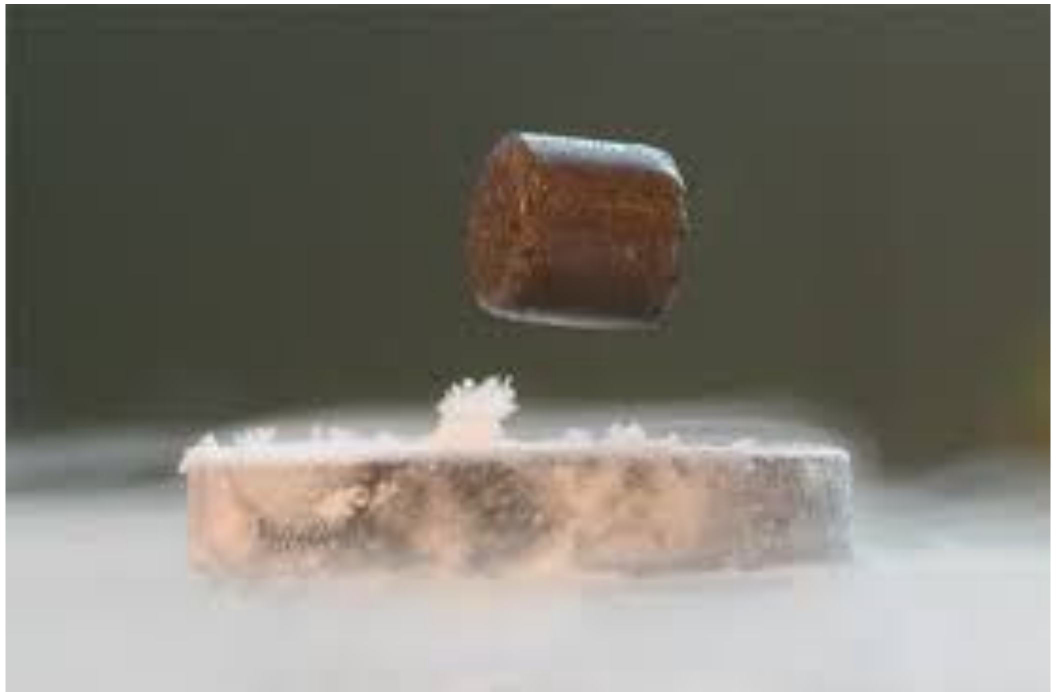
Electronic Matter:
OPs = pairs of fermions

$$\Psi = \langle \hat{\psi}_\uparrow \hat{\psi}_\downarrow \rangle \qquad \vec{M} = \langle \psi^\dagger \vec{\sigma} \psi \rangle$$

BCS

Stoner Hartree Fock



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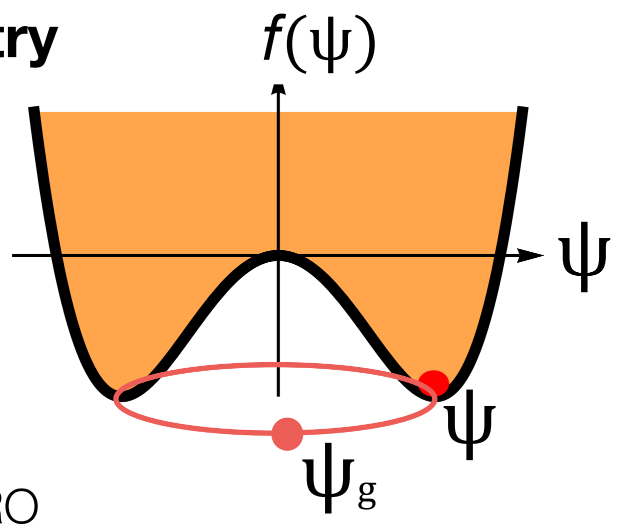

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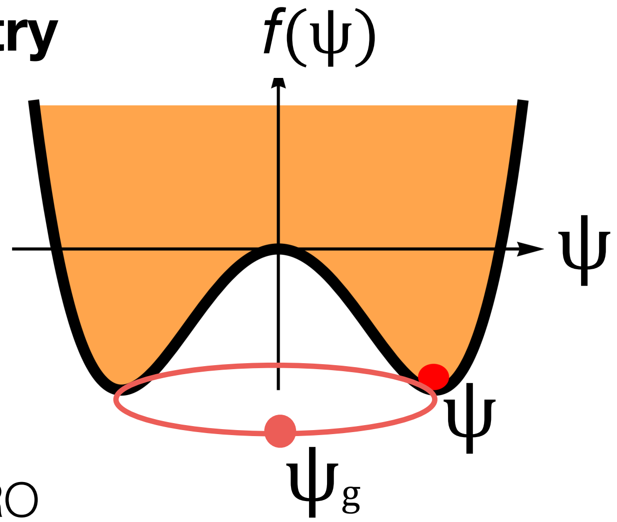

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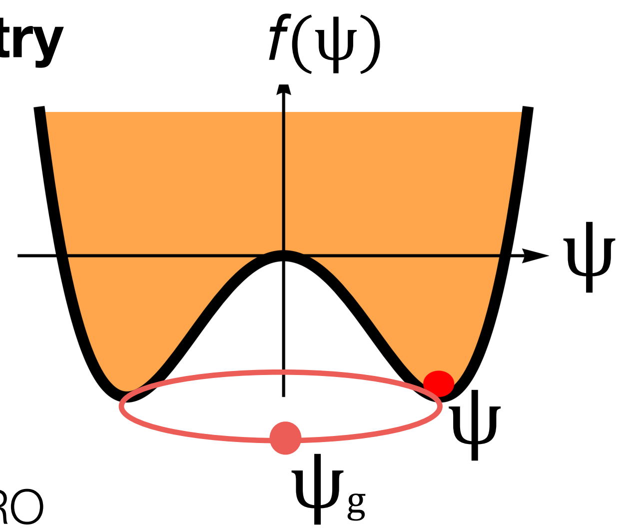

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Operator Fractionalization:
Magnons **fractionalize** into Spinons

Fractionalization

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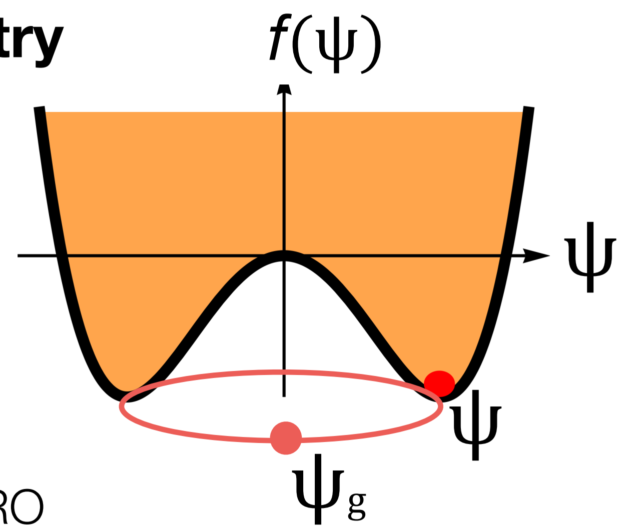

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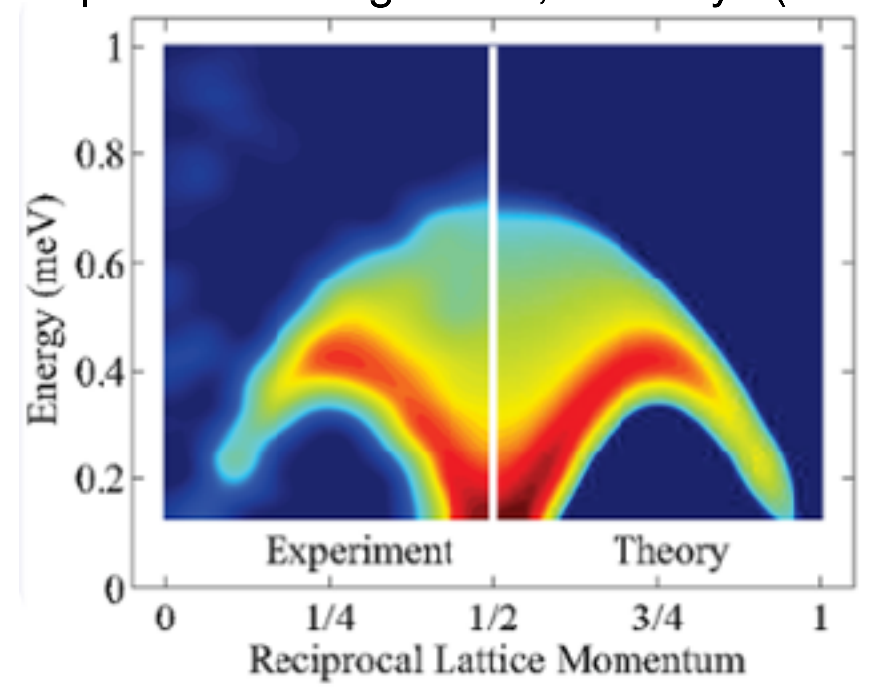

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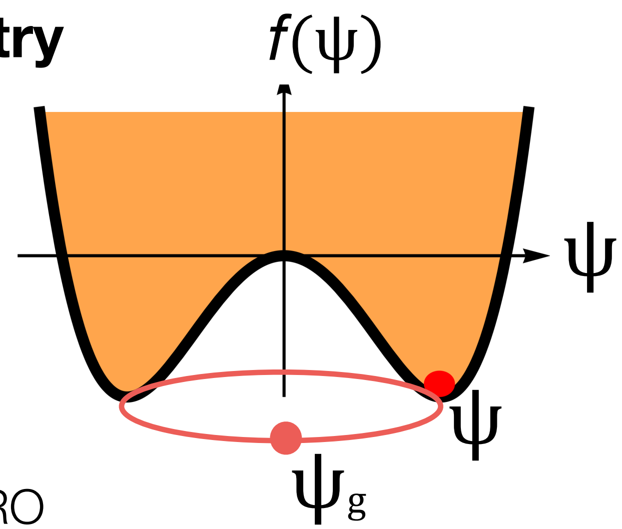

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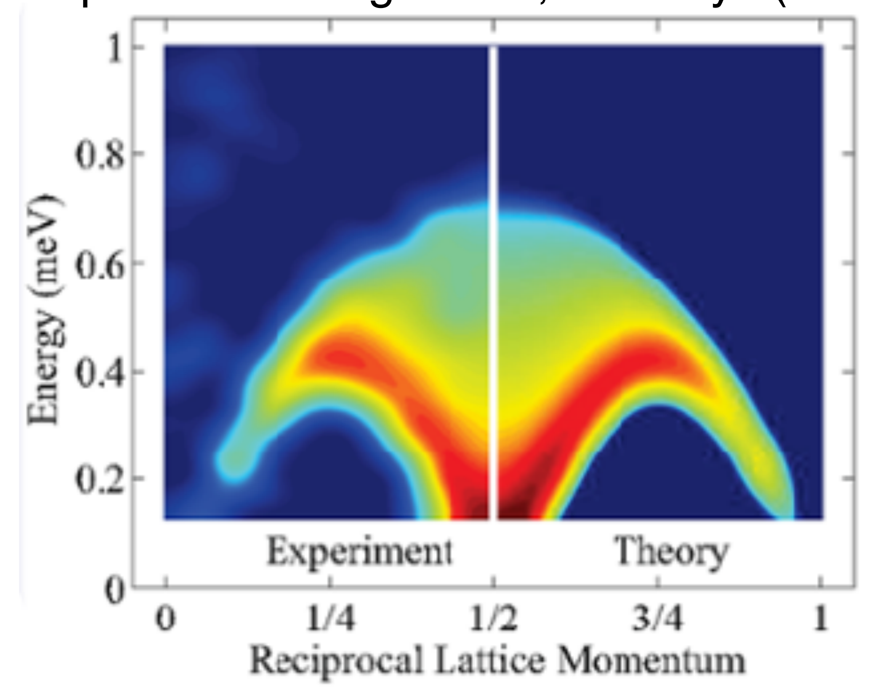

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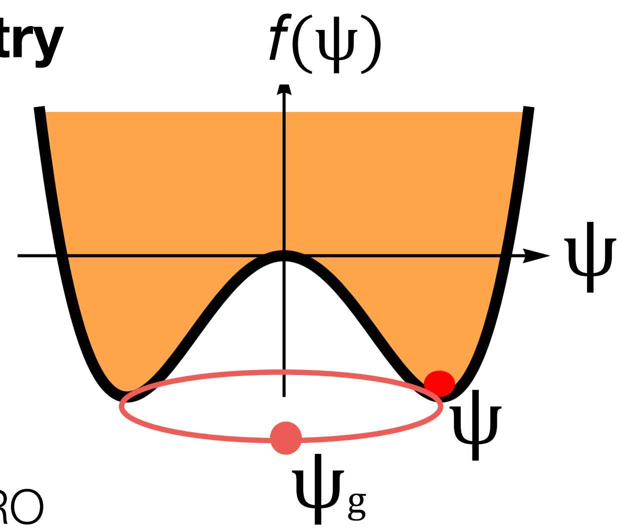

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S=1/2 Heisenberg Chain,
Kitaev Honecomb
Kondo models
Deconfined Criticality

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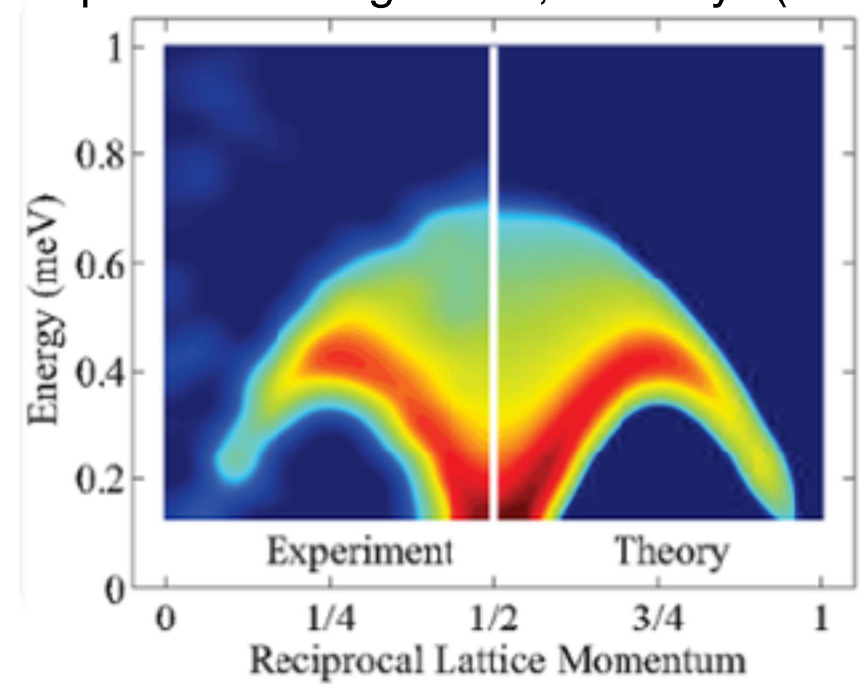

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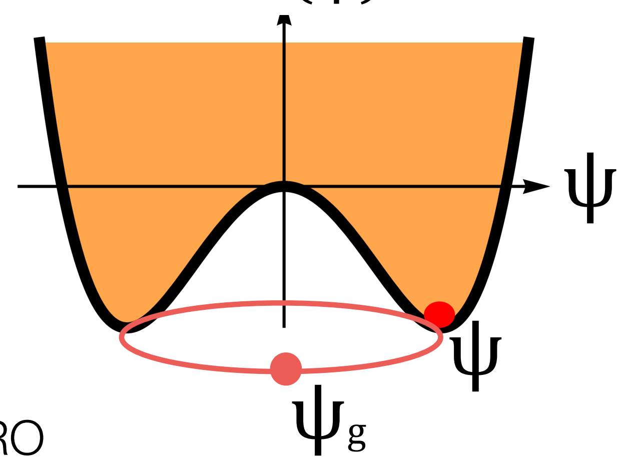

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- *excited* state property

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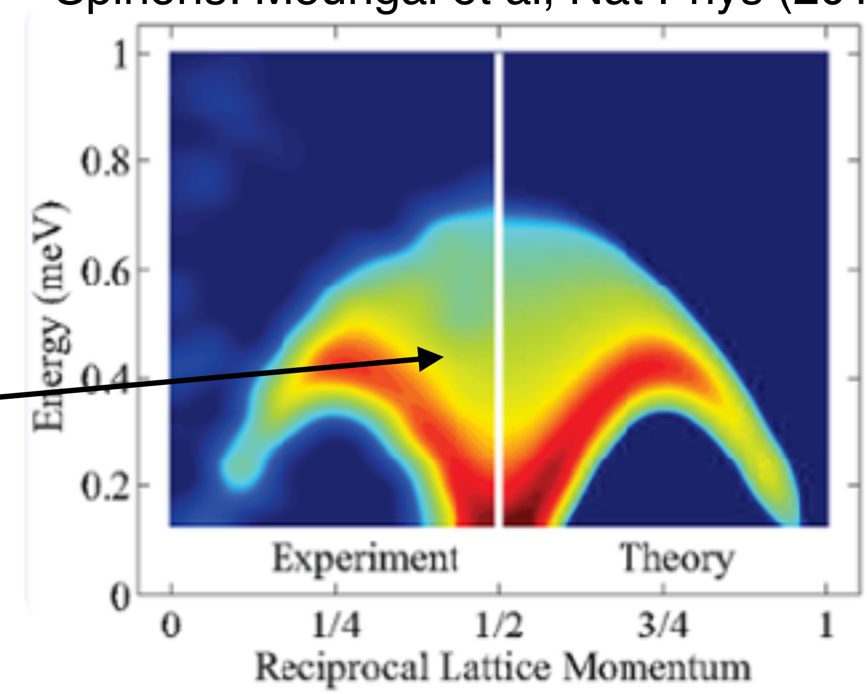

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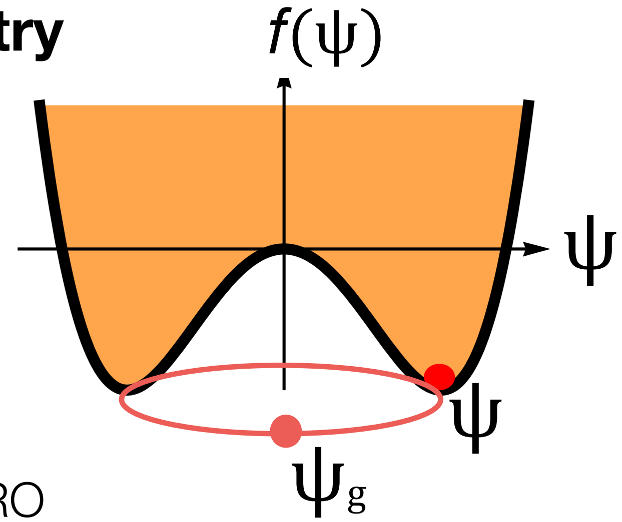

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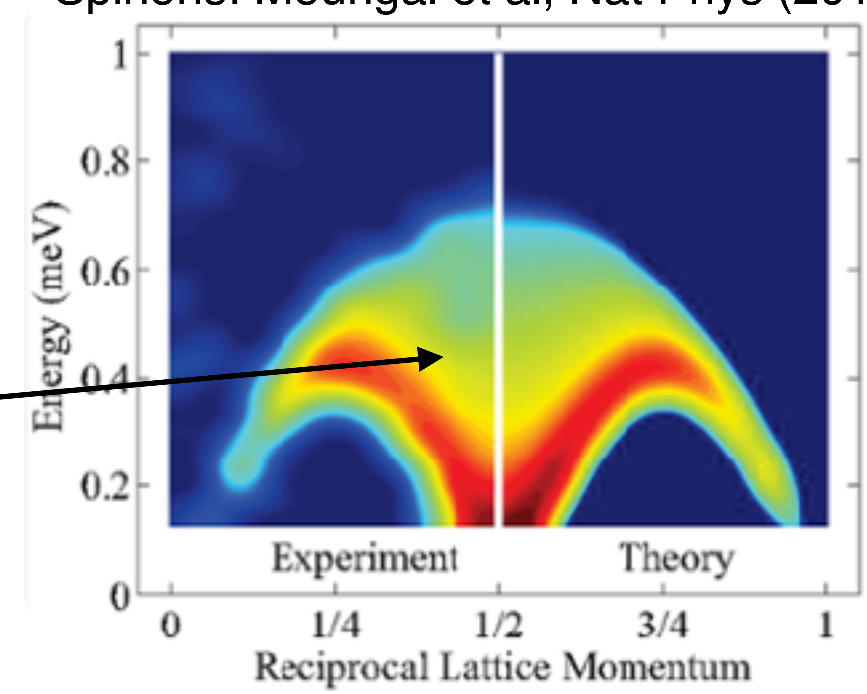

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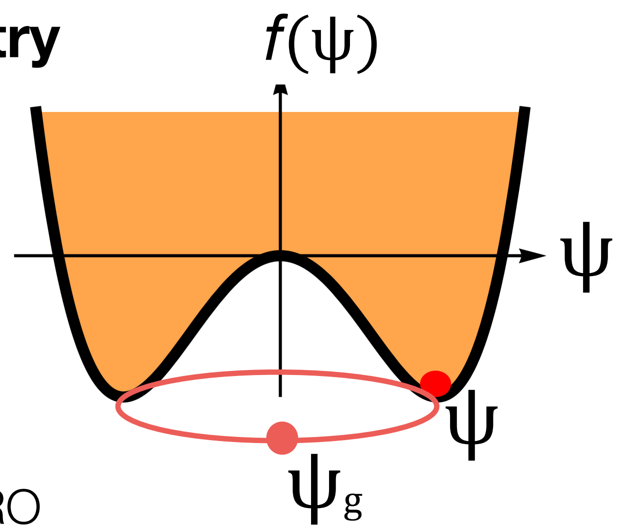

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Operator Fractionalization:

Spins Factorize into Quasiparticles

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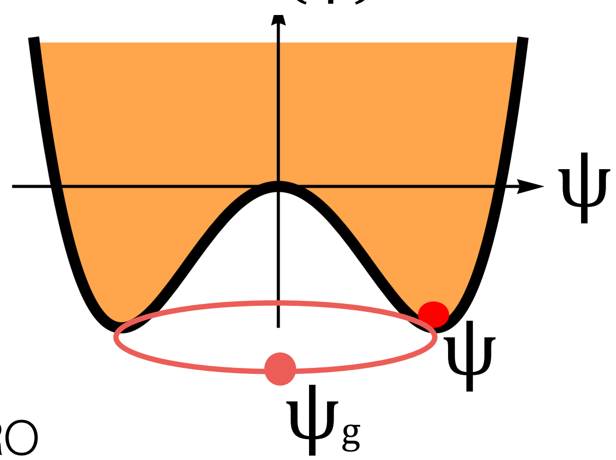
Order + Fractionalization

Broken Symmetry

Landau (1937)
Order Parameter



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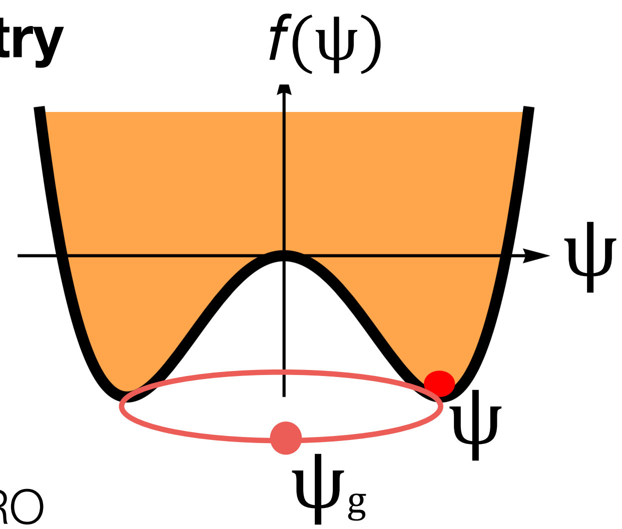

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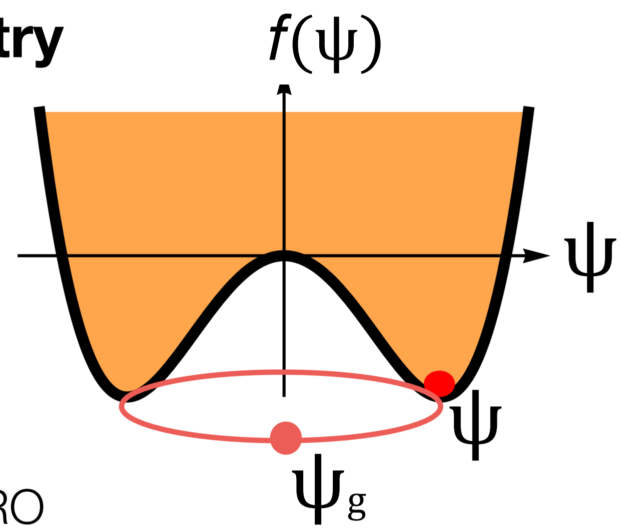
Conjecture:

Order can also fractionalize

Order + Fractionalization

Broken Symmetry

Landau (1937)
Order Parameter



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Conjecture:

Order can also fractionalize

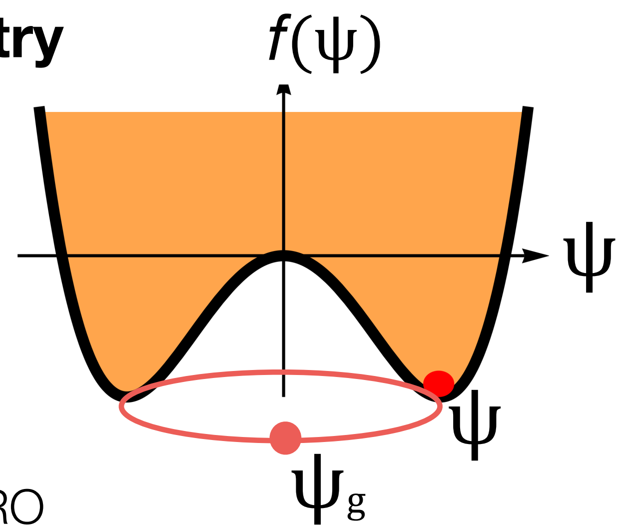
- *excited* state property
- Dynamic property

- Requires an extension of ODRLO into space-time.

Order + Fractionalization

Broken Symmetry

Landau (1937)
Order Parameter



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Motivation: Kondo Lattice Physics

$$H = -t \sum_{(i,j)\sigma} (c^{\dagger}{}_{i\sigma} c_{j\sigma} + \mathrm{H.c}) + J \sum_{j,\alpha\beta} \vec{\sigma}_j \cdot \vec{S}_j,$$

"Platonic" Kondo (Andy Millis)

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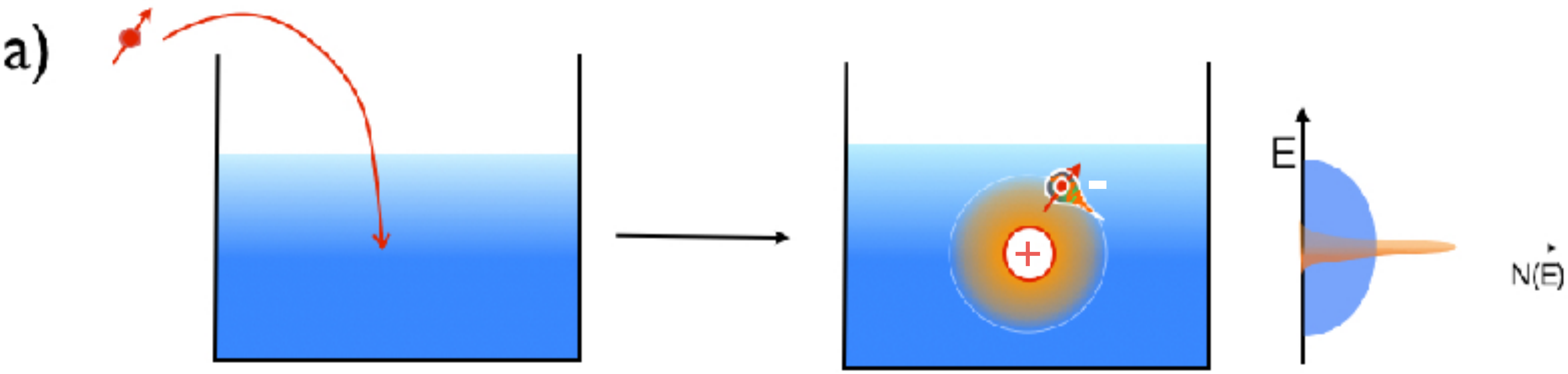

"Many body ionization"

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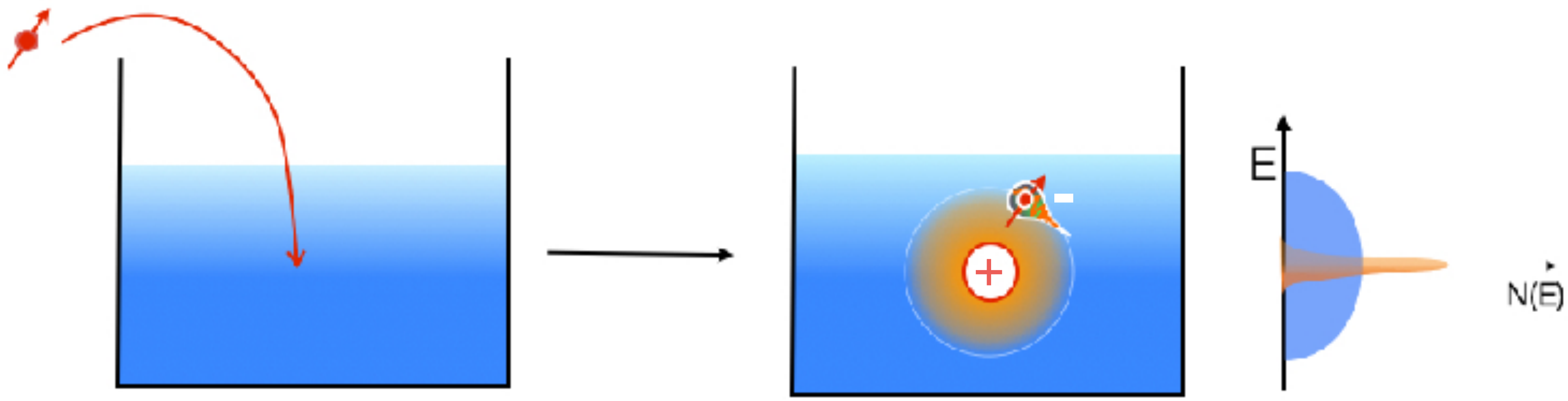

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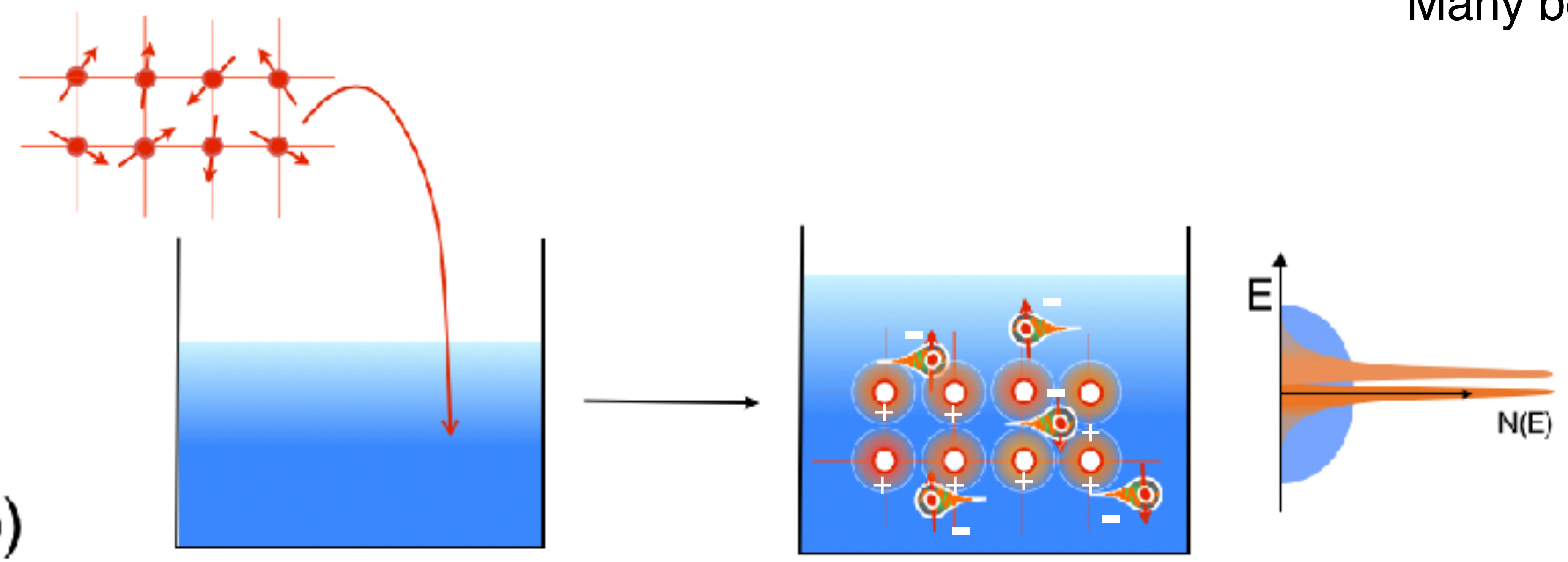

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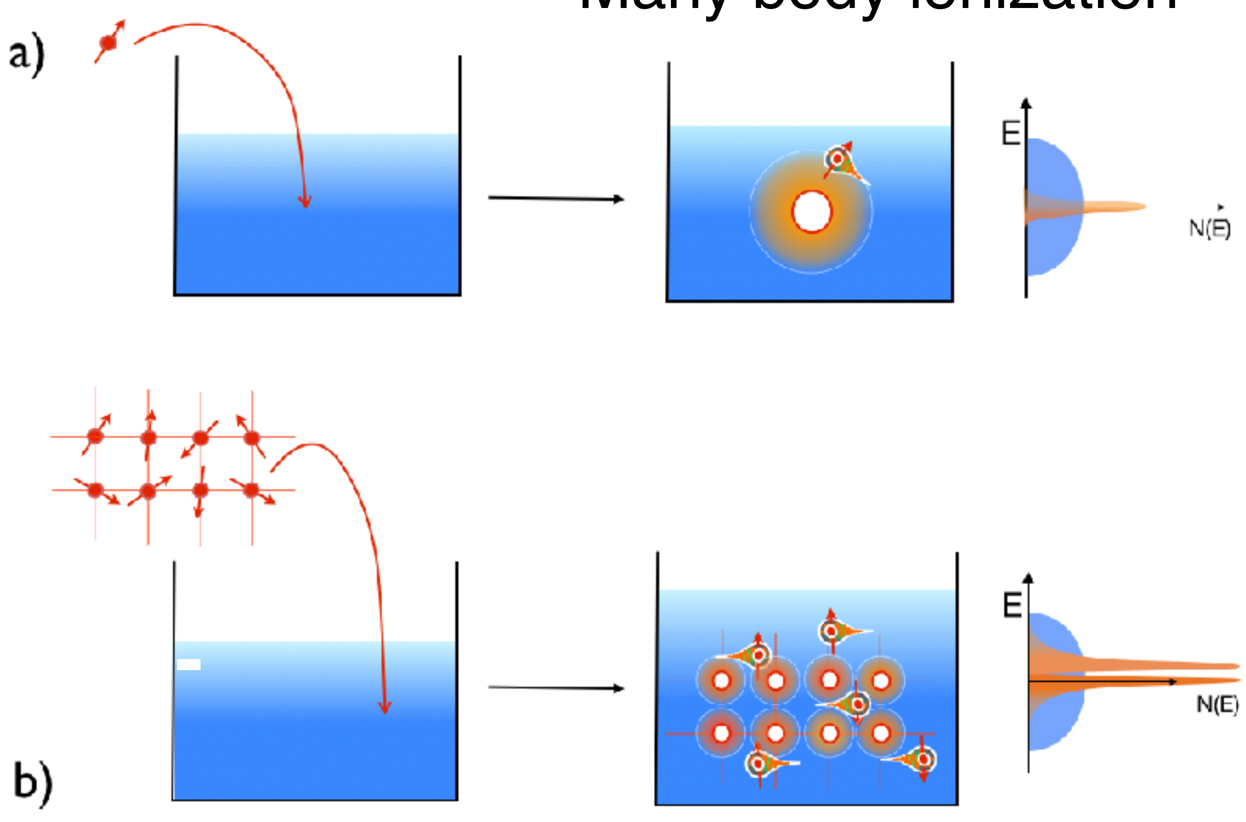

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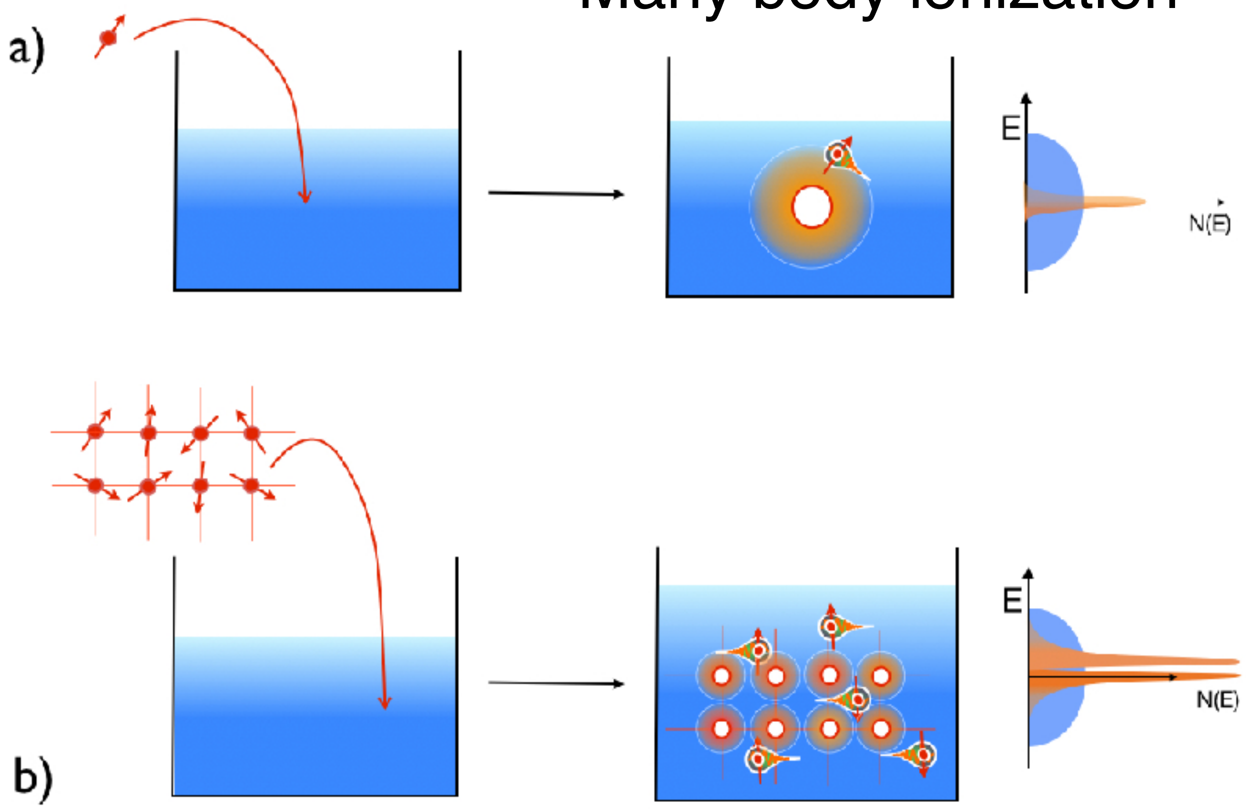

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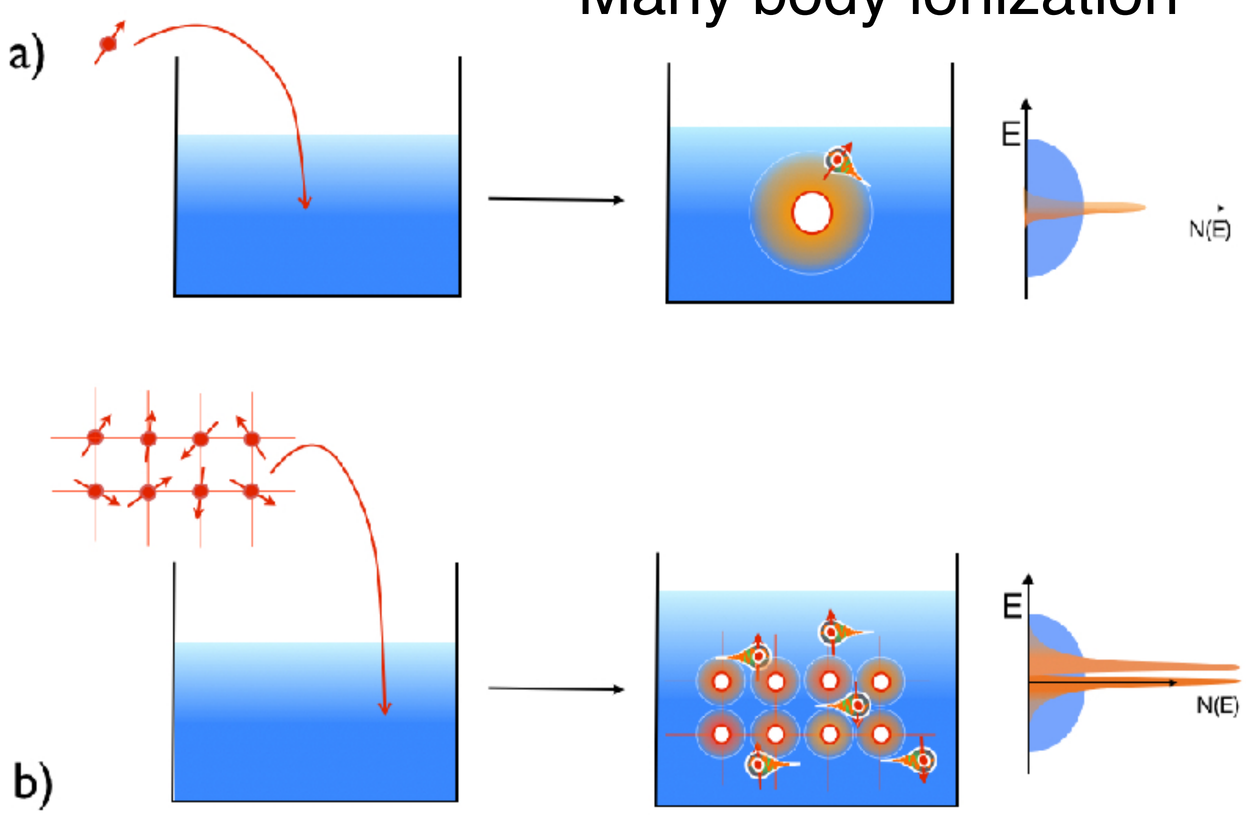

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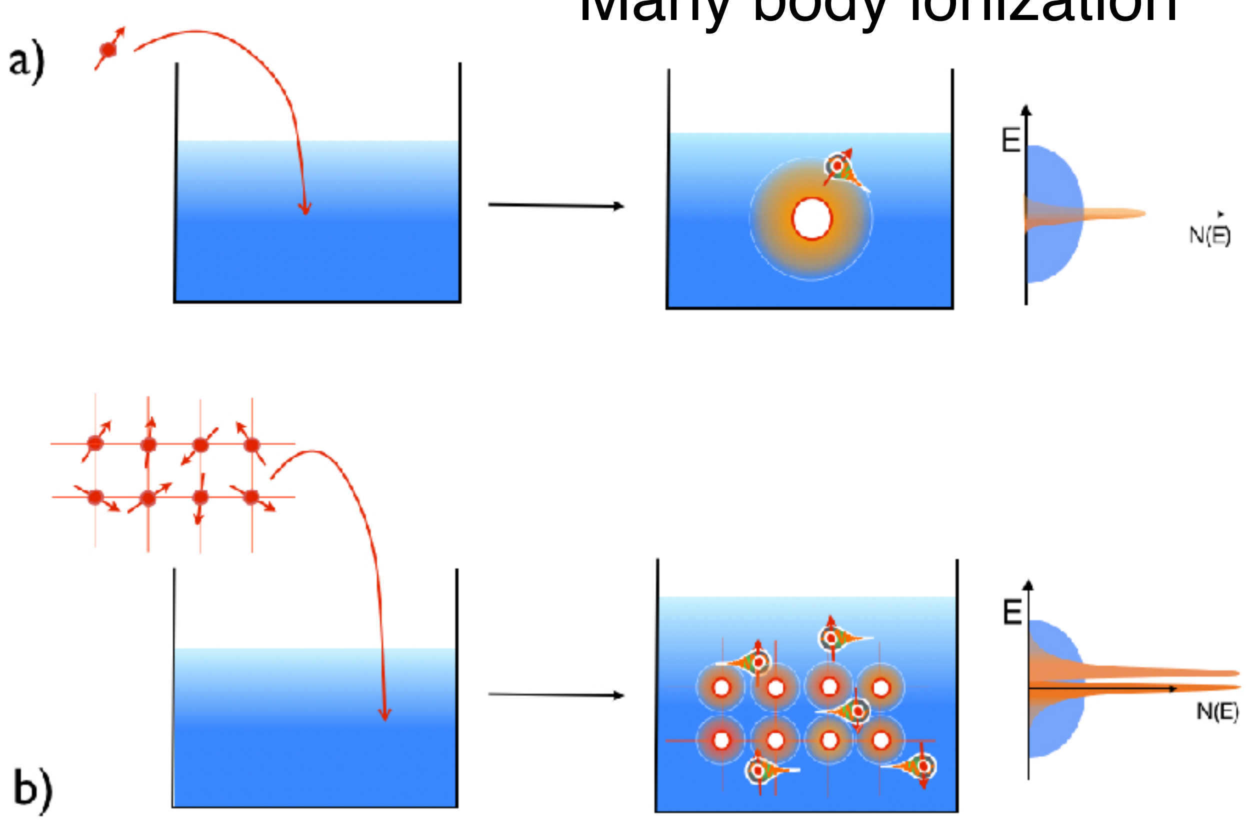
But viewed from the perspective of the Kondo model, the f-electron is an emergent fractionalization of the spin, as a charged Dirac particle.

