SCES in the Quantum Information era: New challenges and paradigms.

Piers Coleman

Center for Materials Theory, Rutgers U, USA
Hubbard Theory Consortium, Royal Holloway, U. London

- Fifty Years of Novel Phases.
- Dark Matter Challenges of the Solid State
- Quantum Information: from NRG to Tensor Networks
- Beyond Hartree Fock/BCS
- Strange insulator.
- Conclusions
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Einstein: Prague 1911-1912
National Geographic. “Genius”. Flynn and Fletcher as Einstein and Grossman

Rutgers

NSF
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**Fig. 6** The principle of equivalence implies the gravitational redshift he clearly states that the equivalence of both systems is certain as far as we restrict ourselves to purely mechanical phenomena, but it will acquire a deeper meaning if we extend the equivalence to all laws of nature. The new principle has a theoretical meaning. In his biography *Einstein: Creator and Rebel*, Banesh Hoffmann writes: "In the paper of 1907... Einstein had already begun his attack on the problem of acceleration, and he returned to it in his Prague paper of 1911. His arguments, particularly in its 1911 form, must rank as one of the most remarkable in the history of science." The principle of equivalence is then applied to derive the gravitational redshift—for the first time in a beautifully pedagogical way (see Fig. 6). As Mark Twain writes: "The nice thing about Science is that one gets such wholesale returns of conjecture from such a trifling investment of fact..."

What is the present-day formulation of the (weak) equivalence principle? Employing Cliff Will’s formulation from his Living Reviews article:

- Test bodies fall with the same acceleration independently of their structure or composition.
- The outcome of any local non-gravitational experiment is independent of: (a) the velocity of the local inertial frame in which it is performed, (b) where and when in the universe it is performed.

From the time of Newton and Eötvös it has been a continuing effort to measure a possible violation of the first item. After the 1950s, it is connected with the names of Dicke in Princeton, Braginskij in Moscow and, most recently, with the group at the University of Washington which used a torsion balance tray to study the accelerations...
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An era of extraordinary opportunity for our field.
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An era of extraordinary opportunity for our field.
Fifty Years of Novel Phases.
SmB$_6$ 1969

Menth, Buehler, Geballe PRL 22, 295 (1969)
Allen, Batlogg, Watchter PRB 20 4807 (1979)
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SmB$_6$ 1969
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UBe$_{13}$ 1974

\[ \frac{1}{T} \]

\[ \log \rho \text{ [\(\Omega\text{cm}\)]} \]

Ott, Rudigier, Fisk and Smith, PRL, 1595 (1983)
This suggests that the superconductivity is not an intrinsic property of UBe$_{13}$, but probably linked with precipitated filaments.


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Discovery often awaits new concepts, new consensus
Fifty Years of Novel Phases.

Discovery often awaits new concepts, new consensus

~5 yrs before Superfluid He-3
10 yrs before Heavy Fermion SC
20 yrs before High Temperature SC
30 yrs before Quantum criticality
40 yrs before Topological insulators, Fe based SC
Dark Matter Challenges of the Solid State.

Abell Cluster 3827 (ESO/R Massey)
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Strange Insulator SmB$_6$

- Pairing Mechanism of Cu/Fe/HFSC
- Quantum Criticality in Metals
- Nature of the pseudogap
- Sign Problem in QMC
- Topology in SCES
- Uemura Scaling $\rho_s \sim T_c$
  (overdoped) cuprates
- Ground-state of Spin Liquids
Dark Matter Challenges of the Solid State.

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These, and many other examples suggest the potential for qualitatively new advances in our understanding of quantum matter.
Dark Matter Challenges of the Solid State.

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Bulk Fermi Surface in a robust insulator?

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Bulk Fermi Surface in a robust insulator?

Cuprates $T_c=11-92$K

Takagi et al, PRL '92

$\rho_{ab}$ vs $T$ (K)

$\log\rho_{ab}$ vs $1/T$

$\rho'$ vs Quantum oscillation frequency (T)

SmB$_6$

Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals

![Graph showing linear resistivity in strange metals](image)

Takagi et al, PRL ‘92

Cuprates \( T_c=11-92 \text{K} \)
• Linear resistivity in strange metals

\begin{center}
\includegraphics[width=\textwidth]{image.png}
\end{center}
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals

![Graph showing linear resistivity in strange metals](T(K) vs. Ω/cm)

Cuprates $T_c = 11-92$K

![Diagram showing magnetic moments and superconductivity](Magnetic moments and superconductivity)

- Increasing localization
- Magnetic moments
- Superconductivity
• Linear resistivity in strange metals

**Diagram:**

Cuprates Tc=11-92K

**Graph:**

- **Cuprates Tc=11-92K**
- **CeCoIn\textsubscript{5}**

**Legend:**

- *heat*
- *charge*
- 5.3 T

**Equations:**

- $\rho_{ab}$
- $T(K)$
- $\rho_c$
- $\rho_a$

**References:**

- Takagi et al, PRL ‘92
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Link with quantum criticality and break-down of Fermi Liquid

Cuprates $T_c=11$-92K

[Graph showing resistivity vs. temperature for Cuprates]

[Graph showing resistivity vs. temperature for CeCoIn$_5$]

CeCoIn$_5$ Tanatar et al (2007)

- $T^2$ dependence of the resistivity
- Recent measurements of the (azimuthal) anisotropy
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Link with quantum criticality and break-down of Fermi Liquid

![Graph of Cuprates Tc=11-92K]

![Graph of BaFe\textsubscript{2}As\textsubscript{2-x}P\textsubscript{x} (x=0.31)]

![Graph of Hayes et al (2015)]
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Link with quantum criticality and break-down of Fermi Liquid

![Graph showing resistivity vs magnetic field](image)

**FIG. 1.** The MR qualitatively looks like a superconducting transition. When plotted this way, the data appear to collapse to a single curve that is well described by a linear behaviors are different; our measure of the scattering rate will be necessary to determine the scale factor, so that only the temperature dependence, as described by Eq. 1.

Although we cannot account for this in our ansatz, the resistance always approaches the same temperature curve so that magnetic field is certain to be dominant energy scale. Comparing the slope of residual resistivity data to determine the scale factor, choosing the lowest temperature and magnetic field scales respectively.

