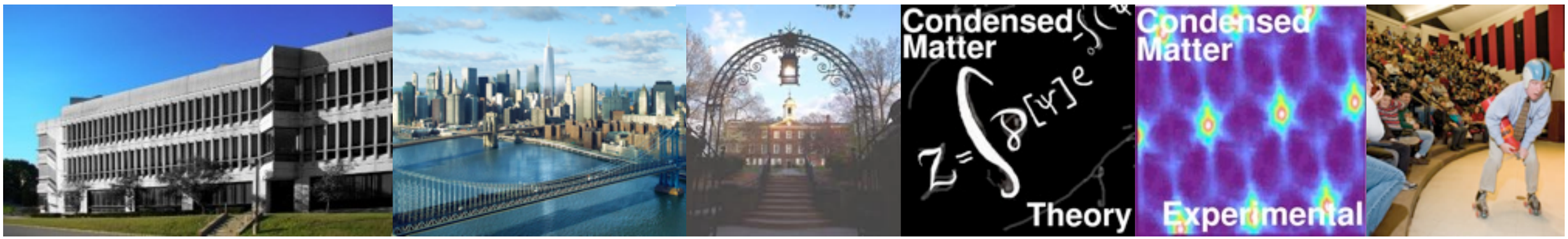
Three Lectures on Heavy Fermion Physics

Lecture III. SOME NEW DEVELOPMENTS



Piers Coleman

Center for Materials Theory, Rutgers.

MaNEP SWITZERLAND

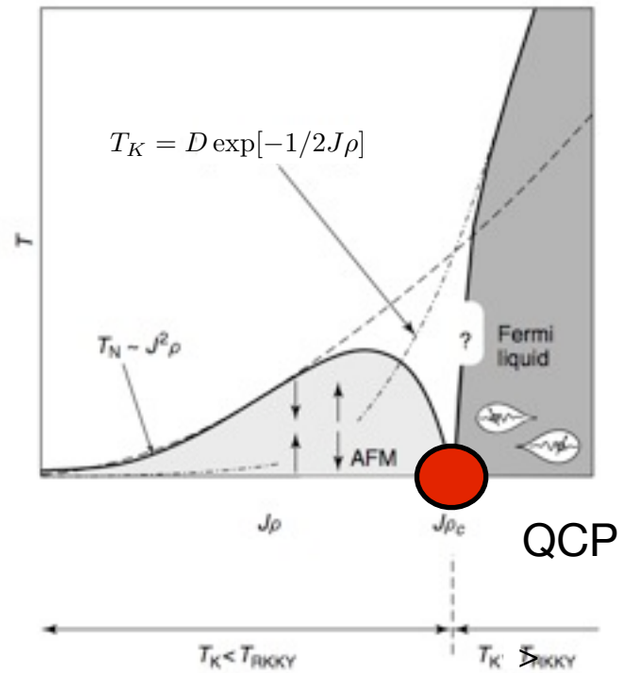
Saas-Fee 5th Winter School

14-15th January 2013

Royal Holloway University of London

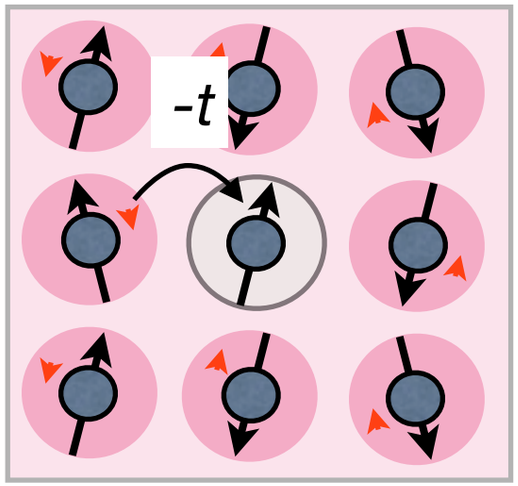
Last time - Lecture II: Kondo Lattice.

Mott/Doniach hypothesis.



$$2\left(\frac{v_{\rm FS}}{(2\pi)^D}\right) = 2 - \delta = n_{\rm spins} + n_e$$

FS sum rule counts spins as charged qp.