$\vec{S} \longrightarrow f^{\dagger}_{\alpha}f_{\beta}$ Spin *Fractionalization*

“Many body ionization”



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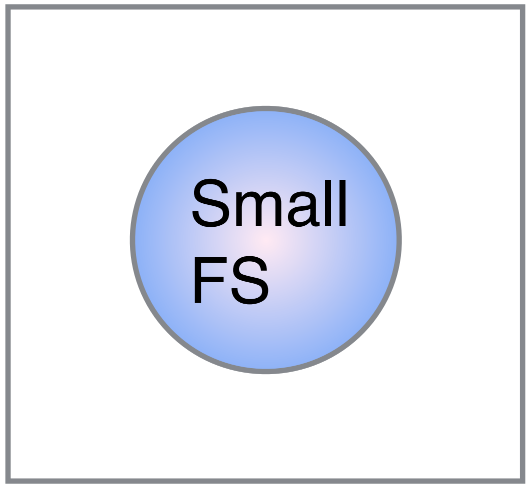
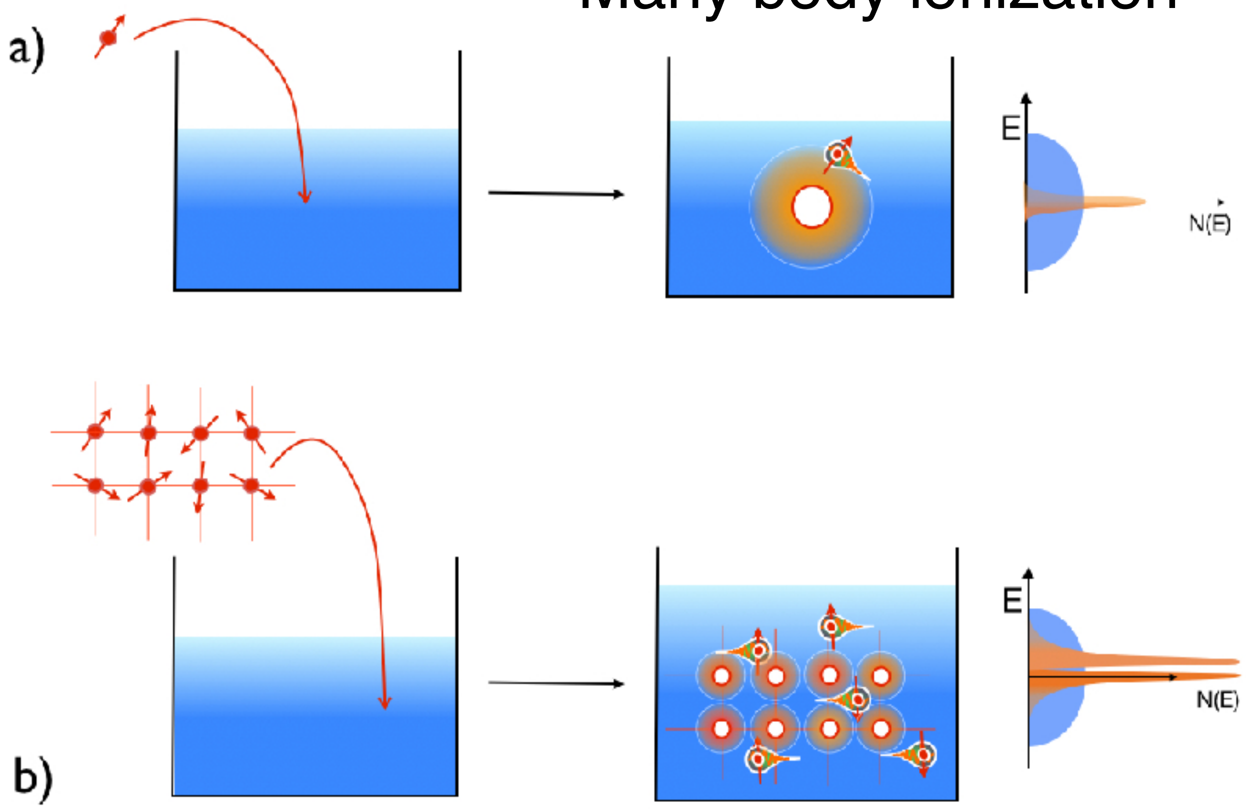


But viewed from the perspective of the Kondo model, the f-electron is an emergent fractionalization of the spin, as a charged Dirac particle.

$\vec{S} \rightarrow f^{\dagger}_{\alpha} f_{\beta}$ Spin *Fractionalization*

"Many body ionization"



Motivation: Kondo Lattice Physics

$$H = -t \sum_{(i,j)\sigma} (c^{\dagger}{}_{i\sigma} c_{j\sigma} + \mathrm{H.c}) + J \sum_{j,\alpha\beta} \vec{\sigma}_j \cdot \vec{S}_j,$$

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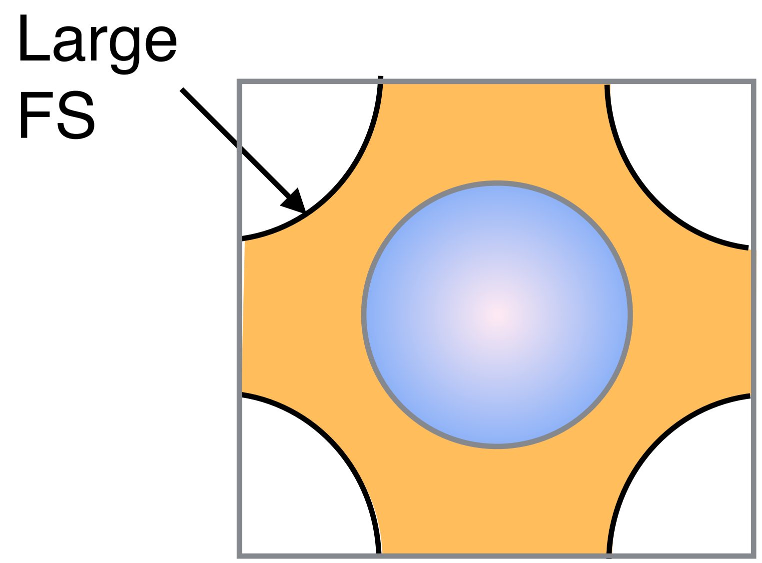

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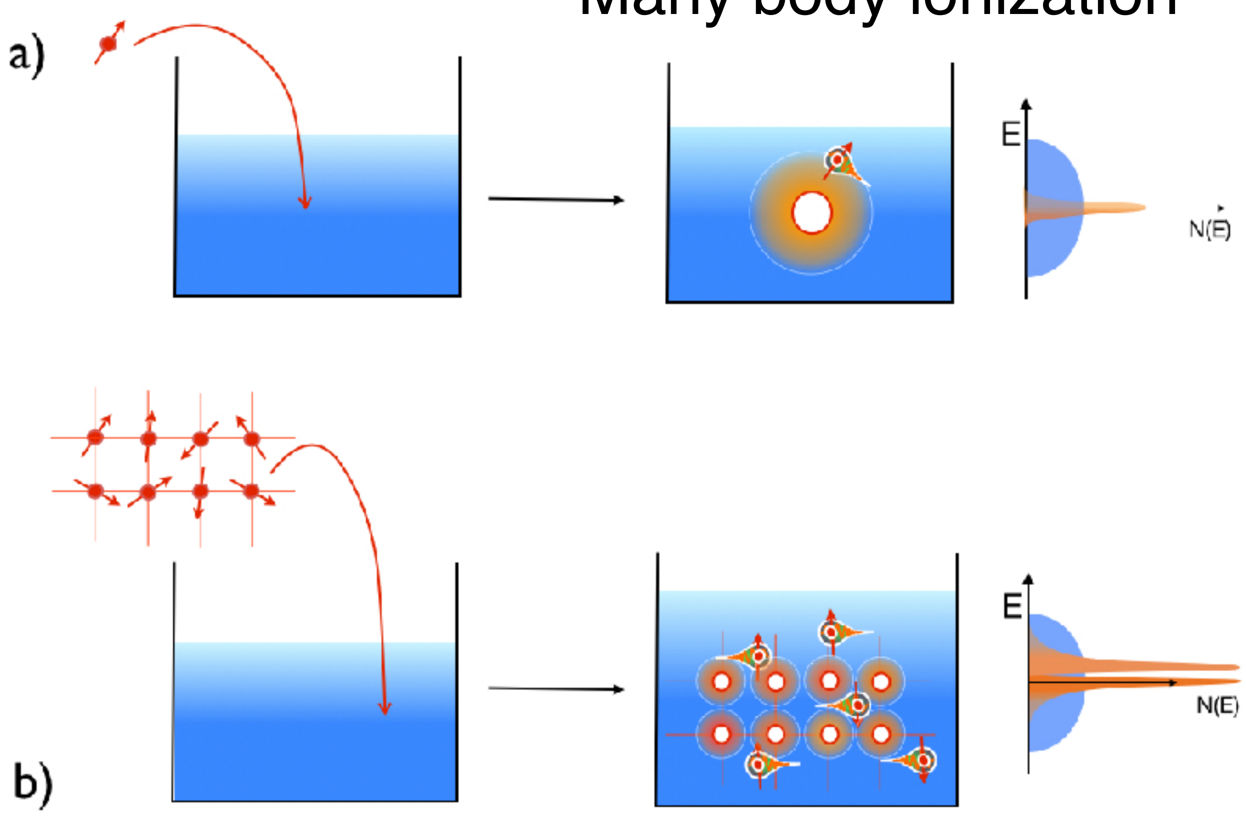

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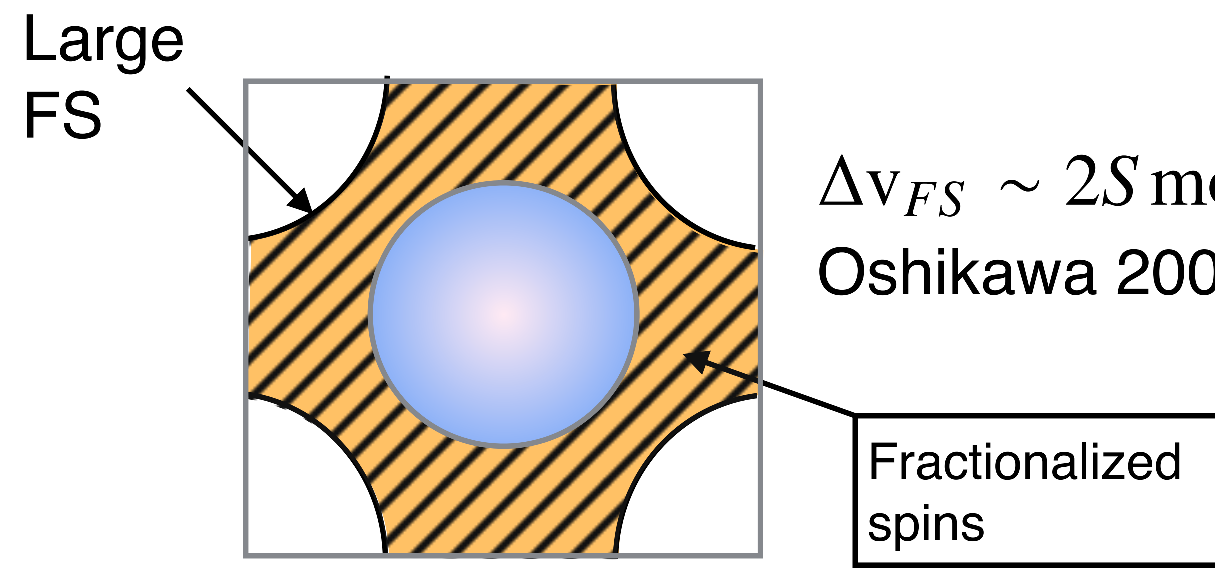

$\Delta v_{FS} \sim 2S \bmod 2$
Oshikawa 2000

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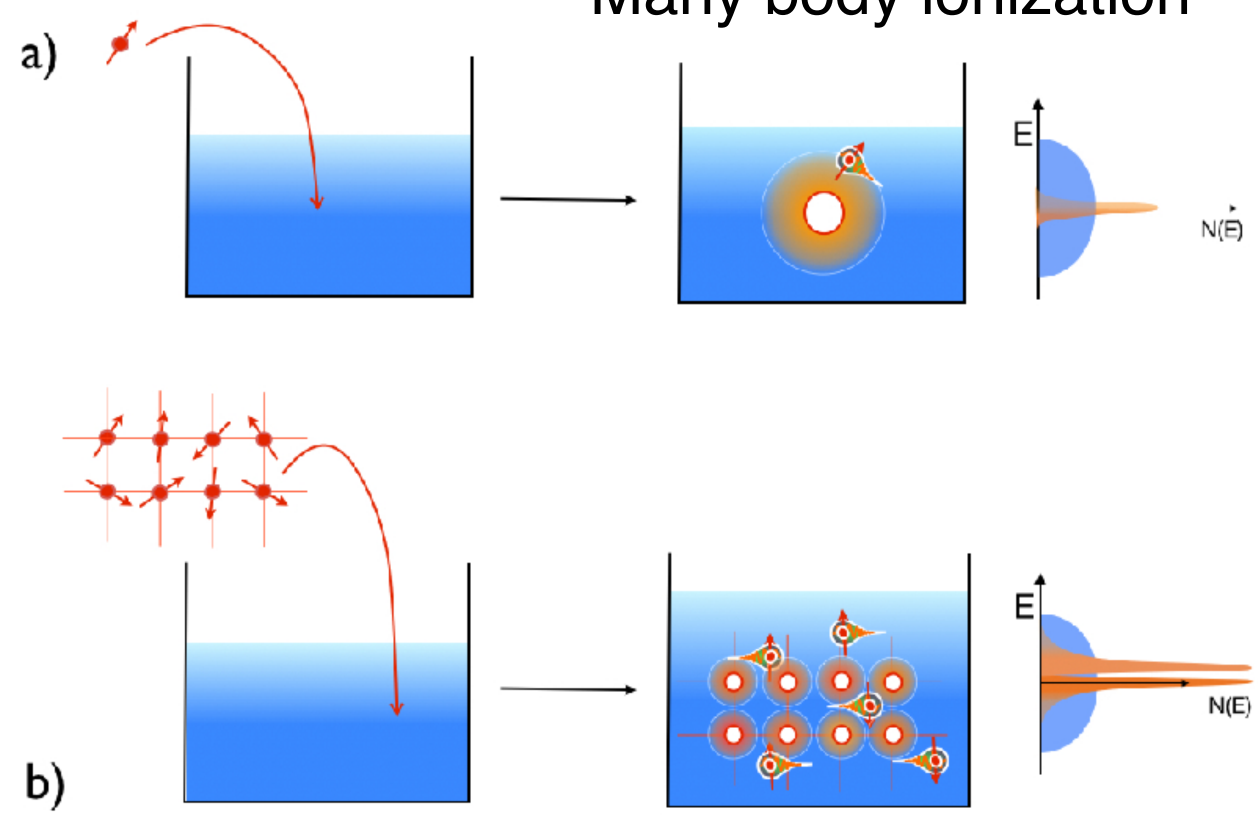

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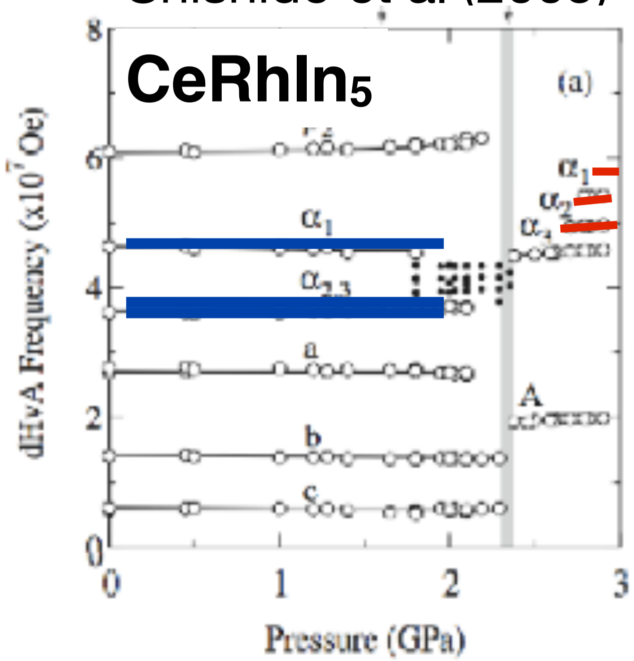
Shishido et al (2005)



Pressure driven Fractionalization

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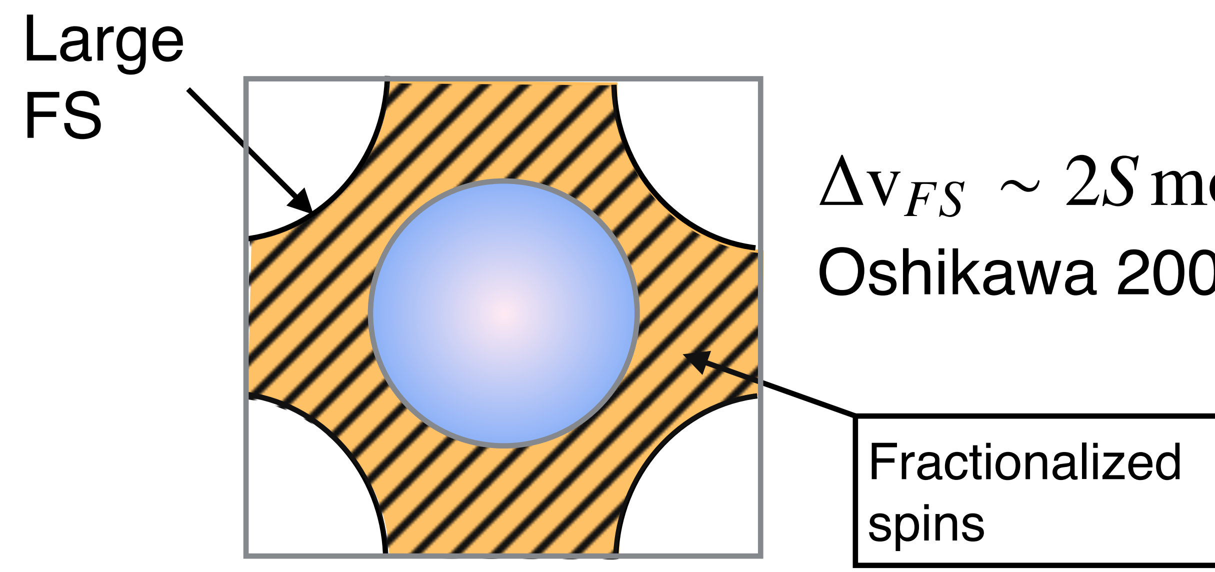

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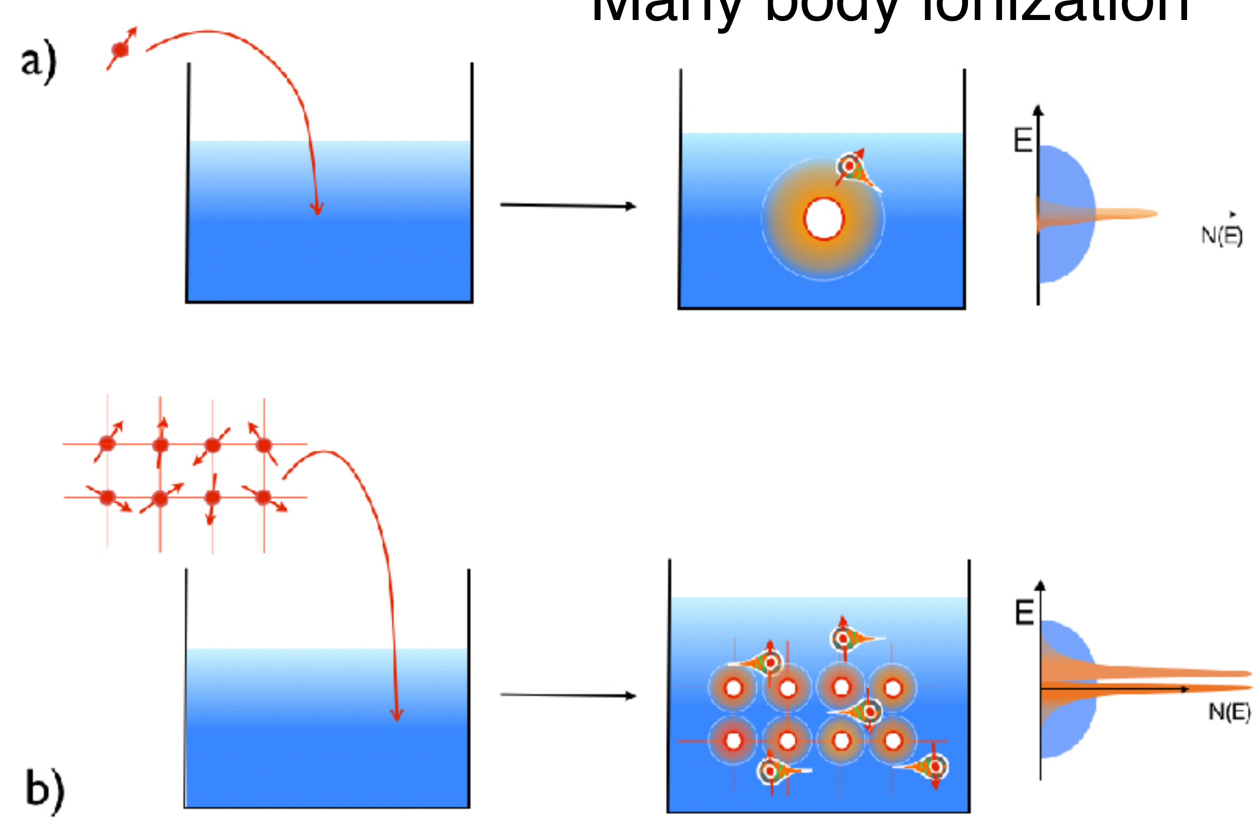

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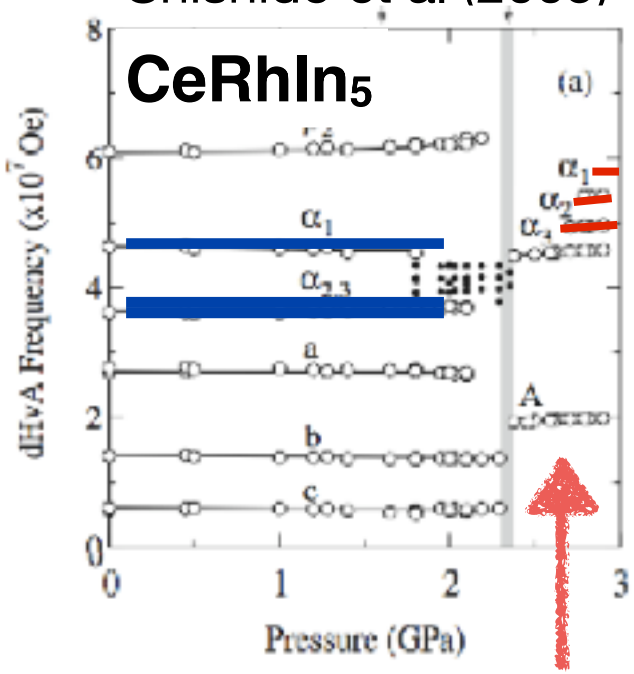

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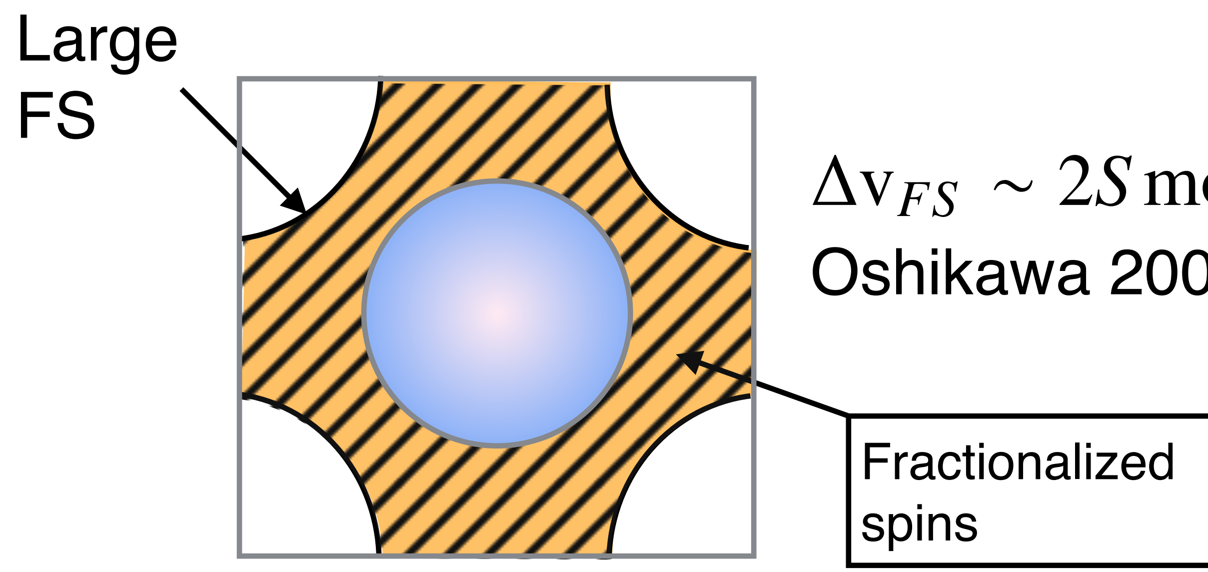
Spin *Fractionalization*



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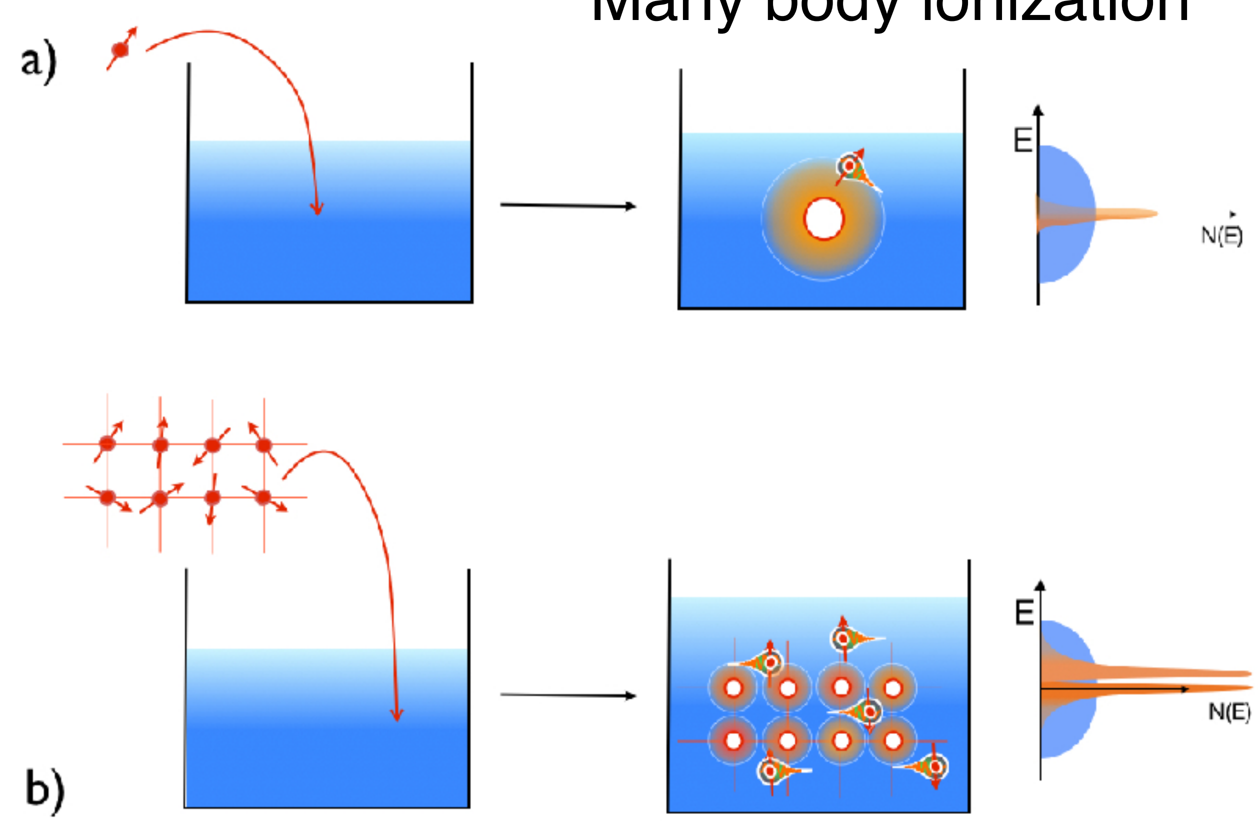

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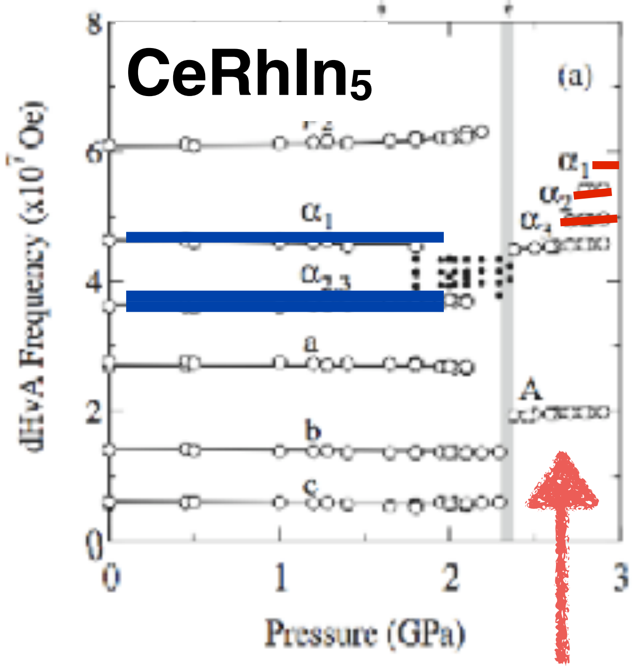

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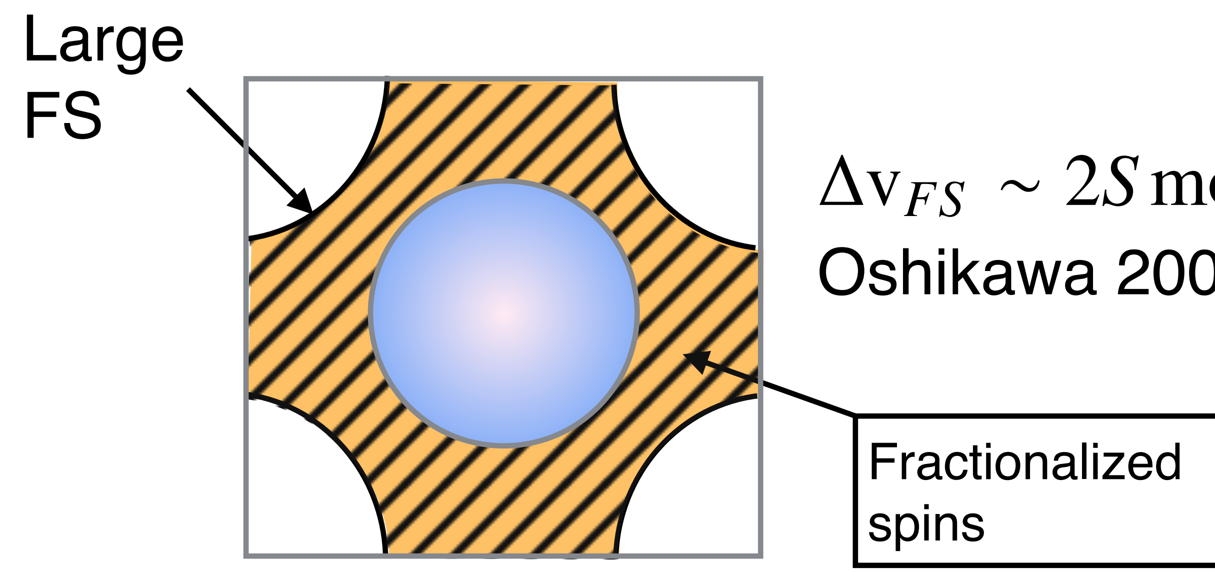

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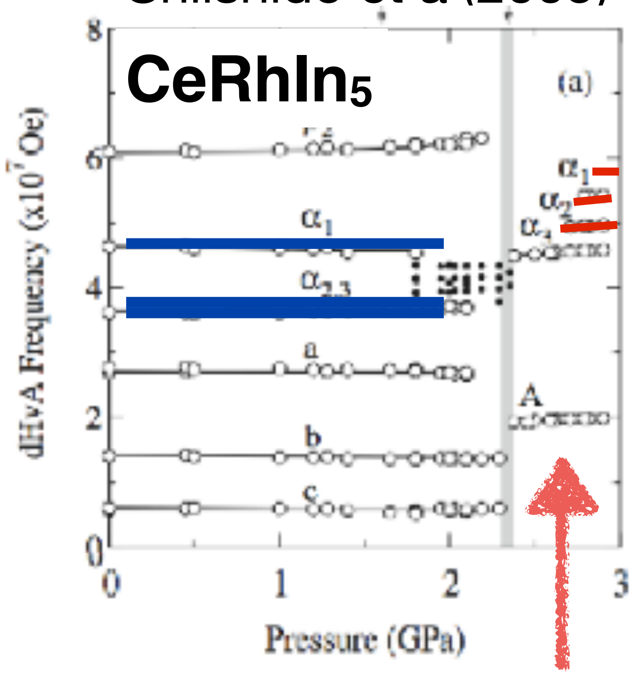

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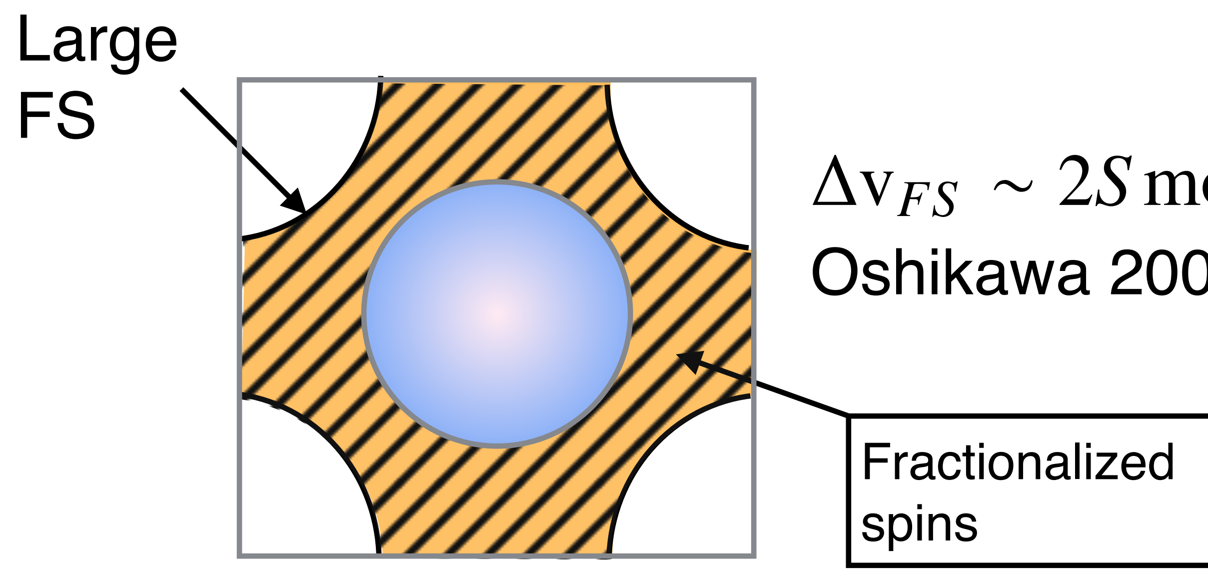

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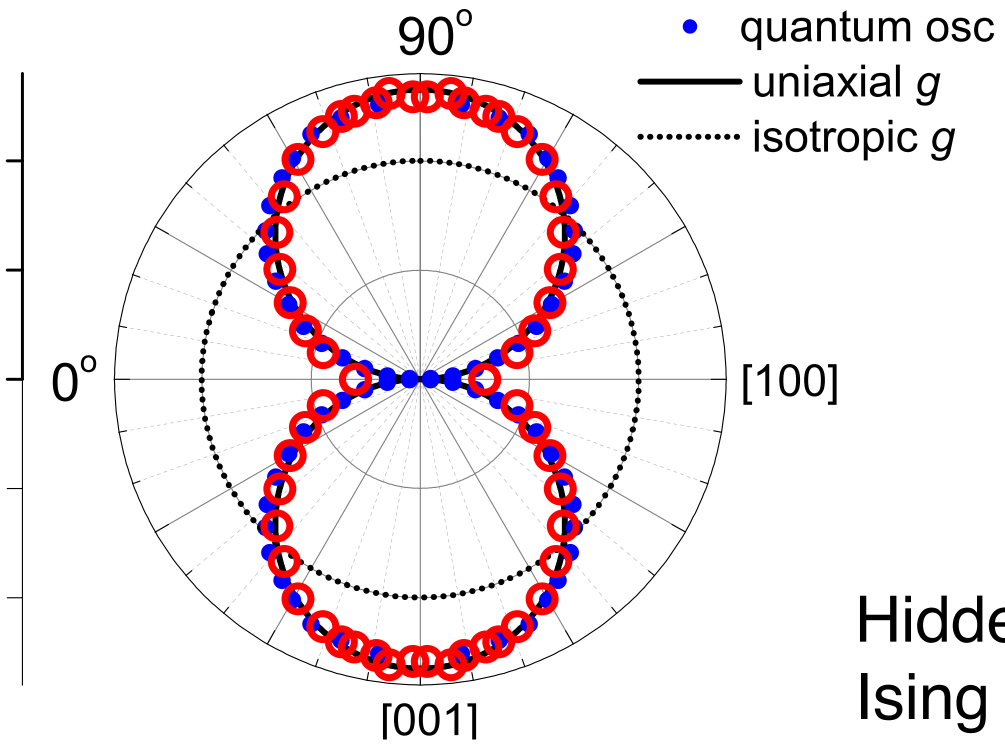
What other kinds of fractionalization are possible?

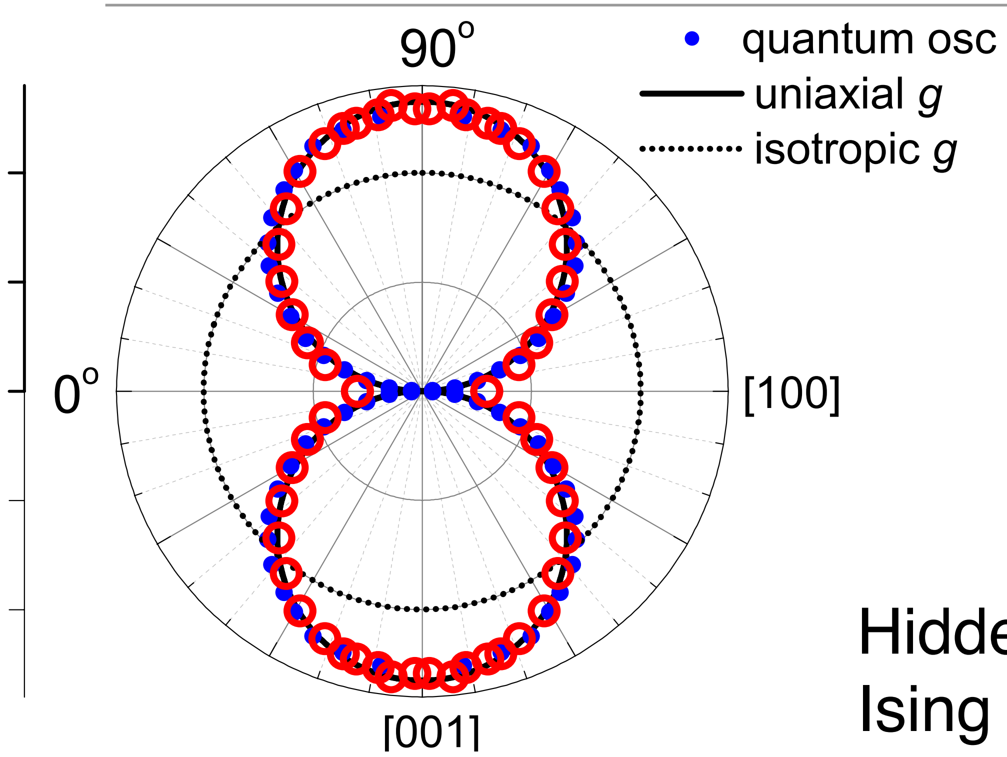
Motivation: Kondo Lattice Physics



Altarawnah et al, 2011

Hidden Order URu_2Si_2
Ising Quasiparticles.

Motivation: Kondo Lattice Physics



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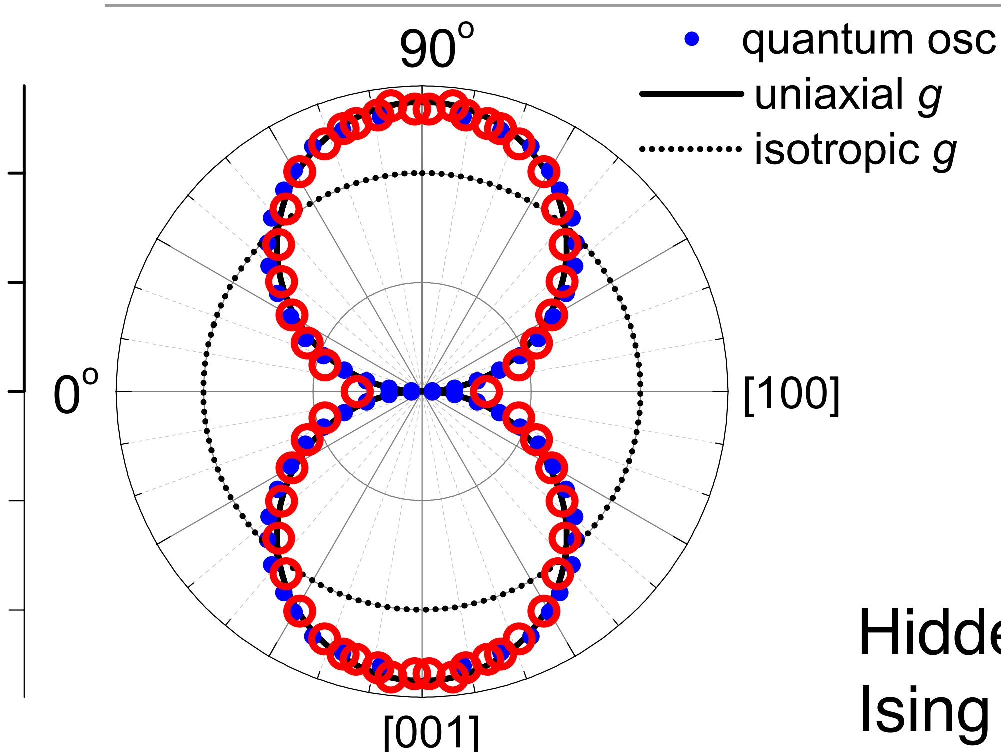

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Integer (J=4) spin fractionalizes into Ising Quasiparticles.

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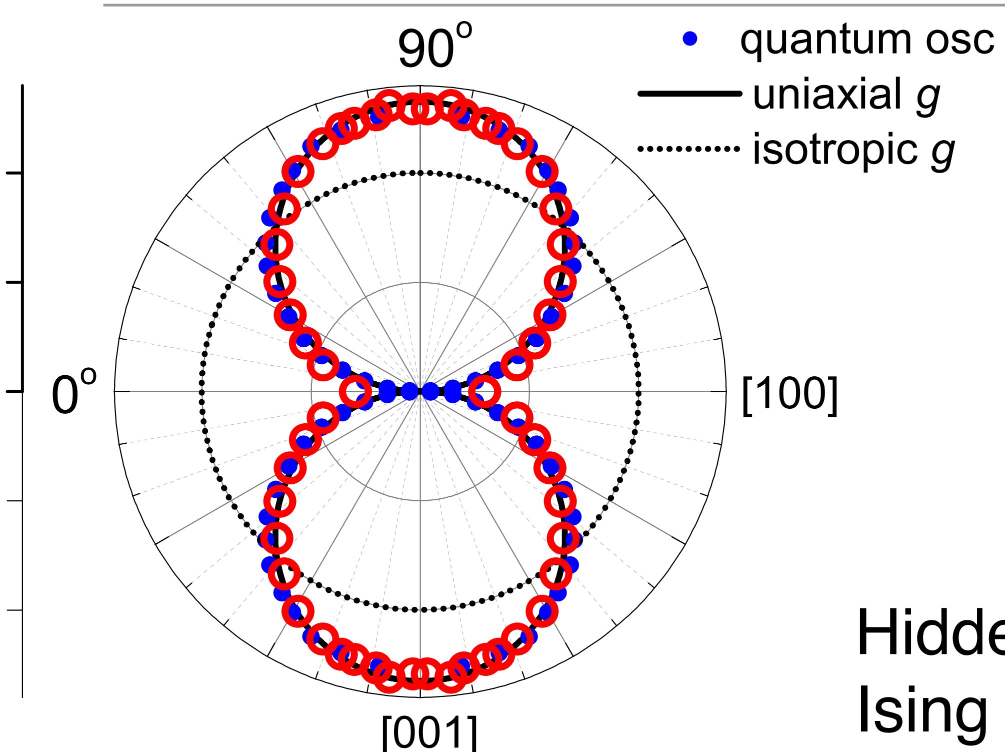

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Hastatic order

$$\Psi = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix}$$

P .Chandra et al, Nature, 493, 621-626 (2013).