For compositions far from the QCP, we expect this behavior to break down completely as the materials crossover to compositions well beyond critical field. Here we remove the temperature dependence, as described by Eq. 1. Remarkably, this behavior holds for a range of dopings near the critical doping-temperature quantum critical phase diagram to reveal a QCP. The vari-

The MR curves were taken at temperatures 4K, 25K, 31K, 38K and 60K. When plotted this way, the data appear as shown in Fig. 4 for compositions well beyond critical field.

A similar ansatz has been used to describe the temperature and magnetic field scales respectively. The vari-

The vari-

The vari-

The vari-
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals \( \Gamma_{tr} \sim k_B T \)
- Link with quantum criticality and break-down of Fermi Liquid
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals $\Gamma_{tr} \sim k_B T$
- Link with quantum criticality and break-down of Fermi Liquid

![Graph showing linear resistivity in strange metals and unidentified critical scattering.](image)

*Cuprates $T_c=11-92$K*

MFL (Varma et al 1989)

Unidentified (local) critical scattering.

Abbamonte: EELS
Kondo effect: Iconic example of Entanglement

\[ \xi = \frac{v_F}{T_K} \]
Kondo effect: Iconic example of Entanglement

\[ \xi = \frac{v_F}{T_K} \]

Spin screened by conduction electrons: entangled

\[ \uparrow \downarrow - \downarrow \uparrow \]
Kondo effect: Iconic example of Entanglement

Spin screened by conduction electrons: **entangled**

\[ \xi = \frac{v_F}{T_K} \]

\[ S(T) = \int_0^T \frac{C_V}{T'} dT' \]

“Spin entanglement entropy”
Kondo effect: Iconic example of Entanglement

Spin screened by conduction electrons: entangled

$\uparrow \downarrow - \downarrow \uparrow$

$S(T) = \int_0^T \frac{C_V}{T'}dT'$

“Spin entanglement entropy”

$\xi = v_F/T_K$

$C_V \frac{R \ln 2}{T_k} \gamma$
Kondo effect: Iconic example of Entanglement

Spin screened by conduction electrons: entangled

\[ S(T) = \int_0^T \frac{C_V}{T'} dT' \]

“Spin entanglement entropy”
Kondo effect: Iconic example of Entanglement

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Spin screened by conduction electrons: *entangled*

$\uparrow \downarrow \uparrow \downarrow$

$S(T) = \int_0^T \frac{C_V}{T'} dT'$

“Spin entanglement entropy”

SCES: What new forms of entanglement are possible?
Kondo effect: QI perspective

\[ \xi = \frac{v_F}{T_K} \]
Kondo effect: QI perspective

\[ \hat{\rho}(r) = \text{Tr}_{r' \rightarrow r} [\psi \langle \psi |] \]

\[ \xi = \frac{v_F}{T_K} \]
Kondo effect: QI perspective

\[ \hat{\rho}(r) = \text{Tr}_{r'} \left[ |\psi\rangle \langle \psi| \right] \]

\[ S(r) = -\text{Tr}\left[ \rho(r) \ln \rho(r) \right] \]

Entanglement Entropy
Kondo effect: QI perspective

\[ S_i(r) = S(r) - S_{BULK}(r) \]

- Density Matrix
  \[ \hat{\rho}(r) = \text{Tr}_{r' > r}[|\psi\rangle\langle\psi|] \]

- Entanglement Entropy
  \[ S(r) = -\text{Tr}[\rho(r) \ln \rho(r)] \]

- Kondo Impurity Entanglement Entropy and the Kondo Screening Cloud

\[ \xi = v_F/T_K \]

\[ r \]

\[ \ln(2) \]

\[ S_{\text{imp}}(r=\text{Constant}) \]

Figure 2.

Universal scaling plot of \( S_{\text{imp}} \) for fixed \( r/R \), (a) for \( R \leq 10^2 \) even, (b) for \( R \leq 10^1 \) odd. DMRG results for the \( J_1 \)–\( J_2 \) chain at \( J_{c2} \) for various couplings \( J'_{K} \). The lines marked \( \pi \xi_K/(12r) \) are the FLT prediction: equation (9).

(c) The location of the maximum, \( (r/\xi_K)_{\text{max}} \), of \( S_{\text{imp}} \) for odd \( R \), plotted versus \( r/R \). Find that \( S_{\text{imp}}(J'_{K}, r, R) \) appears to be a scaling function, depending on the ratio of characteristic lengths when \( r, \xi_K \gg 1 \):

\[ S_{\text{imp}}(J'_{K}, r, R) = S_{\text{imp}}(r/\xi_K, r/R) \]
Kondo effect: QI perspective

Entanglement Entropy

\[ S(r) = -\text{Tr}[\rho(r) \ln \rho(r)] \]

Density Matrix

\[ \hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle \langle \psi|] \]

\[ S_i(r) = S(r) - S_{BULK}(r) \]

\[ S_i(r) = \ln 2 \]

\[ r/\xi_K \]

\[ \ln 2 \]

Sorensen, Chang, LaFlorencie & Affleck, JSM (2007)
Kondo effect: QI perspective

Density Matrix
\[ \hat{\rho}(r) = \text{Tr}_{r' > r}[|\psi\rangle\langle \psi|] \]

Entanglement Entropy
\[ S(r) = -\text{Tr}[\rho(r) \ln \rho(r)] \]

\[ S_i(r) = S(r) - S_{BULK}(r) \]

\[ \xi = v_F/T_K \]

\[ r/\xi_K \]

Region A

2-channel: Quantum Critical
Kondo effect: QI perspective

\[ S(r) = -\text{Tr}[\rho(r) \ln \rho(r)] \]

Entanglement Entropy

\[ S_i(r) = S(r) - S_{BULK}(r) \]

1-channel

\[ S_i(r) \sim (1/2) \ln 2 \]

2-channel

\[ S_i(r) \sim \ln 2 \]

Screening cloud of 2-channel Kondo model is infinitely large

Alkurtass et al, PRB 93,081106R (2016)
From Wilsonian NRG to Tensor Networks

NRG Wilson 1973
DMRG White 1992
From Wilsonian NRG to Tensor Networks

Wavefunction

Big tensor $\Psi_{i_1i_2i_3i_4i_5i_6}$

$t$  $-t\Lambda^{-1}$  $-t\Lambda^{-(N-1)}$
From Wilsonian NRG to Tensor Networks

NRG Wilson 1973
DMRG White 1992

Wavefunction

$\sim e^N$

$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6}$
From Wilsonian NRG to Tensor Networks