Strong Coupling Limit

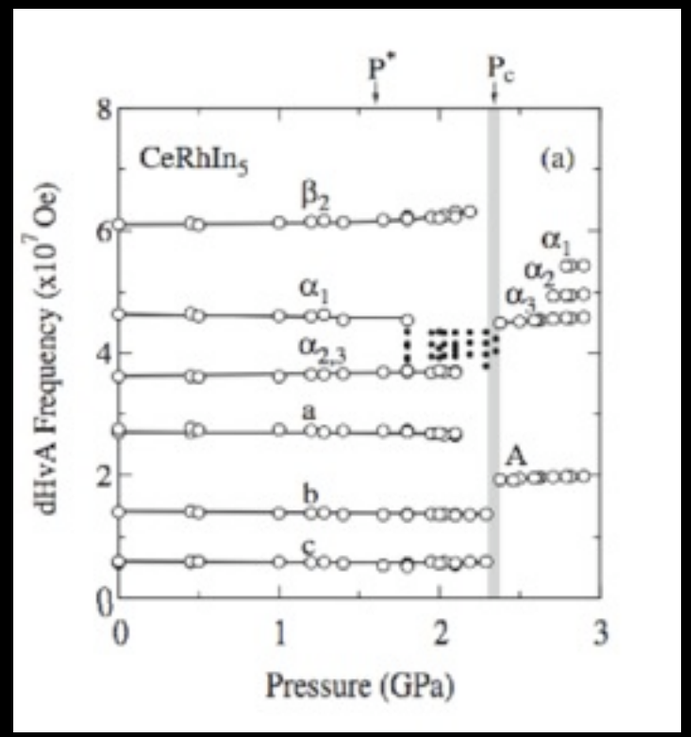
Large N Approach

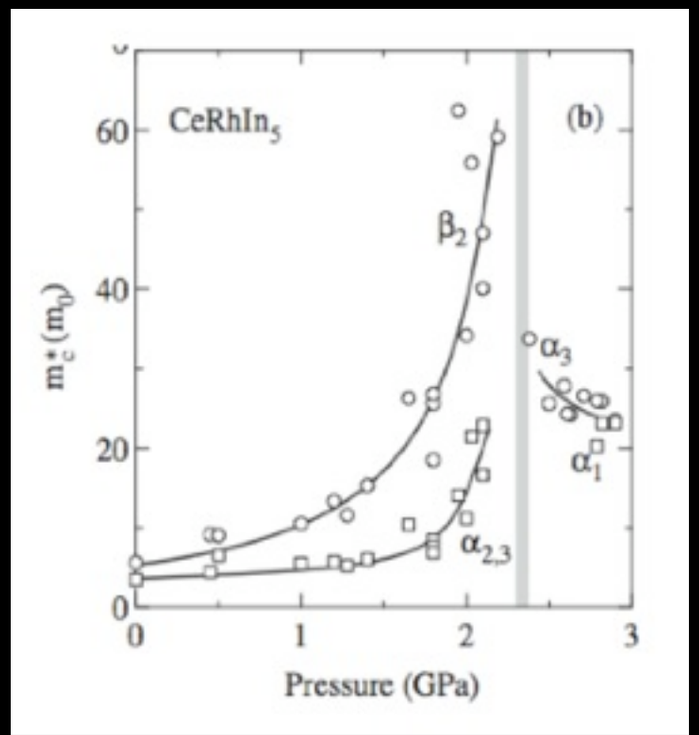
$$\frac{J}{N} c_{j\alpha} S_{\alpha\beta} \equiv \bar{V}_j f_{j\beta}$$

Composite Fermion

Read & Newns 1983.