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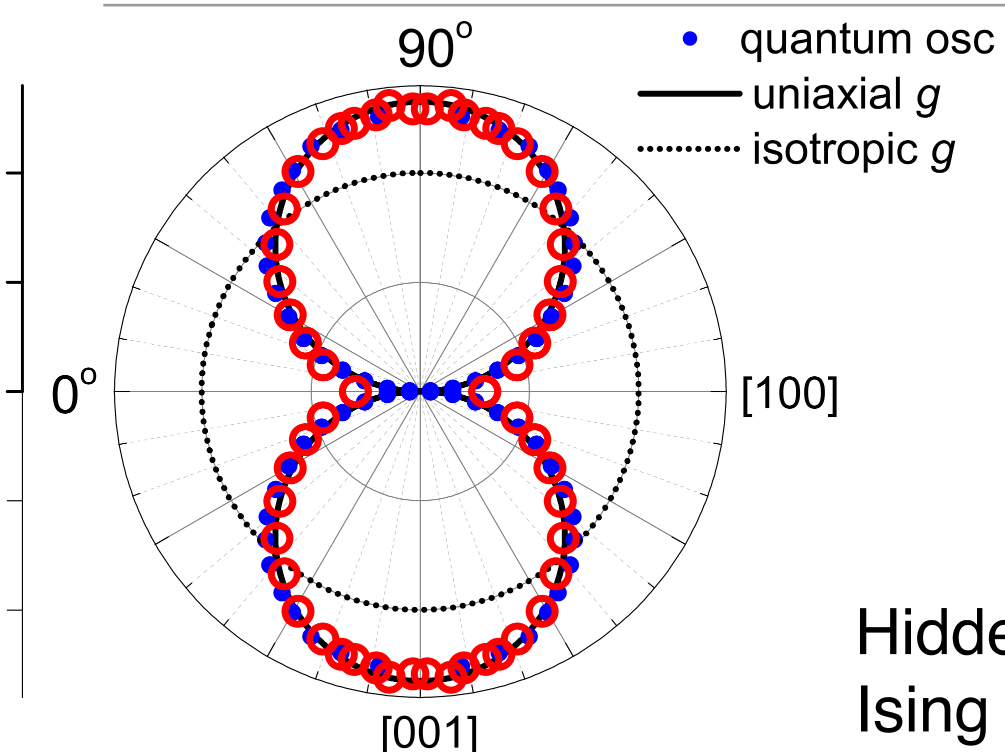

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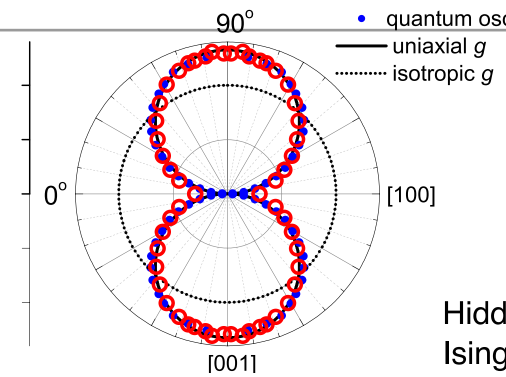

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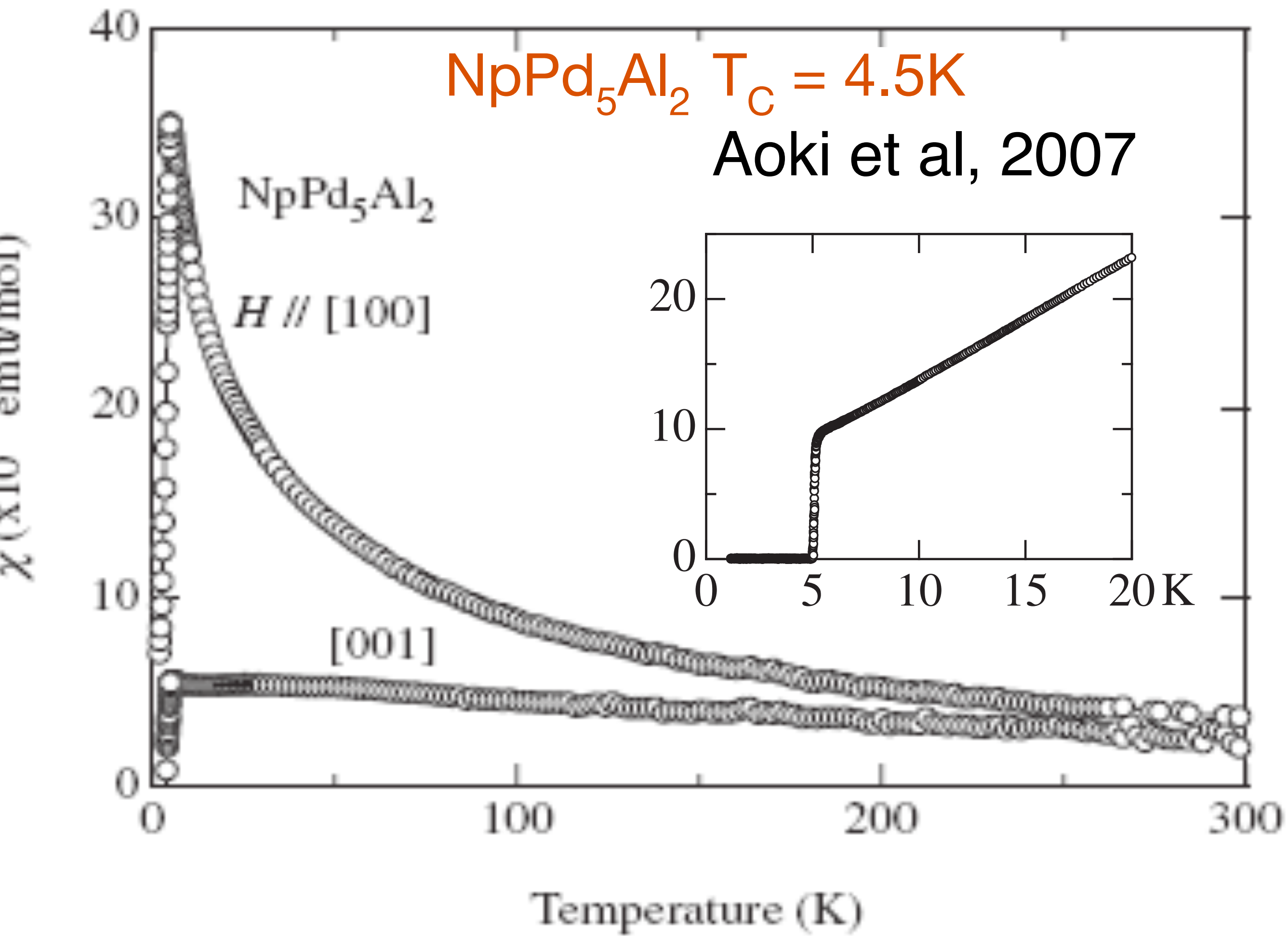
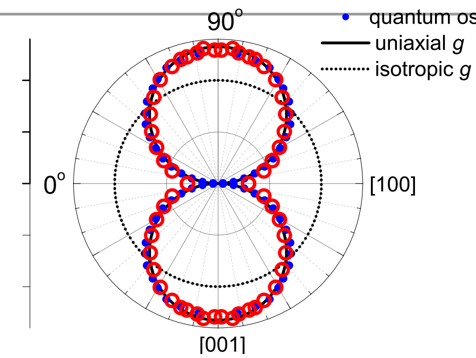


NpPd$_5$Al$_2$ T$_C$ = 4.5K



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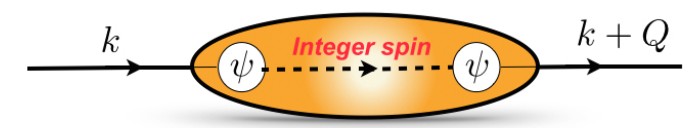

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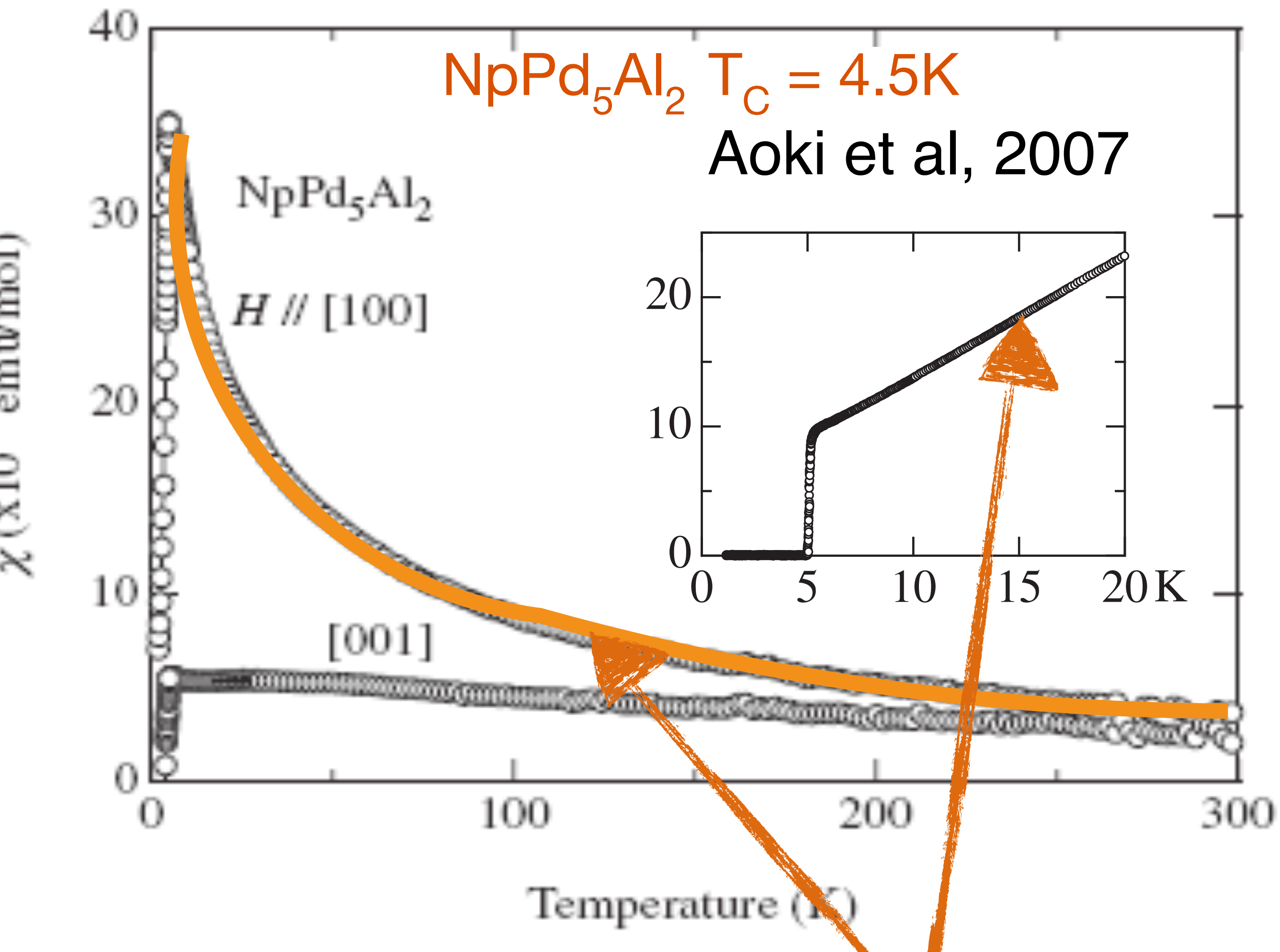
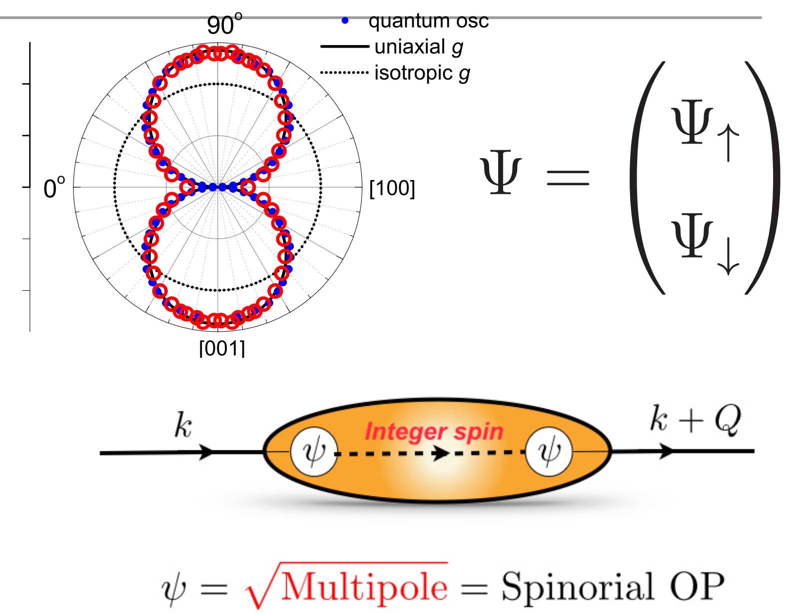
URu$_2$Si$_2$
Hastatic order?

Motivation: Kondo Lattice Physics



$NpPd_5Al_2$ T_C = 4.5K
Curie Law Superconductor



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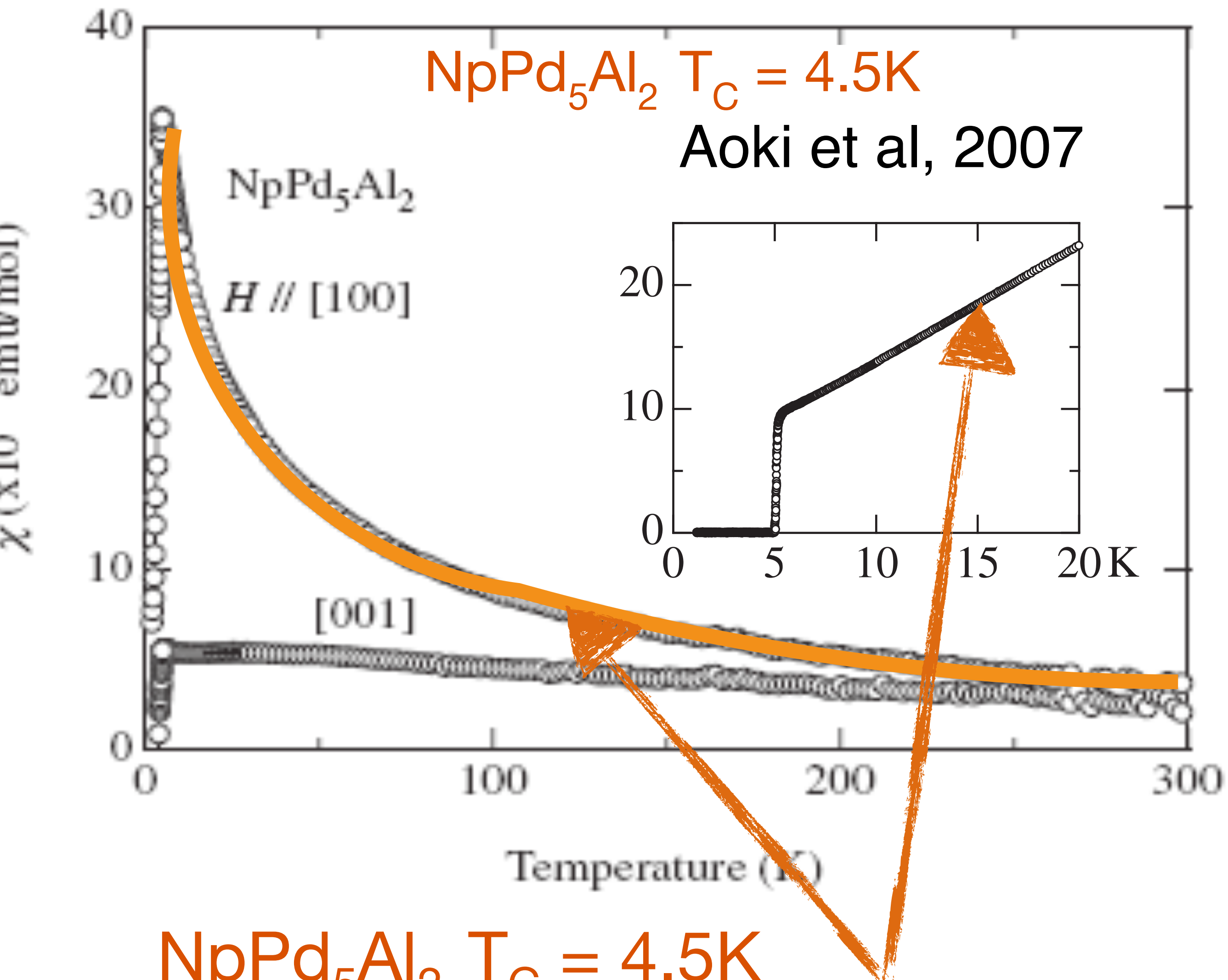

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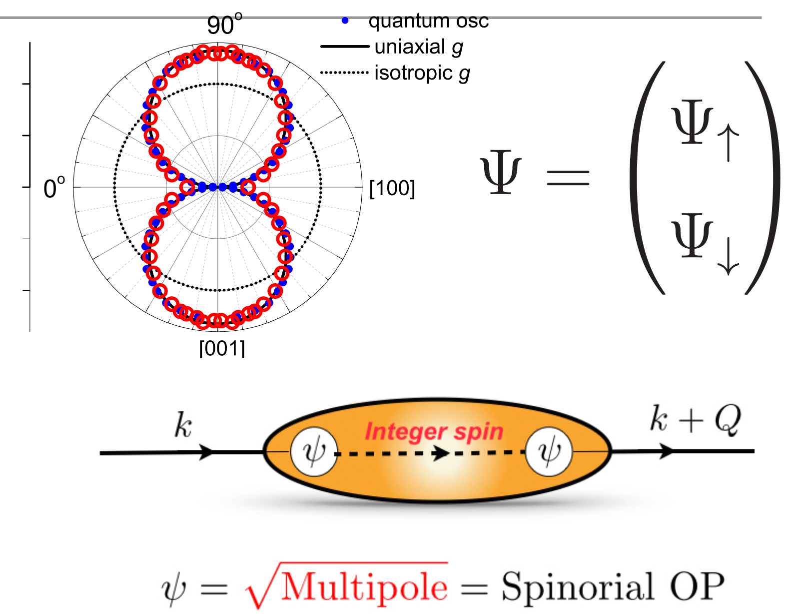

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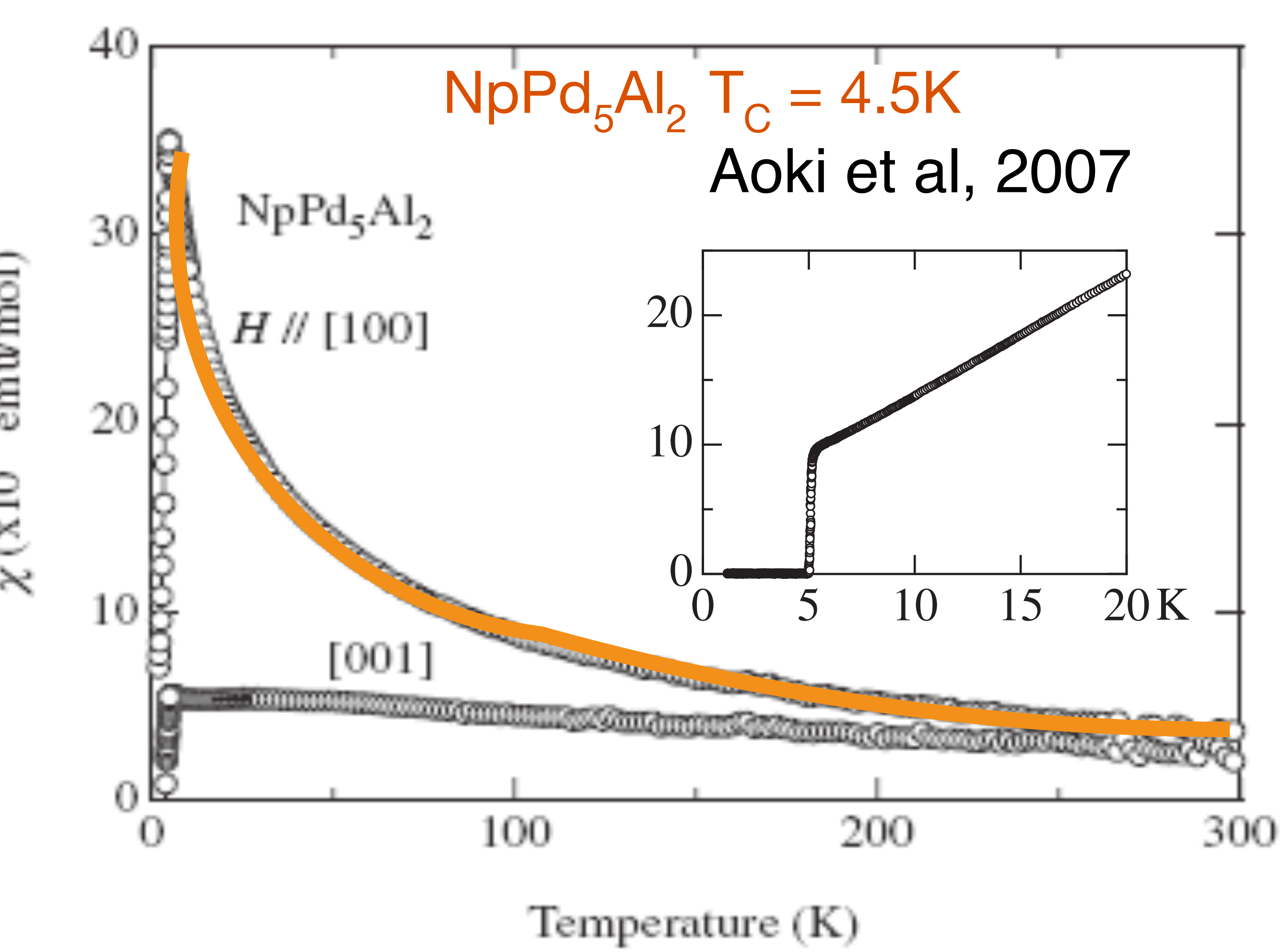
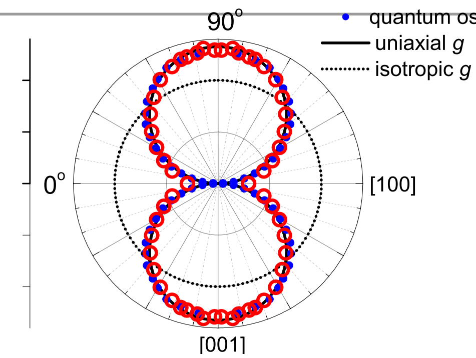
Sometimes, spin fractionalization appears to coincide with a phase transition.

Motivation: Kondo Lattice Physics





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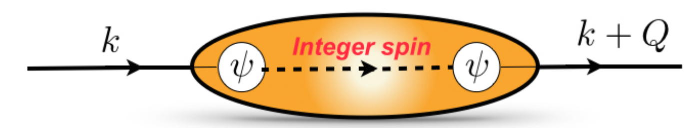

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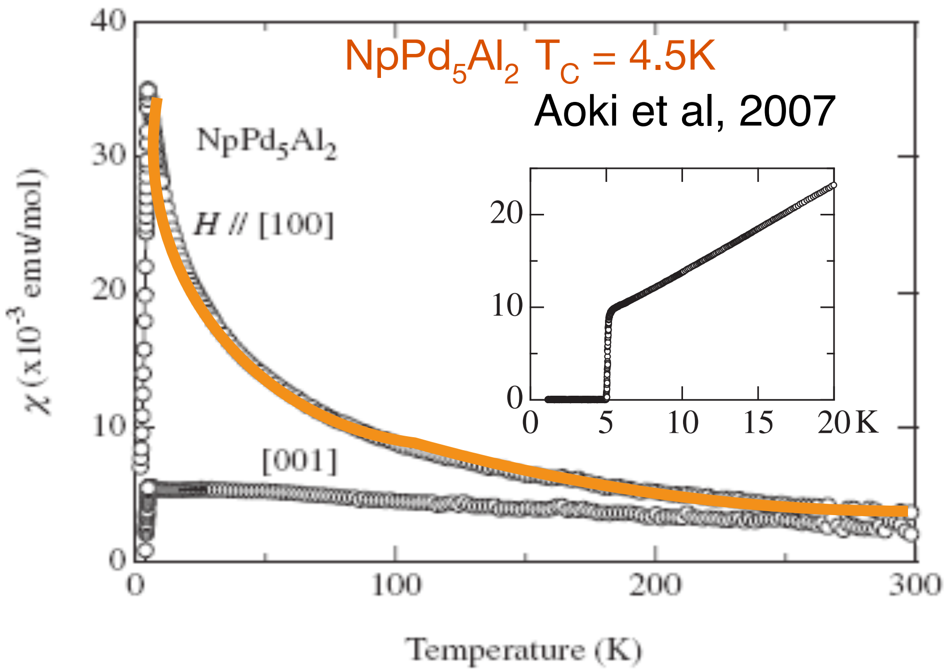
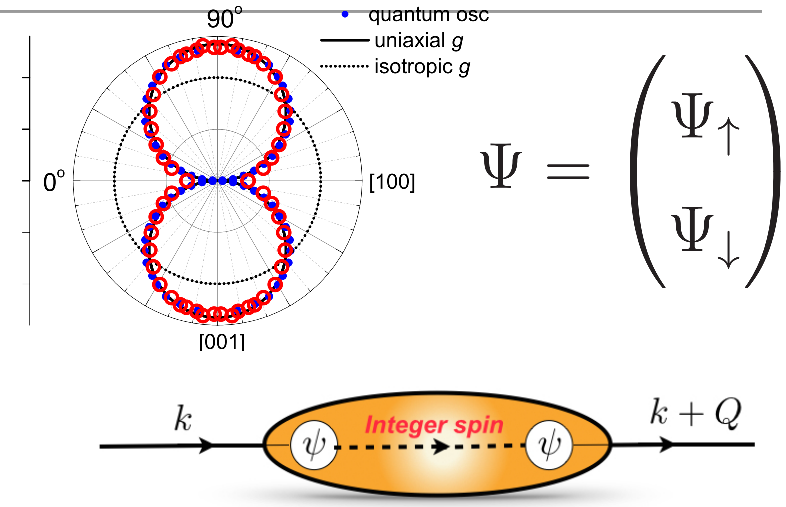


Kondo effect and SC coincide.



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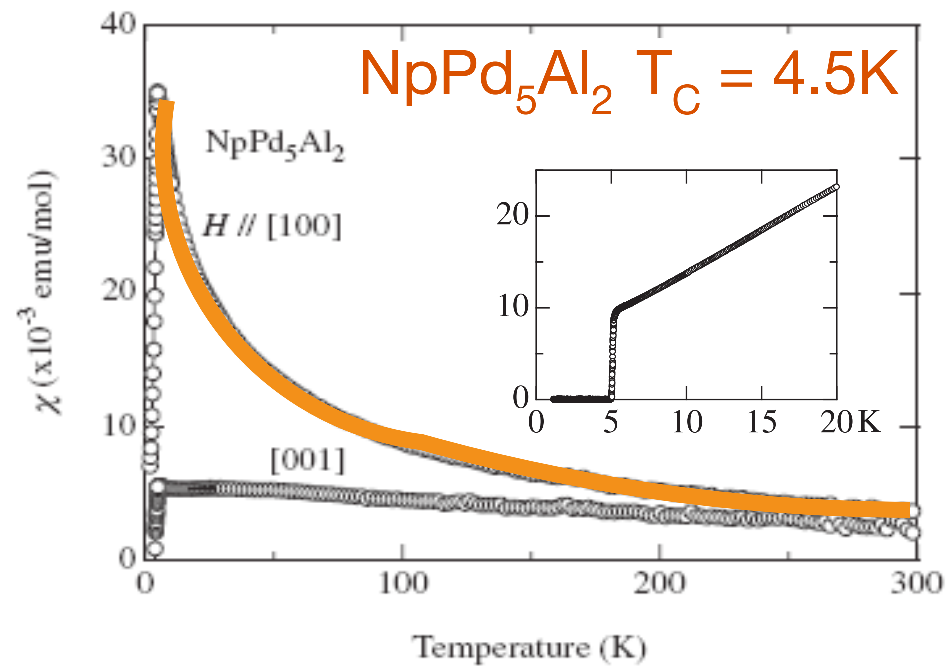

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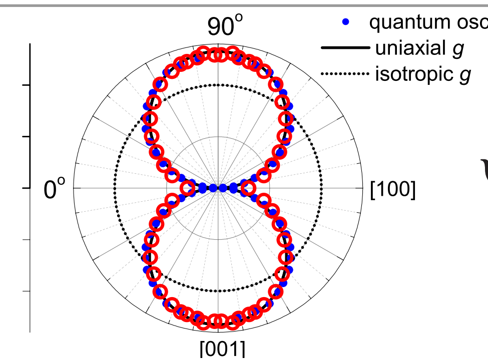

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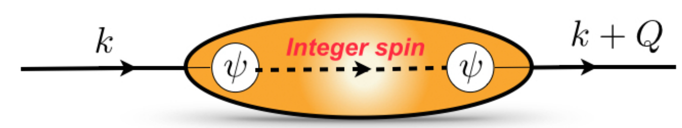

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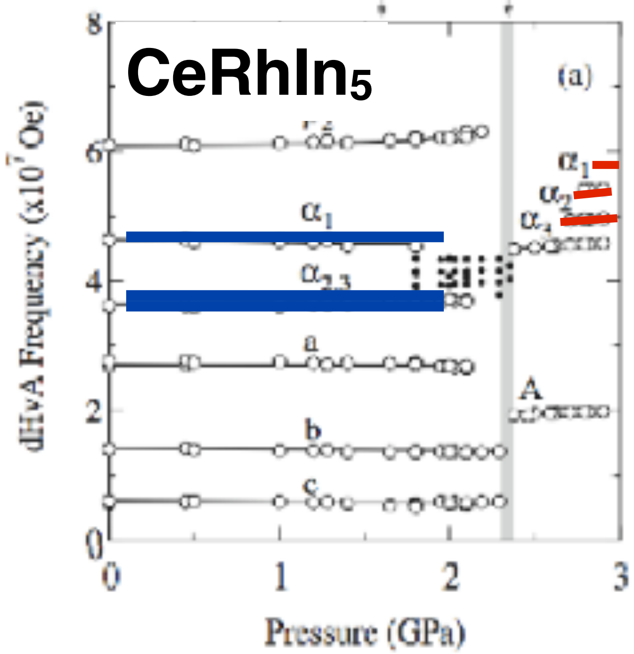

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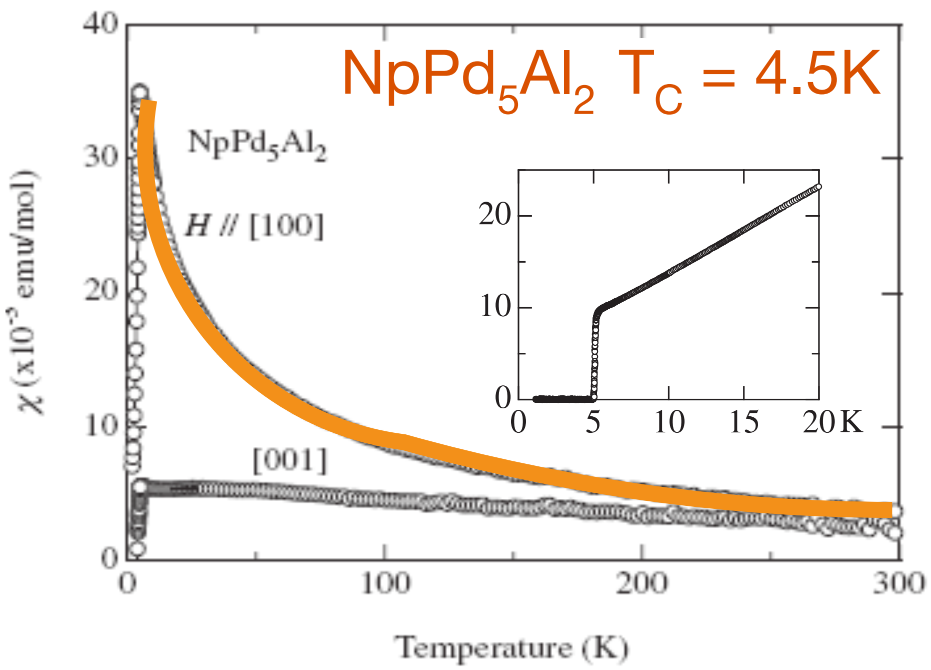
Fractionalization



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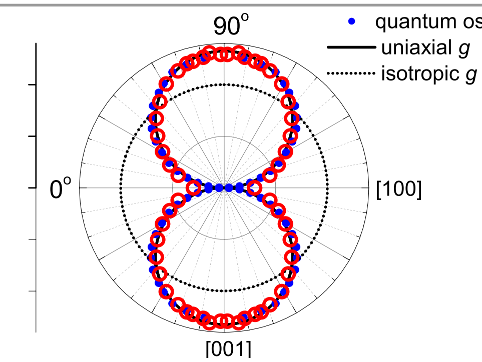

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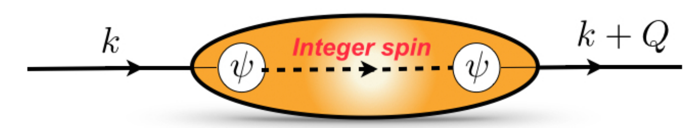

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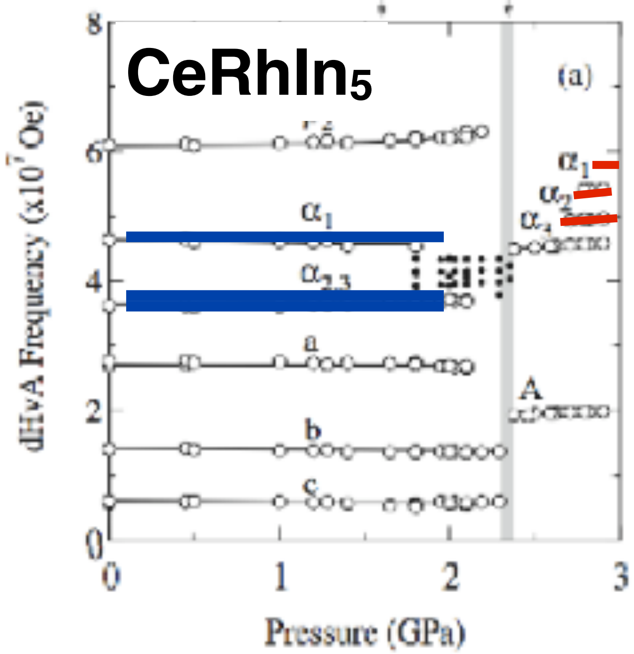

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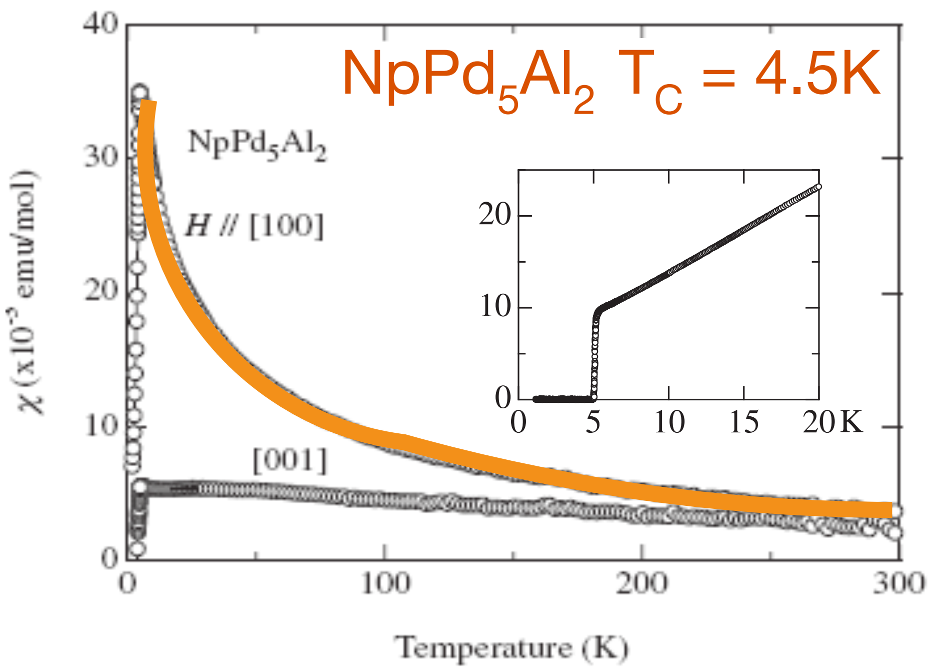

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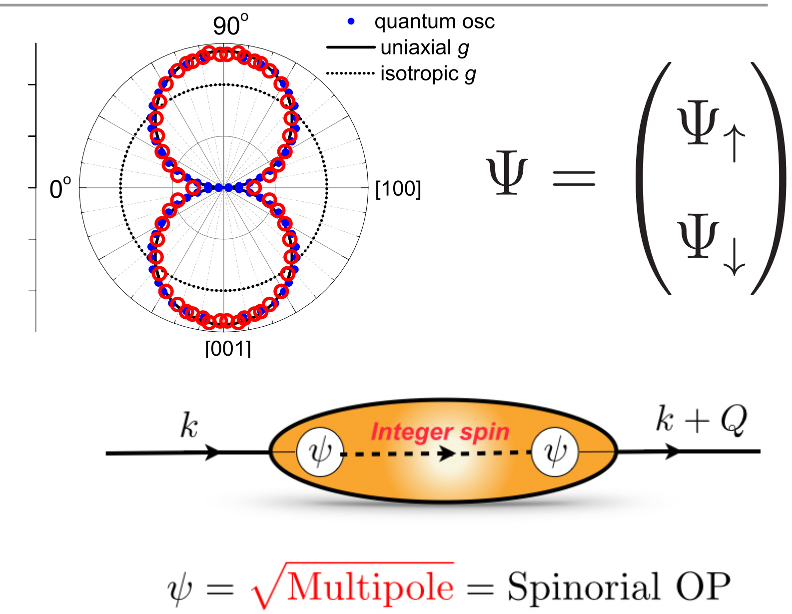

URu_2Si_2
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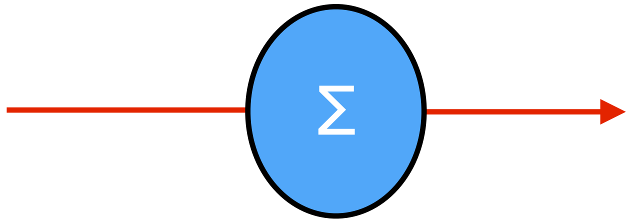
Conjecture:

Order can fractionalize

The Link with fermions

Dyson self-energy

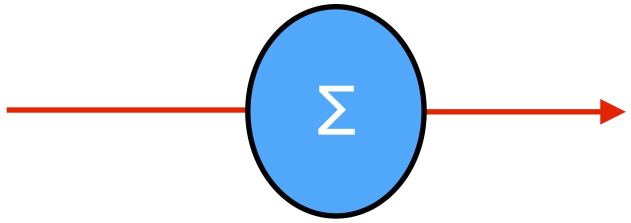


Conventional Broken Symmetry: Local in time

$$\Sigma_{\alpha\beta}(2,1) = M_{\alpha\beta}\delta(2-1)$$

The Link with fermions

Dyson self-energy



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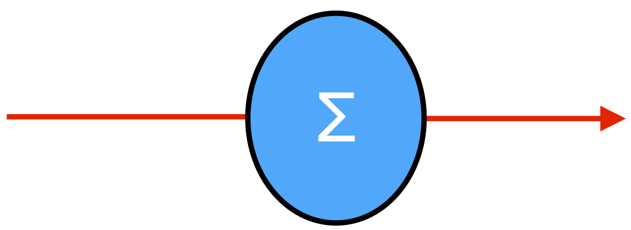

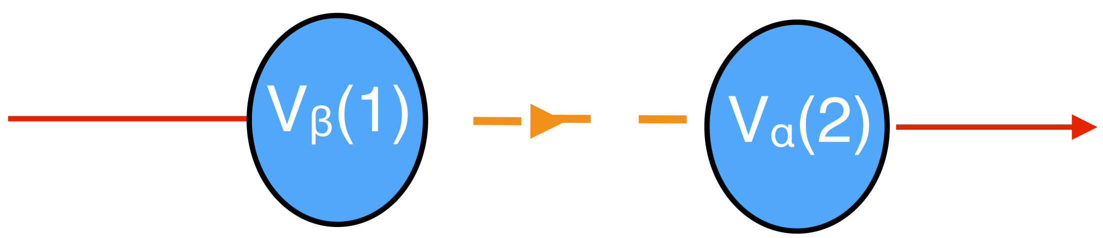

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Order Fractionalization: non-local in time.

The Link with fermions

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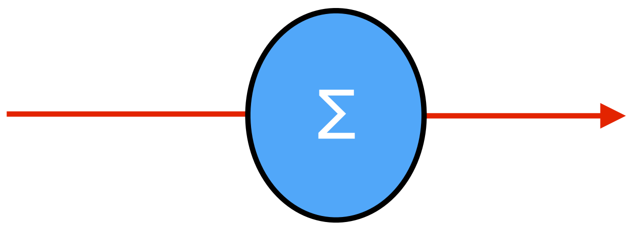

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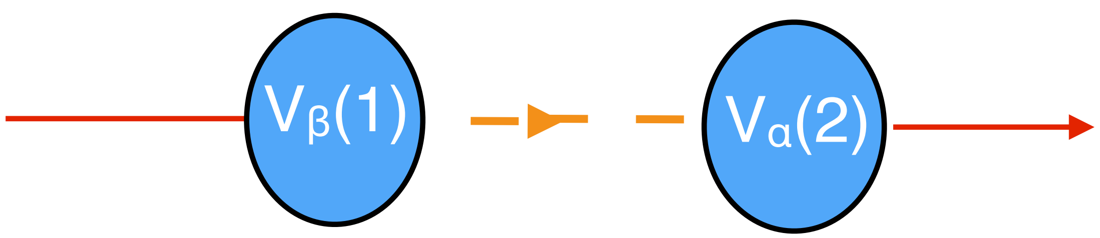

Order Fractionalization: non-local in time.

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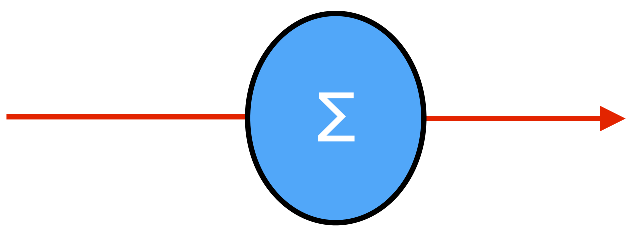
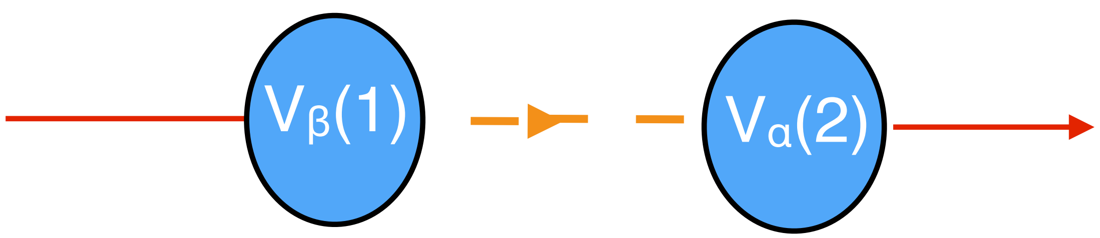
The Link with fermions

Dyson self-energy





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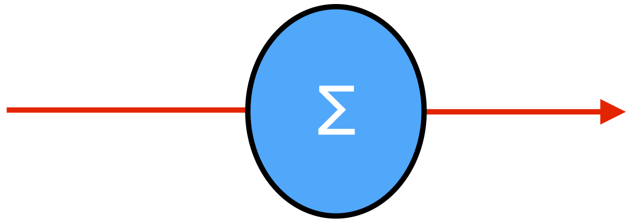
- *REDUCIBLE* product rep.