Wavefunction

Big tensor $\Psi_{i_1 i_2 i_3 i_4 i_5 i_6}$

$\sim e^N$

Tensor network: matrix product state (MPS)

$\begin{array}{cccccccc}
A & a & B & b & C & c & D & d \\
i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\
\end{array}$

$D$ bond dimension

$A_{ijk}$ rank-3 tensor

NRG Wilson 1973
DMRG White 1992
From Wilsonian NRG to Tensor Networks

Wavefunction

Big tensor $\Psi_{i_1 i_2 i_3 i_4 i_5 i_6}$

$\sim e^N$

Tensor network: matrix product state (MPS)

Polynomial in N

$\sim e^N$
From Wilsonian NRG to Tensor Networks

\[ -t \quad -t^{\Lambda^{-1}} \quad -t^{\Lambda^{-(N-1)}} \]

Variational MPS is equivalent to Density Matrix Renormalization Group

\[ \sim e^N \]

Polynomial in N

\[ \text{Tensor network: matrix product state (MPS)} \]

\[ A \quad a \quad B \quad b \quad C \quad c \quad D \quad d \quad E \quad e \quad F \]

\[ i_1 \quad i_2 \quad i_3 \quad i_4 \quad i_5 \quad i_6 \]

\[ D \text{ bond dimension} \]

\[ \text{rank-3 tensor} \]

Kenneth Wilson 1936-2013

NRG Wilson 1973
DMRG White 1992
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (MPS)
From Wilsonian NRG to Tensor Networks

**Tensor network**: matrix product state (MPS)

A

a b c d e f

i_1 i_2 i_3 i_4 i_5 i_6

D bond dimension

poly(N) numbers vs \( \exp(N) \) many numbers

Tensor network ansatz for a wave function
From Wilsonian NRG to Tensor Networks

**Tensor network**: matrix product state (MPS)

PEPS (2D)
projected entangled-pair state

$D$ bond dimension
From Wilsonian NRG to Tensor Networks

**Tensor network**: matrix product state (MPS)

![Tensor Network Diagram](image)

- Capability for detailed study of spectral functions in impurity, 1 & 2D.

**PEPS (2D)**

- Projected entangled-pair state

![Projected Entangled-Pair State Diagram](image)
From Wilsonian NRG to Tensor Networks

**Tensor network**: matrix product state (MPS)

---

**PEPS (2D)**

projected entangled-pair state

---

Capability for detailed study of spectral functions in impurity, 1 & 2D.

---

Weichelsbaum et al (2009)
Tensor/multidimensional array

Tensor network ansatz for a wave function

State: \( \exp(N) \) many numbers

Big tensor

Tensor network functions in impurity, 1 & 2D.

Capability for detailed study of spectral functions in impurity, 1 & 2D.

PEPS (2D)
projected entangled-pair state

From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (MPS)

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**Tensor network**: matrix product state (MPS)

- **PEPS (2D)**: projected entangled-pair state

Capacity for detailed study of spectral functions in impurity, 1 & 2D.
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PEPS (2D)
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FM Kondo-Heisenberg
From Wilsonian NRG to Tensor Networks

**Tensor network**: matrix product state (MPS)

![Diagram of Tensor Network]

- **Capability for detailed study of spectral functions in impurity, 1D & 2D**

**PEPS (2D)**
projected entangled-pair state

![Diagram of PEPS (2D)]

- **FM Kondo-Heisenberg**
- **Critically screened**
- **Fully screened Luttinger Liquid**
- **FM**
- **$T_{K/J_H}$**

Put some numbers: if with an incredibly large number of quantum states. In order to give a quantitative idea, let us imagine that you are given a quantum many-body wave-function. Specifying its coefficients in some local basis is an effective representation. The Hilbert space of a quantum many-body system is really big and its constituents. It is expected that this structure is different depending on the dimensionality of the system: this should be different for 1D and 2D systems, and so on. But it should also be different depending on the dimensionality of Nature. For a system of e.g. 3.4 Hilbert space is far too large to be tractable, and the language of TN is precisely the correct one to pursue this kind of connection.

As we shall see, a TN has this information directly available in its description in terms of a network of quantum correlations. In a way, we can think of TN states as quantum states given in an intuitions, ideas and results.
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (MPS)

PEPS (2D)
projected entangled-pair state

Capability for detailed study of spectral functions in impurity, 1D & 2D

FM Kondo-Heisenberg

2-channel Kondo

2-channel Kondo Lattice
Fig. 12.5 Dependence of Free energy on order parameter for (a) an Ising order parameter $\psi = \psi_1$, showing two degenerate minima and (b) complex order parameter $\psi = \psi_1 + i \psi_2 = |\psi| e^{i\phi}$, where the Landau free energy forms a “Mexican Hat Potential” in which the free energy minimum forms a rim of degenerate states with energy that is independent of the phase $\phi$ of the uniform order parameter.

is an essential component of broken continuous symmetry. In superfluids, the emergence of a well-defined phase associated with the order parameter is intimately related to persistent currents, or superflow. We shall shortly see that when we “twist” the phase, a superflow develops.

\[ \mathbf{j} \propto \mathbf{\nabla} \phi. \]

To describe this rigidity, we need to take the next step, introducing a term into energy functional that keeps track of the energy cost of a non-uniform order parameter. This leads us onto Landau Ginzburg theory.

12.3 Ginzburg Landau theory I: Ising order

Landau theory describes the energy cost of a uniform order parameter: a more general theory needs to account for inhomogeneous order parameters in which the amplitude varies or the direction of the order parameter.
Beyond Hartree Fock/BCS

Order parameter $\psi$

- Order Parameters are bosons, and must contain an even number of fermions: even charge, integer spin.

$$\Psi = \langle \hat{\psi}_\uparrow \hat{\psi}_\downarrow \rangle \quad \vec{M} = \langle \psi^\dagger \vec{\sigma} \psi \rangle$$

cf BCS theory, Stoner Magnetism.
Beyond Hartree Fock/BCS

- Order Parameters are bosons, and must contain an even number of fermions: even charge, integer spin.

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cf BCS theory, Stoner Magnetism.