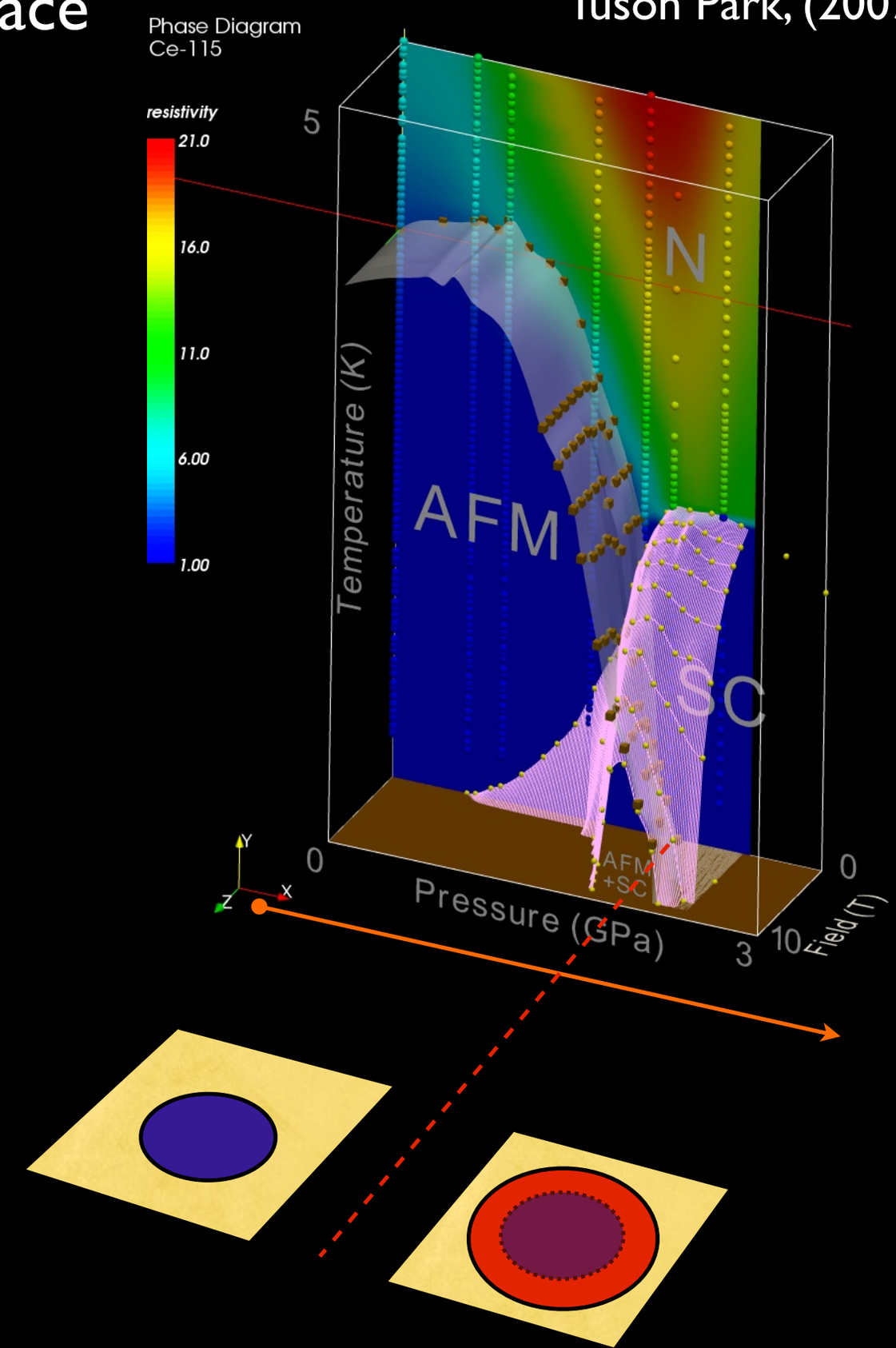
Reconstruction of the Fermi Surface and mass divergence





Shimuzu et al (2006)

Tuson Park, (2007).



Three Lectures on HF Lecture III:

1. Topological Kondo Insulators

2. Composite pairing.

3. Hidden Order in URu2Si2

Rutgers
Center for Materials Theory

Three Lectures on HF Lecture III:

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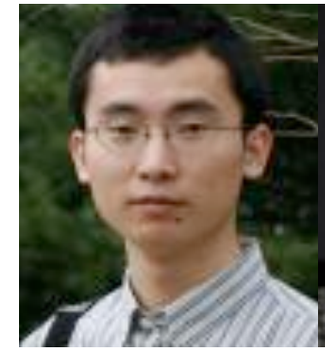
Rutgers
Center for Materials Theory

Collaborators

Maxim Dzero	Kent State
Kai Sun	Michigan
Victor Galitski	Maryland


Kai


Maxim


Victor

Topological Kondo Insulators, Dzero, Sun, Galitski, PC Phys. Rev. Lett. **104**, 106408 (2010)