The Link with fermions

Dyson self-energy



Conventional Broken Symmetry: Local in time

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- *Irreducible* rep, e.g. $M_{\alpha\beta} = m.\sigma_{\alpha\beta}$

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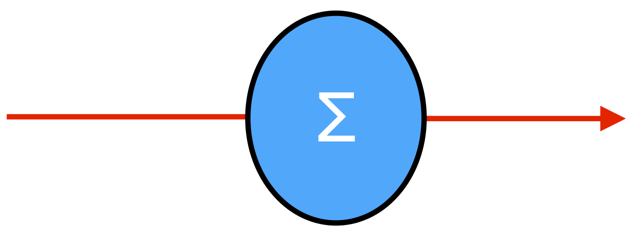

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Order fractionalization, if it occurs, is linked to the formation of fermionic bound-states

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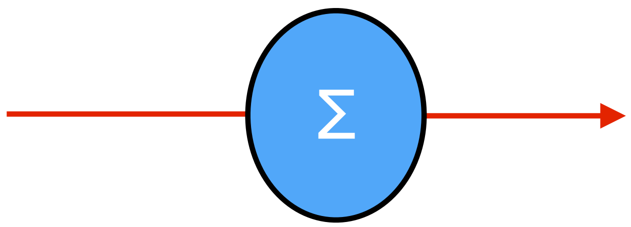

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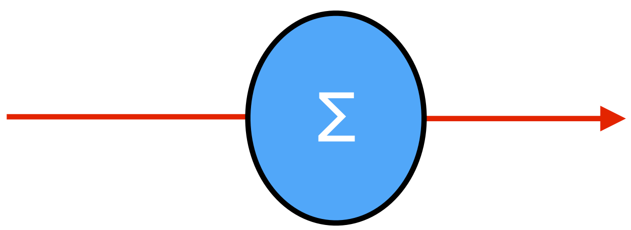

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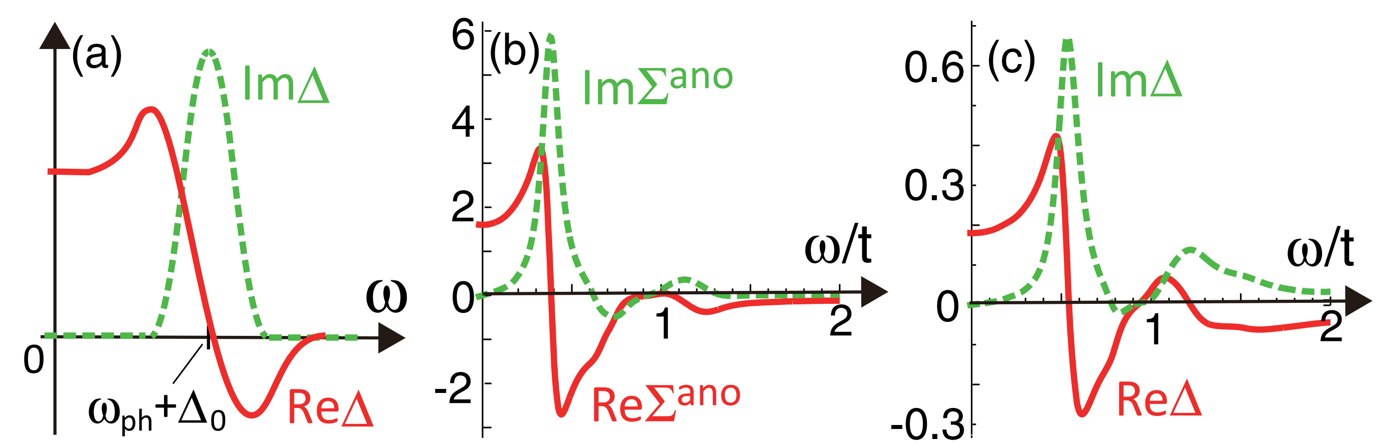

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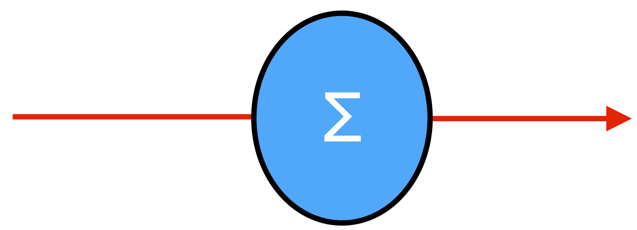

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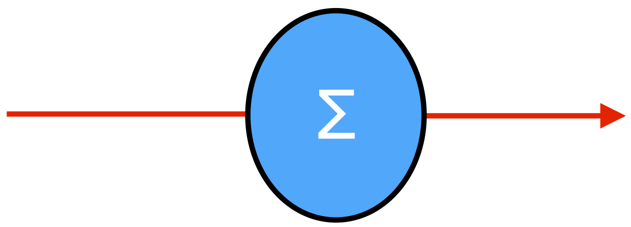

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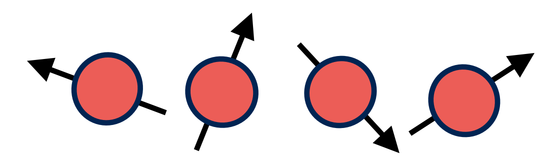
From Weiss to Kondo

Curie-Weiss Magnetism



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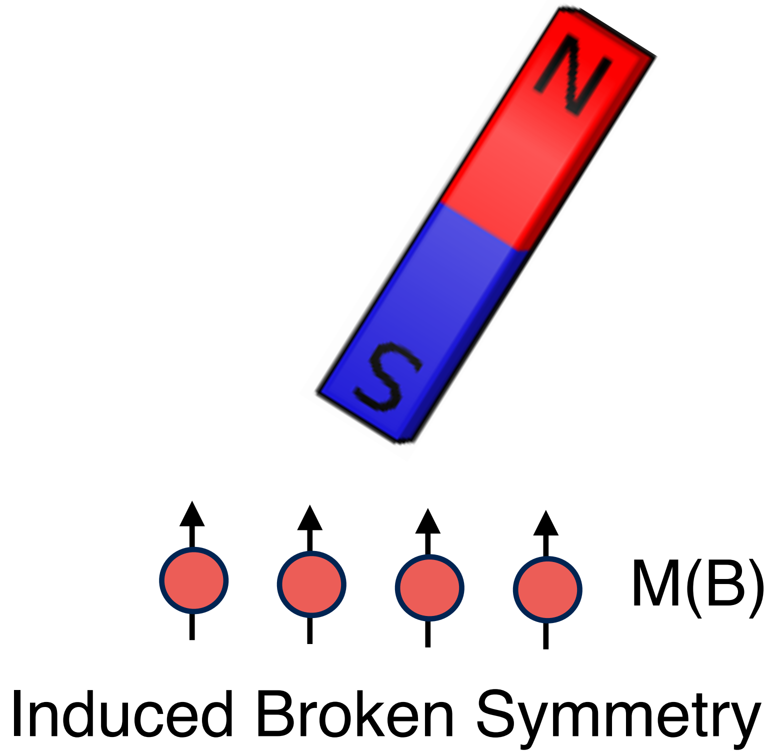

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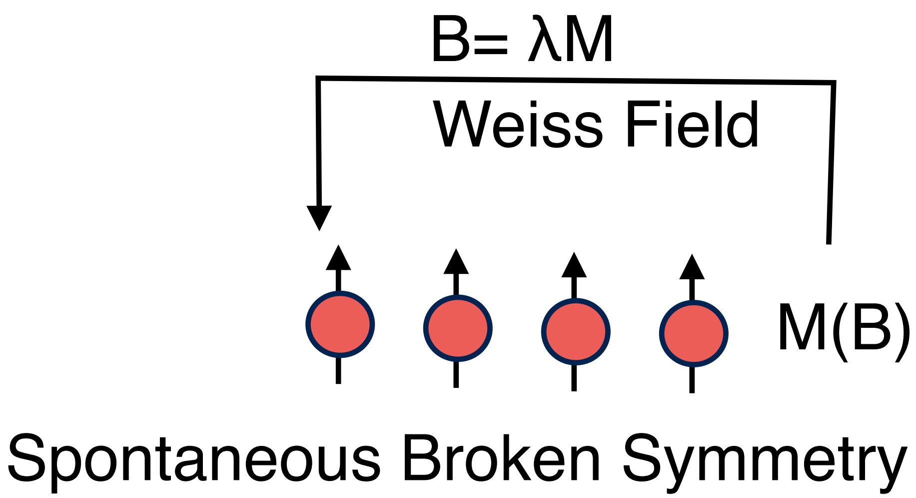

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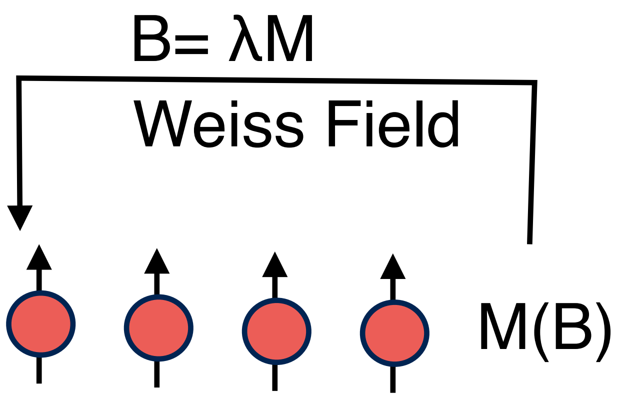
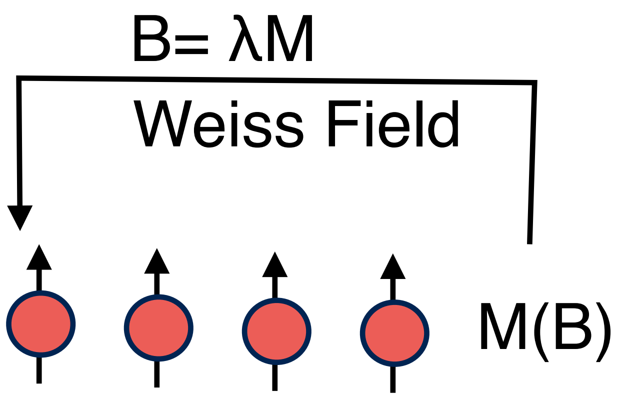


Pre-requisite:

From Weiss to Kondo



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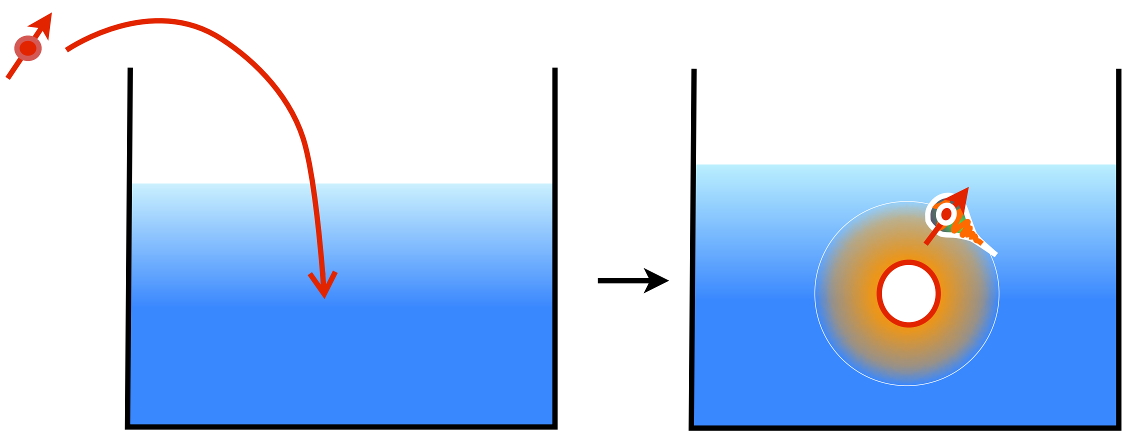
- Find an impurity model where we can *induce* Order Fractionalization with an external field.

From Weiss to Kondo

Kondo Model: ideal setting



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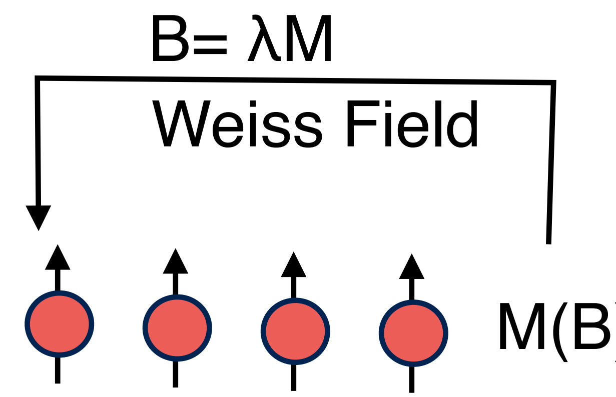

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Fractionalization and Hybridization

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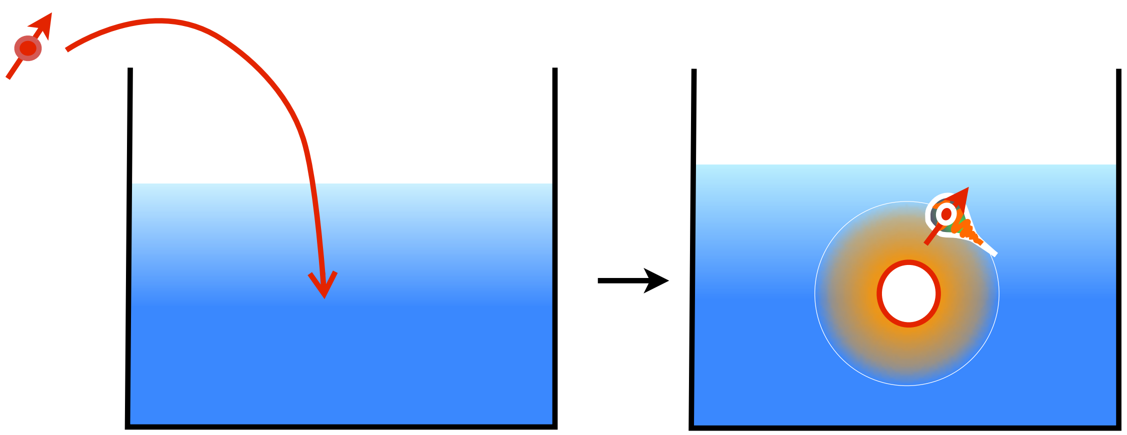

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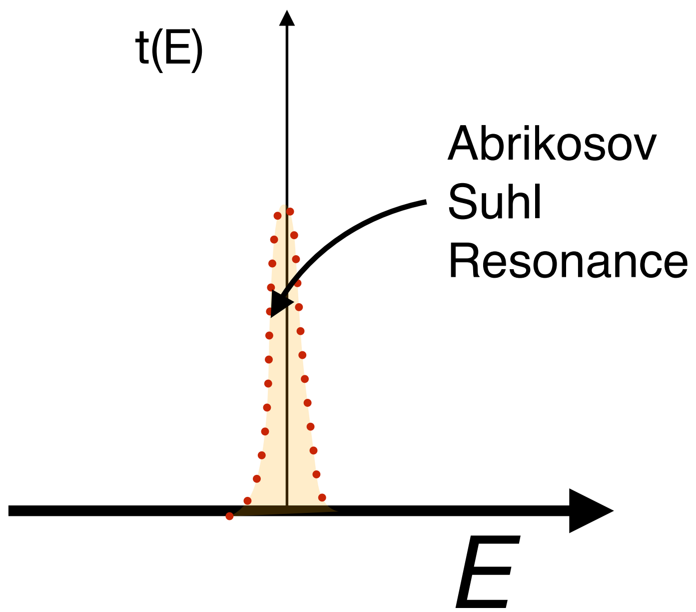

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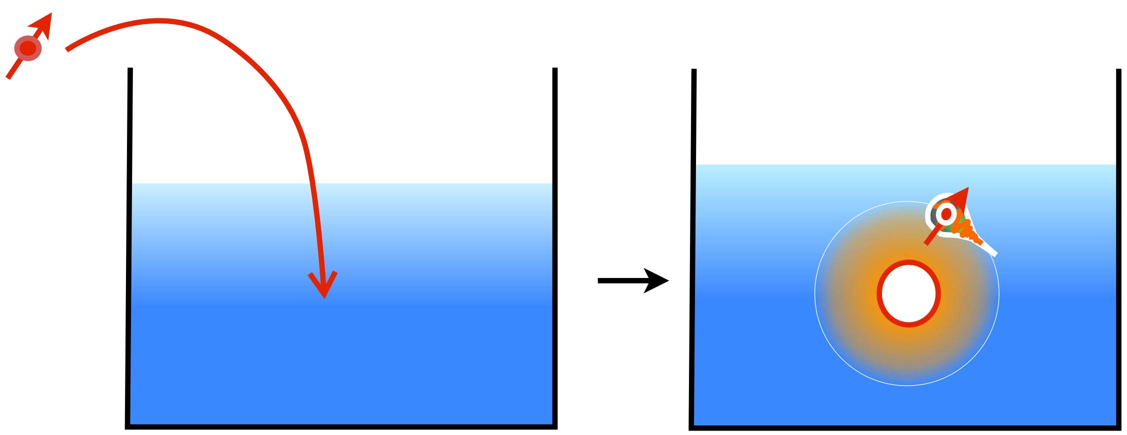

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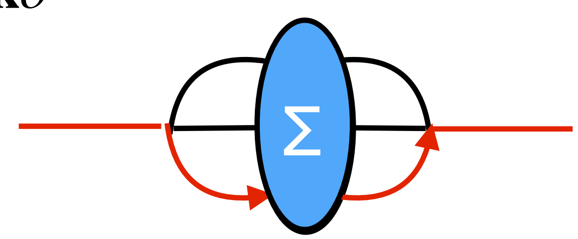

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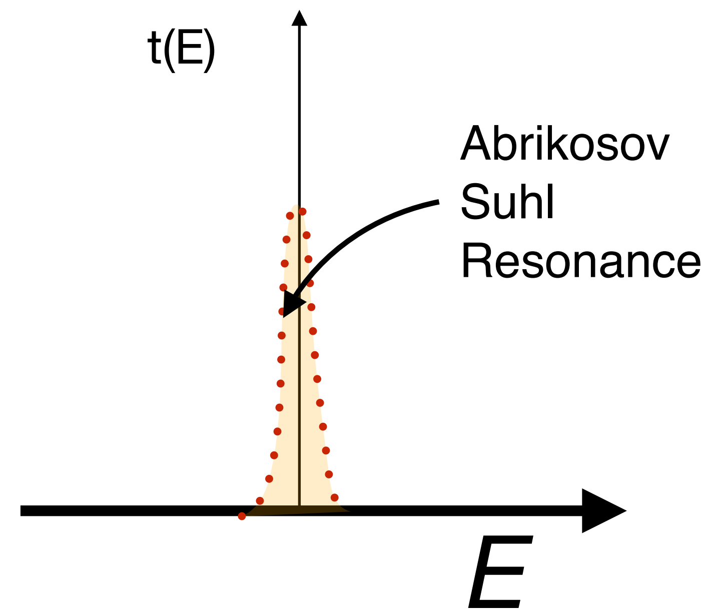

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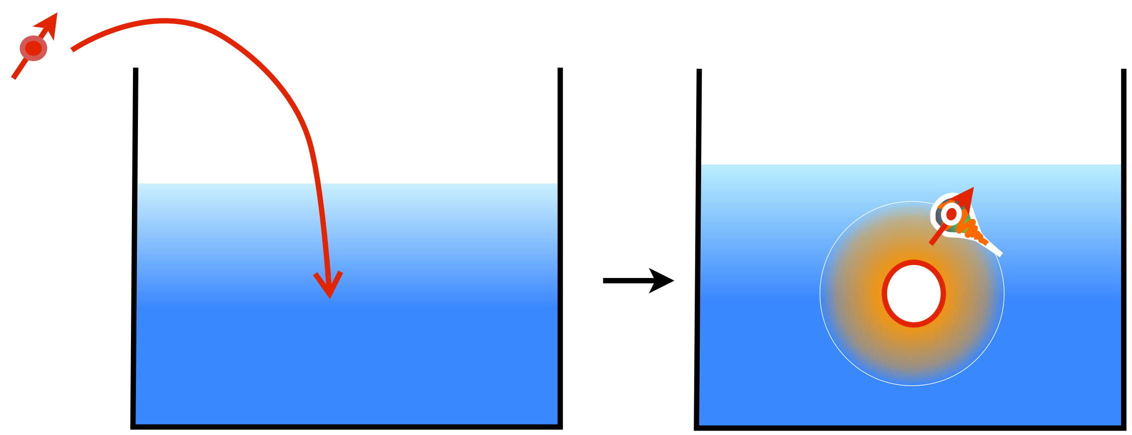

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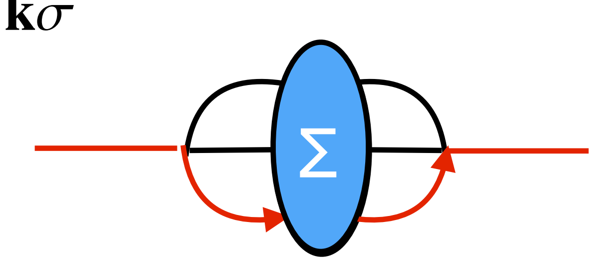

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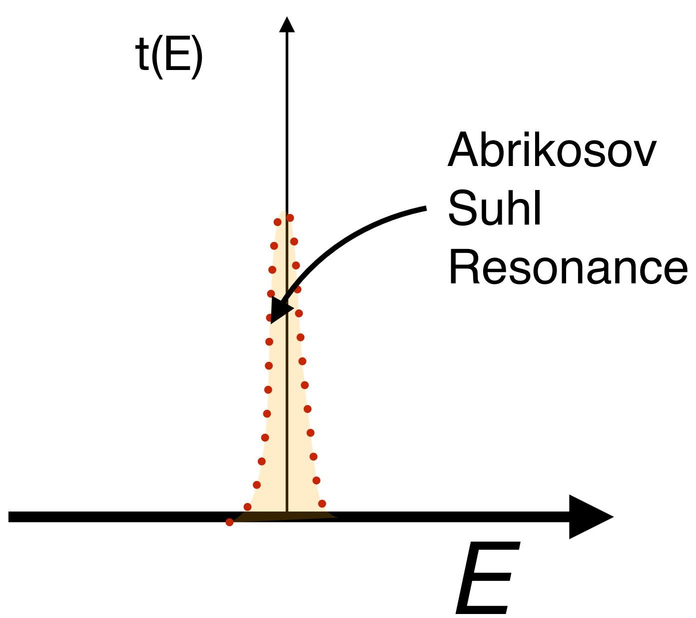
Nozieres Local Fermi Liquid
Large N Mean-Field Theory



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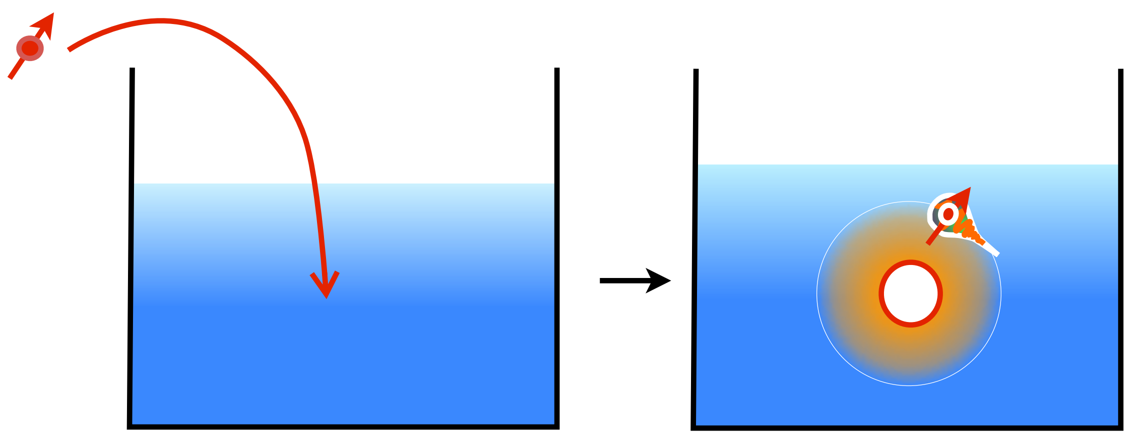

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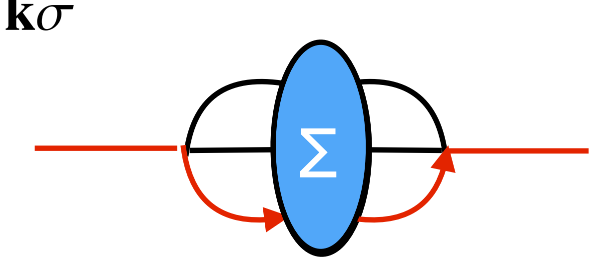

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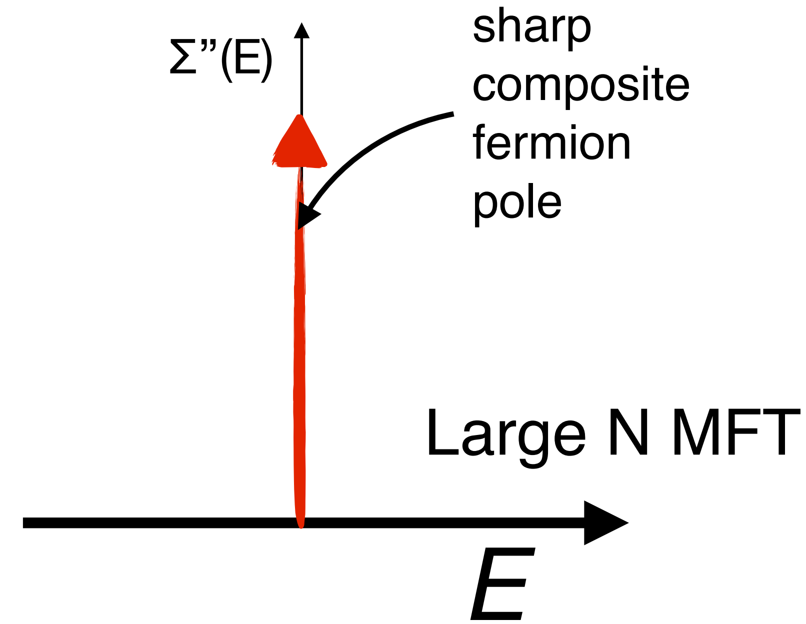

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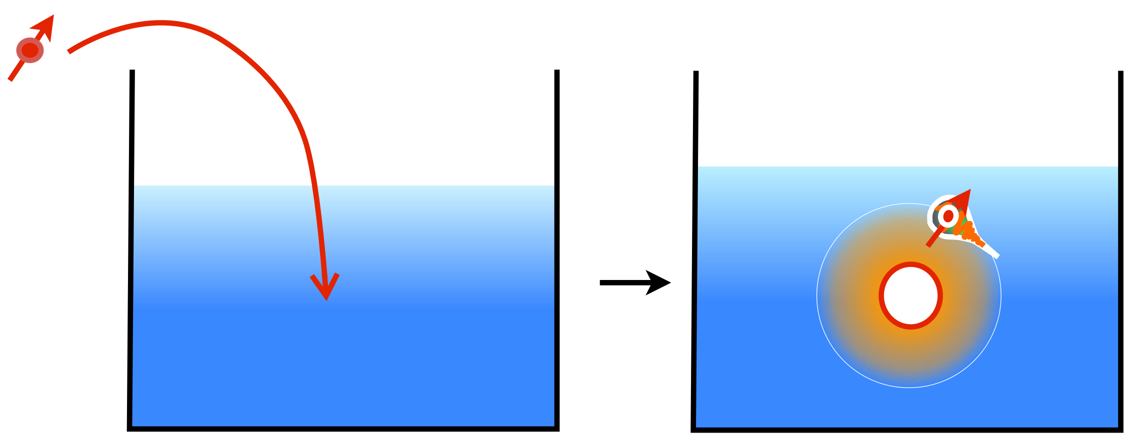

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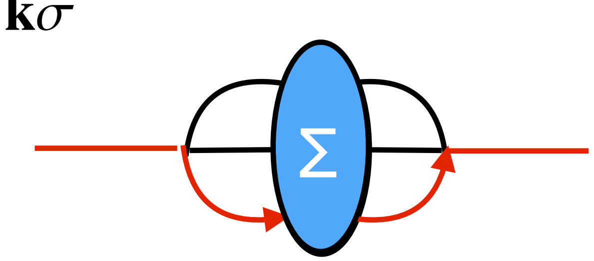

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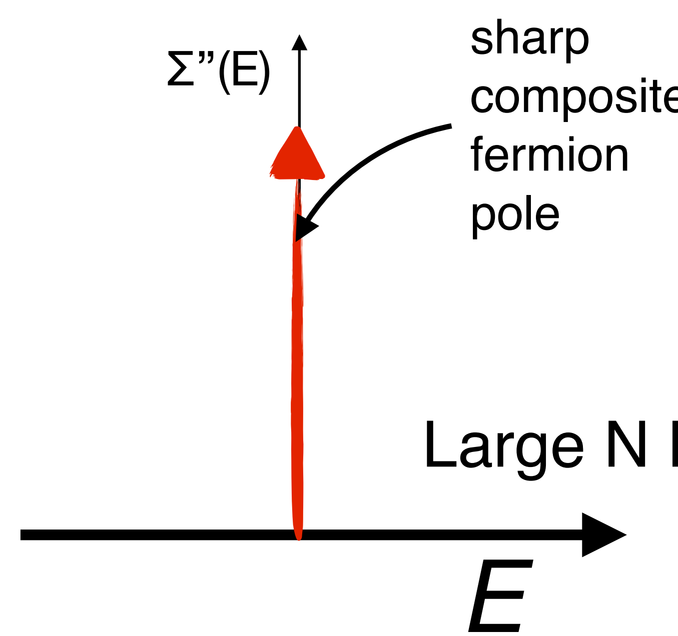

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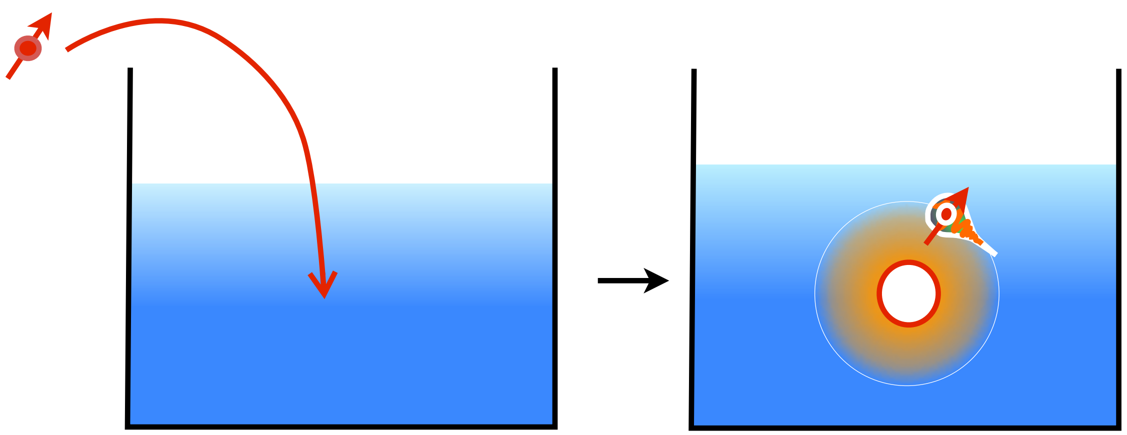

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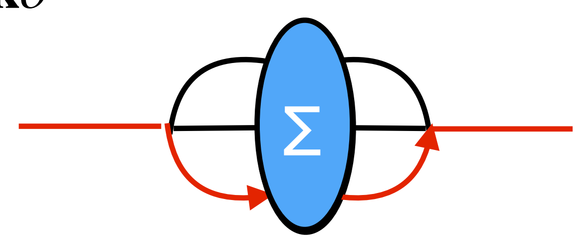

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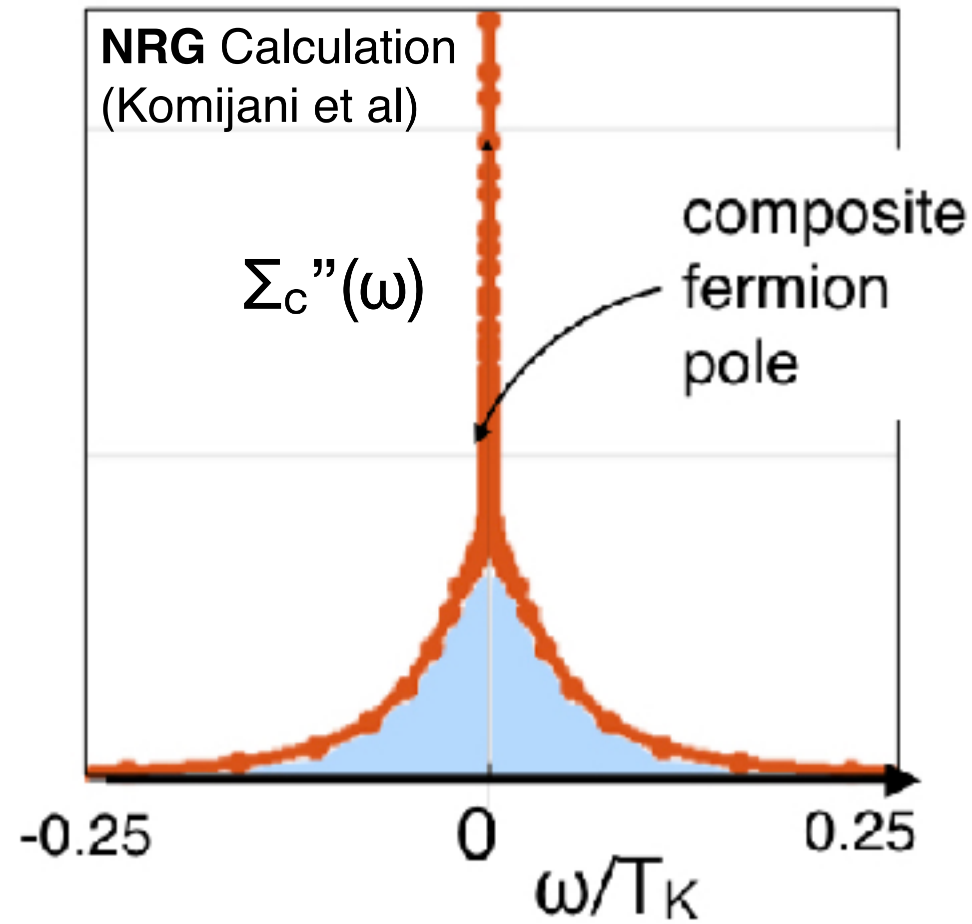

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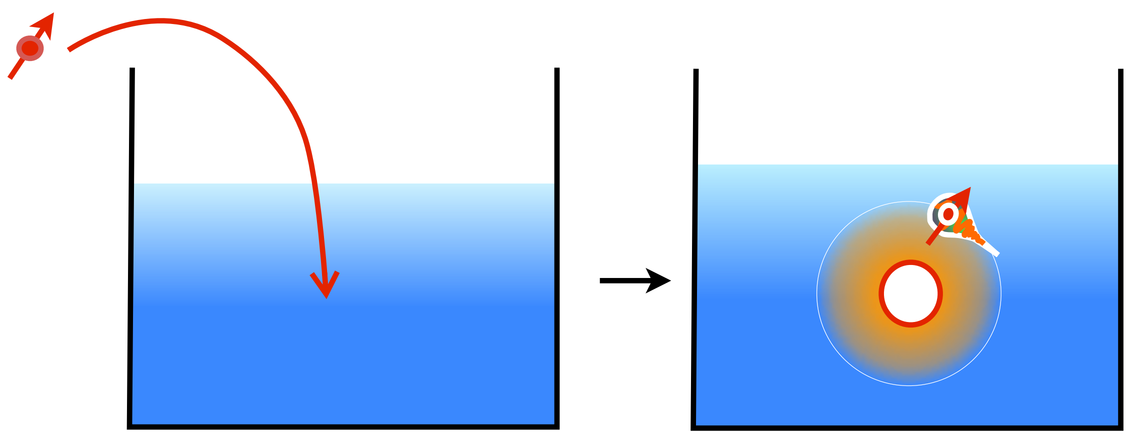

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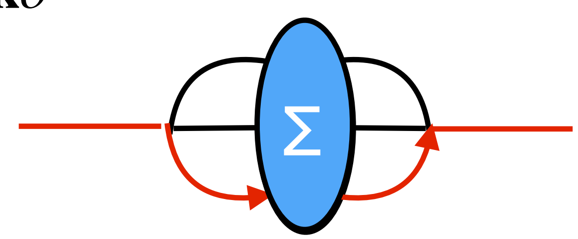

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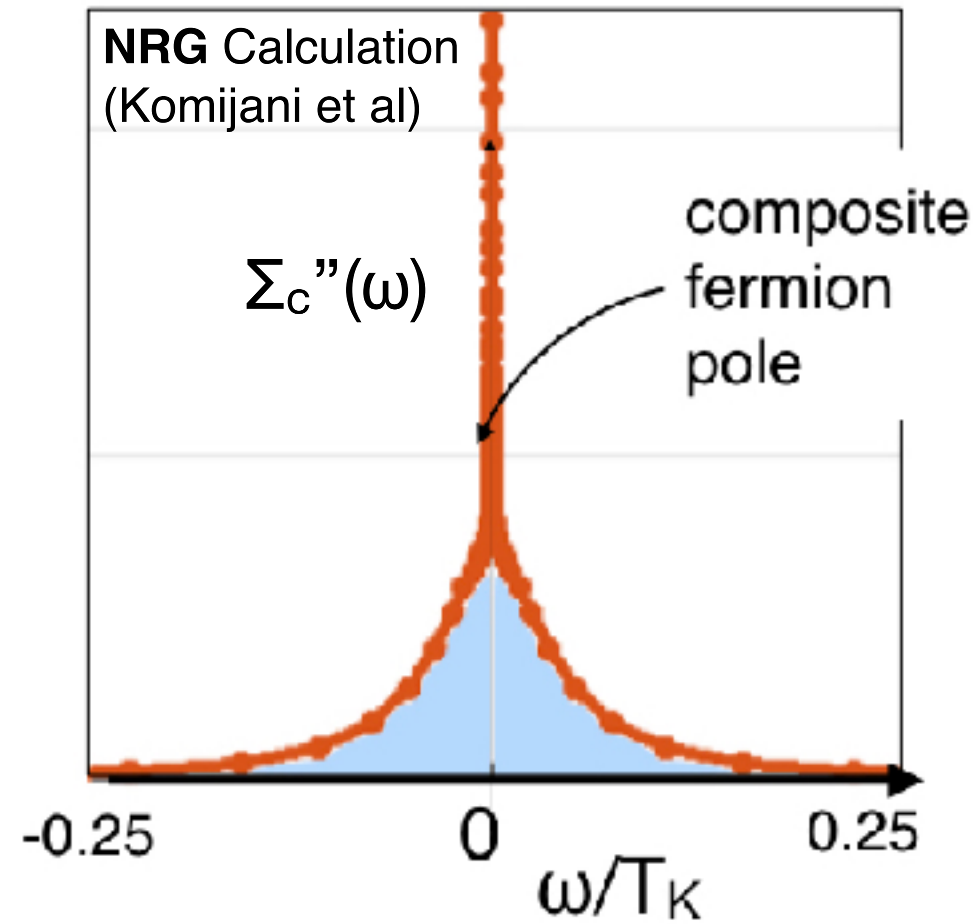

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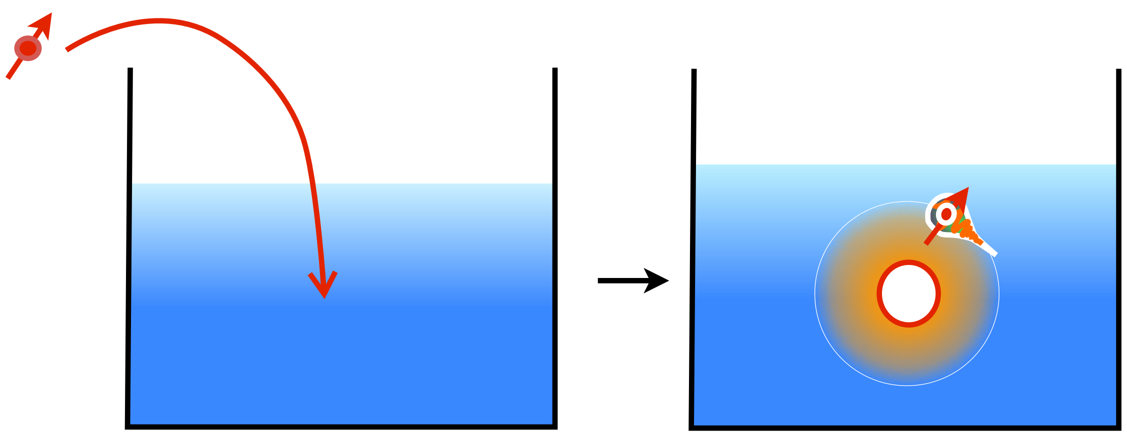

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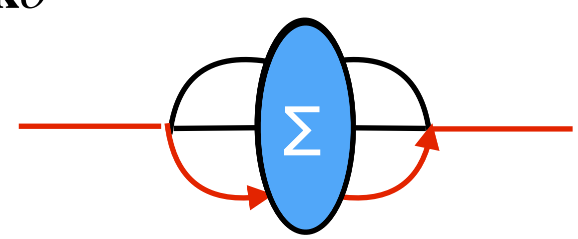

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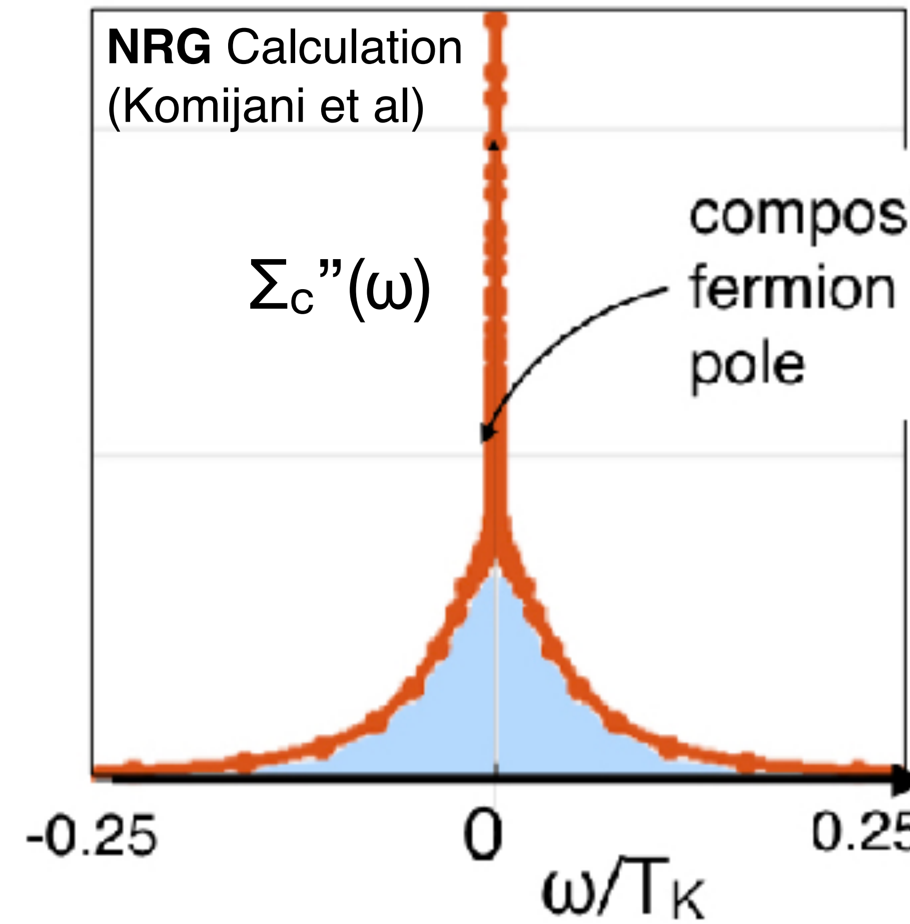

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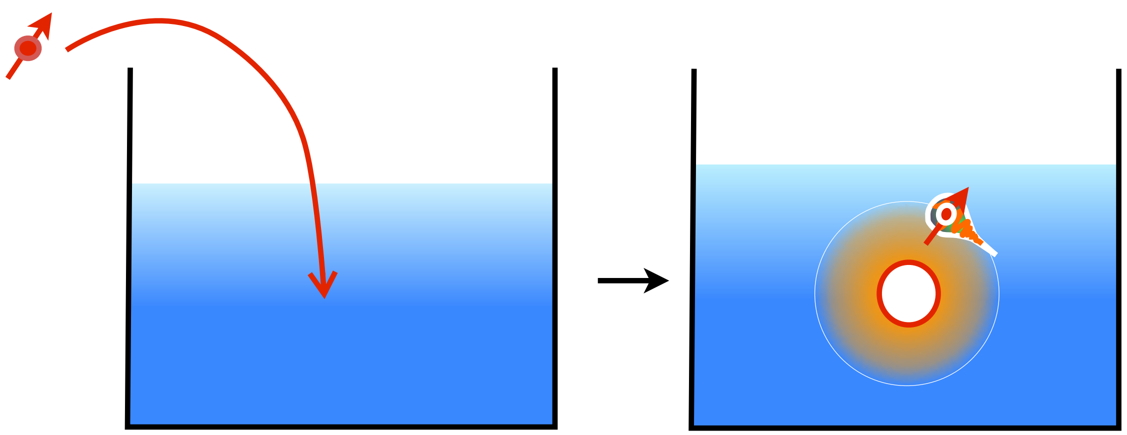

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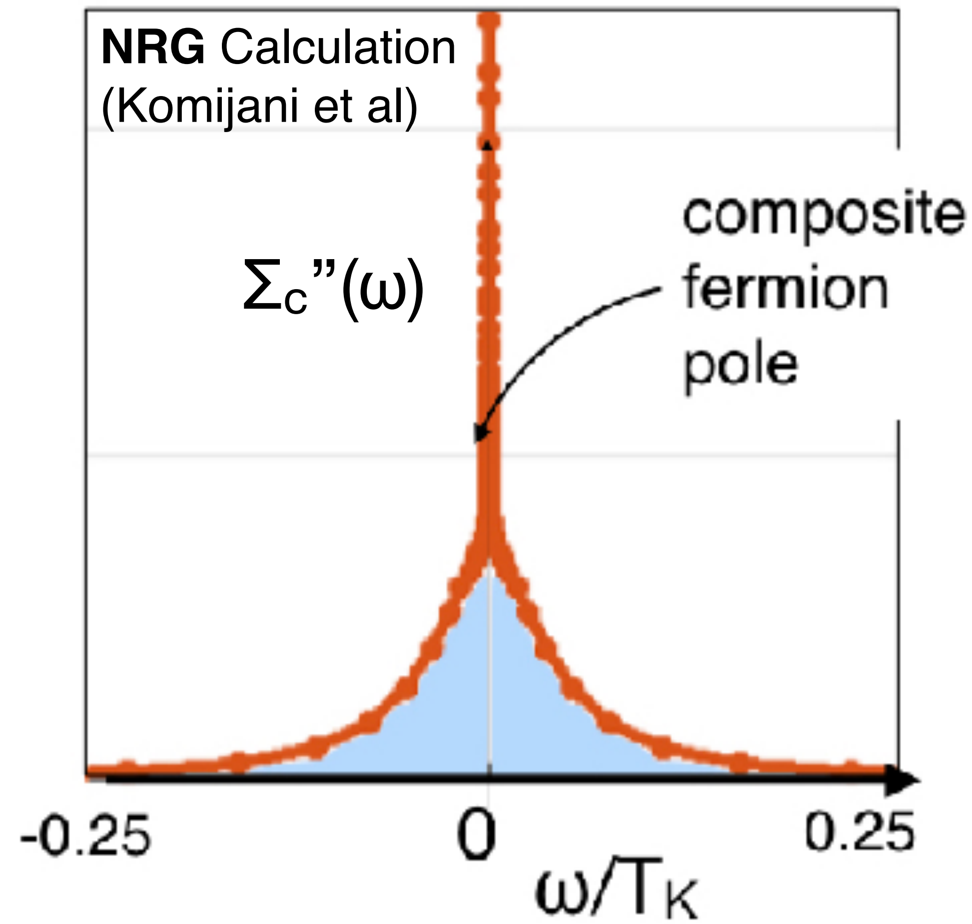