- But can order parameters, like excitations, fractionalize?

\[ \psi = \sqrt{\text{Multipole}} = \text{Spinorial OP} \]


Beyond Hartree Fock/BCS: 3 body Bound States

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \psi_0^\dagger \vec{\sigma} \psi_0 \cdot \vec{S}_0 \]
Beyond Hartree Fock/BCS: 3 body Bound States

\[
H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \psi_0^\dagger \vec{\sigma} \psi_0 \cdot \vec{S}_0
\]

Irreducible t-matrix
\[
t(E) = \Sigma(E) + t(E) G_0(E) \Sigma(E)
\]
Beyond Hartree Fock/BCS: 3 body Bound States

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J\psi_0^\dagger \vec{\sigma} \psi_0 \cdot \vec{S}_0 \]

Irreducible t-matrix
\[ t(E) = \Sigma(E) + t(E)G_0(E)\Sigma(E) \]

\[ \Sigma(\omega) \sim \frac{V^2}{\omega} \]
Beyond Hartree Fock/BCS: 3 body Bound States

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\[ \Sigma(\omega) \sim \frac{V^2}{\omega} \]

\[ \mathcal{F}_\alpha = J(\vec{\sigma} \cdot \vec{S}_0) \psi_{0\alpha} \rightarrow V f_\alpha(0) \]

Read and Newns, 1983

*Three-body* Bound State
Beyond Hartree Fock/BCS: 3 body Bound States

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + V(\psi_j^{\dagger}f_j + H.c) \]

\[ \Sigma(\omega) \sim \frac{V^2}{\omega} \]

\[ \mathcal{F}_\alpha = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow Vf_\alpha(0) \]

Read and Newns, 1983

*Three-body* Bound State
Beyond Hartree Fock/BCS: 3 body Bound States

\[ H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + V(\psi_j^\dagger f_j + \text{H.c}) \]

\[ \Sigma(\omega) \sim \frac{V^2}{\omega} \]

\[ F_\alpha = J(\vec{\sigma} \cdot \vec{S}_0)\psi_{0\alpha} \rightarrow V f_\alpha(0) \]

Read and Newns, 1983

*Three-body* Bound State
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

2-channel Kondo
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

2-channel Kondo

Critical Non Fermi Liquid
Order Parameter Fractionalization Hypothesis

2-channel Kondo

Critical Non Fermi Liquid

J+δ

J-δ

J

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Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

2-channel Kondo

Strongest Channel

\( J + \delta \)

\( J - \delta \)

\( 2 \)-channel Kondo

Channel specific composite fermion

\( \Sigma_1(E) \)

\( \Sigma_2(E) \)
Order Parameter Fractionalization Hypothesis

\[ \text{Strongest Channel} \]

\[ \Sigma_1(E) \]

\[ \Sigma_2(E) \]

\[ (\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_0) \begin{pmatrix} \psi_{1\beta} \\ \psi_{2\beta} \end{pmatrix} \rightarrow \begin{pmatrix} V \\ 0 \end{pmatrix} f_\beta \]
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Strongest Channel

Channel specific composite fermion

Hybridization Weiss Field
Order Parameter Fractionalization Hypothesis

2-channel Kondo Lattice

$J + \delta$

$J - \delta$

Strongest Channel

Channel specific composite fermion

$\Sigma_1(E)$

$\Sigma_2(E)$

$\begin{pmatrix} \psi_{1\beta} \\ \psi_{2\beta} \end{pmatrix} \rightarrow \begin{pmatrix} V \\ 0 \end{pmatrix} f_\beta$

Hybridization Weiss Field

P. Chandra, P. Coleman, Y. Komijani
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Strongest Channel

\[ J + \delta \]
\[ J - \delta \]

2-channel Kondo

Channel specific composite fermion

\[ \Sigma_1(E) \]
\[ \Sigma_2(E) \]

Hybridization Weiss Field

\[
(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_0) \begin{pmatrix} \psi_{1\beta} \\ \psi_{2\beta} \end{pmatrix} \rightarrow \begin{pmatrix} V \\ 0 \end{pmatrix} f_\beta
\]

Spinor OP Forms Spontaneously

\[
(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_j)\psi_{\lambda\beta} \rightarrow V_{\lambda} f_\alpha(0)
\]
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Strongest Channel

Channel specific composite fermion

Hybridization Weiss Field

Spinor OP Forms Spontaneously

Fractionalization of Bound-State into Fermion+OP
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Composite order

\[ \Psi = \langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle \]

\[ \propto |V_1|^2 - |V_2|^2 \]

Emery and Kivelson 1993
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Composite order

\[ \Psi = \left\langle \left( \psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2 \right) \cdot \vec{S} \right\rangle \]

\[ \propto |V_1|^2 - |V_2|^2 \]

Emery and Kivelson 1993

2-channel Kondo Lattice

Hisono, Otsuki & Kuromoto, PRL 107, 247202 (2011)

Spinor OP Forms Spontaneously

Fractionalization of Bound-State into Fermion+OP
### Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

<table>
<thead>
<tr>
<th>Kondo Majorana</th>
<th>2-channel Kondo Lattice</th>
<th>Composite Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \vec{S} \cdot \vec{\sigma} \rangle_{\alpha \beta} \psi_{\beta} )</td>
<td>( Vf_{\alpha} )</td>
<td>HF</td>
</tr>
<tr>
<td>( \langle \vec{S} \cdot \vec{\sigma} \rangle_{\alpha \beta} \psi_{\beta} )</td>
<td>( V\lambda f_{\alpha} )</td>
<td>Composite Multipole</td>
</tr>
<tr>
<td>( \langle \vec{S} \cdot \vec{\sigma} \rangle_{\alpha \beta} \psi_{\beta} )</td>
<td>( V\lambda f_{\alpha} + \Delta \lambda \bar{a} f_{-\alpha}^\dagger )</td>
<td>Composite Pair</td>
</tr>
<tr>
<td>( \Psi_{\alpha} \hat{\chi}_{\lambda} )</td>
<td>Hystatic</td>
<td></td>
</tr>
</tbody>
</table>

**Odd-w triplet/Skyrme Insulator**

\[ \langle \psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2 \rangle \cdot \vec{S} \]

**Composite Pair**

\[ \langle (\psi_1 \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle \]

**Hystatic**

\[ \langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{\tau}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi \]

---

**Emery and Kivelson 1993**

\[ (\vec{\sigma}_{\alpha \beta} \cdot \vec{S})_{j} \psi_{\lambda \beta} \rightarrow V_{\lambda} f_{\alpha}(0) \]