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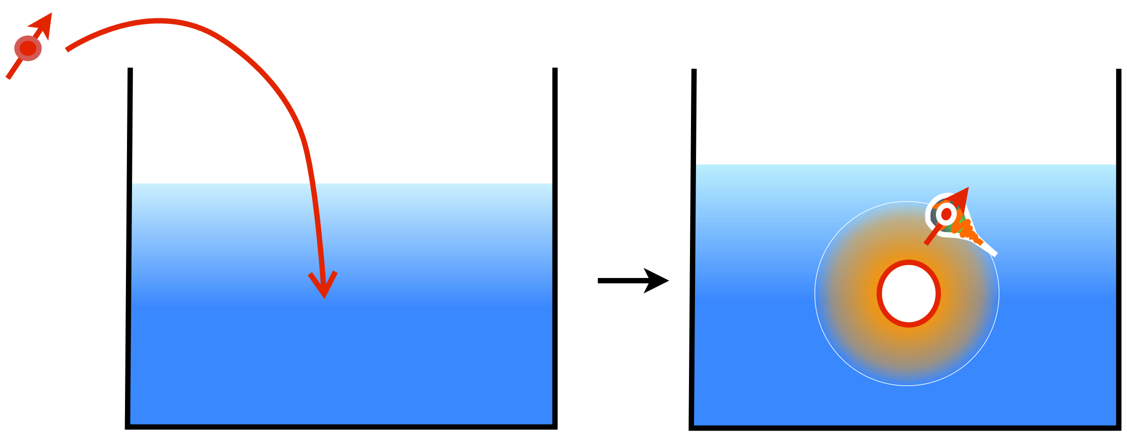

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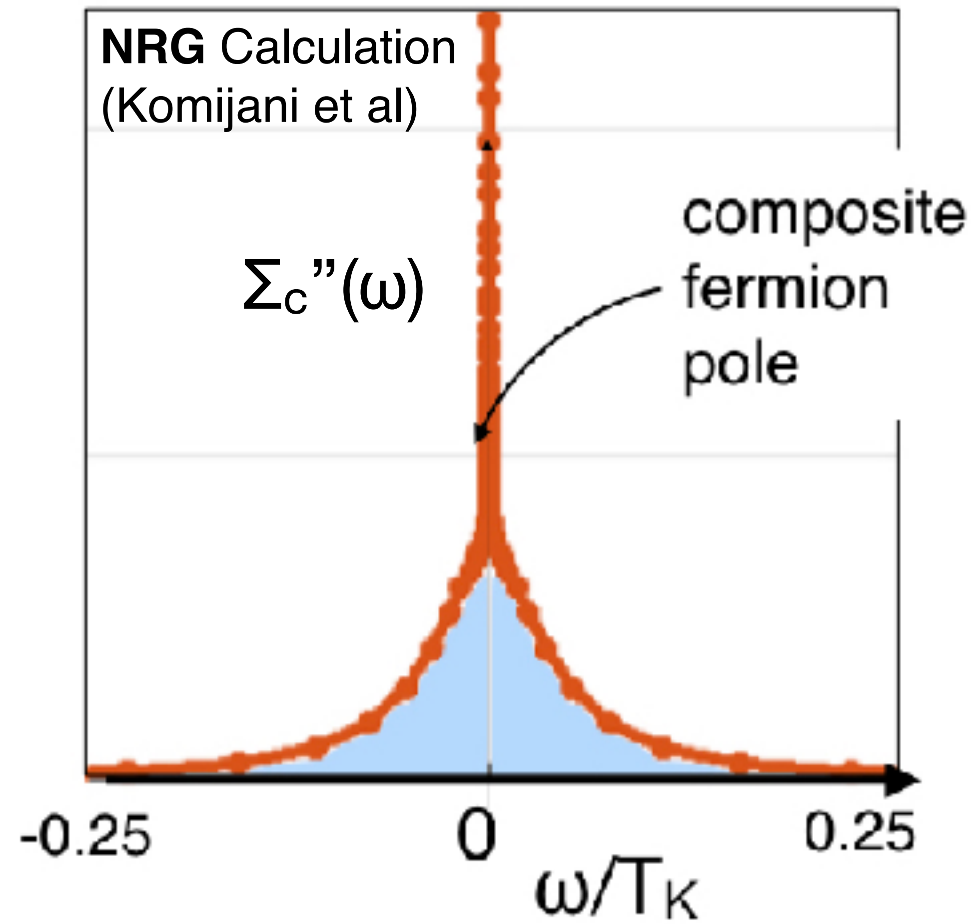

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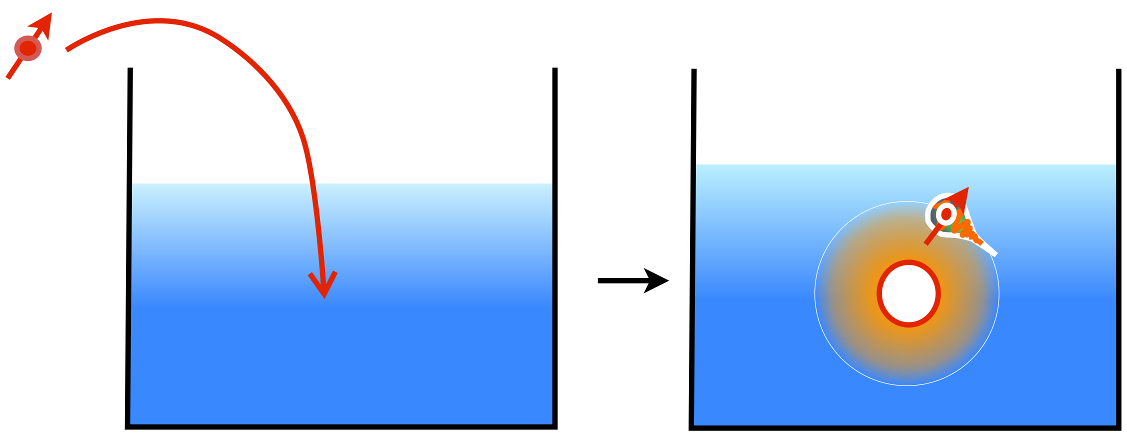

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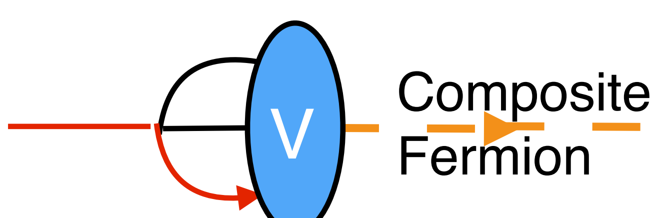

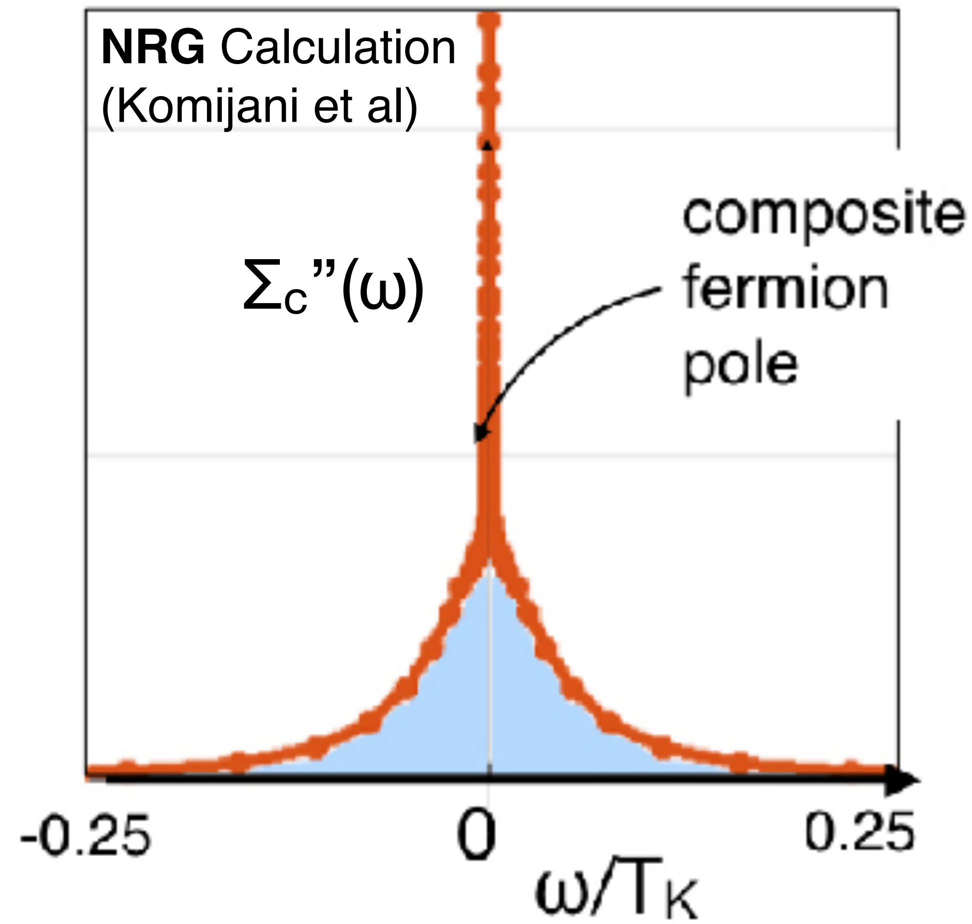

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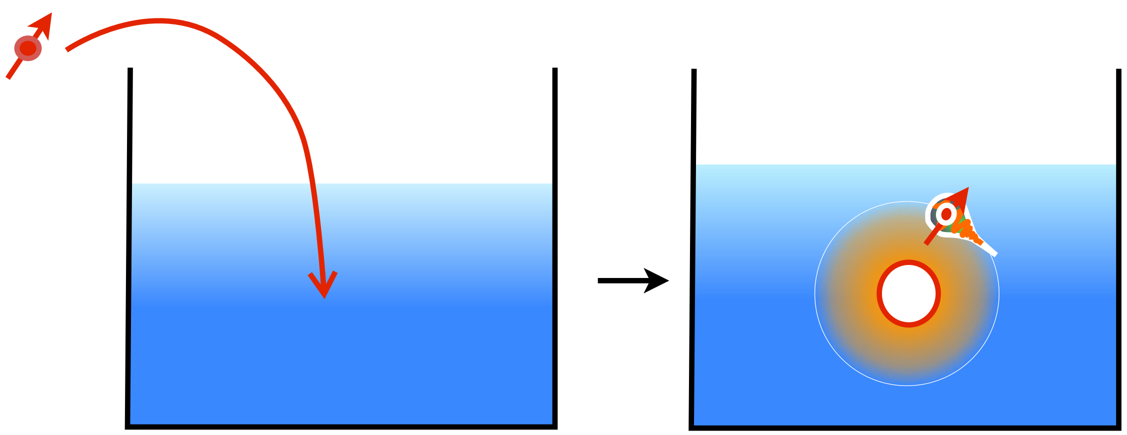

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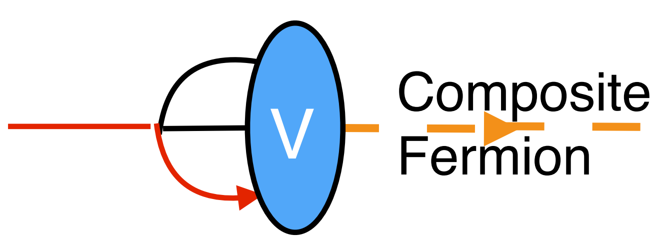

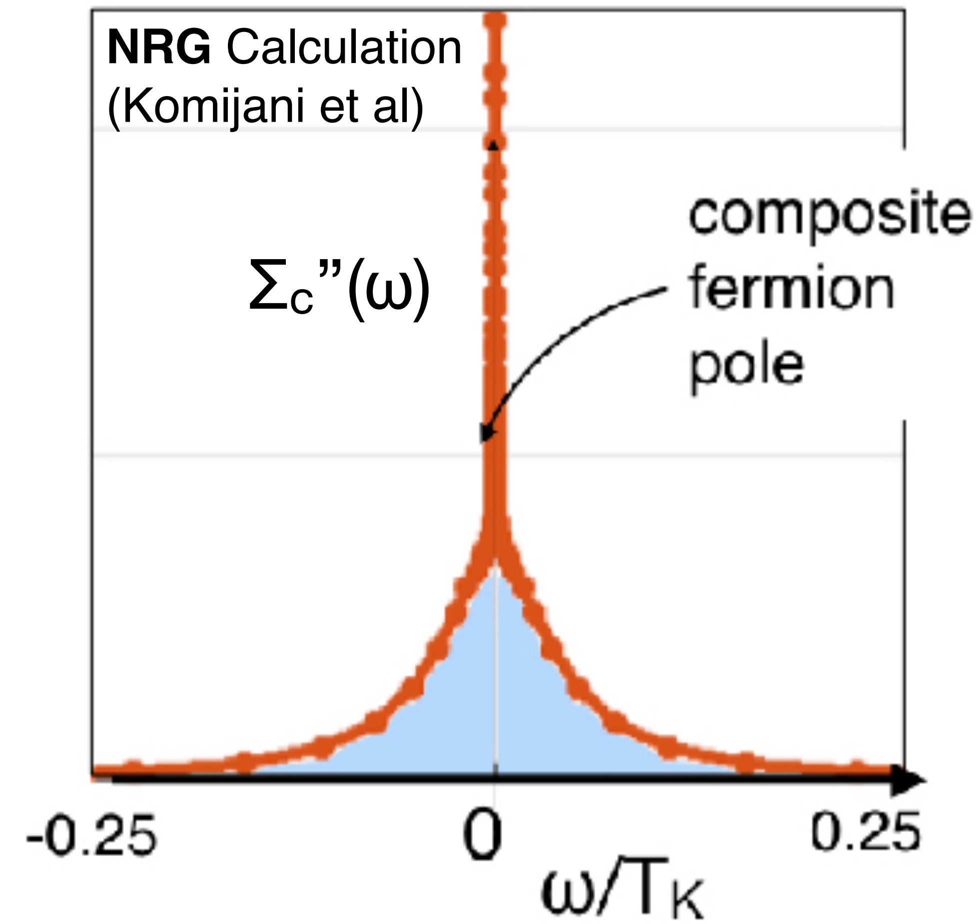

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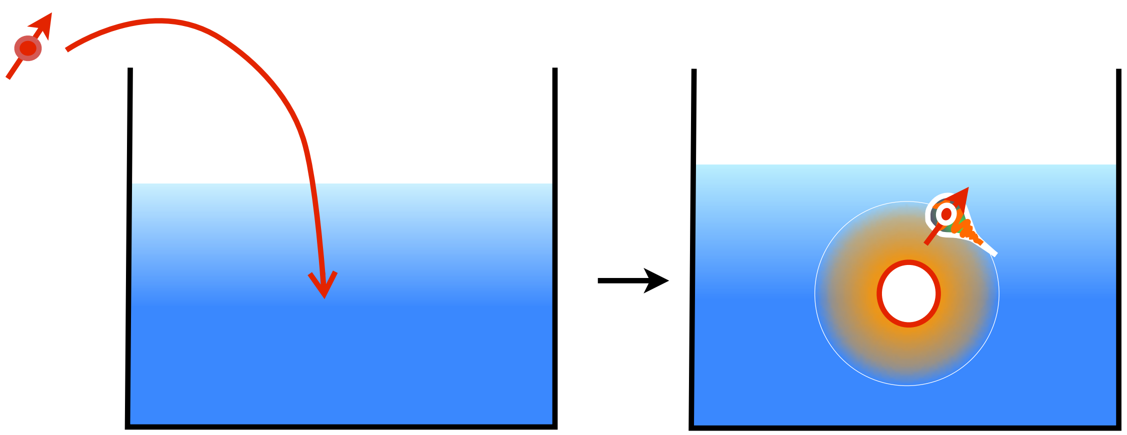
$$\mathcal{F}_\alpha = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow V f_\alpha(0)$$

Three-body Bound State

Fractionalization and Hybridization

Kondo Model: ideal setting



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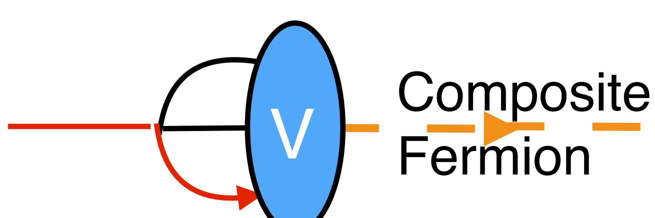
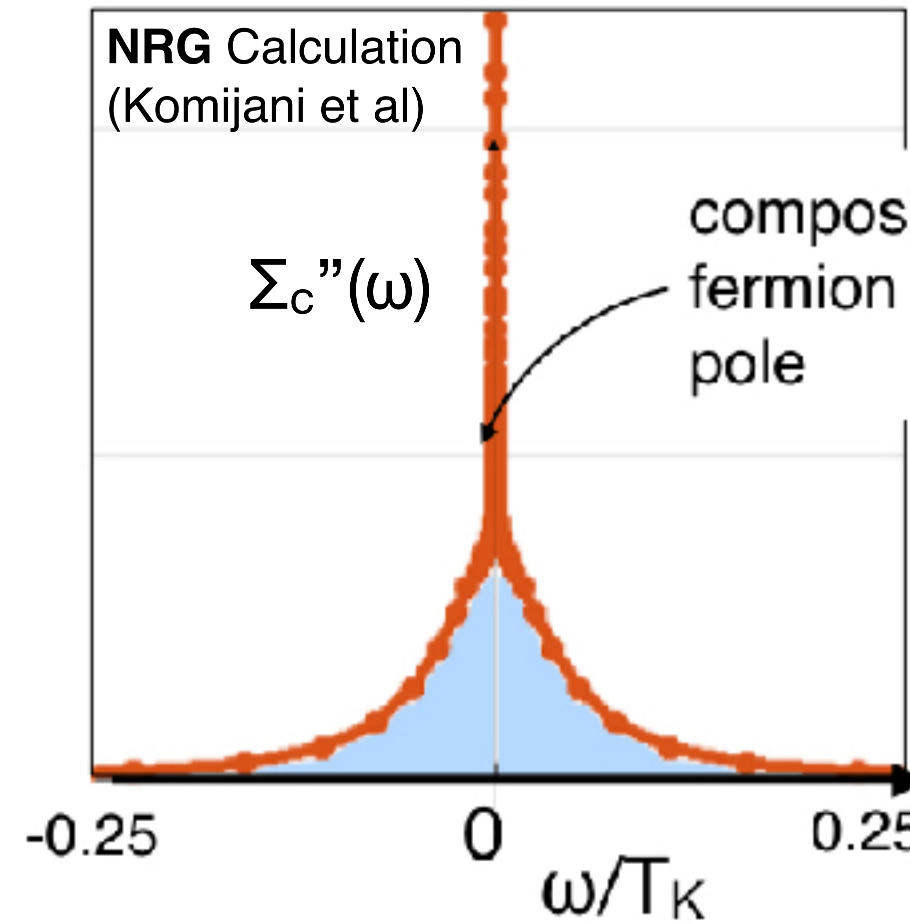




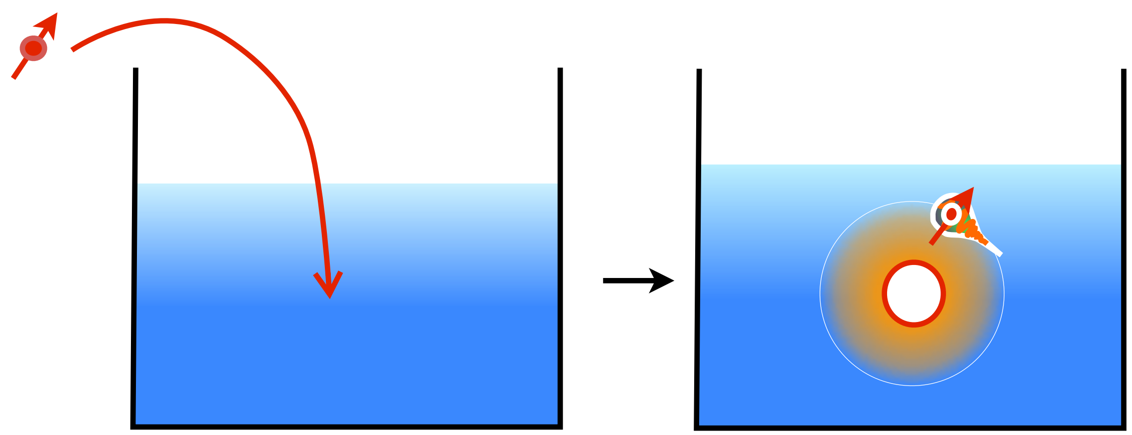
But large N assumes fractionalization,
Does it happen at S=1/2? Confirmed.

$$\mathcal{F}_\alpha = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow V f_\alpha(0)$$

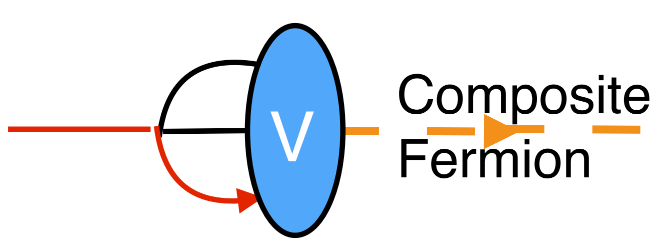
Three-body Bound State

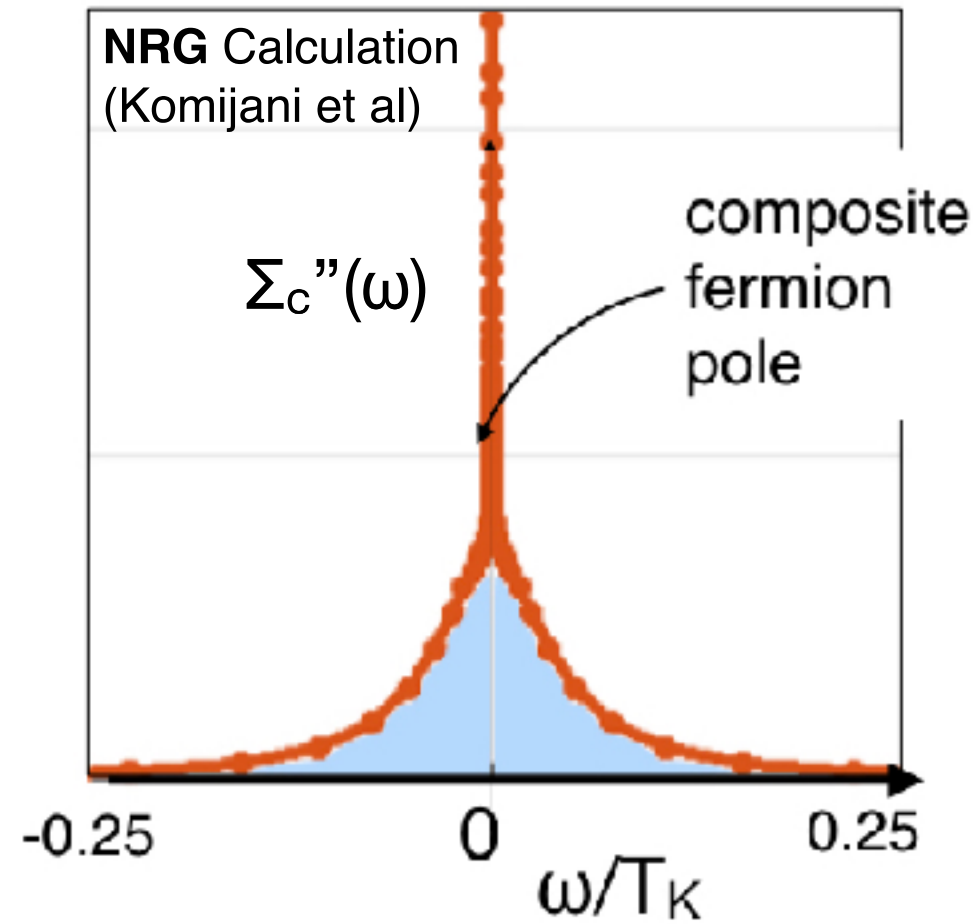
Fractionalization and Hybridization

Kondo Model: ideal setting



$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + V(\psi^{\dagger}_{\sigma} f_{\sigma} + \mathrm{H.c})$$





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$$\mathcal{F}_{\alpha} = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow V f_{\alpha}(0)$$

<u>*Three-body*</u> Bound State

Fractionalization and Hybridization

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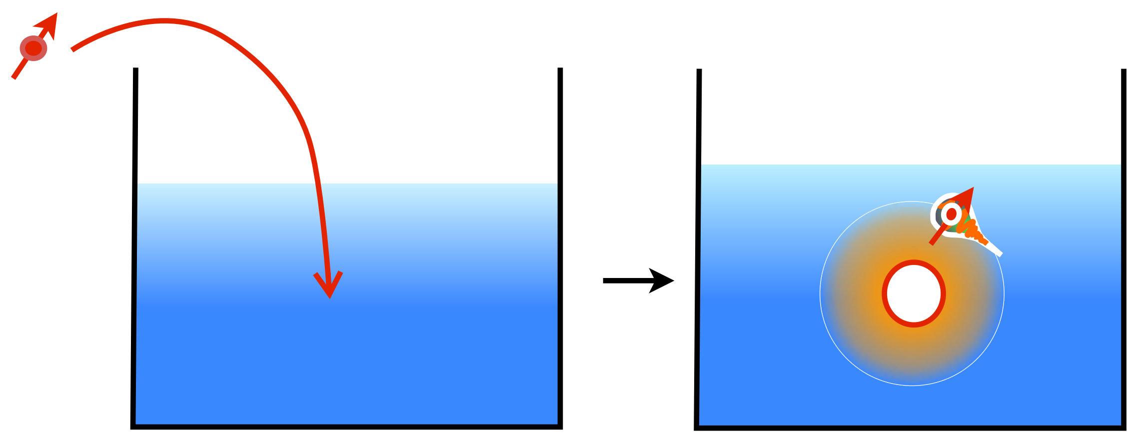
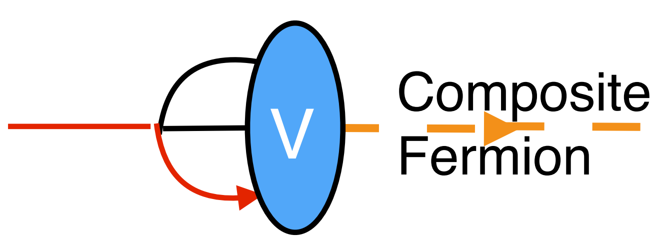
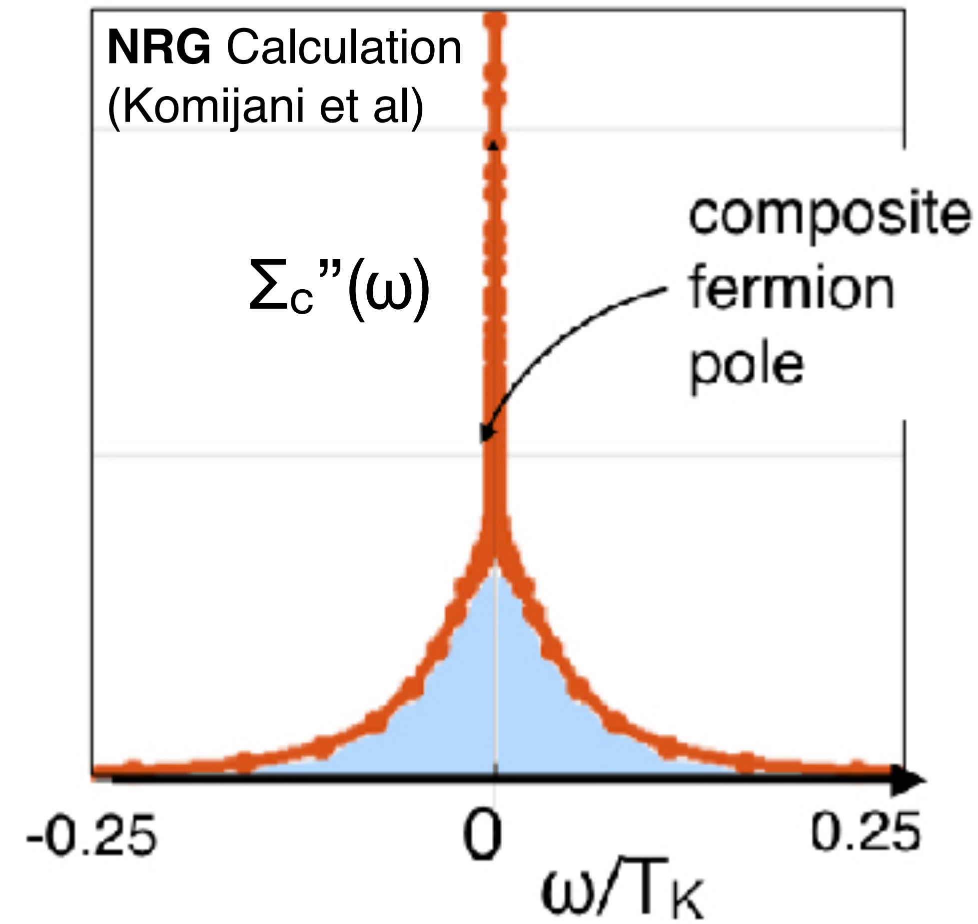


$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + V(\psi^{\dagger}_{\sigma} f_{\sigma} + \mathrm{H.c})$$

The hybridization is a Higgs field for the spinon which absorbs its U(1) gauge field into the EM field, giving the f-electron charge



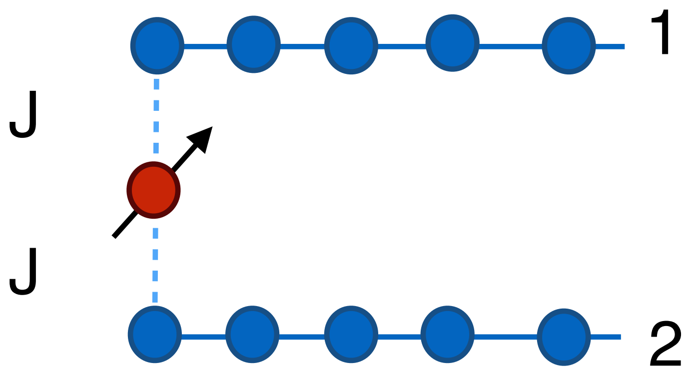


But large N assumes fractionalization, Does it happen at S=1/2? Confirmed.

$$\mathcal{F}_{\alpha} = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow V f_{\alpha}(0)$$

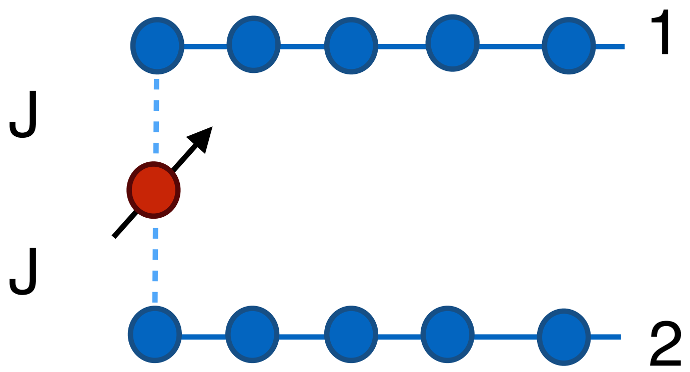
Three-body Bound State

Order Parameter Fractionalization (Induced)



2-channel Kondo

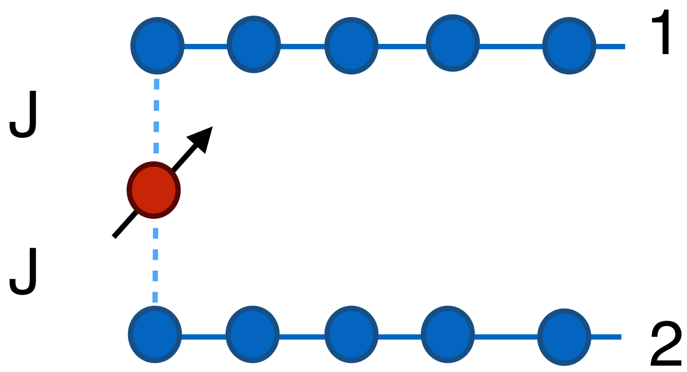
Order Parameter Fractionalization (Induced)



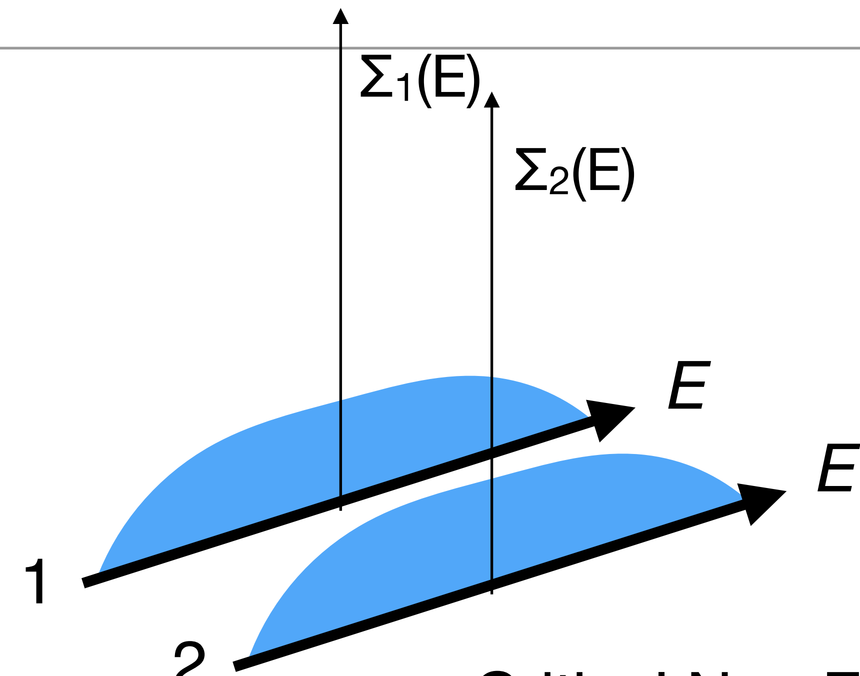
2-channel Kondo

$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\;\sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S}$$

Order Parameter Fractionalization (Induced)



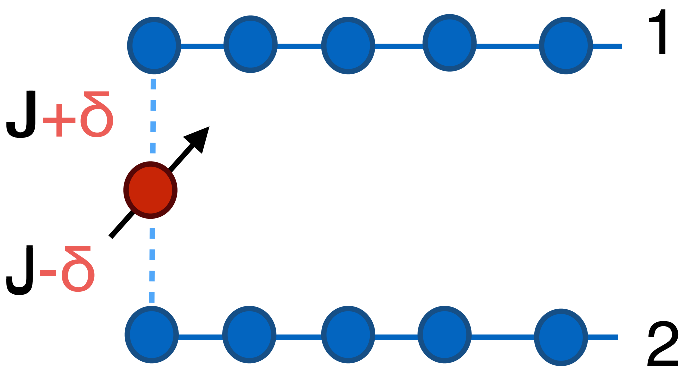
2-channel Kondo



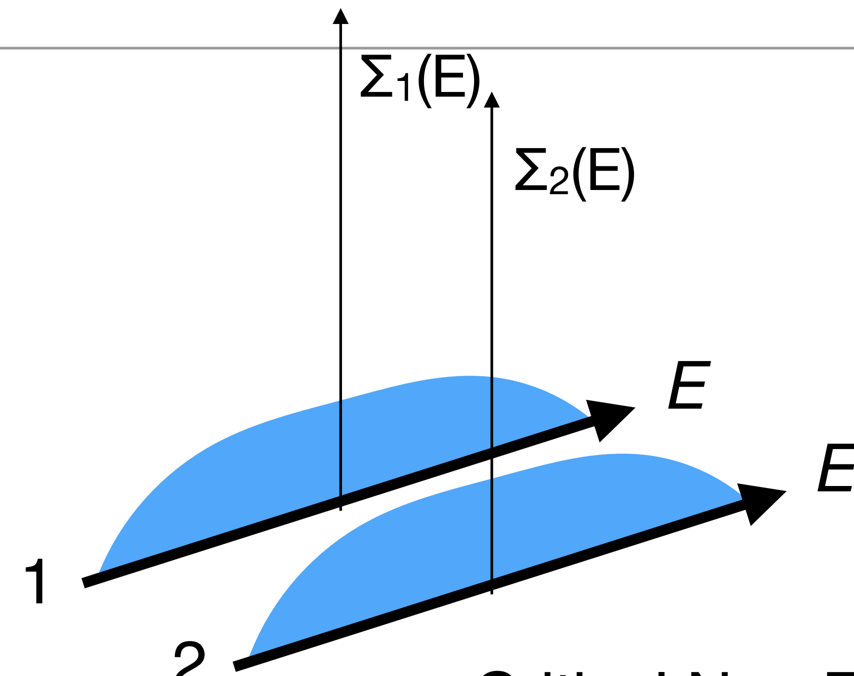
Critical Non Fermi LIquid

$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\ \sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S}$$

Order Parameter Fractionalization (Induced)



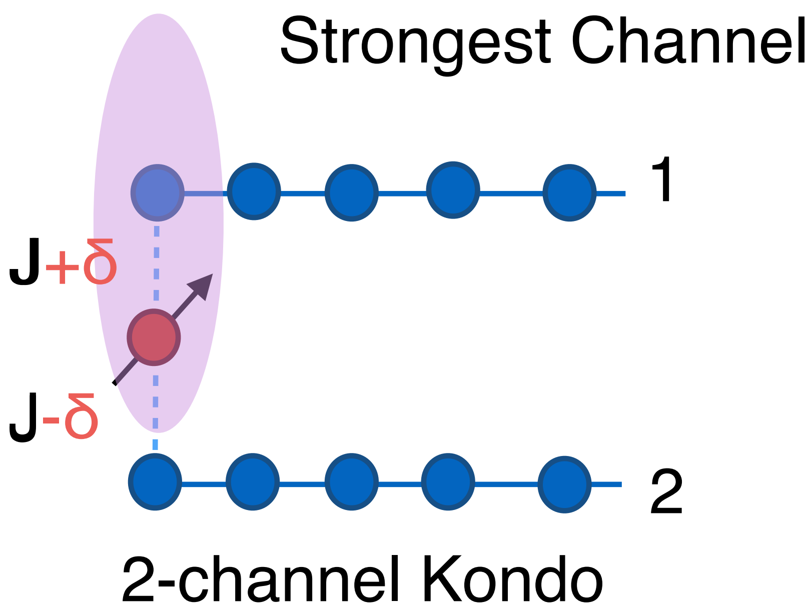
2-channel Kondo

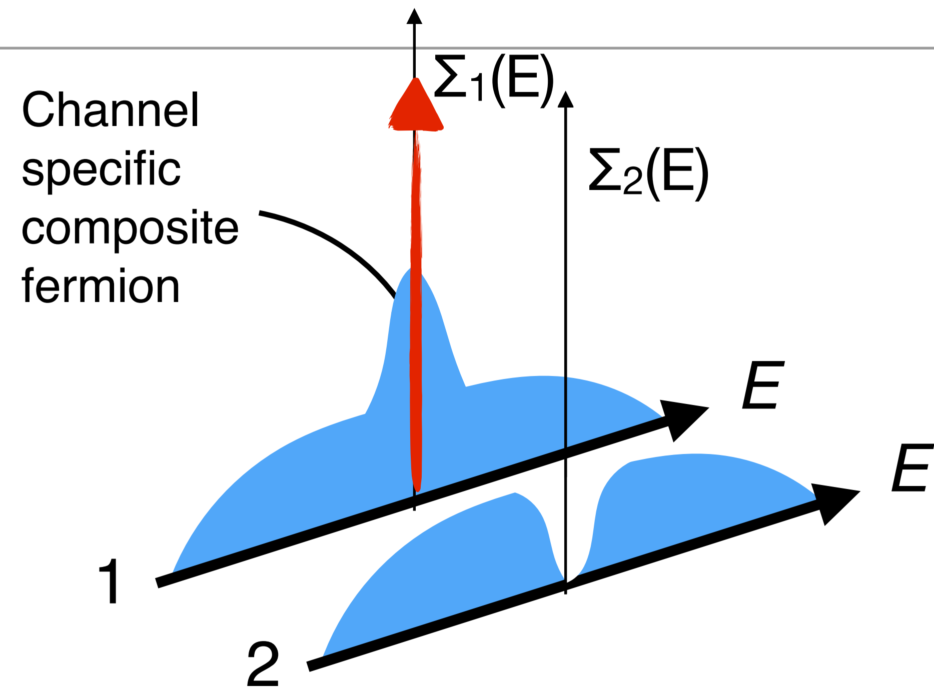


Critical Non Fermi LIquid

$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\ \ \sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S} + \delta J [\vec{\sigma}_1(0) - \vec{\sigma}_2(0)] \cdot \vec{S}$$

Order Parameter Fractionalization (Induced)





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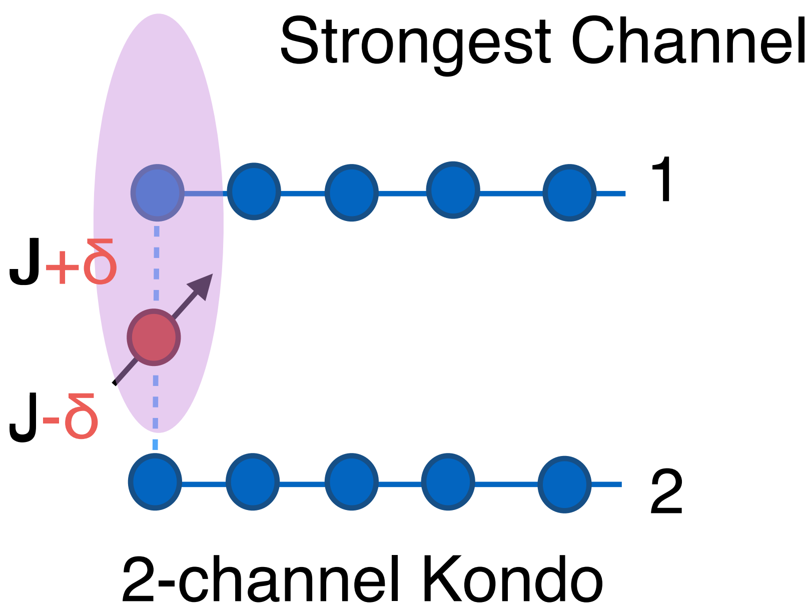