**Spinor OP Forms Spontaneously**

**Fractionalization of Bound-State into Fermion+OP**
<table>
<thead>
<tr>
<th>Kondo Majorana</th>
<th><strong>Order Parameter Fractionalization Hypothesis</strong></th>
<th>Composite Order</th>
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<tr>
<td>$(\vec{S} \cdot \vec{S})<em>{\alpha \beta} \psi</em>{\beta}$</td>
<td>$V f_{\alpha}$</td>
<td>HF</td>
</tr>
<tr>
<td></td>
<td>$(\vec{\sigma} \cdot \vec{\eta})<em>{\alpha \beta} V</em>{\beta}$</td>
<td>Odd-w triplet/Skyrme Insulator</td>
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<tr>
<td>2-channel</td>
<td>$V_{\lambda} f_{\alpha}$</td>
<td>Composite Multipole</td>
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<td>$V_{\lambda} f_{\alpha} + \Delta_{\lambda} \bar{\alpha} f_{-\alpha}^{\dagger}$</td>
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</tr>
<tr>
<td></td>
<td>$\Psi_{\alpha} \hat{\chi}_{\lambda}$</td>
<td>Hastatic</td>
</tr>
</tbody>
</table>

$\langle \psi^{\dagger} (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$

$\langle \psi_{\uparrow} \psi_{\downarrow} \vec{S} \rangle \propto V^T \vec{\sigma} \sigma_2 V$

$\langle \left( \psi_{\uparrow} \vec{\sigma} \psi_{\downarrow} - \psi_{\downarrow} \vec{\sigma} \psi_{\uparrow} \right) \cdot \vec{S} \rangle$

$\langle \left( \psi_{\downarrow} \vec{\sigma} \sigma_2 \psi_{\uparrow} \right) \cdot \vec{S} \rangle$

$\langle c^{\dagger} \vec{\sigma} (\vec{I} \cdot \tilde{\tau}) c \rangle \propto \Psi^{\dagger} \vec{\sigma} \Psi$

**Hidden Order**

[Graph showing $C_{\text{Si}}/T$ vs. $T(K)$ with $T_c$ and $T_0$ labels.]

(Palstra et al., 85)
# Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

| Kondo Majorana | $V f_\alpha$ | HF | $\langle \psi^\dagger (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|----------------|-------------|----|----------------------------------|
| 2-channel      | $V_\lambda f_\alpha$ | Composite Multipole | $\langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$ |
|                | $V_\lambda f_\alpha + \Delta_\lambda \bar{\alpha} f_\alpha^\dagger$ | Composite Pair | $\langle (\psi_1^\dagger \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle$ |
|                | $\Psi_\alpha \hat{\chi}_\lambda$ | Hastatic | $\langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{\tau}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$ |

**Hidden Order**

![Graph showing specific data points for URu2Si2](Palstra et al., 85)
# Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

## Hidden Order

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Operator</th>
<th>Hamiltonian</th>
<th>Symmetry</th>
<th>Order Parameter</th>
</tr>
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<td>Kondo Majorana</td>
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<td>$V f_{\alpha}$</td>
<td>HF</td>
<td>$\langle \psi^\dagger (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto</td>
</tr>
<tr>
<td>2-channel</td>
<td>$(\vec{S} \cdot \vec{\sigma})<em>{\alpha\beta} \psi</em>{\beta}$</td>
<td>$V_\lambda f_{\alpha}$</td>
<td>Composite Multipole</td>
<td>$\langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$</td>
</tr>
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<td>$V_\lambda f_{\alpha} + \Delta \lambda \bar{\alpha} f_{-\alpha}^\dagger$</td>
<td>Composite Pair</td>
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<td>$\Psi_{\alpha} \hat{\chi}_{\lambda}$</td>
<td>Hastatic</td>
<td>$\langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{c}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$</td>
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</tbody>
</table>

### Multipole Expansion

- $\Psi_{\alpha} \hat{\chi}_{\lambda}$

### Key References


---

**Figure Legend**

- **URu$_2$Si$_2$** (Palstra et al., 85)
- **C$_S/T$$^*$** (mJ/K/mol)
- **$T_c$**
- **$T_0$**
- **$k + Q$**

---

*See eg:*
**Order Parameter Fractionalization Hypothesis**

P. Chandra, PC, Y. Komijani

| Kondo Majorana | $V f_\alpha$ | HF | $\langle \psi^\dagger (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|----------------|--------------|----|----------------------------------|
| 2-channel      | $V \lambda f_\alpha$ | Composite Multipole | $\langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$ |
|                | $V \lambda f_\alpha + \Delta \lambda \tilde{\alpha} f_\alpha^\dagger$ | Composite Pair | $\langle (\psi_1^\dagger \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle$ |
|                | $\Psi_\alpha \hat{\chi}_\lambda$ | Hastatic | $\langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{\tau}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$ |

**Hidden Order**

**URu$_2$Si$_2$**

(Palstra et al., 85)

See eg:

### Order Parameter Fractionalization Hypothesis

| Kondo      | $V f_\alpha$             | HF               | $\langle \psi^* (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|------------|--------------------------|------------------|-----------------------------------------------------|
| Majorana   | $(\vec{S} \cdot \vec{\sigma})_{\alpha \beta} \psi_\beta$ | Odd-w triplet/Skyrme Insulator | $\langle \psi^*_\uparrow \psi_{\downarrow} \vec{S} \rangle \propto V^T \vec{\sigma} \sigma_2 V$ |
| 2-channel  | $V_\lambda f_\alpha$     | Composite Multipole | $\langle \left( \psi^*_1 \sigma_2 \psi_2 - \psi^*_2 \sigma_2 \psi_2 \right) \cdot \vec{S} \rangle$ |
|            | $(\vec{S} \cdot \vec{\sigma})_{\alpha \beta} \psi_\beta$ | Composite Pair | $\langle \left( \psi^*_1 \sigma_2 \psi_2 \right) \cdot \vec{S} \rangle$ |
|            | $V_\lambda f_\alpha + \Delta_\lambda \vec{\alpha} f^*_\alpha$ |               | $\langle c^* \vec{\sigma} \left( \vec{I} \cdot \vec{t} \right) c \rangle \propto \Psi^* \vec{\sigma} \Psi$ |

#### Hidden Order

**URu$_2$Si$_2$** (Palstra et al., 85)

- **Ising Anisotropy**
- **Multipole**
- **Composite Order**

- **Hastatic**

**Chandra, Coleman, Flint, Nature (2013)**
indicating a non-Fermi liquid character. At linearly below 10 K, as shown in the inset of Fig. 2(a), the resistivity decreases with respect to the direction of the magnetic field. The superconducting transition is defined as the zero-resistivity at a temperature $T_c$. Superconductivity is stable against the magnetic field, as shown in Fig. 2(b), and is found to be highly anisotropic with respect to the [001] direction. On the other hand, the upper critical field is strongly suppressed with decreasing temperature. The value of $H_{c2}$ at 0 K is extremely large, but for $H$ between 20 and 50 kOe, a value of $H_{c2}$ is obtained as $7 \pm 1$, which is compared with $35 \pm 9$ kOe at 80 mK.

**Order Parameter Fractionalization Hypothesis**

| Kondo Majorana | $V f_{\alpha}$ | HF | $\langle \psi^\dagger (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|----------------|----------------|----|----------------------------------|
| 2-channel      | $V_{\lambda} f_{\alpha}$ | Composite Multipole | $\langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$ |
|                | $V_{\lambda} f_{\alpha} + \Delta_{\lambda} \vec{f}_{\alpha}^\dagger$ | Composite Pair | $\langle (\psi_1^\dagger \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle$ |
|                | $\Psi_{\alpha} \hat{\chi}_{\lambda}$ | Hastatic | $\langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{r}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$ |


**NpPd$_5$Al$_2$**

$T_C = 4.5K$

Curie Law SC
indicating a non-Fermi liquid character. At linearly below 10 K, as shown in the inset of Fig. 2(a), the resistivity decreases paramagnetic e.

The value of \( H \) is highly anisotropic and large for \([001]\). The value of \( m_c \) is the mass anisotropy ratio for \([001]\) and \([100]\). The value of \( m_c \) is extremely large, but

Next we show in Fig. 5 the temperature dependence of the resistivity under various constant magnetic fields in NpPd(1)\(_2\).


\( \text{NpPd}_5\text{Al}_2 \ T_c = 4.5\text{K} \)

Curie Law SC
### Order Parameter Fractionalization Hypothesis

| Kondo | $V f_\alpha$ | HF | $\langle \psi^\dagger (\vec{S} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|---|---|---|---|
| Majorana | $(\vec{S} \cdot \vec{\sigma})_{\alpha\beta} \psi_\beta$ | Odd-w triplet/Skyskine Insulator | $\langle \psi^\dagger \downarrow \vec{s} \psi \uparrow \downarrow \vec{s} \psi \rangle \propto V^T \vec{\sigma} \sigma_2 \vec{V}$ |
| 2-channel | $(\vec{S} \cdot \vec{\sigma})_{\alpha\beta} \psi_\lambda$ | Composite Multipole | $\langle (\psi^\dagger \vec{\sigma} \psi_1 - \psi^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$ |
| | $V_\lambda f_\alpha$ | Composite Pair | $\langle (\psi^\dagger \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle$ |
| | $V_\lambda f_\alpha + \Delta \vec{\alpha} f^\dagger_{-\alpha}$ | Hystatic | $\langle c^\dagger \vec{\sigma} (\vec{I} \cdot \vec{s}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$ |

### Aoki et al. 2007

#### Composite order

$$\langle (\psi^\dagger \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle \propto (V_1 \Delta_2 - V_2 \Delta_1)$$


### NpPd$_5$Al$_2$

$T_C = 4.5K$

Curie Law SC
Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

| Majorana | $\mathbf{(S \cdot \tilde{S})_{\alpha\beta}}$ | $V f_\alpha$ | HF | $\langle \psi^\dagger (\tilde{\sigma} \cdot \tilde{S}) \psi \rangle \propto |V|^2$ |
| --- | --- | --- | --- | --- |
| Kondo | $\mathbf{(\sigma \cdot \tilde{\eta})_{\alpha\beta}}$ | $V f_\alpha$ | Odd-w triplet/Skyrme Insulator | $\langle \psi^\dagger \psi \tilde{S} \rangle \propto V^T \tilde{\sigma} \sigma_2 V$ |
| 2-channel | $\mathbf{(S \cdot \tilde{S})_{\alpha\beta}}$ | $V \lambda f_\alpha$ | Composite Multipole | $\langle (\psi^\dagger \tilde{\sigma} \psi_1 - \psi^\dagger \tilde{\sigma} \psi_2) \cdot \tilde{S} \rangle$ |
| | $V_\lambda f_\alpha + \Delta \lambda \tilde{\alpha} f^\dagger_{-\alpha}$ | Composite Pair | $\langle (\psi_1 \tilde{\sigma} \sigma_2 \psi_2) \cdot \tilde{S} \rangle$ |
| | $\Psi_{\alpha} \hat{\chi}_\lambda$ | Hstatic | $\langle c^\dagger \tilde{\sigma} (\tilde{I} \cdot \tilde{\sigma}) c \rangle \propto \Psi^\dagger \tilde{\sigma} \Psi$ |

### Order Parameter Fractionalization Hypothesis

| Kondo              | $V f_\alpha$         | HF                  | $\langle \psi^\dagger (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto |V|^2$ |
|--------------------|----------------------|---------------------|--------------------------------------------------|
| Majorana           | $(\vec{S} \cdot \vec{\sigma})_{\alpha \beta} \psi_{\beta}$ | Odd-w triplet/Skyrme Insulator | $\langle \psi^\dagger \psi \downarrow \vec{S} \rangle \propto V^T \vec{\sigma} \sigma_2 V$ |
| 2-channel          | $(\vec{S} \cdot \vec{\sigma})_{\alpha \beta} \psi_{\beta}$ | Composite Multipole | $\langle \left( \psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2 \right) \cdot \vec{S} \rangle$ |
|                    | $V_\lambda f_\alpha$ | Composite Pair      | $\langle \left( \psi_1 \vec{\sigma} \sigma_2 \psi_2 \right) \cdot \vec{S} \rangle$ |
|                    | $V_\lambda f_\alpha + \Delta_\lambda \alpha f_\alpha^\dagger$ | Hastatic            | $\langle c^\dagger \vec{\sigma} (\vec{l} \cdot \vec{\tau}) c \rangle \propto \Psi^\dagger \vec{\sigma} \Psi$ |