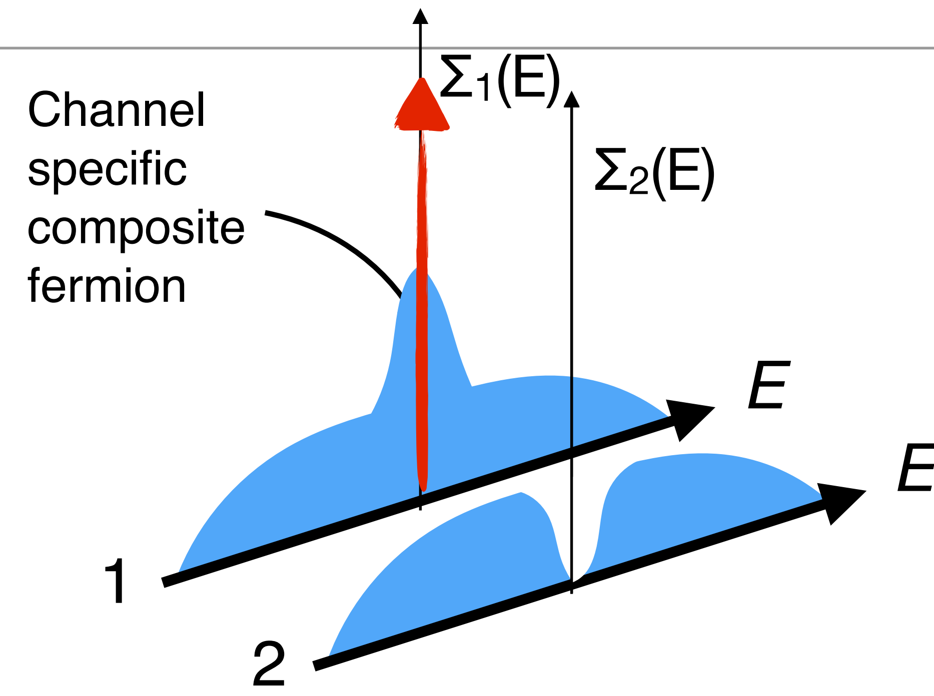

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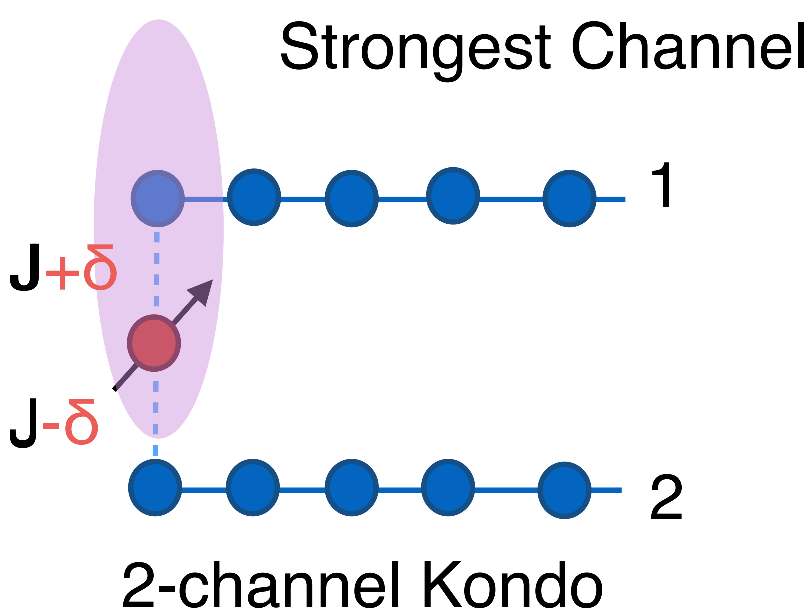
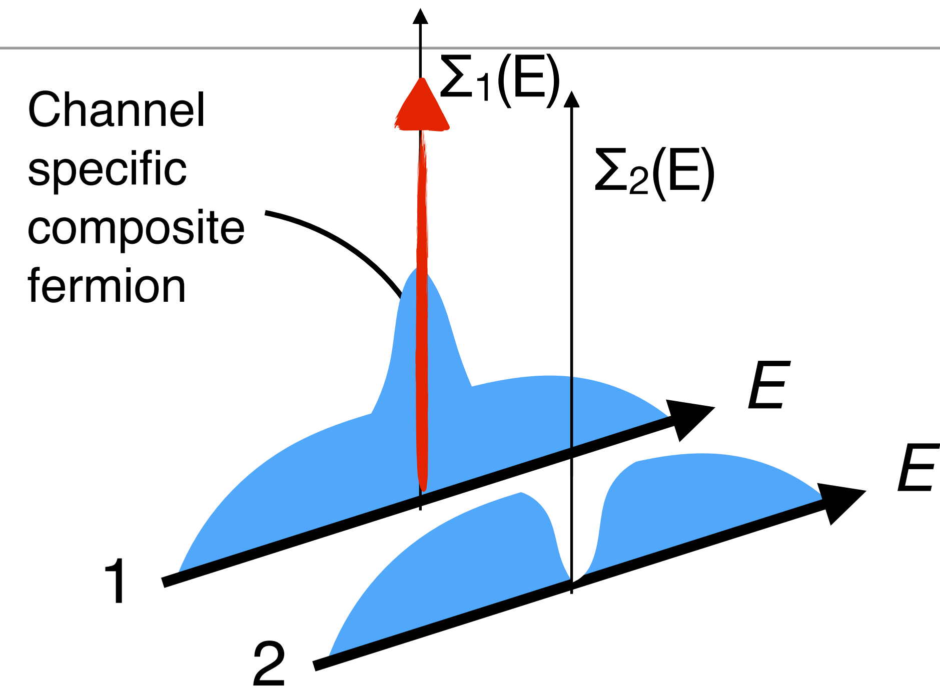
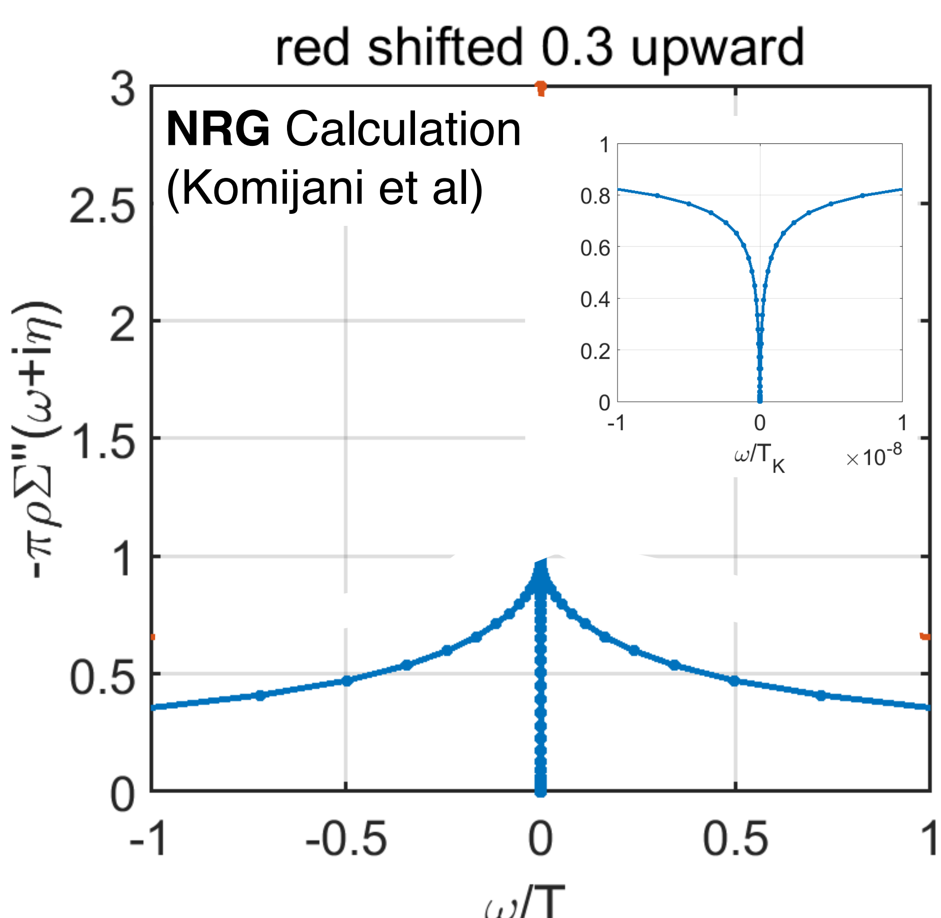
$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\lambda\sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S} + \delta J [\vec{\sigma}_1(0) - \vec{\sigma}_2(0)] \cdot \vec{S}$$







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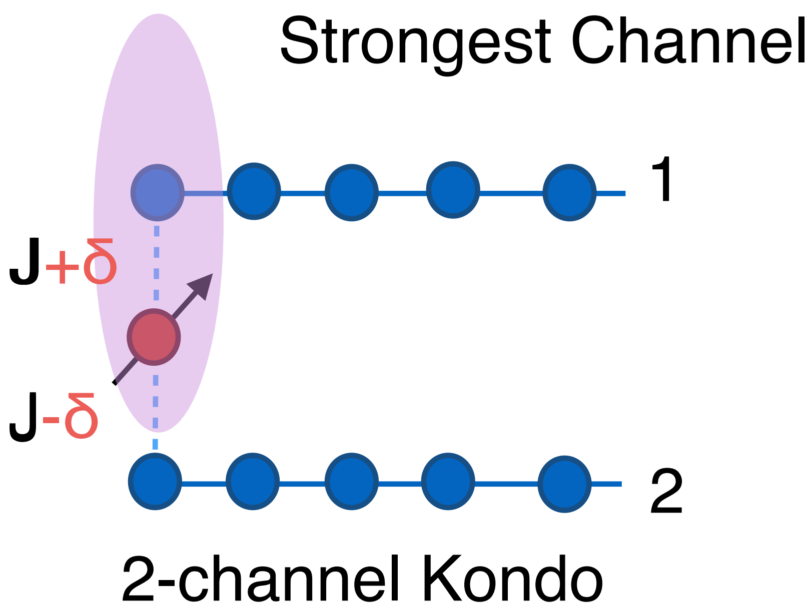
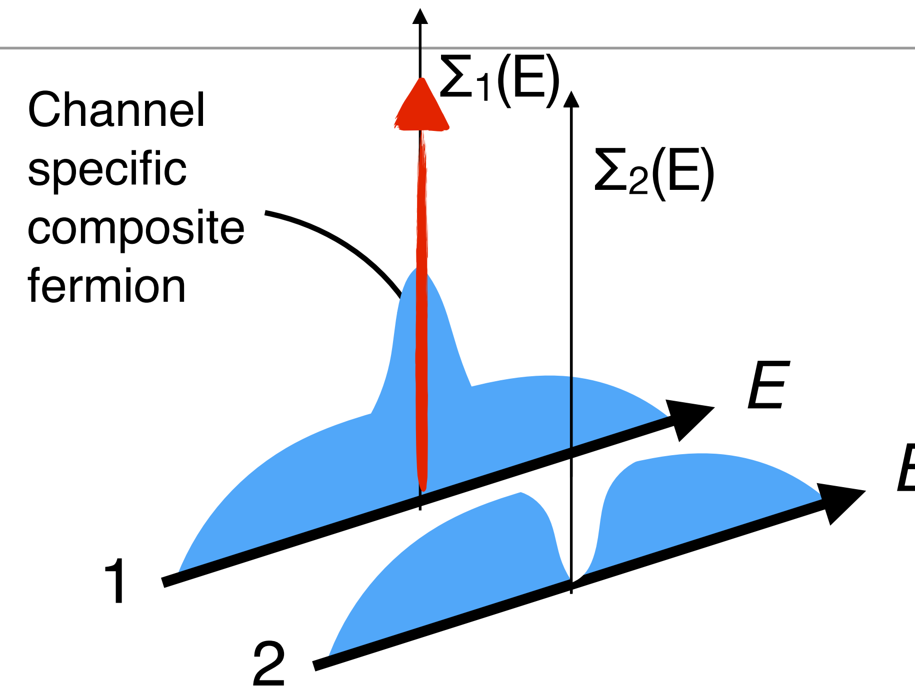
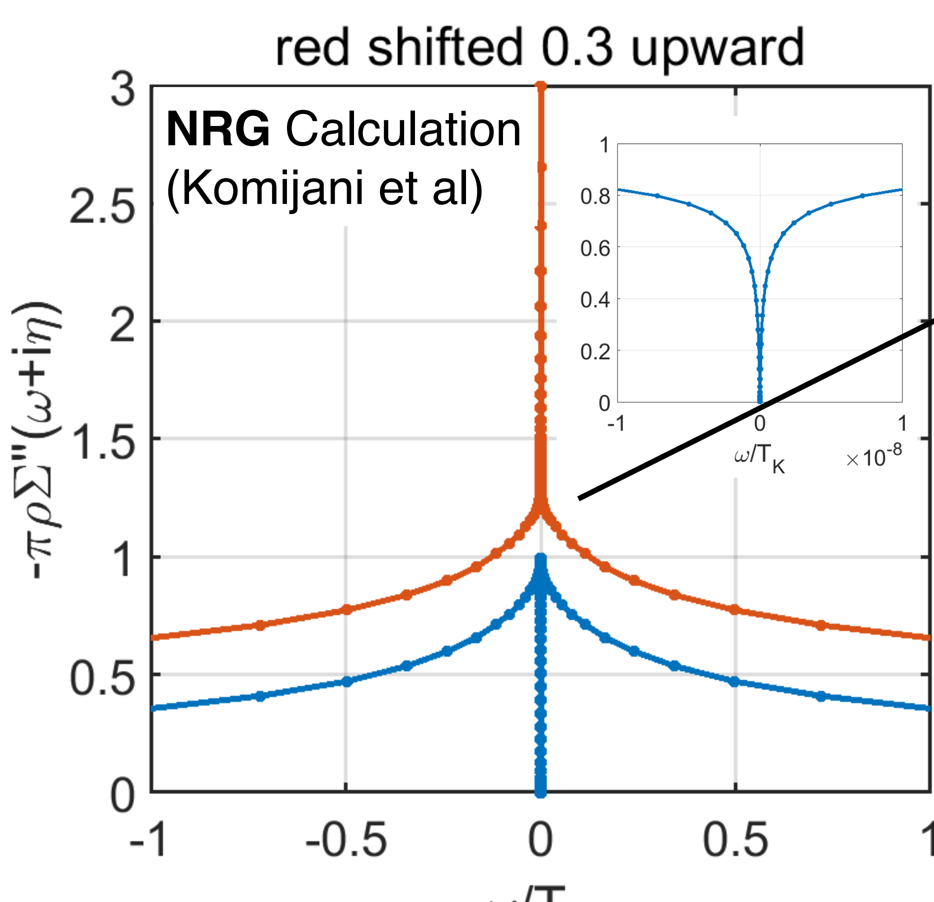
$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\lambda\sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S} + \delta J [\vec{\sigma}_1(0) - \vec{\sigma}_2(0)] \cdot \vec{S}$$





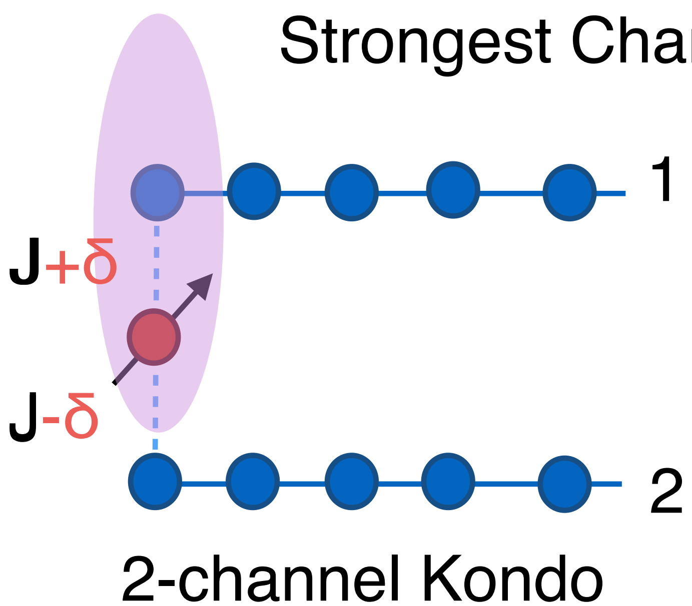
$$V_{\lambda} = \begin{pmatrix} V \\ 0 \end{pmatrix}$$

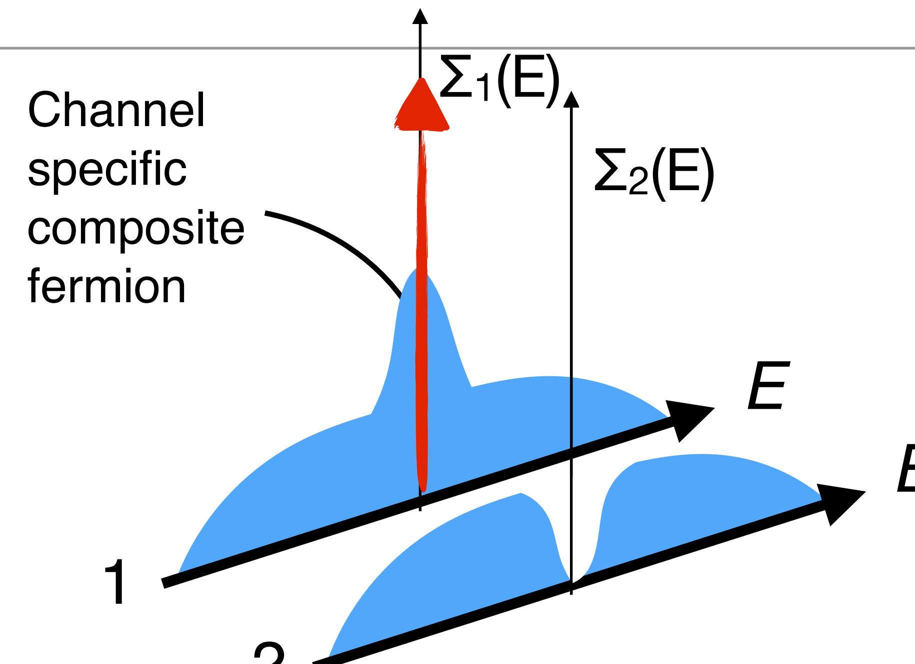


$$\Sigma_{\lambda\lambda'}(\omega - i\delta) = V_{\lambda} V^{*}_{\lambda'} \frac{1}{\omega - i\delta}$$

Order Parameter Fractionalization (Induced)

$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\lambda\sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0) \cdot \vec{S} + \delta J [\vec{\sigma}_1(0) - \vec{\sigma}_2(0)] \cdot \vec{S}$$





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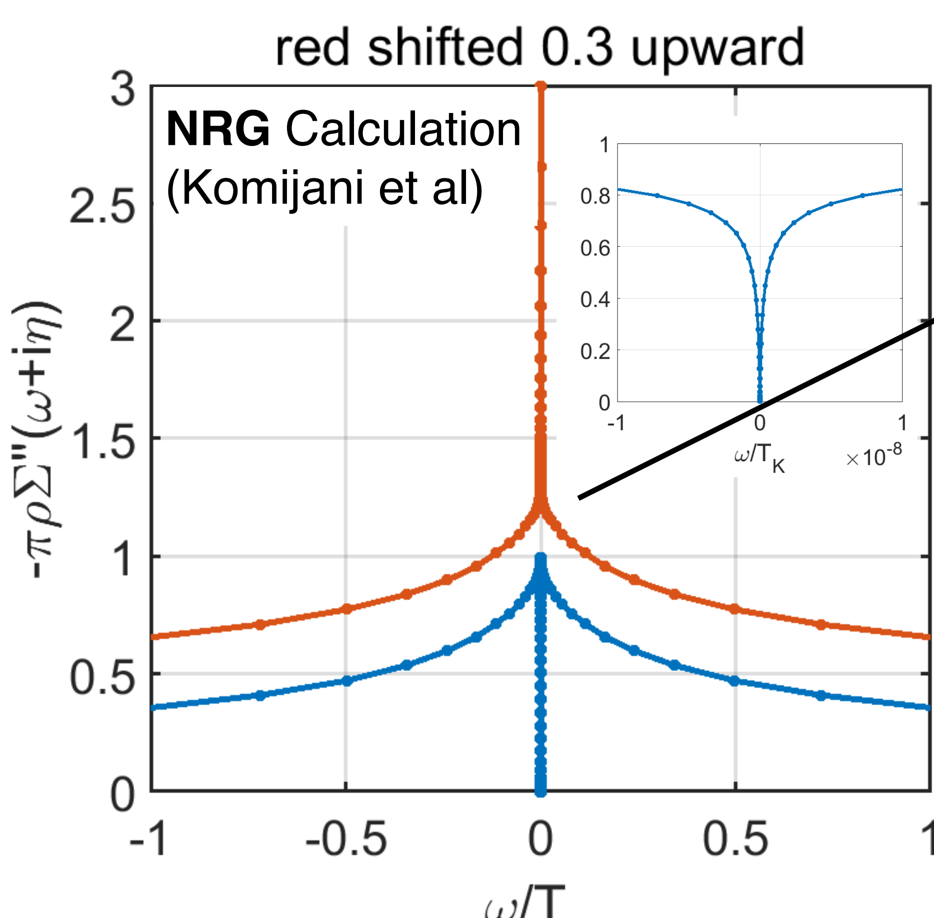


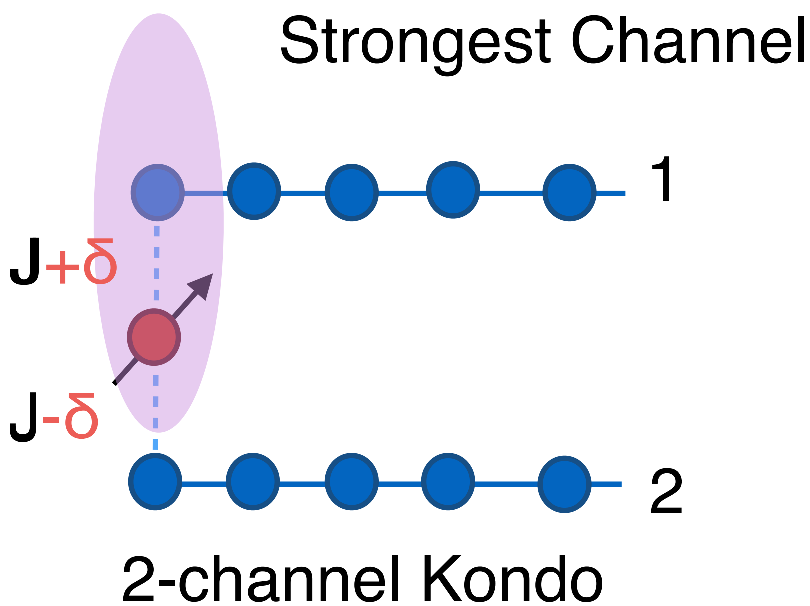
$$\Sigma_{\lambda\lambda'}(\omega - i\delta) = V_{\lambda} V^{*}_{\lambda'} \frac{1}{\omega - i\delta}$$

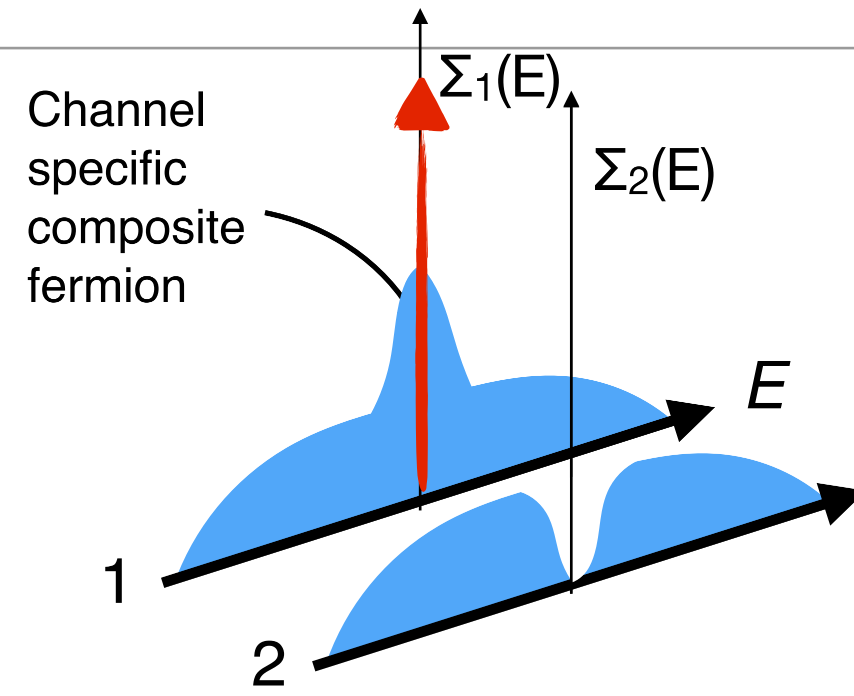
$$\Sigma_{\lambda\lambda'}(2,1) \xrightarrow{|t_2 - t_1| \to \infty} V_{\lambda}(2) V_{\lambda'}(1) \mathrm{sgn}(t_2 - t_1)$$

ODLRO in Time

Order Parameter Fractionalization (Induced)

$$H = \sum_{\vec{k}\sigma\lambda} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\lambda\sigma} c_{\vec{k}\lambda\sigma} + J \sum_{\lambda=1,2} \vec{\sigma}_{\lambda}(0)\cdot\vec{S} + \delta J[\vec{\sigma}_1(0) - \vec{\sigma}_2(0)]\cdot\vec{S}$$





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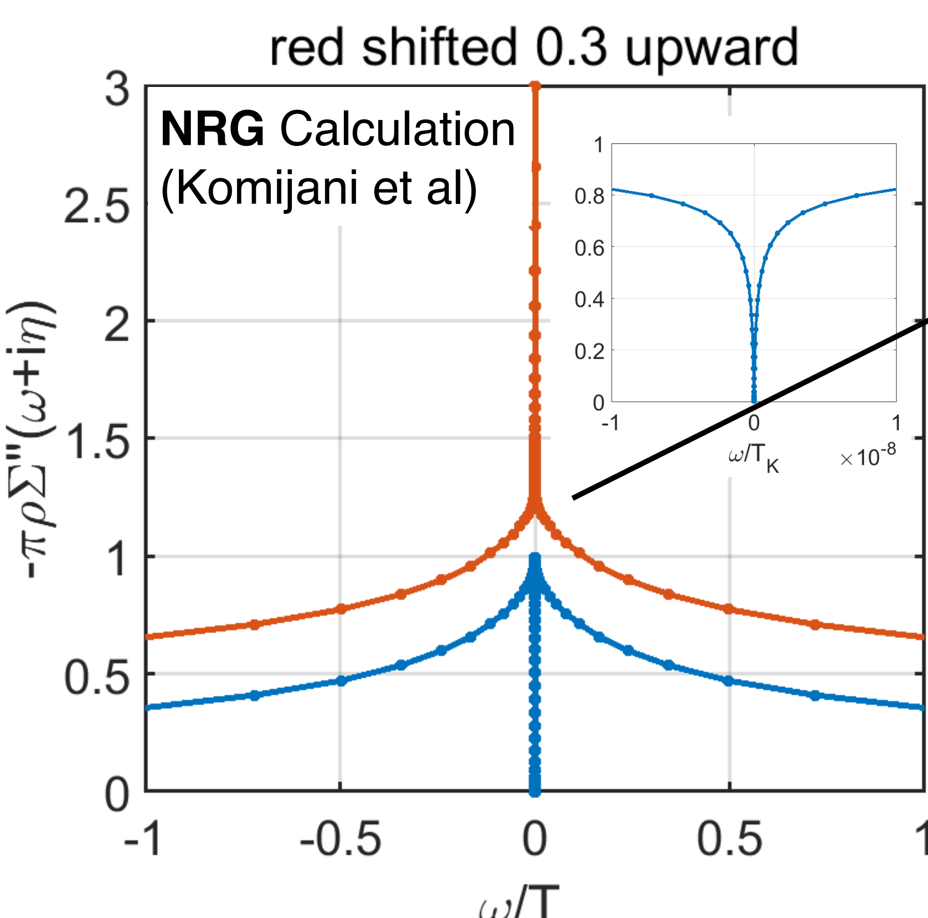


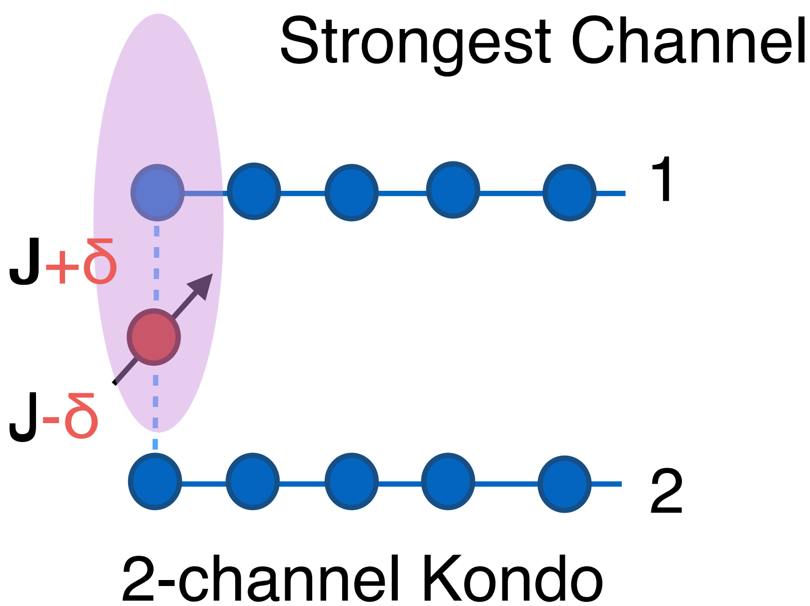
$$(\vec{\sigma}_{\alpha\beta}\cdot\vec{S}\;)\psi_{\lambda\beta} \rightarrow V_{\lambda} f_{\alpha}(0)$$

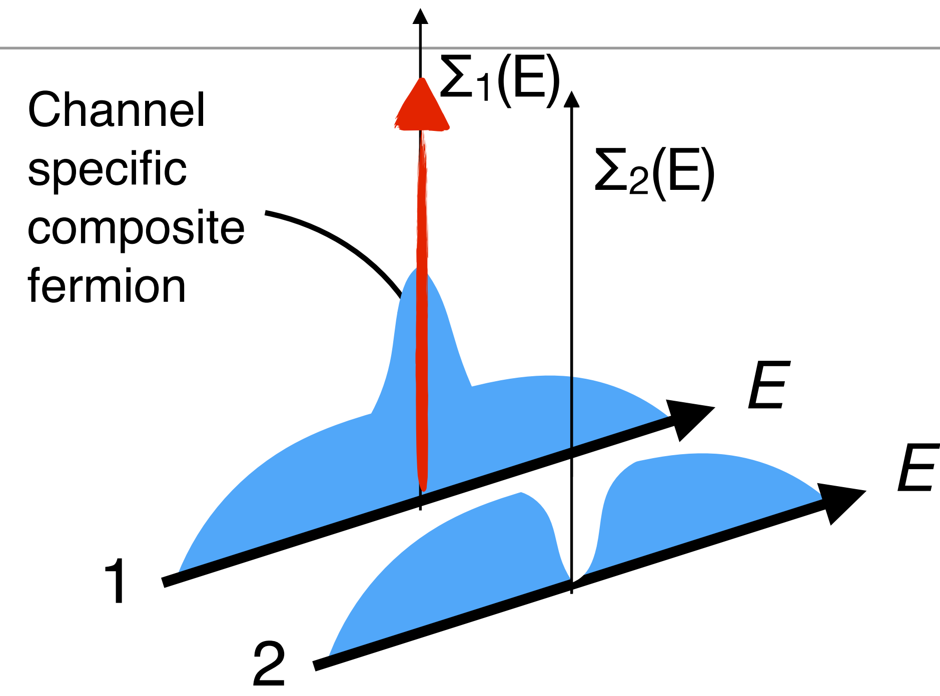
Fractionalization of Composite into Fermion+OP

$$\Sigma_{\lambda\lambda'}(2,1) \xrightarrow{|t_2-t_1|\to\infty} V_{\lambda}(2)V_{\lambda'}(1)\mathrm{sgn}(t_2-t_1)$$

ODLRO in Time

Order Fractionalization (Spontaneous)

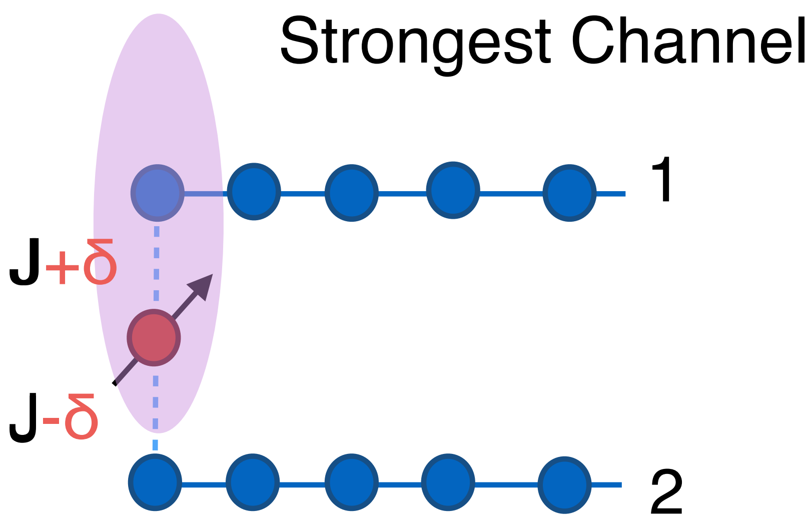




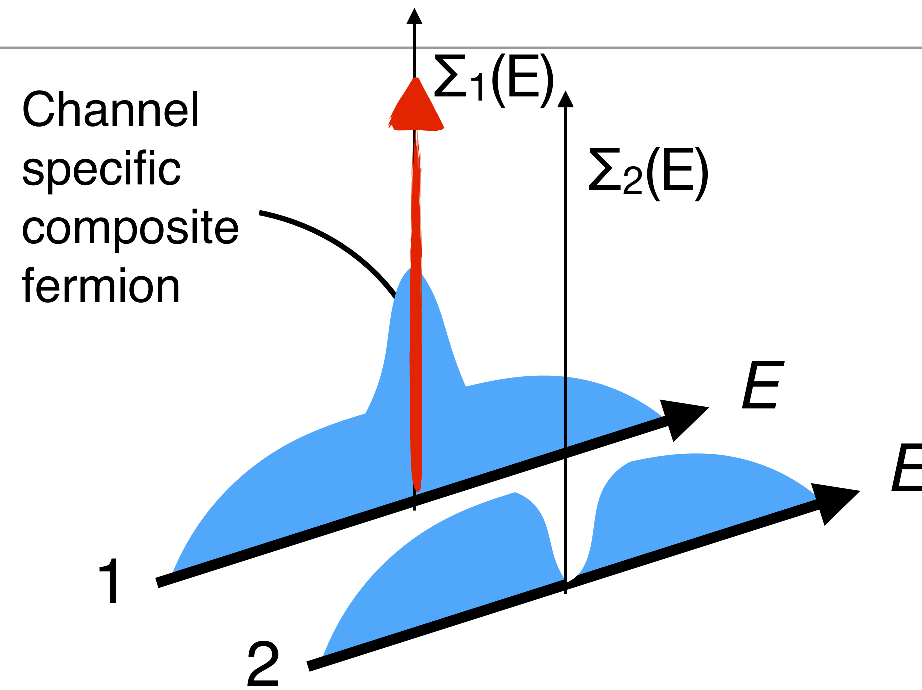
$$V_\lambda = \begin{pmatrix} V \\ 0 \end{pmatrix}$$

$$(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}\ )\psi_{\lambda\beta} \rightarrow V_\lambda f_\alpha(0)$$

Order Fractionalization (Spontaneous)

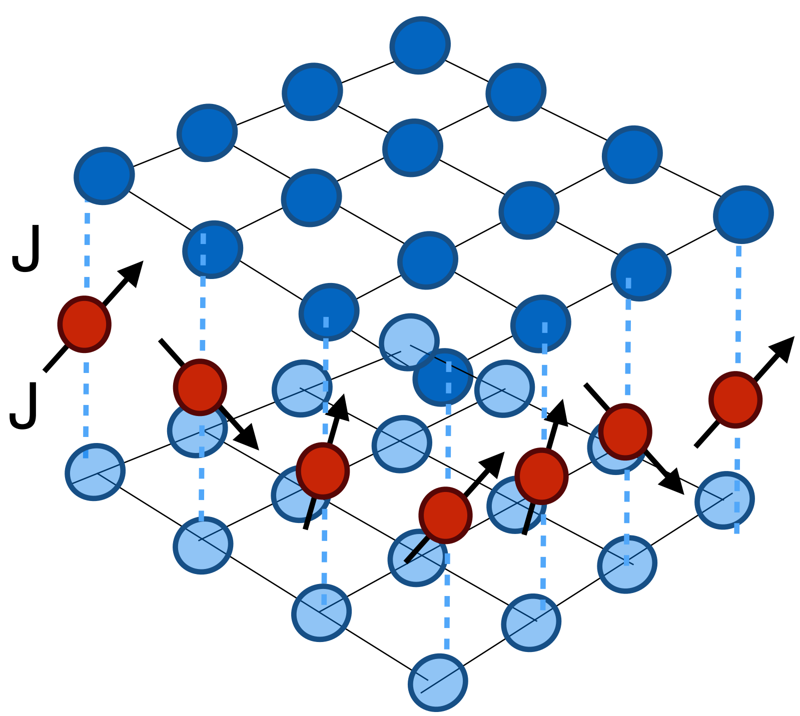


2-channel Kondo



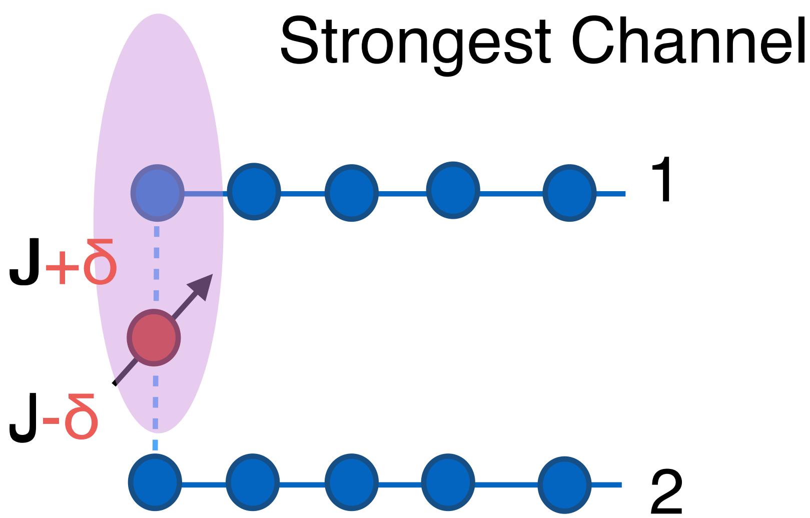
$$V_\lambda = \begin{pmatrix} V \\ 0 \end{pmatrix}$$

$$(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}\ )\psi_{\lambda\beta} \rightarrow V_\lambda f_\alpha(0)$$

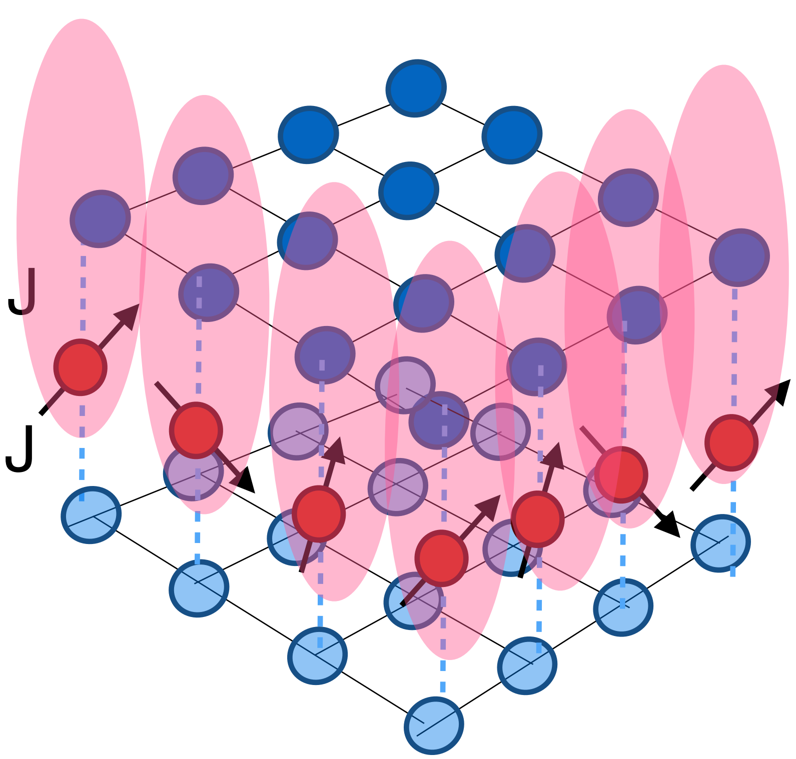


2-channel Kondo Lattice

Order Fractionalization (Spontaneous)



2-channel Kondo



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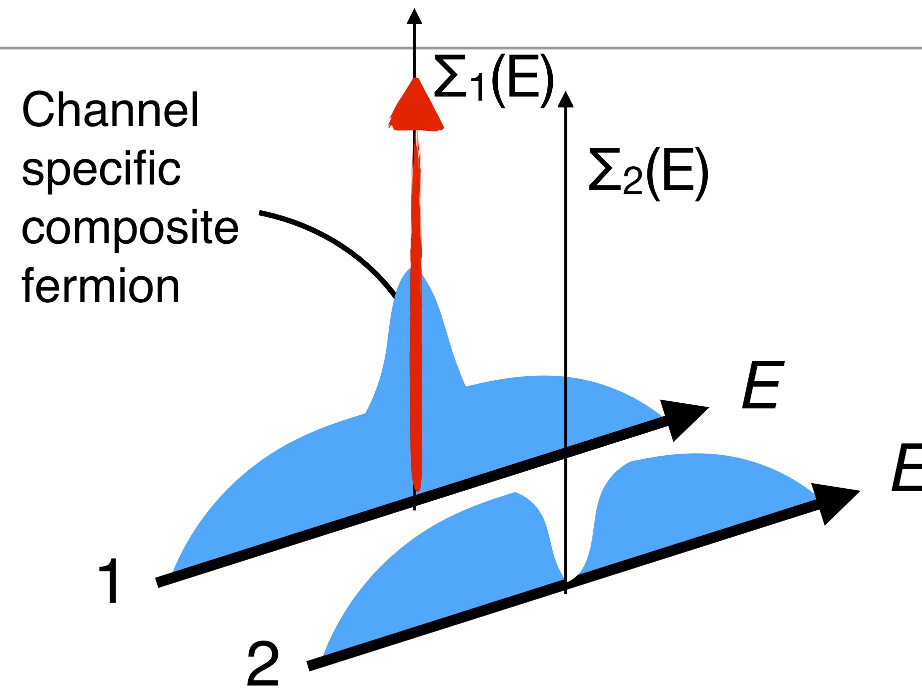
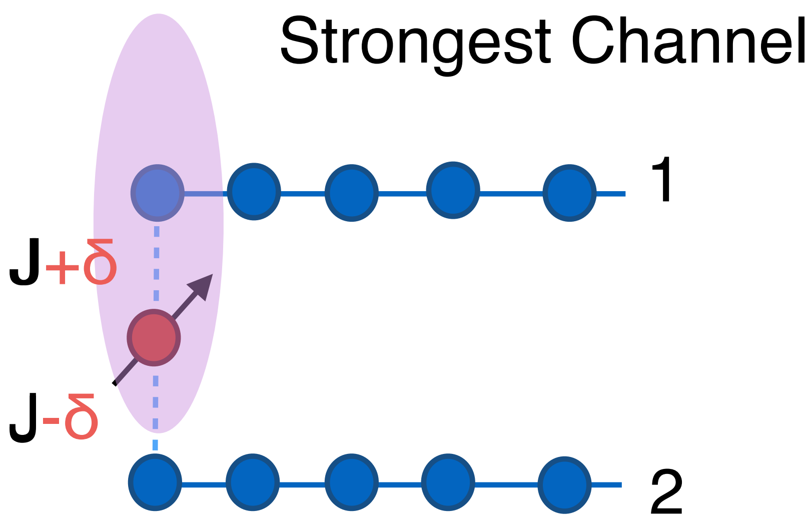


$$V_\lambda = \begin{pmatrix} V \\ 0 \end{pmatrix}$$

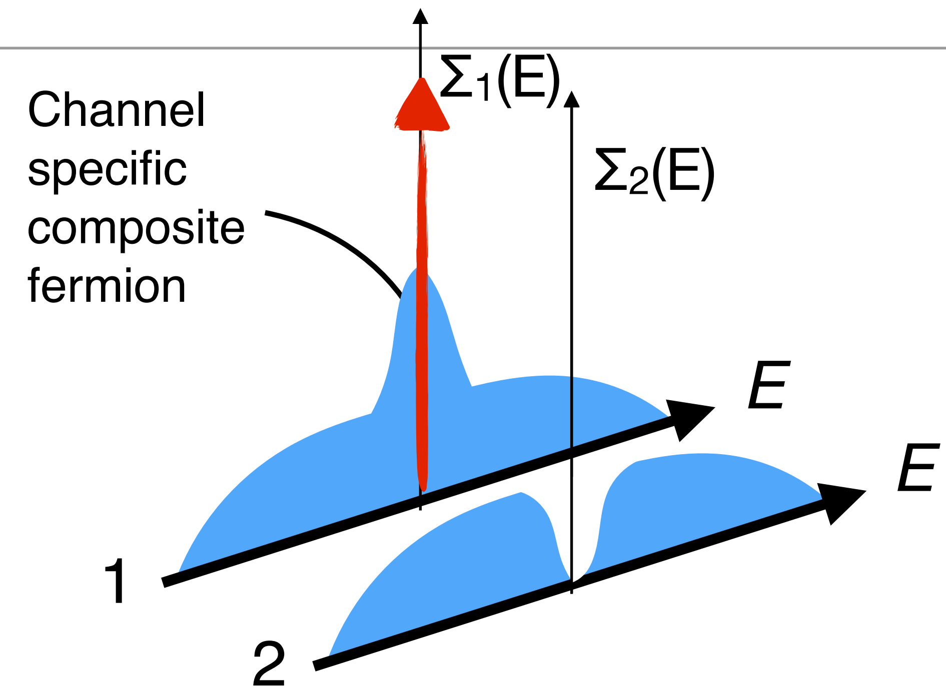
$$(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}\ )\psi_{\lambda\beta} \rightarrow V_\lambda f_\alpha(0)$$

Spinor OP Forms Spontaneously

Order Fractionalization (Spontaneous)



2-channel Kondo



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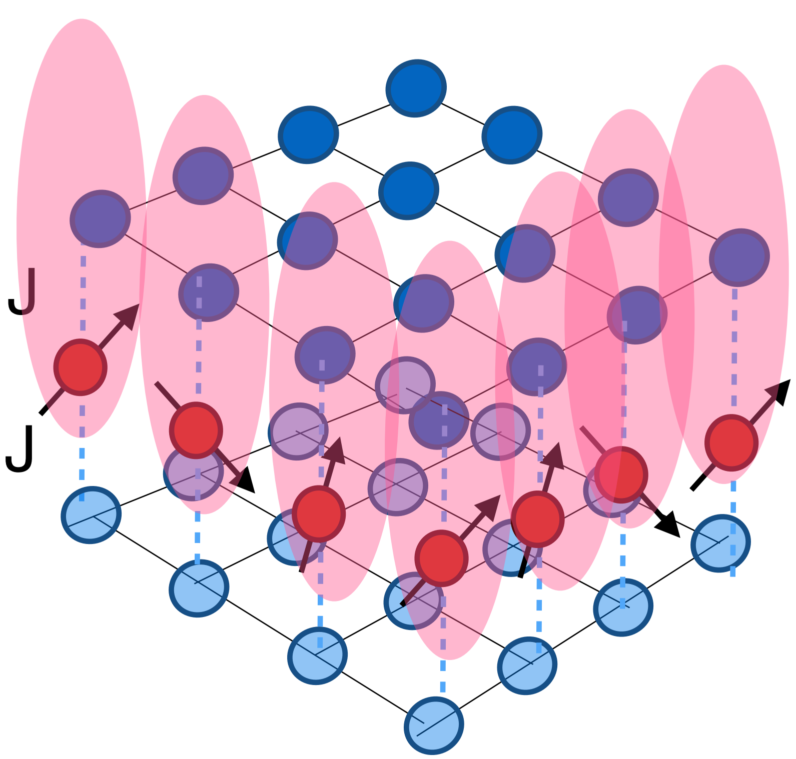


2-channel Kondo Lattice

$$\Sigma_{\lambda\lambda'}(2,1) \xrightarrow{|2-1|\rightarrow\infty} V_\lambda(2)V_{\lambda'}(1)g(2-1)$$