#### SmB$_6$

Order Parameter Fractionalization Hypothesis

<table>
<thead>
<tr>
<th>Kondo</th>
<th>$V f_\alpha$</th>
<th>HF</th>
<th>Composite Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majorana</td>
<td>$(\hat{S} \cdot \hat{\sigma})<em>{\alpha \beta} \psi</em>\beta$</td>
<td>$(\hat{\sigma} \cdot \hat{n})<em>{\alpha \beta} V</em>\beta$</td>
<td>Odd-w triplet/Skyrme Insulator</td>
</tr>
<tr>
<td>2-channel</td>
<td>$(\hat{S} \cdot \hat{\sigma})<em>{\alpha \beta} \psi</em>\beta$</td>
<td>$V_\lambda f_\alpha$</td>
<td>Composite Multipole</td>
</tr>
<tr>
<td></td>
<td>$V_\lambda f_\alpha + \Delta \tilde{\alpha} f^{\dagger}_{\alpha}$</td>
<td>Composite Pair</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Psi_\alpha \hat{\chi}_\lambda$</td>
<td>hastatic</td>
<td>$\langle \psi^{\dagger}_1 \hat{\sigma} \psi_1 - \psi^{\dagger}_2 \hat{\sigma} \psi_2 \rangle \cdot \hat{S}$</td>
</tr>
</tbody>
</table>

FIG. 1. Quantum oscillation frequency ($T$) vs. temperature ($T$) for SmB$_6$. The blue curve shows the experimental data, and the red curve is the best fit to the data. The inset shows a 3D representation of the quantum oscillations.

FIG. 2. (a) Hybridization of 3 localized Majorana fermions per spin with 4 Majorana fermions of the composite order parameter. The stiffness coefficients are $\alpha, \lambda, \xi$. The last term is the field energy, where $\psi$ represents the order parameter. The Landau quantization of the projected Majorana Fermi surface.

$V = \frac{\alpha}{1/T}$

"$\tilde{\alpha}$" is the effective mass, which is a function of temperature and magnetic field. The effective mass is important for understanding the behavior of the Majorana fermions in the composite order parameter.

$T = 40$ mK

$\rho = 10^6$ [Q cm]

$\psi^{\dagger}_1 \hat{\sigma} \psi_1 - \psi^{\dagger}_2 \hat{\sigma} \psi_2$ is the composite multipole term.

$\langle \psi^{\dagger}_1 \hat{\sigma} \psi_1 \rangle$ is the composite pair term.

$\langle \psi^{\dagger}_1 \hat{\sigma} \psi_2 \rangle$ is the composite pair term.

$\langle c^{\dagger} \hat{\sigma} (\vec{I} \cdot \vec{r}) c \rangle$ is the hastatic term.


$\text{SmB}_6$ : Strange Insulator
A possible Topological Insulator?

- Insulating gap opens at $T_K \sim 50K$
- Resistivity plateau below $T \sim 3K$

Topological surface states?

Dzero, Sun, Galitski, Coleman, PRL 104, 106408 (2010)

Menth, Buehler, Geballe PRL 22, 295 (1969)
Allen, Batlogg, Watchter PRB 20 4807 (1979)
A possible Topological Insulator?

- Insulating gap opens at $T_K \sim 50 K$
- Resistivity plateau below $T \sim 3 K$

Topological surface states?

Dzero, Sun, Galitski, Coleman, PRL 104, 106408 (2010)

![Diagram of Topological Insulator](image)

- $d$-$f$ band inversion at the X point
- 3 Dirac cones on the surface
A possible Topological Insulator?

- Insulating gap opens at $T_K \sim 50$K
- Resistivity plateau below $T \sim 3$K

Topological surface states?

Dzero, Sun, Galitski, Coleman, PRL 104, 106408 (2010)


- d-f band inversion at the X point
- 3 Dirac cones on the surface

Nonlocal transport $\rightarrow$ surface cond.

A possible Topological Insulator?

**ARPES → Surface states**

Xu et. al PRB 88, 121102(R) (2013)

Nonlocal transport → surface cond.


- d-f band inversion at the X point
- 3 Dirac cones on the surface

**Topological Phase Transition**

\[ Z_2(a) = +1 \]

\[ Z_2(b) = (-1)^3 \]

**Surface**
Robust bulk insulator
(Kurdak et al, APS 2017)
Seemingly Topological, yet
Linear Sheat
Phalen, PRX 4 031012 (2014)

Robust bulk insulator
(Kurdak et al, APS 2017)
Seemingly Topological, yet

• Bulk Linear SHeat $10 \times \text{LaB}_6$
Robust bulk insulator
(Kurdak et al, APS 2017)
Seemingly Topological, yet

- Bulk Linear SHeat $10x \text{LaB}_6$
- Optical conductor, DC insulator
Robust bulk insulator (Kurdak et al, APS 2017)
Seemingly Topological, yet
- Bulk Linear SHeat 10x LaB$_6$
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
The SmB$_6$ Conundrum

Robust bulk insulator
(Kurdak et al, APS 2017)
Seemingly Topological, yet
- Bulk Linear SHeat 10x LaB$_6$
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
- Bulk dHvA + Lifschitz Kosevitch

**The SmB$_6$ Conundrum**

Linear SHeat
Phalen, PRX 4 031012 (2014)

AC Metal

$\kappa/T \sim B$
M. Hartstein, M Sutherland
S. Sebastian et al. (Preprint)

Bulk dHvA

Lifschitz Kosevich = FT of Fermi Function.

$F = 5.3 \, kT$
$m^*/m_e = 0.58 \pm 0.01$
The SmB$_6$ Conundrum

**Bulk is anomalous**: electronically insulating but hosts seemingly gapless excitations.

- Bulk Linear SHeat $10x$ LaB$_6$
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
- Bulk dHvA + Lifschitz Kosevich
The SmB$_6$ Conundrum

Bulk is anomalous: electronically insulating but hosts seemingly gapless excitations.

Excitonic/Breakdown Theory

J. Knolle and N. Cooper PRL 118, 096604 (2017)

Fractionalization

The SmB₆ Conundrum

**Bulk is anomalous:** electronically insulating but hosts seemingly gapless excitations.

Robust bulk insulator (Kurdak et al, APS 2017)
Seemingly Topological, yet
- Bulk Linear SHeat 10x LaB₆
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
- Bulk dHvA + Lifschitz Kosevich
The SmB₆ Conundrum

**Bulk is anomalous:** electronically insulating but hosts seemingly gapless excitations.

- Neutral but exhibit orbital diamagnetism,
- Obey Fermi-Dirac statistics.
- Quasiparticles do not couple to E but couple to B.

Robust bulk insulator (Kurdak et al, APS 2017)
Seemingly Topological, yet
- Bulk Linear SHeat 10x LaB₆
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
- Bulk dHvA + Lifschitz Kosevich

$$F = q(E + \mathbf{v} \times \mathbf{B})$$
The SmB₆ Conundrum

Bulk is anomalous: electronically insulating but hosts seemingly gapless excitations.

- Neutral but exhibit orbital diamagnetism,
- Obey Fermi-Dirac statistics.

- Quasiparticles do not couple to E but couple to B.
- Yet microscopically particles couple to the vector potential:

\[ F = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]

\[ p \rightarrow p - eA \]

\[ E = -\frac{\partial A}{\partial t} - \nabla \phi \]

\[ B = \nabla \times A \]
The SmB₆ Conundrum

**Bulk is anomalous:** electronically insulating but hosts seemingly gapless excitations.

- Neutral but exhibit orbital diamagnetism,
- Obey Fermi-Dirac statistics.
- Quasiparticles do not couple to E but couple to B.
- Yet microscopically particles couple to the vector potential:
  - a differential coupling to E and B is prevented by gauge invariance.

Robust bulk insulator
(Kurdak et al, APS 2017)
Seemingly Topological, yet
- Bulk Linear SHeat 10x LaB₆
- Optical conductor, DC insulator
- B-dependent Thermal Conductivity
- Bulk dHvA + Lifschitz Kosevich

\[ F = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]

\[ p \rightarrow p - eA \]

\[ E = -\frac{\partial A}{\partial t} - \nabla \phi \]

\[ B = \nabla \times A \]
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Bulk is anomalous: electronically insulating but hosts seemingly gapless excitations.

- Neutral but exhibit orbital diamagnetism,
- Obey Fermi-Dirac statistics.
- Quasiparticles do not couple to E but couple to B.
- Yet microscopically particles couple to the vector potential:
  - a differential coupling to E and B is prevented by gauge invariance.

Can we have broken gauge invariance without superconductivity?
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**Superconductivity**: requires Meissner *and* topological rigidity.

\[ \pi_1(S^1) = \mathbb{Z} \quad \pi_1(S^2) = 0 \]

Hanson, Oganesyan and Sondhi, Annals Of Physics vol. 313, 497 (2004)

→ Can we have broken gauge invariance without superconductivity?
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s-wave odd-frequency triplet.

\((-1)^{i+j+k} \langle \psi_\uparrow \psi_\downarrow S(x) \rangle \propto \hat{l}(x) + i \hat{m}(x),\)

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\]
SmB₆: a Skyrme Dielectric?
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Superconductivity: requires Meissner and topological rigidity.

s-wave odd-frequency triplet.

$$(-1)^{i+j+k} \langle \psi^\dagger \psi S(x) \rangle \propto \hat{l}(x) + i \hat{m}(x),$$

$$\nabla^2 B = \frac{1}{\lambda^2 L} (B - \phi_0 \rho(x))$$

London equation

$$\rho(x) = B/\phi_0$$

Skyrmion density

Fluid of Coreless skyrmions of the n field allow field penetration.
Superconductivity: requires Meissner and topological rigidity.

s-wave odd-frequency triplet.

\((-1)^{i+j+k} \langle \psi_\uparrow \psi_\downarrow S(x) \rangle \propto \hat{I}(x) + i\hat{m}(x),\)

\(\nabla^2 B = \frac{1}{\chi_L^2} (B - \phi_0 \rho(x))\)

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Skyrmion density

\(\kappa \sim B\)

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London equation

\[ \mathbf{\rho}(x) = \frac{B}{\phi_0} \]

Skyrmion density

\[ H_{c1} \sim \frac{1}{137c} \left( \frac{V_K}{a} \right) \sim 1 \text{ Gauss} \]
Superconductivity: requires Meissner and topological rigidity.

**s-wave odd-frequency triplet.**

\[ (-1)^{i+j+k} \langle \psi_\uparrow \psi_\downarrow S(x) \rangle \propto \hat{I}(x) + i\hat{m}(x), \]

\[ \hat{S}_j = -\left(\frac{i}{2}\right) \hat{n}_j \times \hat{n}_j \]

Majorana spin representation

Meissner phase
perfect diamagnetism

Skyrmion fluid
coreless, mobile

\[ n_S \sim B \rightarrow \kappa \sim B \]

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**Superconductivity:** requires Meissner and topological rigidity.

**s-wave odd-frequency triplet.**

$$(-1)^{i+j+k} \langle \psi_\uparrow \psi_\downarrow \mathbf{S}(\mathbf{x}) \rangle \propto \mathbf{l}(\mathbf{x}) + i\mathbf{m}(\mathbf{x}),$$

$$\mathbf{S}_j = -\left(\frac{i}{2}\right) \mathbf{n}_j \times \mathbf{\bar{n}}_j$$

Majorana spin representation

(b) $\kappa \sim B$

Meissner phase
perfect diamagnetism

Skyrmion fluid
coreless, mobile

$n_S \sim B \rightarrow \kappa \sim B$

$H_{c1} \sim \frac{1}{137c} \left(\frac{V_K}{a}\right) \sim 1$ Gauss

FIG. 2. (a) Hybridization of 3 localized Majorana fermions per spin with 4 Majorana fermions of the superconductor. (c) Landau quantization of the projected Majorana Fermi surface.

6
-6
(0, 0, 0)

-6

k

(π, π,

(k)

0.7

DOS

ε$_{k-eA}$, $\epsilon^M = (\epsilon_{k-eA} + \epsilon_{k+eA})/2$

0

-4

-4

4

Energy

Energy

6

Hermitian square of the order parameter

$\mathbf{t}$

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FIG. 2 (a) Hybridization of 3 localized Majorana fermions per spin with 4 Majorana fermions of the superconductor.
Conclusions

- **50 Years of Novel Phases.** Discovery often awaits new concepts.

- **Dark Matter Challenges of the Solid State.** Potential for qualitatively new advances in our understanding of quantum matter.

- **Quantum I: MPS, PEPS.** Tools to manipulate and explore new mechanisms of entanglement.

- **Beyond Hartree Fock/BCS:** Order parameter fractionalization hypothesis.

- **Fermi surface in an insulator.** Skyrme Dielectric?
Thank you!