ODLRO in Space Time

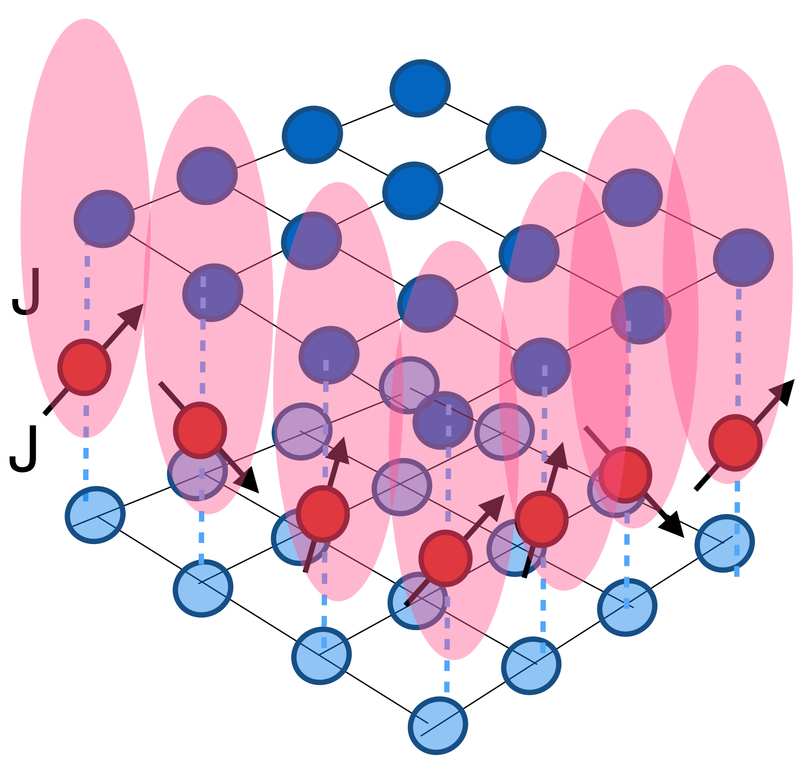
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Composite order Fractionalized

$$\Psi = \langle \left(\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2 \right) \cdot \vec{S} \rangle$$
$$\propto |V_1|^2 - |V_2|^2$$

cf Emery and Kivelson 1993



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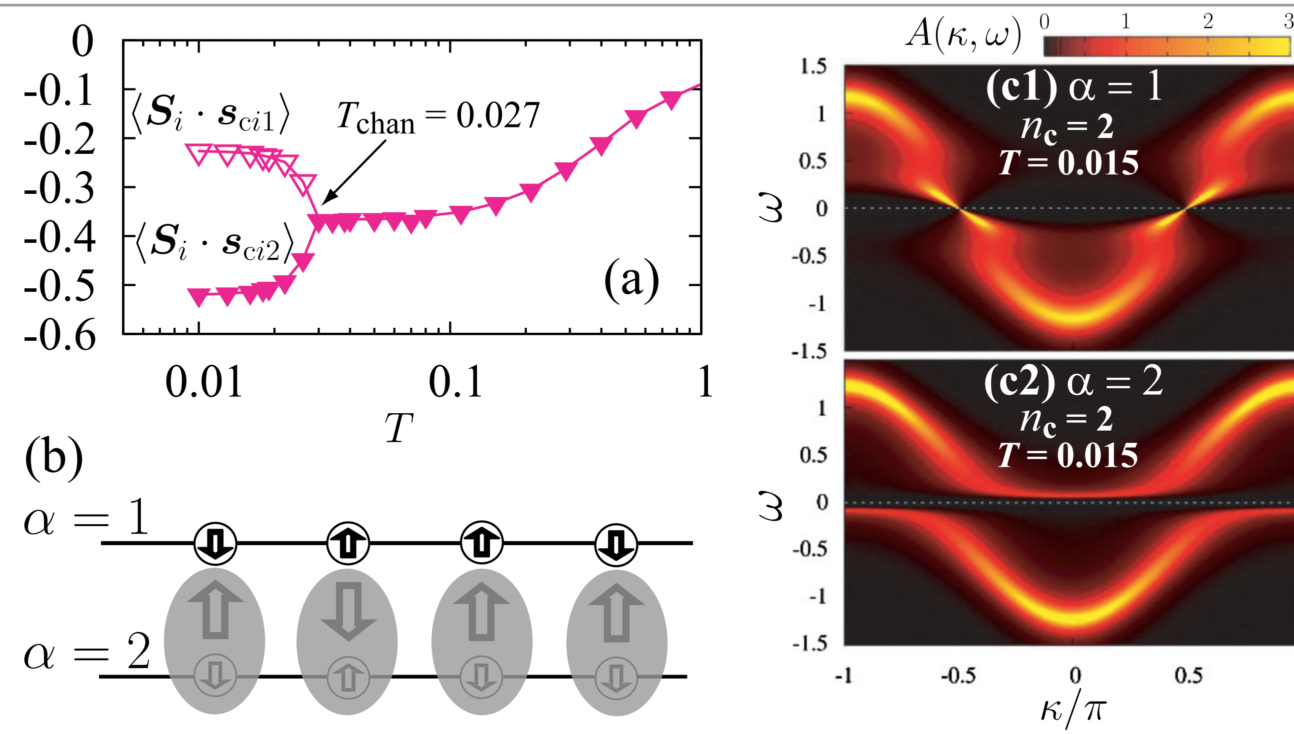
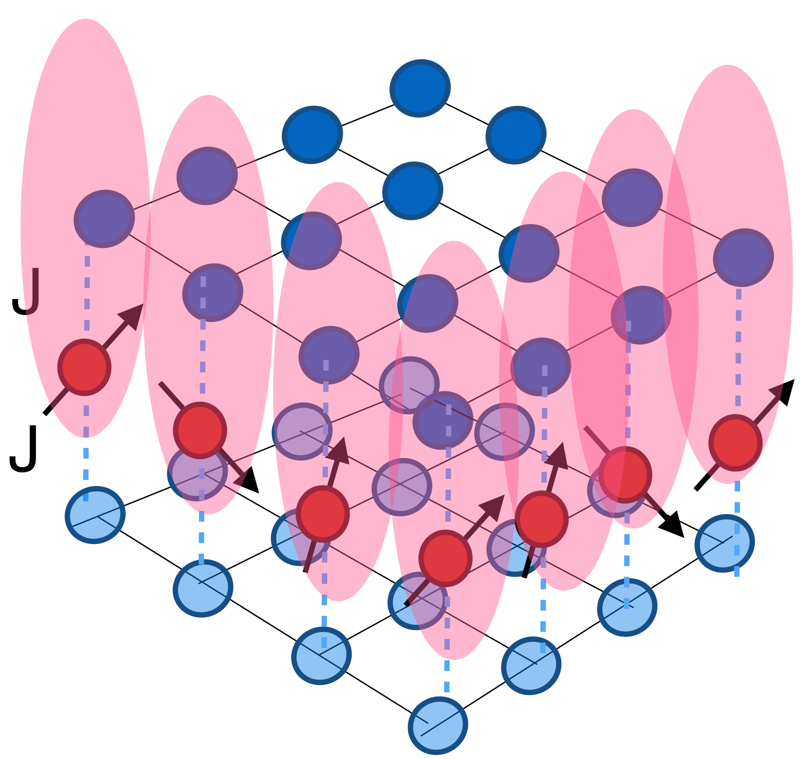


Hisono, Otsuki & Kuromoto, PRL 107, 247202 (2011)



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ODLRO in Space Time

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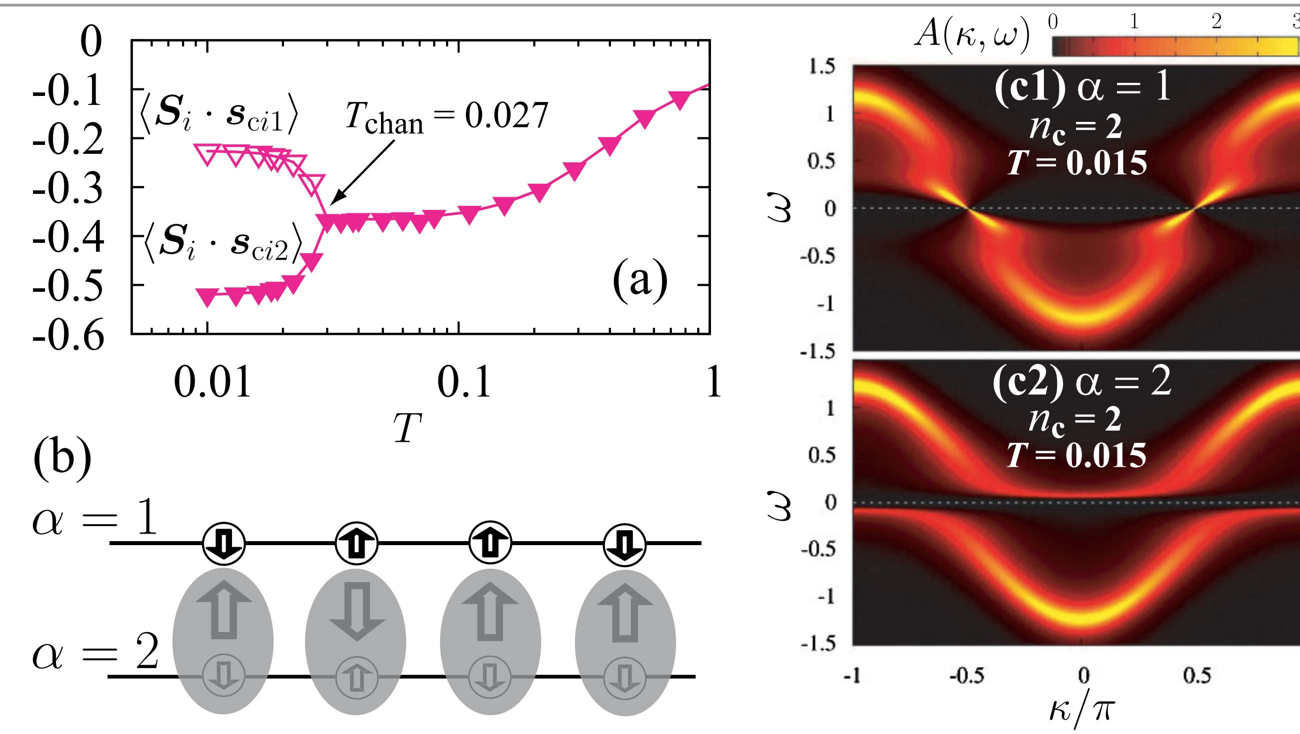

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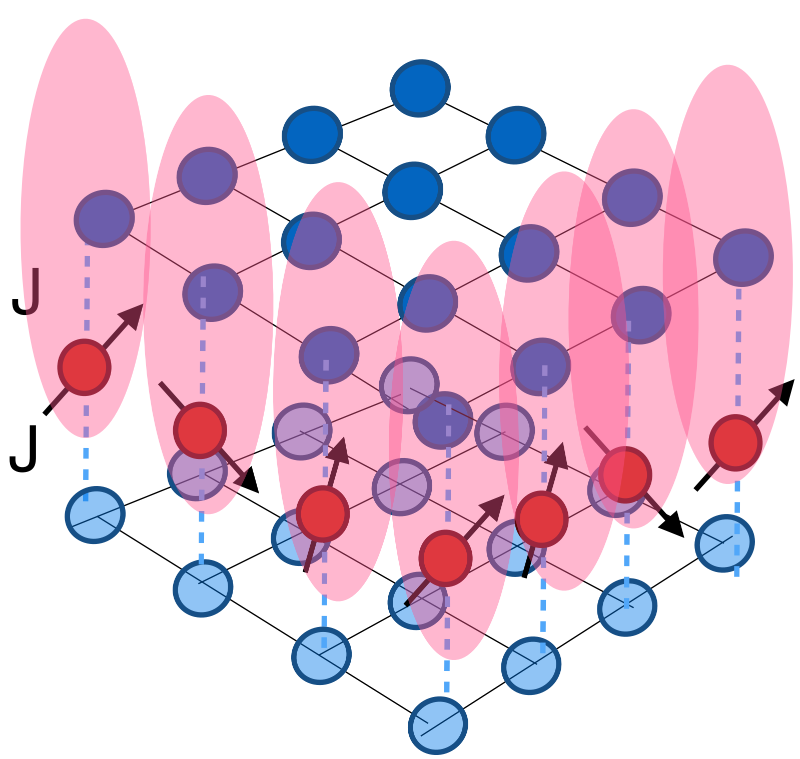


2-channel Kondo Lattice

$$(\overline{\psi\psi\psi})_\Lambda(x) = V^\lambda_{\alpha\alpha'}(x) f_{\alpha'}(x)$$

$$\Sigma_{\lambda\lambda'}(2,1) \xrightarrow{|2-1|\to\infty} V_\lambda(2) V_{\lambda'}(1) g(2-1)$$

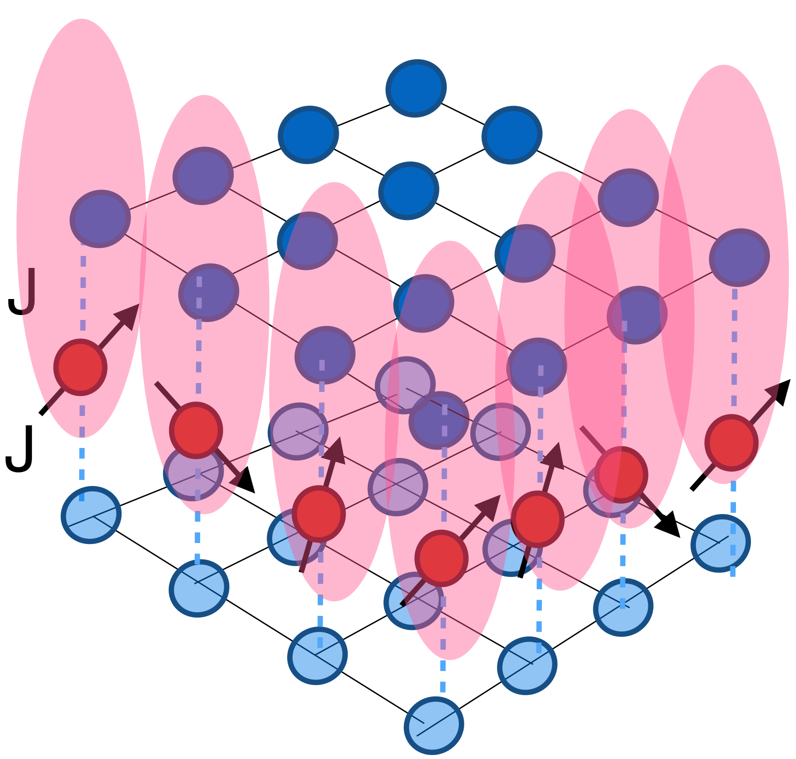
ODLRO in Space Time

Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani, A. Toth

Composite Order

2-channel	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_{\lambda\beta}$	$V_{\lambda} f_{\alpha}$	Composite Multipole	$\langle\left(\psi_1^{\dagger}\vec{\sigma}\psi_1-\psi_2^{\dagger}\vec{\sigma}\psi_2\right)\cdot\vec{S}\rangle$



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$$(\overline{\psi\psi\psi})_{\Lambda}(x) = V^{\lambda}_{\alpha\alpha'}(x) f_{\alpha'}(x)$$

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ODLRO in Space Time

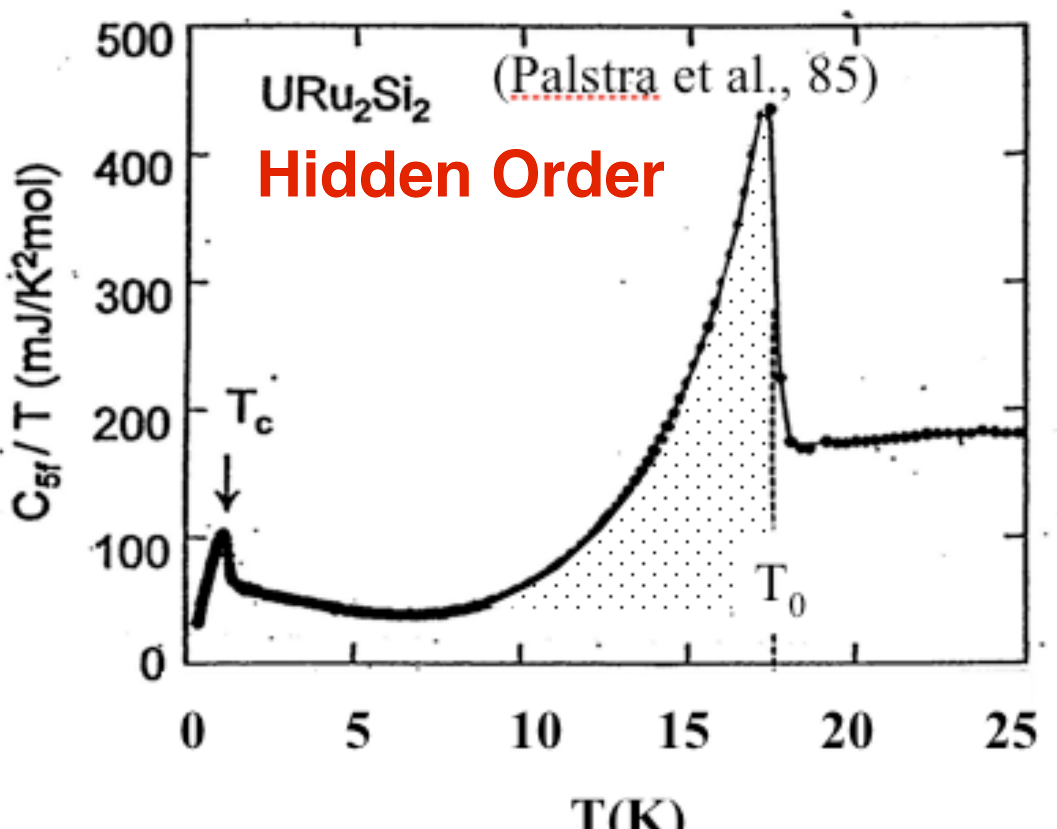
Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

$$(\psi\psi\psi)_\Lambda(x) = V^\lambda_{\alpha\alpha'}(x) f_{\alpha'}(x)$$

Composite Order

2-channel	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_{\lambda\beta}$	$V_\lambda f_\alpha$	Composite Multipole	$\langle(\psi_1^\dagger\vec{\sigma}\psi_1 - \psi_2^\dagger\vec{\sigma}\psi_2)\cdot\vec{S}\rangle$



Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

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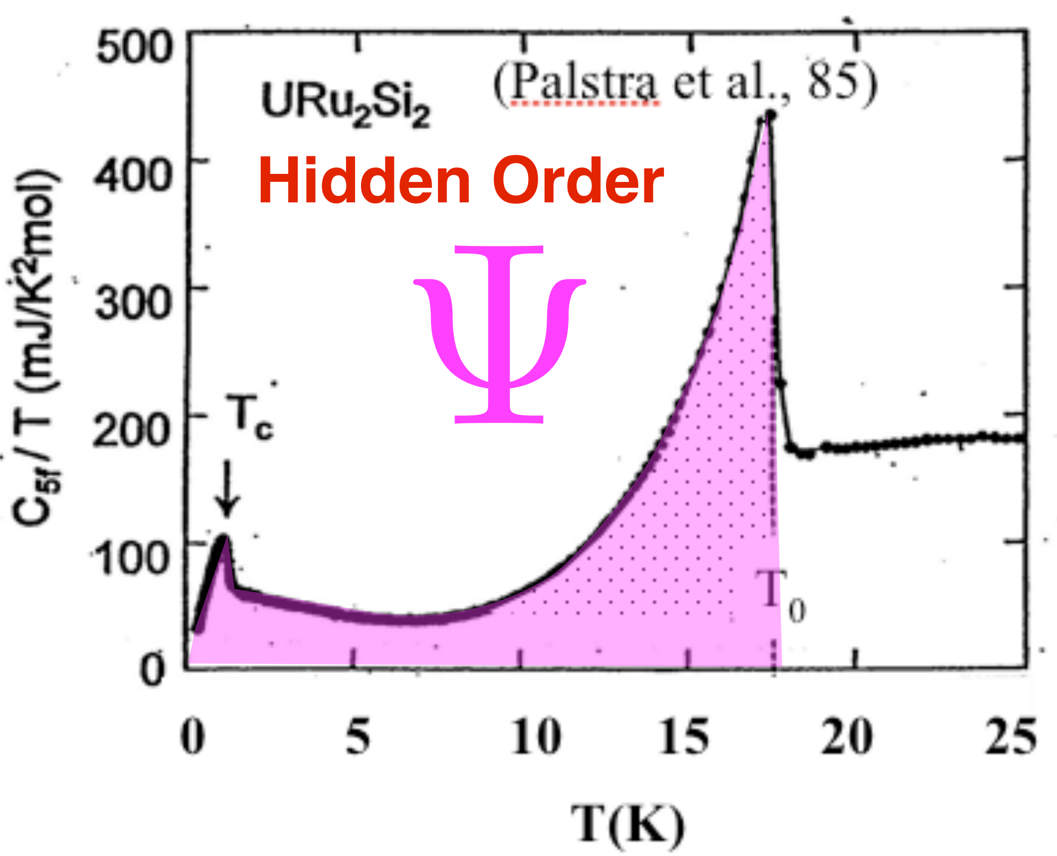

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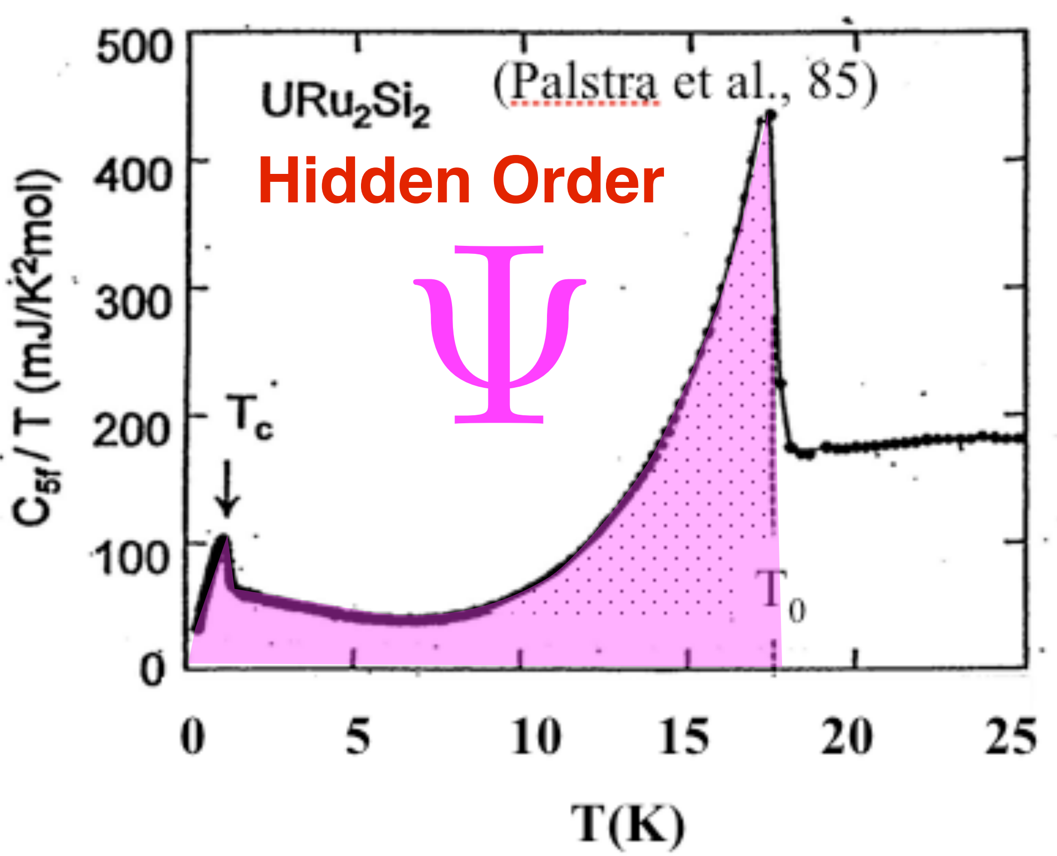

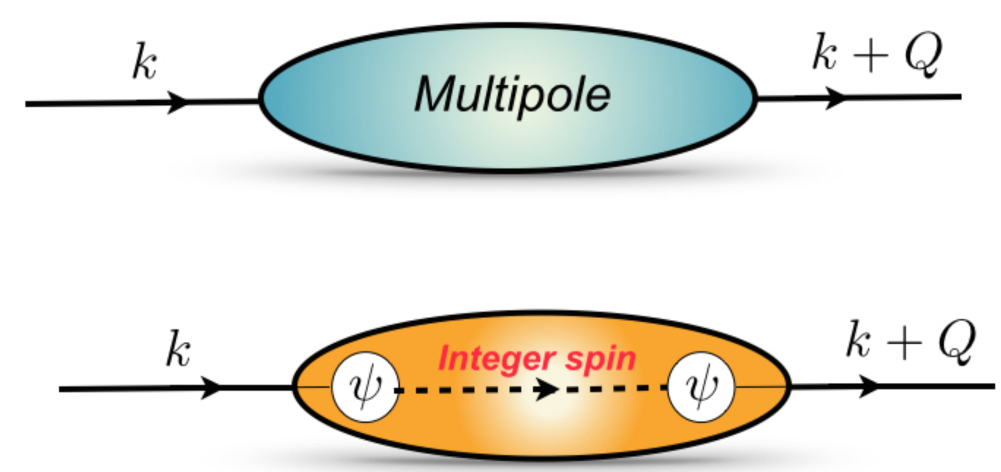

See eg:

J=3 Kiss & Fazekas, Phys Rev B, (2005)
J=4 Kotliar & Haule, Nat Phys, (2009)
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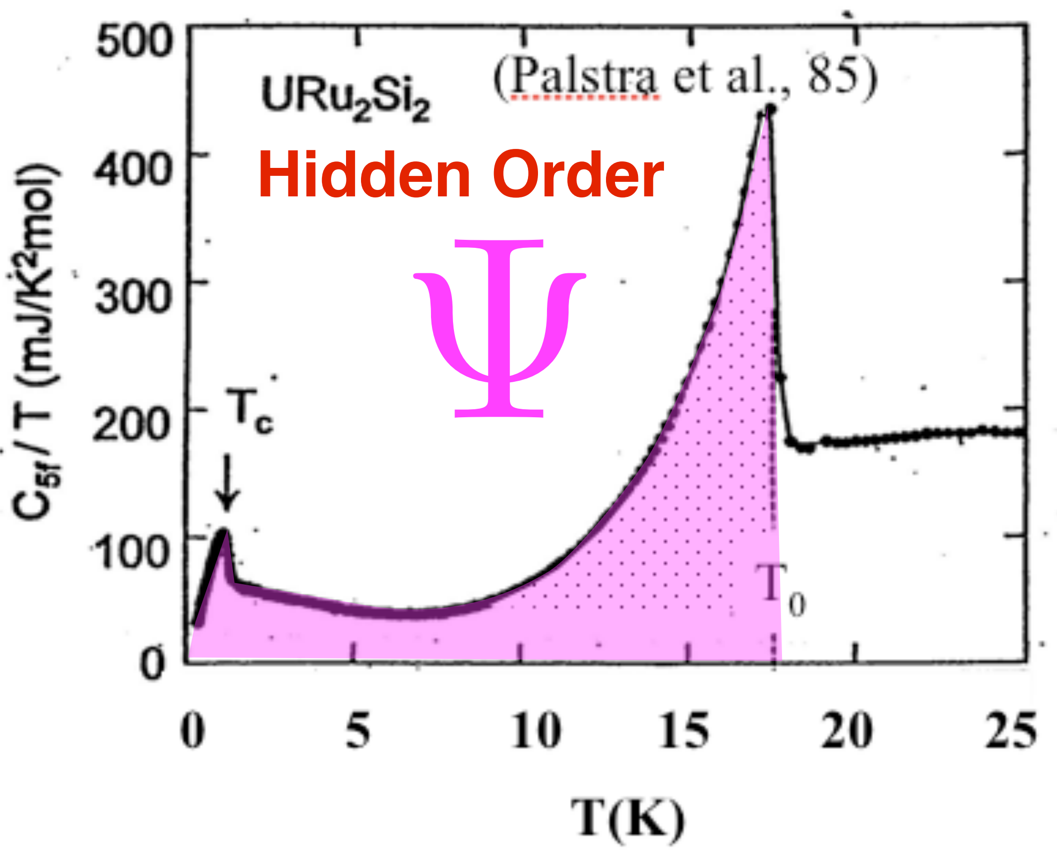
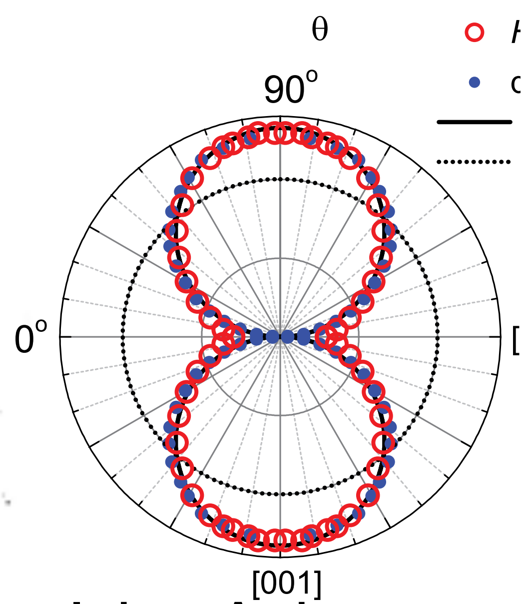
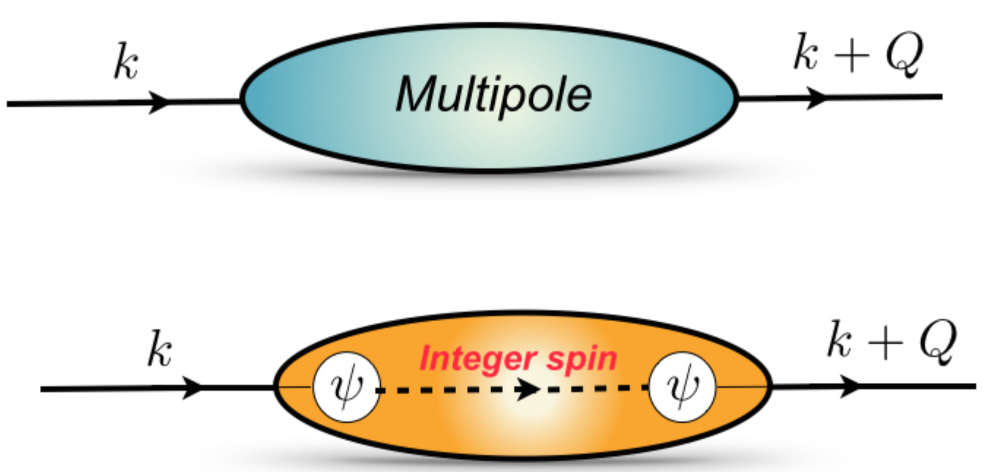




Ising Anisotropy



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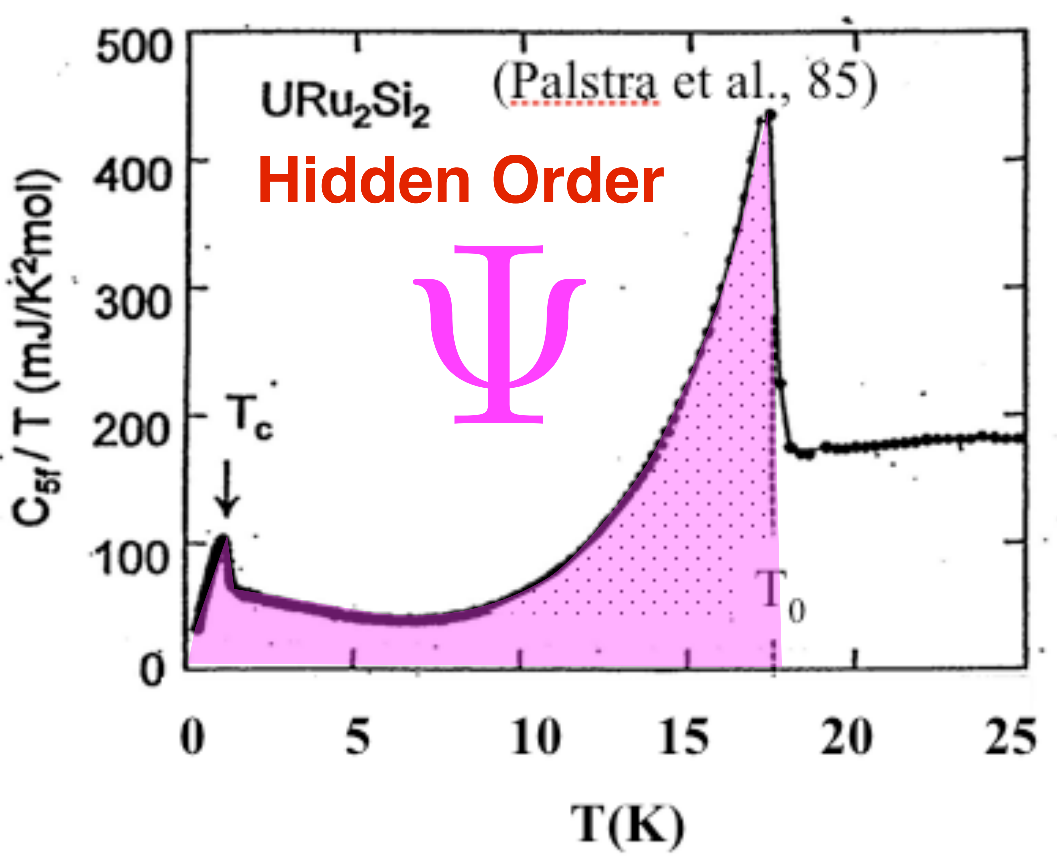

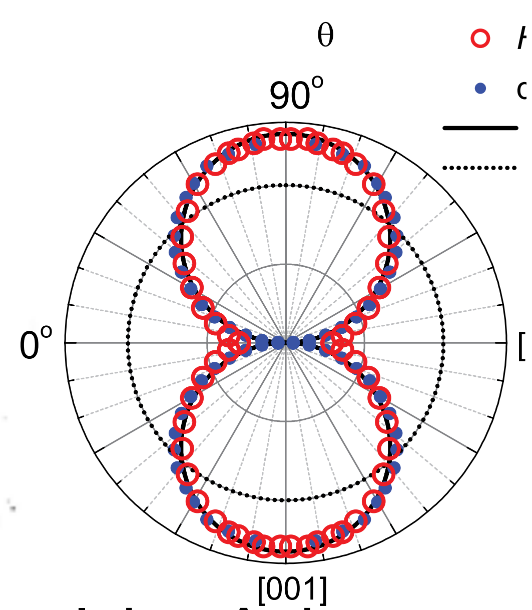

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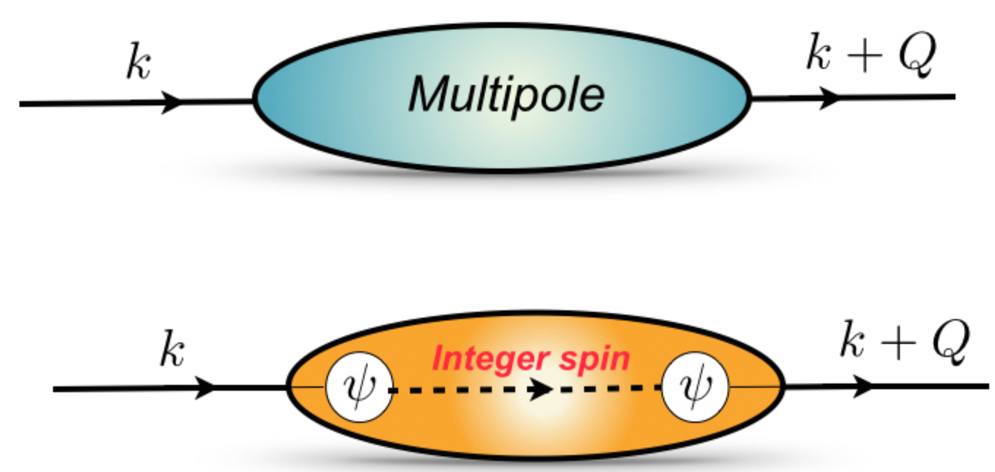

$$\psi = \sqrt{\text{Multipole}} = \text{Spinorial OP}$$

Chandra, Coleman, Flint, Nature (2013)

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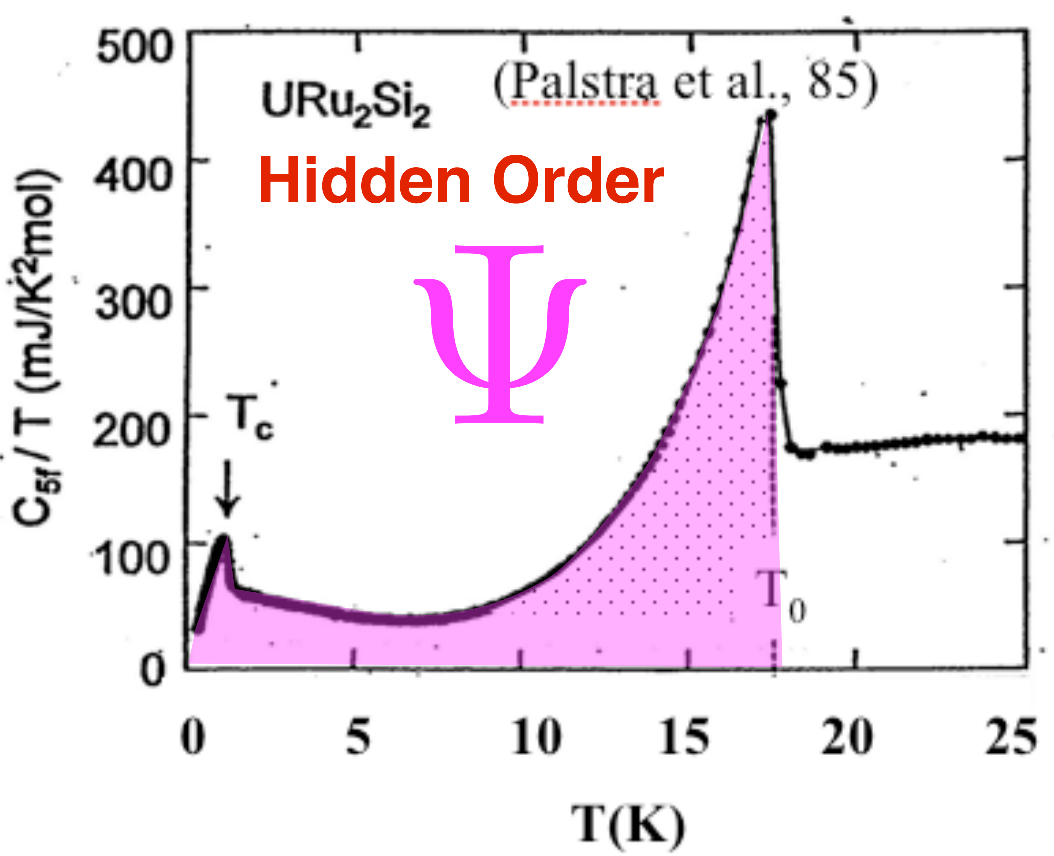
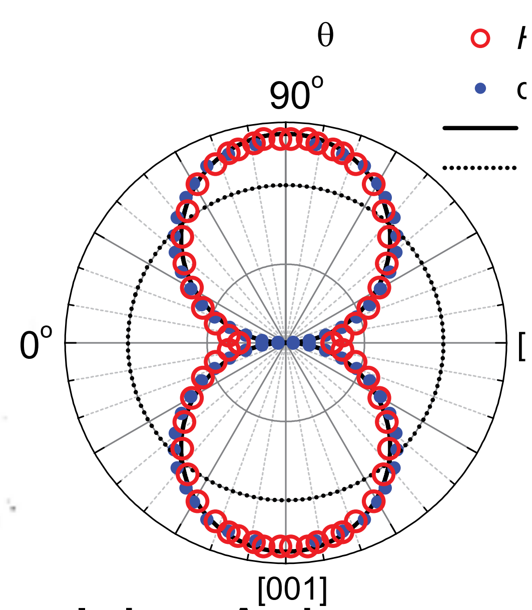
		$\Psi_\alpha\hat{\chi}_\lambda$	Hastatic	$\langle c^\dagger\vec{\sigma}\left(\vec{I}\cdot\vec{\tau}\right)c\rangle \propto \Psi^\dagger\vec{\sigma}\,\Psi$





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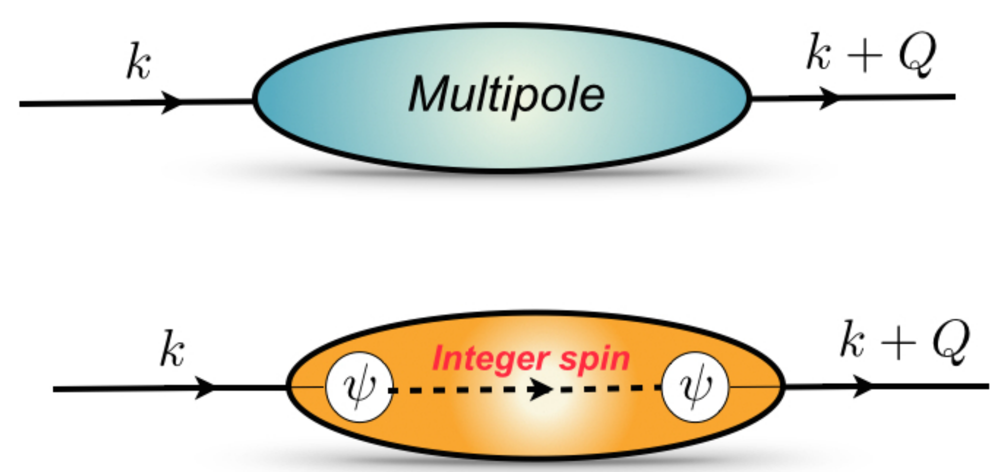

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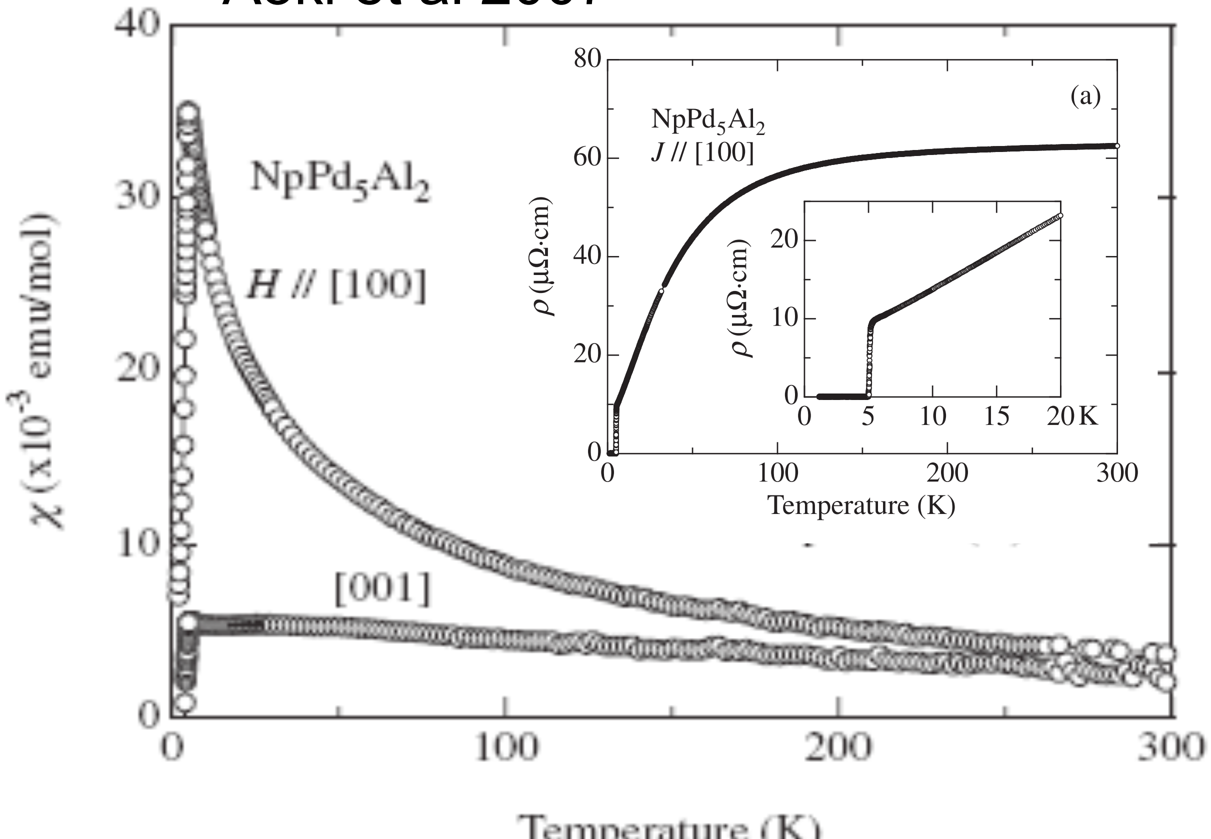
Aoki et al 2007



See: Flint, Dzero, PC, Nat Phys. (2008)

$NpPd_5Al_2$ T_C = 4.5K

Curie Law SC

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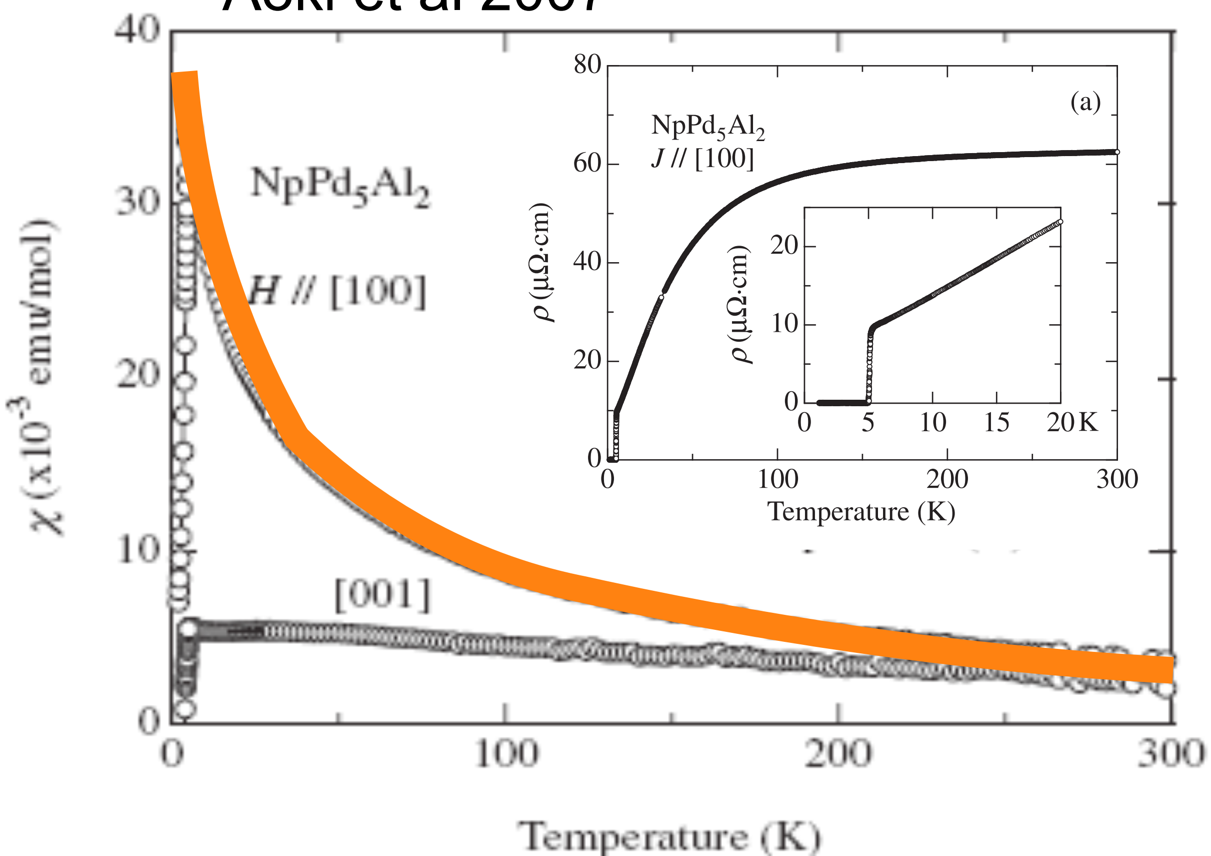

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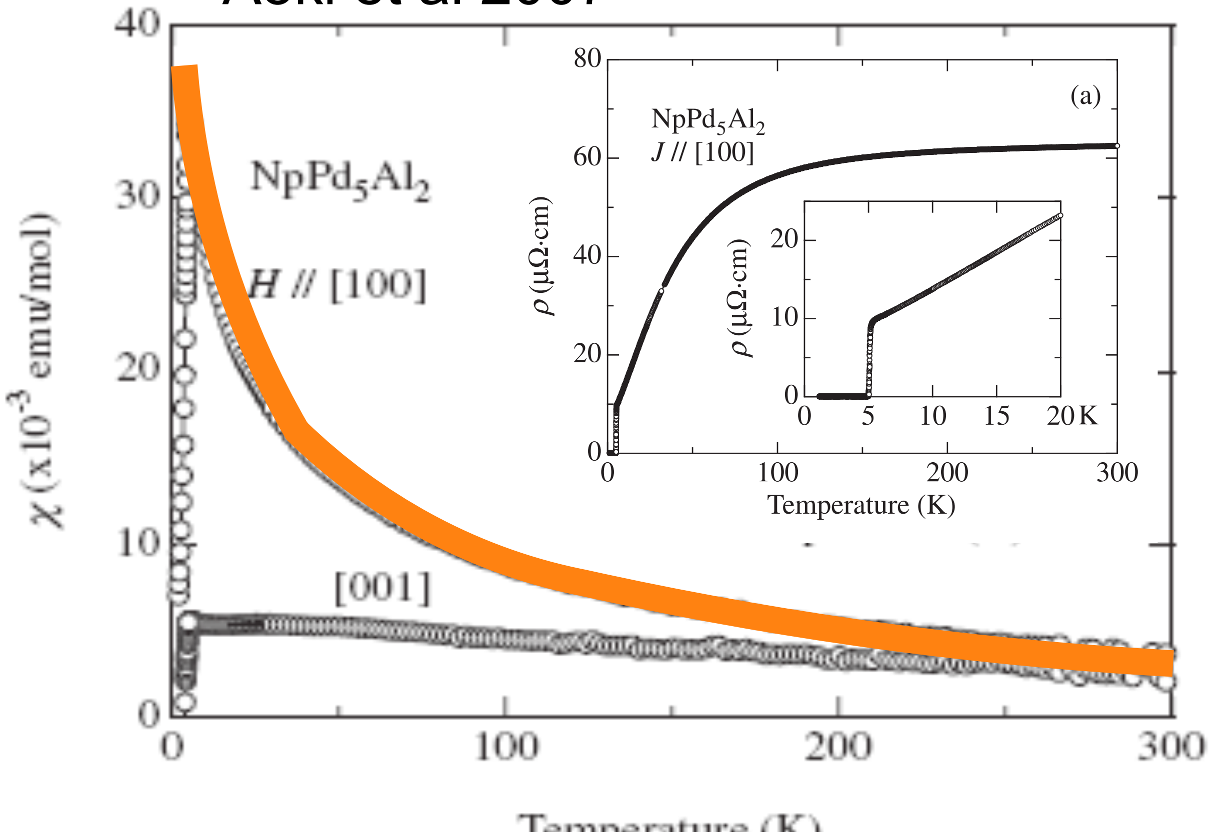

Composite order

$$\langle(\psi_1\vec{\sigma}\sigma_2\psi_2)\cdot\vec{S}\rangle \propto (V_1\Delta_2 - V_2\Delta_1)$$

See: Flint, Dzero, PC, Nat Phys. (2008)

$NpPd_5Al_2$ $T_C = 4.5K$

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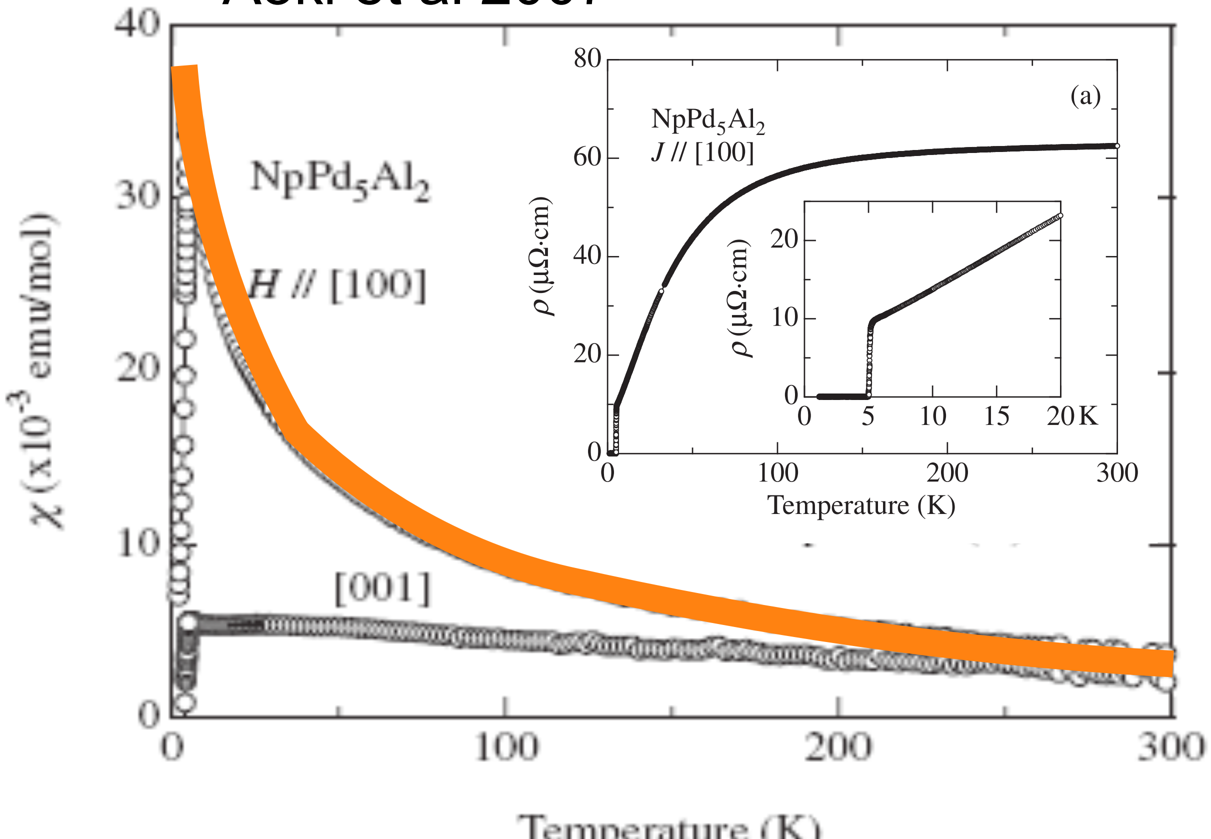
Composite Order

		$V_\lambda f_\alpha$	Composite Multipole	$\langle\left(\psi_1^\dagger\vec{\sigma}\psi_1 - \psi_2^\dagger\vec{\sigma}\psi_2\right)\cdot\vec{S}\rangle$
2-channel	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_{\lambda\beta}$	$V_\lambda f_\alpha + \Delta_\lambda \bar{\alpha} f^\dagger_{-\alpha}$	Composite Pair	$\langle(\psi_1\vec{\sigma}\sigma_2\psi_2)\cdot\vec{S}\rangle$
		$\Psi_\alpha \hat{\chi}_\lambda$	Hastatic	$\langle c^\dagger\vec{\sigma}\left(\vec{I}\cdot\vec{\tau}\right)c\rangle \propto \Psi^\dagger\vec{\sigma}\,\Psi$

Aoki et al 2007



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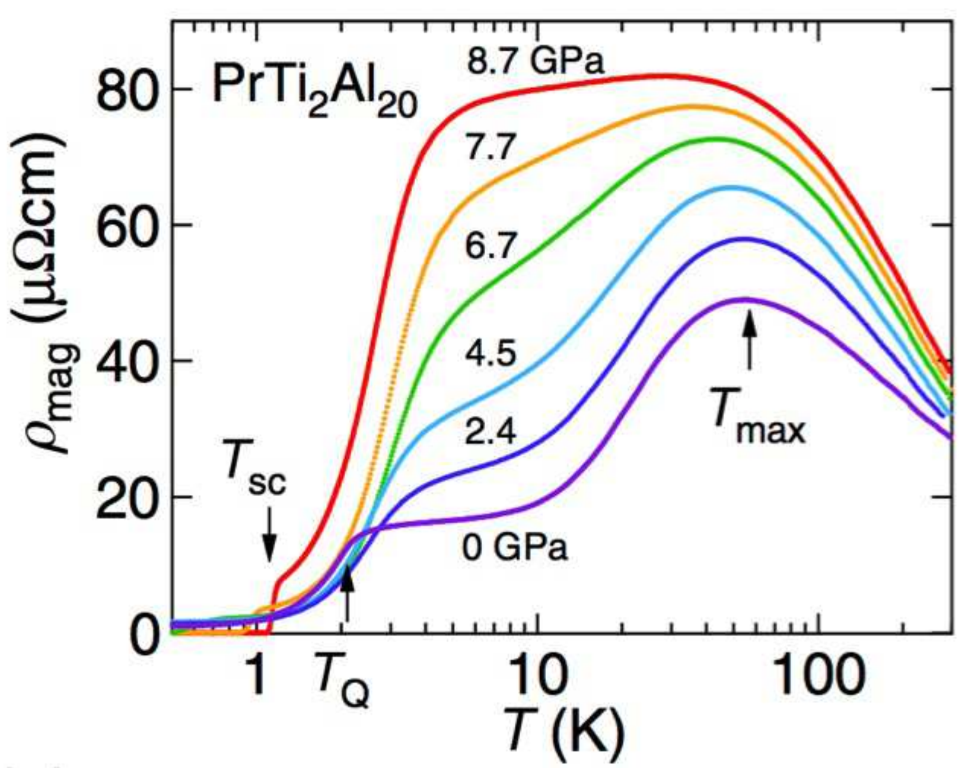
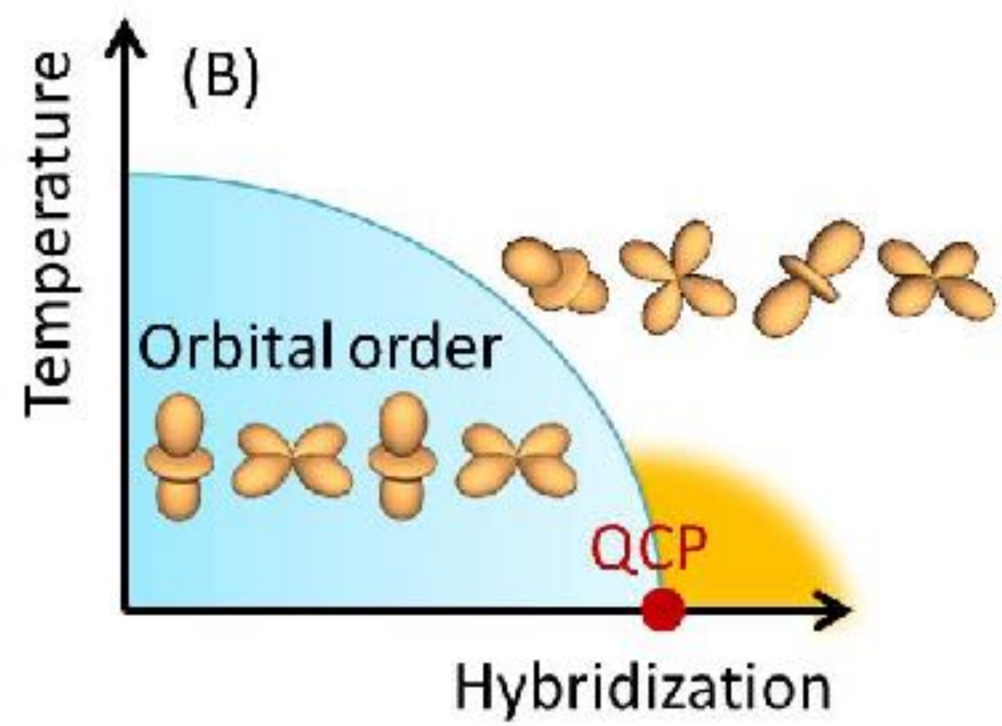
A. Sakai, K. Kuga, and S. Nakatsuji, J. Phys. Soc. Jpn. **81**, 083702 (2012).





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Composite Order

Kondo	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_\beta$	$V f_\alpha$	HF	$\langle \psi^\dagger(\vec{\sigma}\cdot\vec{S})\psi\rangle \propto \lvert V\rvert^2$
Majorana		$(\vec{\sigma}\cdot\vec{\eta})_{\alpha\beta}\mathcal{V}_\beta$	Odd-w triplet/ Skyrme Insulator	$\langle \psi_\uparrow\psi_\downarrow\vec{S}\rangle \propto \mathcal{V}^T\vec{\sigma}\sigma_2\mathcal{V}$
2-channel	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_{\lambda\beta}$	$V_\lambda f_\alpha$	Composite Multipole	$\langle\left(\psi_1^\dagger\vec{\sigma}\psi_1 - \psi_2^\dagger\vec{\sigma}\psi_2\right)\cdot\vec{S}\rangle$
		$V_\lambda f_\alpha + \Delta_\lambda \bar{\alpha} f^\dagger_{-\alpha}$	Composite Pair	$\langle(\psi_1\vec{\sigma}\sigma_2\psi_2)\cdot\vec{S}\rangle$
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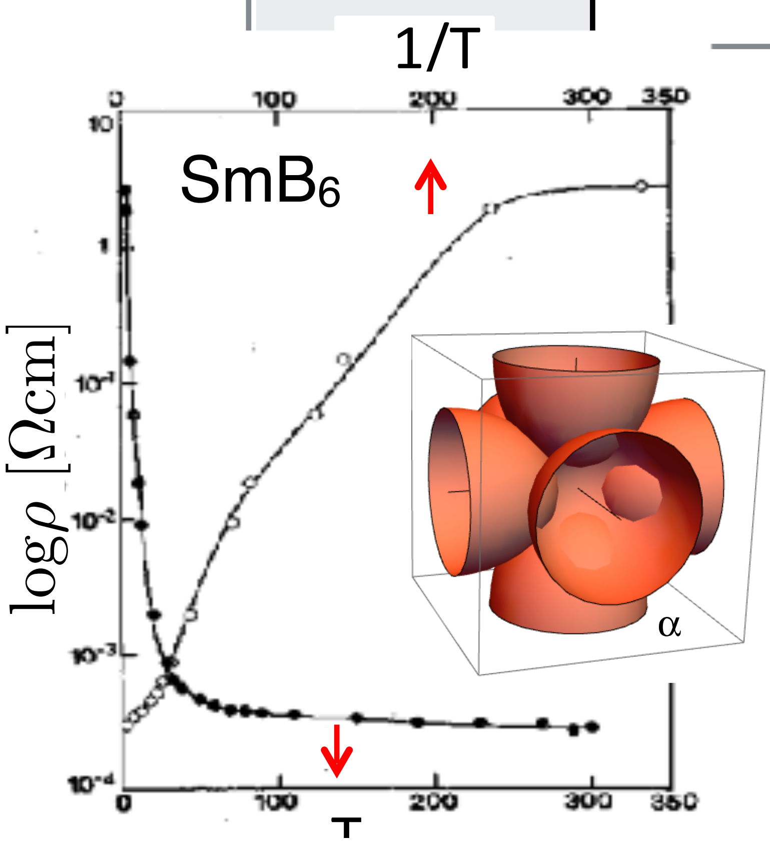

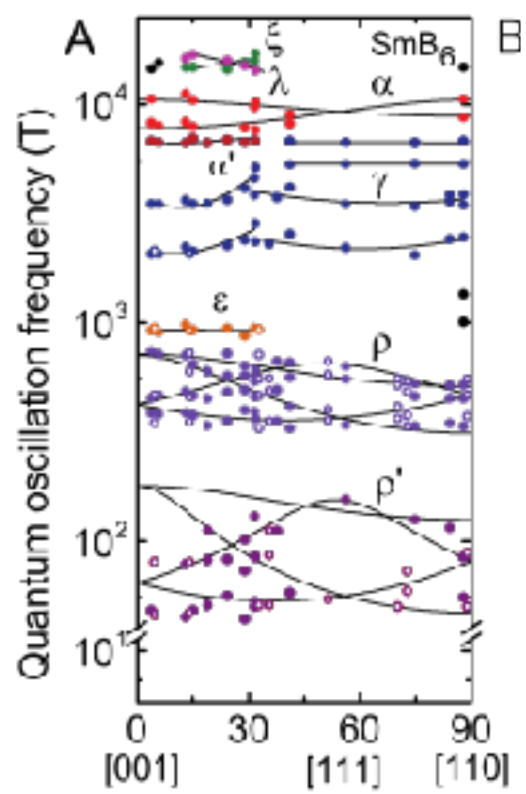

Tan *et al.* Science 349, 287 (2015)

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2-channel	$(\vec{S}\cdot\vec{\sigma})_{\alpha\beta}\psi_{\lambda\beta}$	$V_\lambda f_\alpha$	Composite Multipole	$\langle\left(\psi_1^\dagger\vec{\sigma}\psi_1 - \psi_2^\dagger\vec{\sigma}\psi_2\right)\cdot\vec{S}\rangle$
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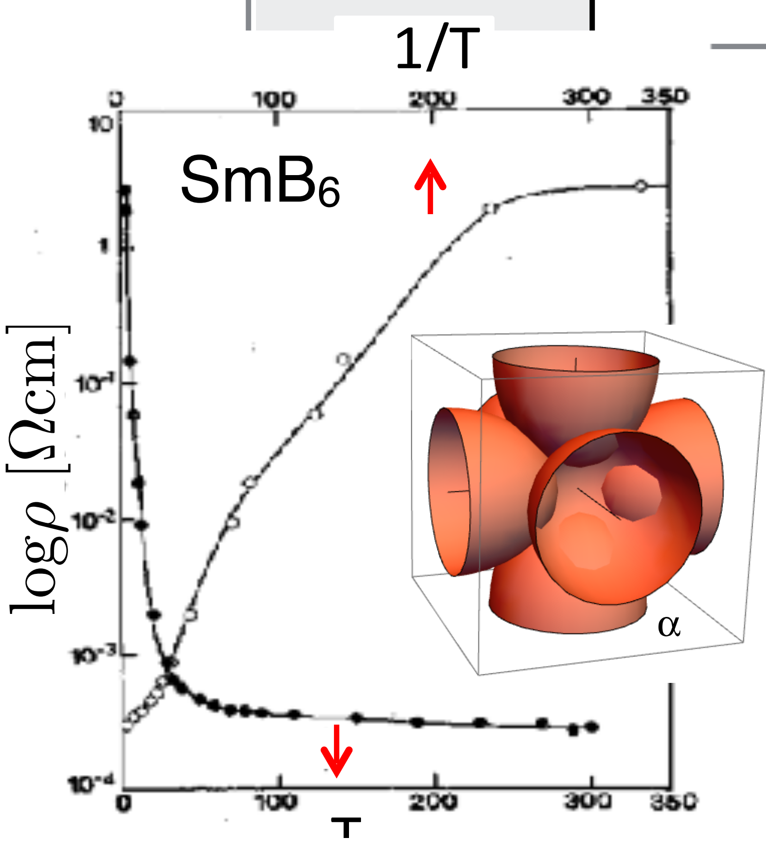

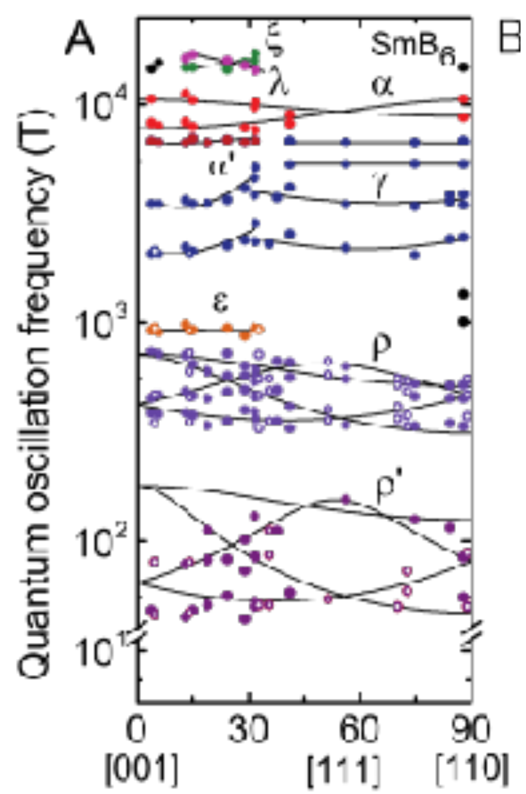

Tan *et al.* Science 349, 287 (2015)

$$\vec{S} = -\frac{i}{2}\vec{\eta}\times\vec{\eta}$$

Majorana Fractionalization
(cf Kitaev)

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		$V_\lambda f_\alpha + \Delta_\lambda \bar{\alpha} f^\dagger_{-\alpha}$	C F	$)\cdot\vec{S}\rangle$
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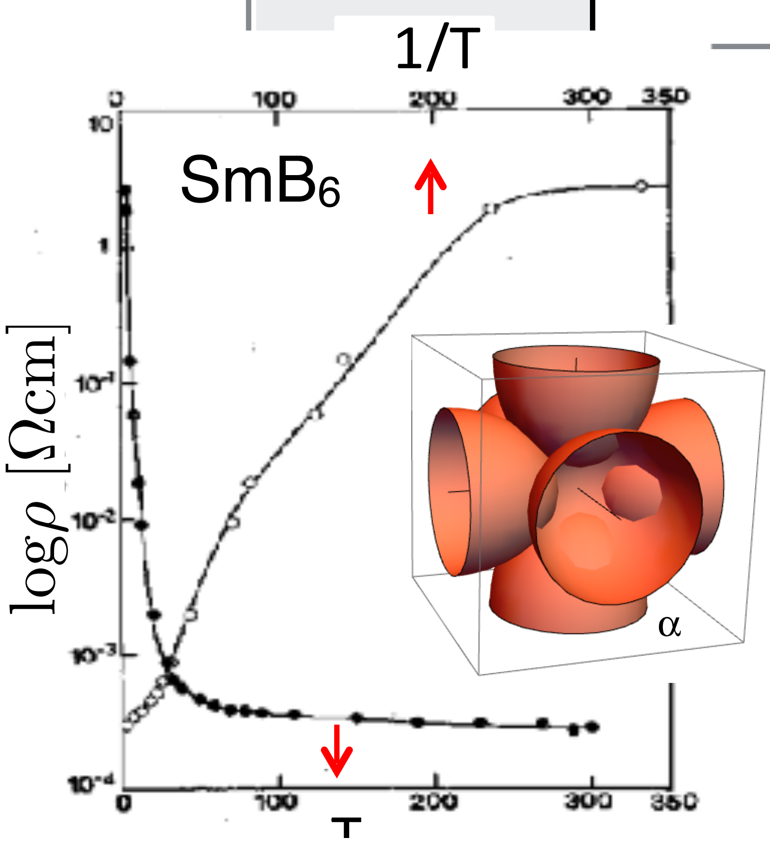

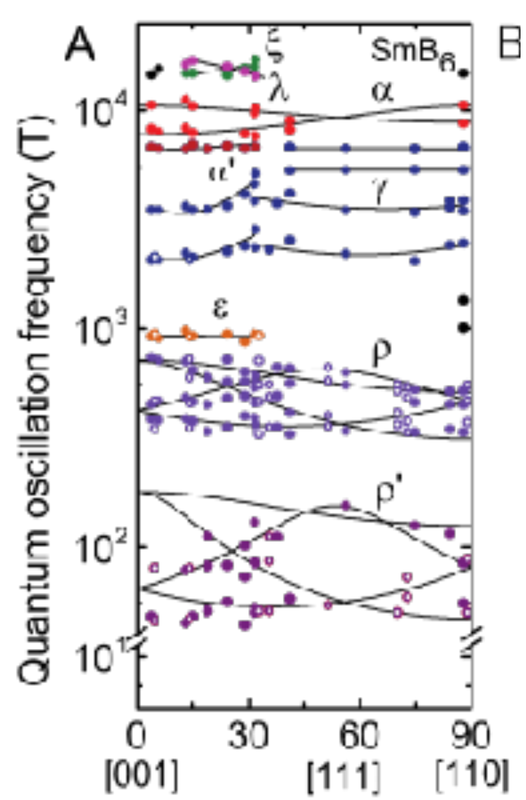

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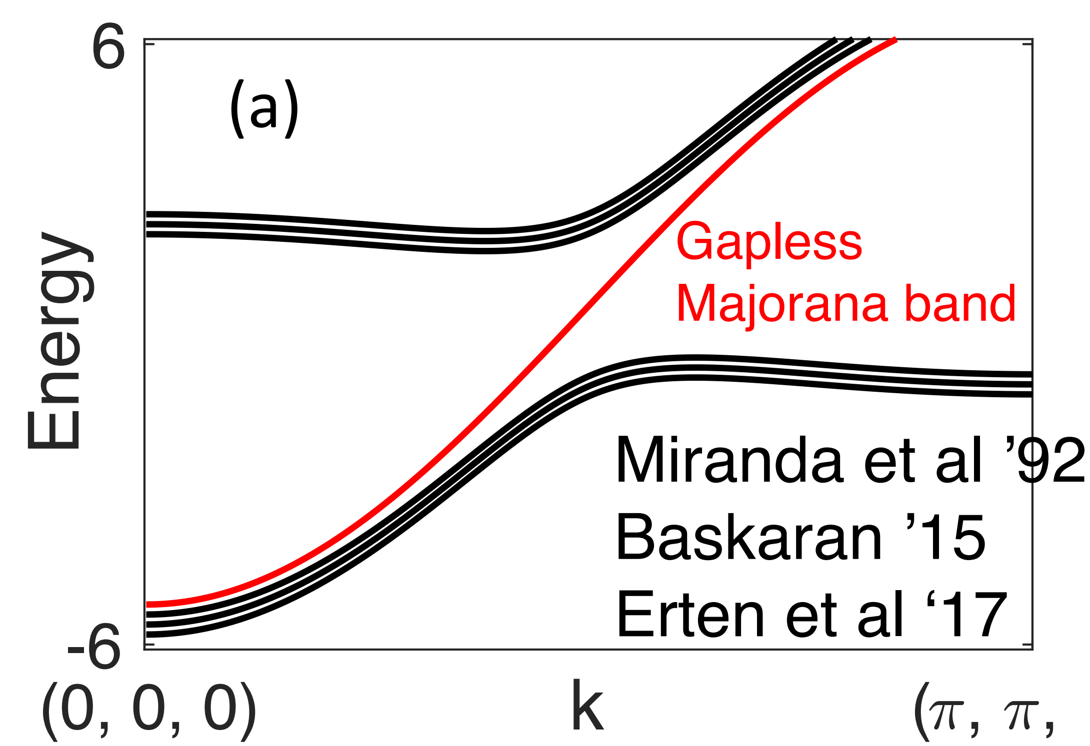

Conclusions

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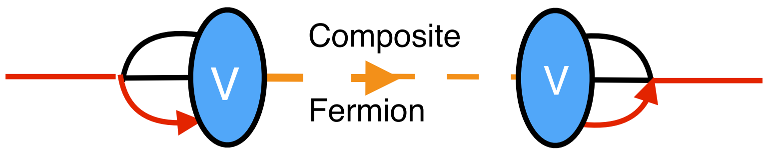
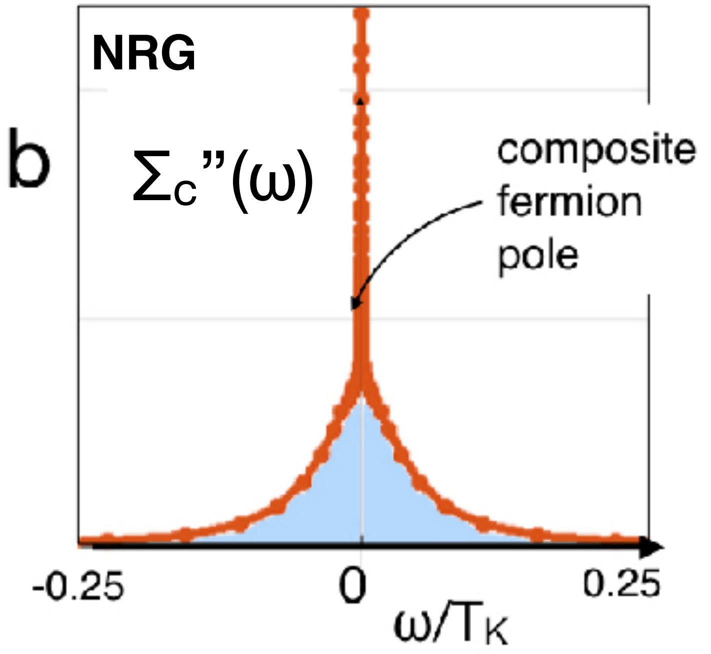


- Kondo effect is a hitherto unrecognized form of *spin fractionalization.*



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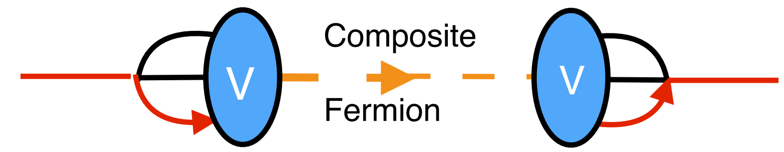

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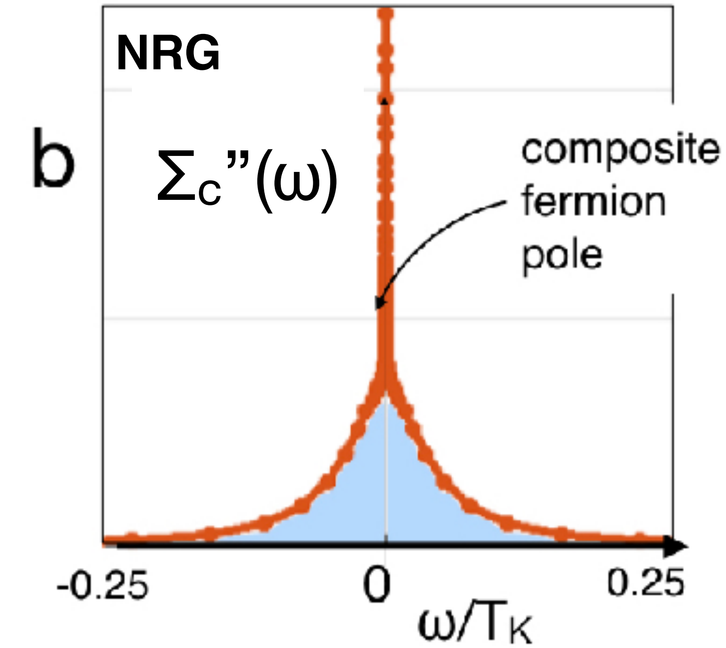
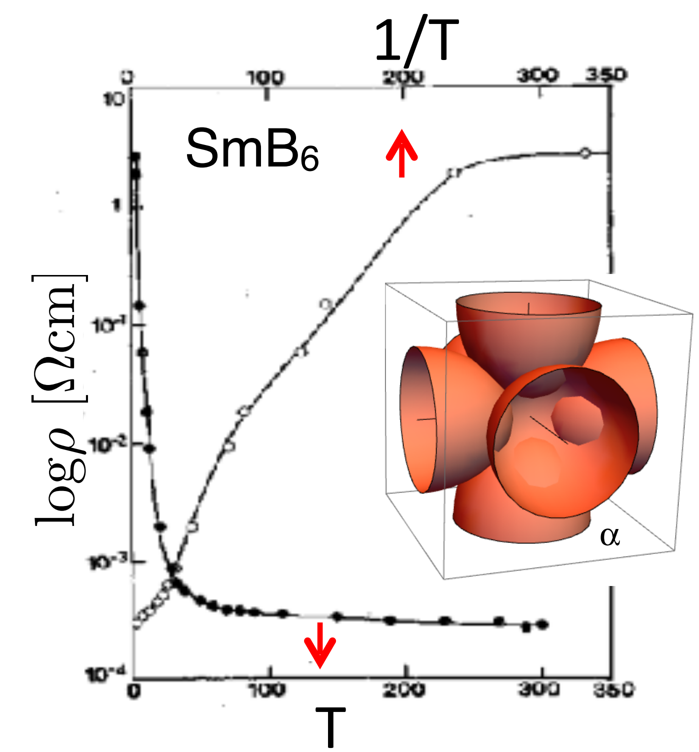
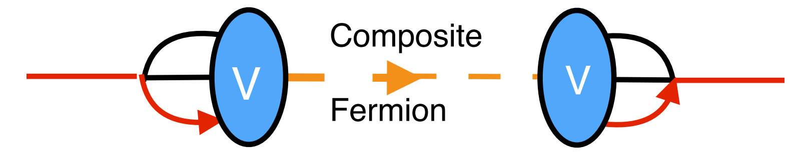




$$\vec{S} = -\frac{i}{2}\vec{\eta}\times\vec{\eta}$$

Majorana
Fractionalization ?

Conclusions



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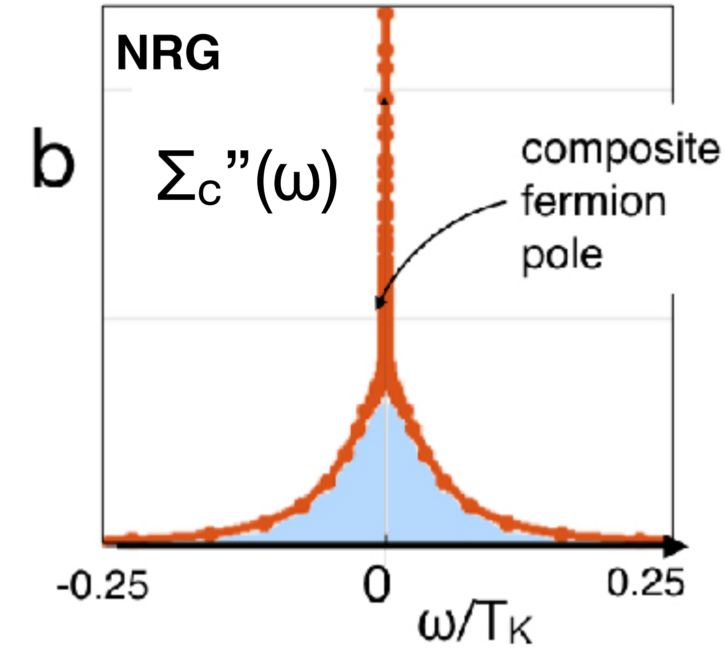

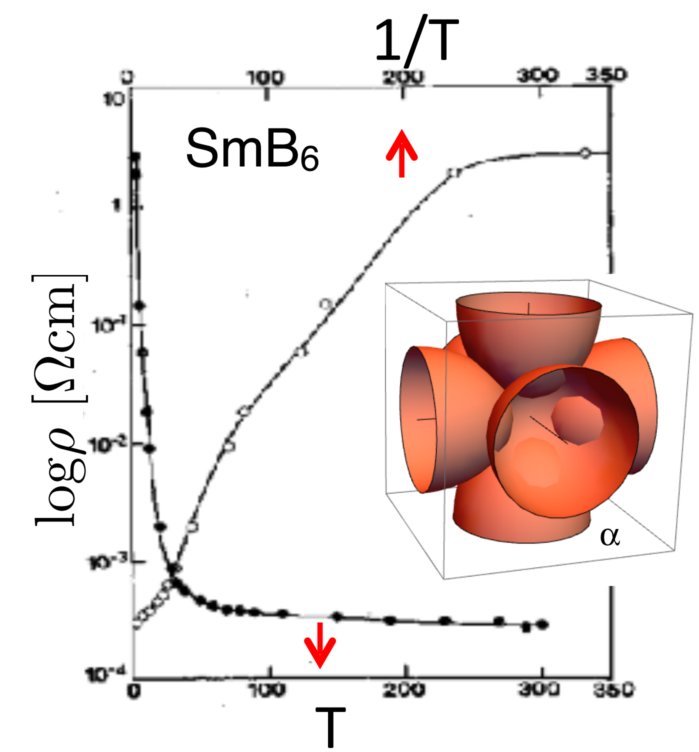

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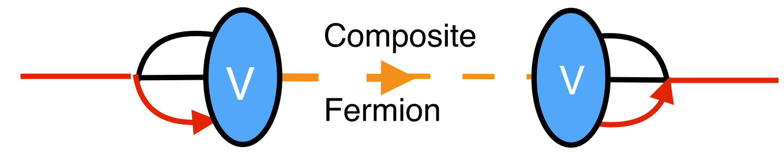
Majorana Fractionalization ?

Conclusions



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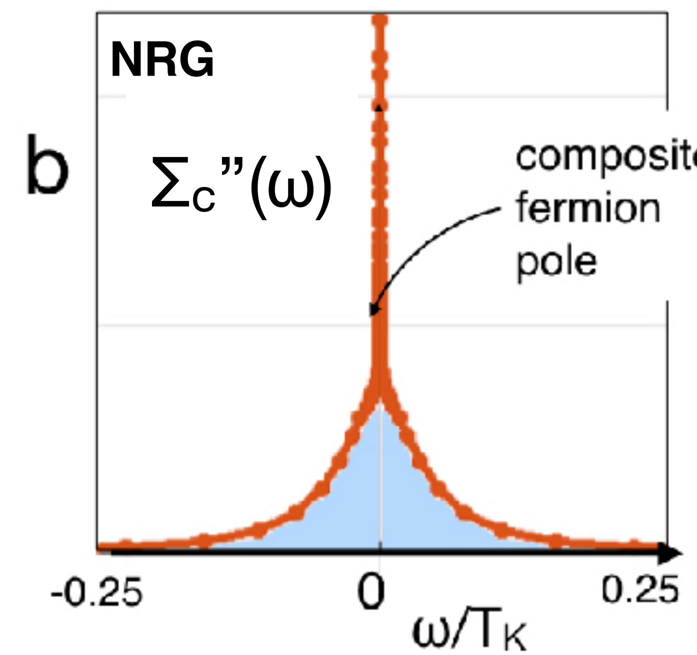
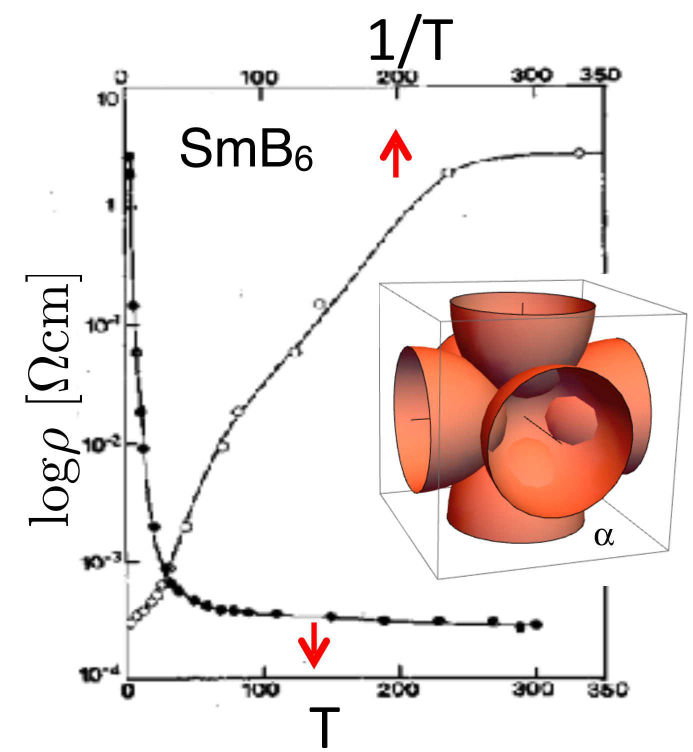
- Simultaneous development of Kondo and Order suggests order Fractionalization
- Order fractionalization can be induced in the two channel Kondo model





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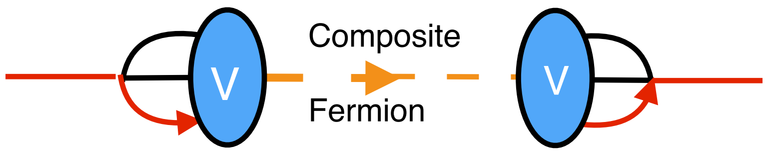
Majorana Fractionalization ?

Conclusions

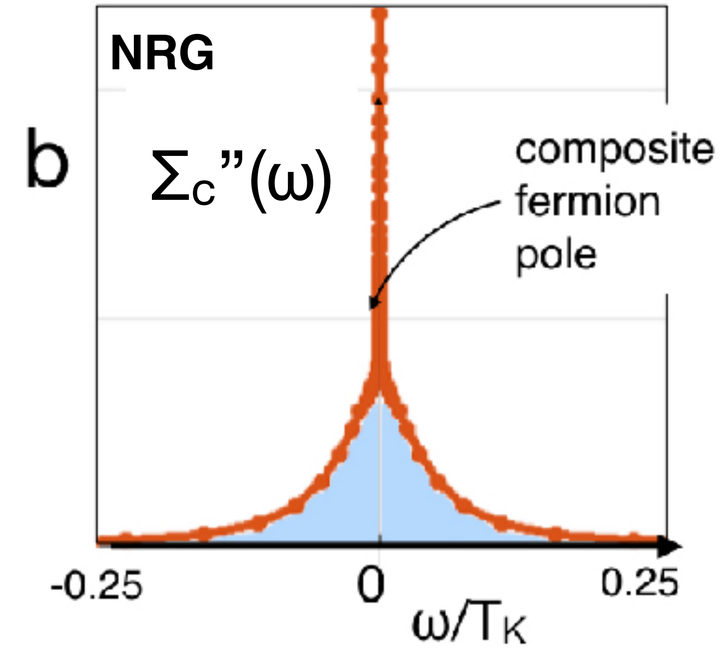


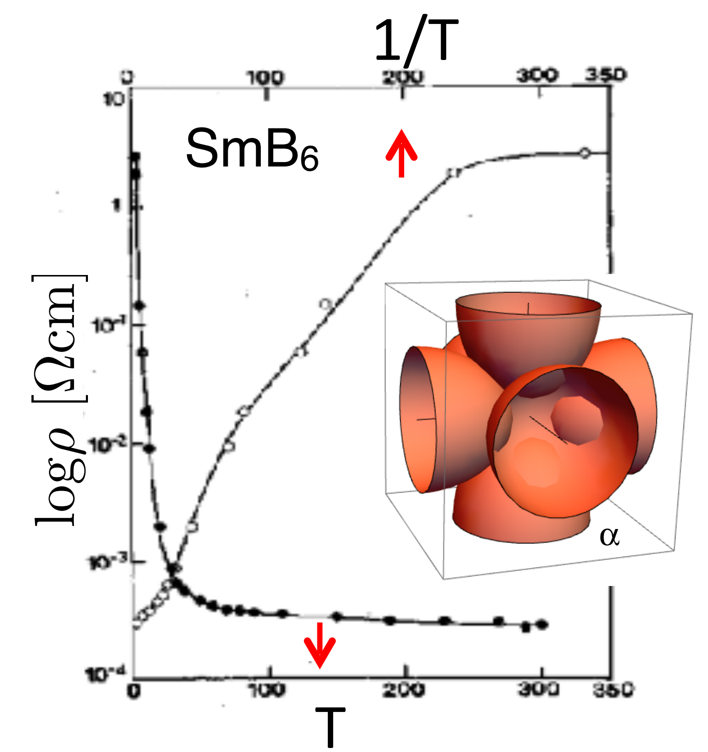
- Kondo effect is a hitherto unrecognized form of *spin fractionalization.*
- *Other forms of spin fractionalization are possible (cf Kitaev)*
- Simultaneous development of Kondo and Order suggests order Fractionalization
- Order fractionalization can be induced in the two channel Kondo model
- Order fractionalization conjecture

$$(\overline{\psi\psi\psi})_\Lambda(x) = V^\lambda_{\alpha\alpha'}(x) f_{\alpha'}(x)$$

$$\Sigma_{\lambda\lambda'}(2,1) \xrightarrow{|2-1|\to\infty} V_\lambda(2)V_{\lambda'}(1)g(2-1)$$

ODLRO in Space Time





$$\vec{S} = -\frac{i}{2}\vec{\eta}\times\vec{\eta}$$

Majorana Fractionalization ?

Thank You!