Maxim Dzero, Kai Sun, Piers Coleman and Victor Galitski, Phys. Rev. B 85 , 045130-045140 (2012).

arXiv.org > cond-mat > arXiv:1211.5104

Condensed Matter > Strongly Correlated Electrons

Discovery of the First Topological Kondo Insulator: Samarium Hexaboride

Steven Wolgast, Cagliyan Kurdak, Kai Sun, J. W. Allen, Dae-Jeong Kim, Zachary Fisk

(Submitted on 21 Nov 2012 (v1), last revised 27 Nov 2012 (this version, v2))

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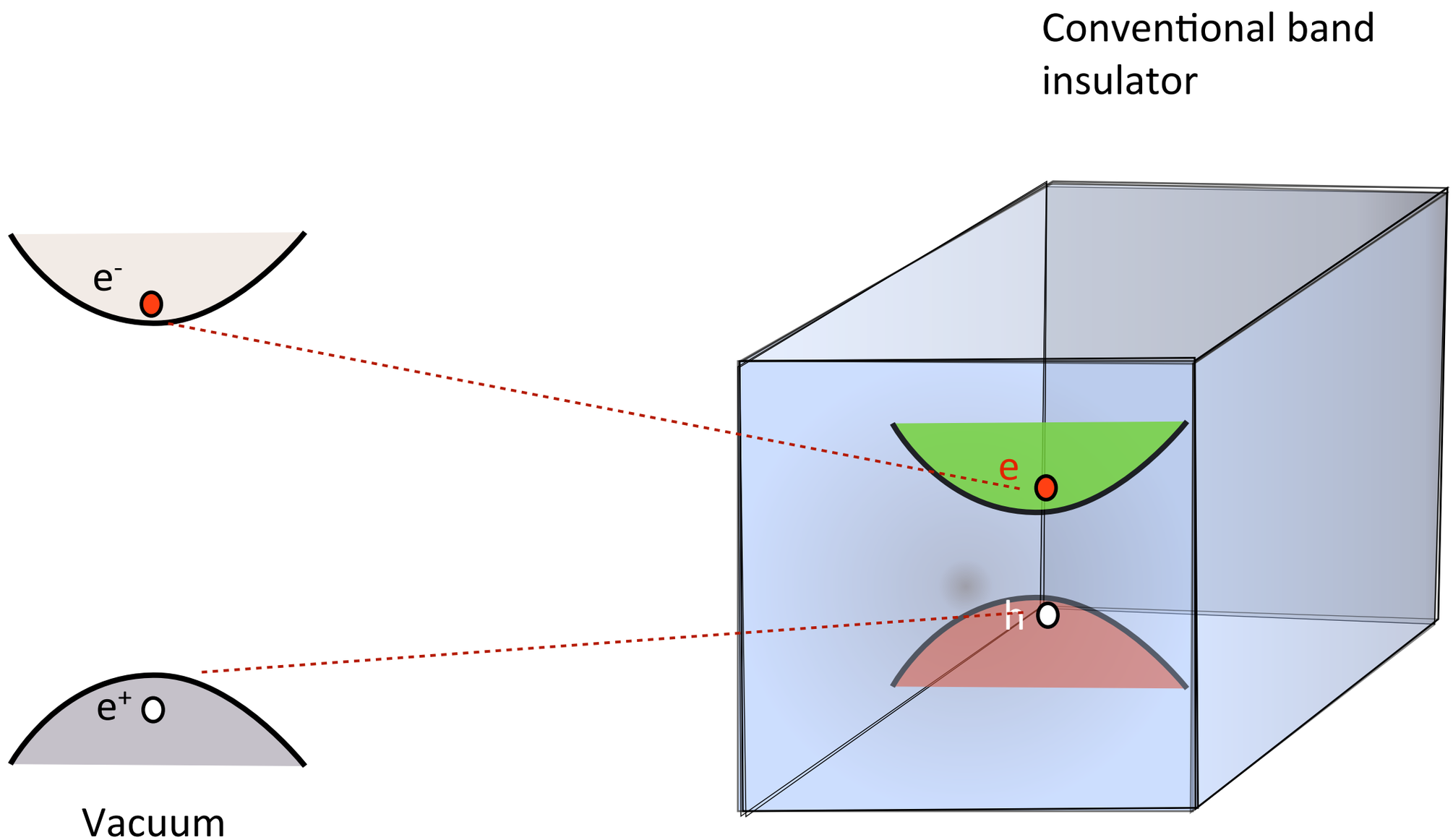
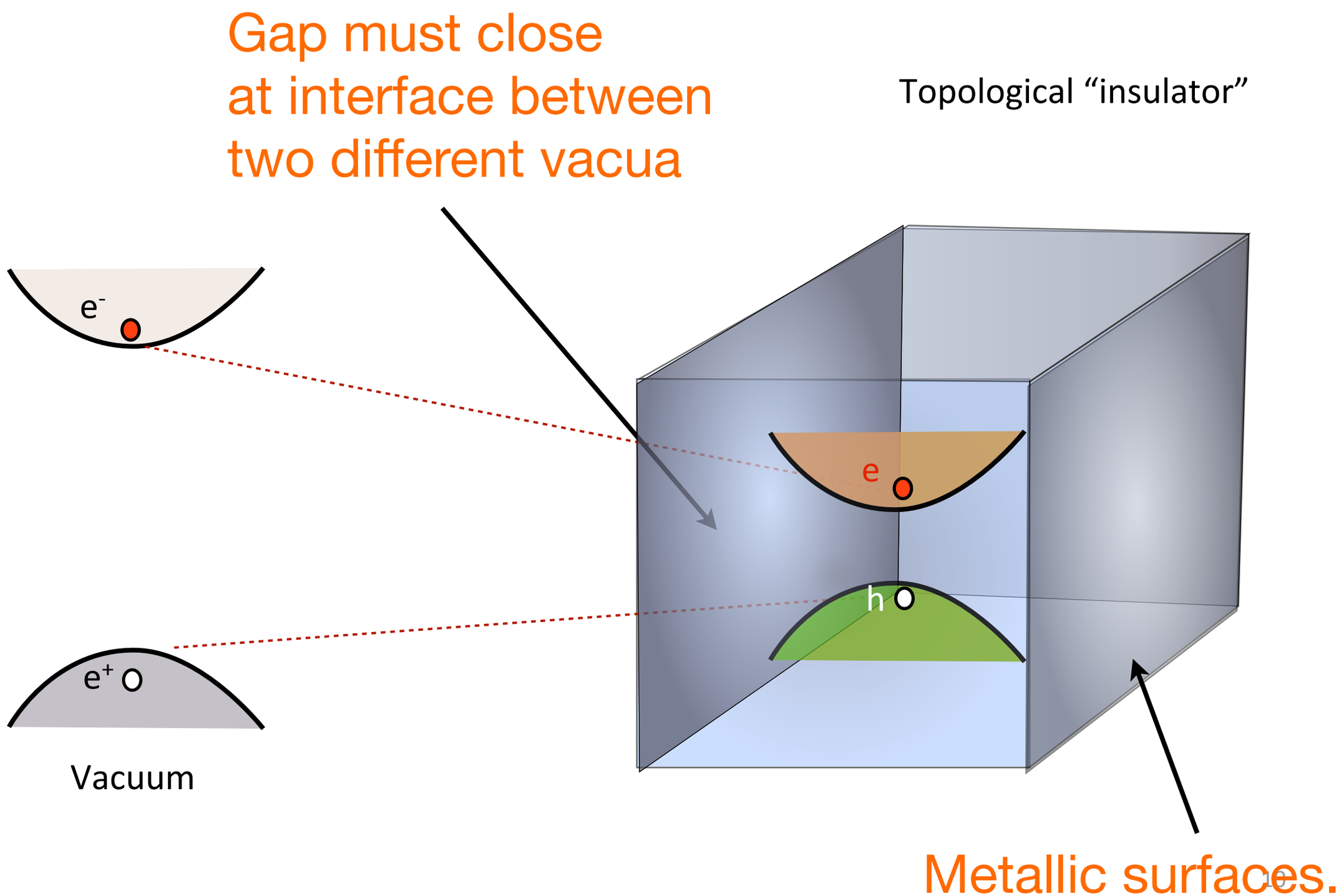
Topological insulators

Hasan and Kane (RMP 2009)
Qi and Zhang (RMP 2010)

Conventional band insulator: adiabatic continuation of the vacuum.



Topological insulator: adiabatically disconnected to vacuum.



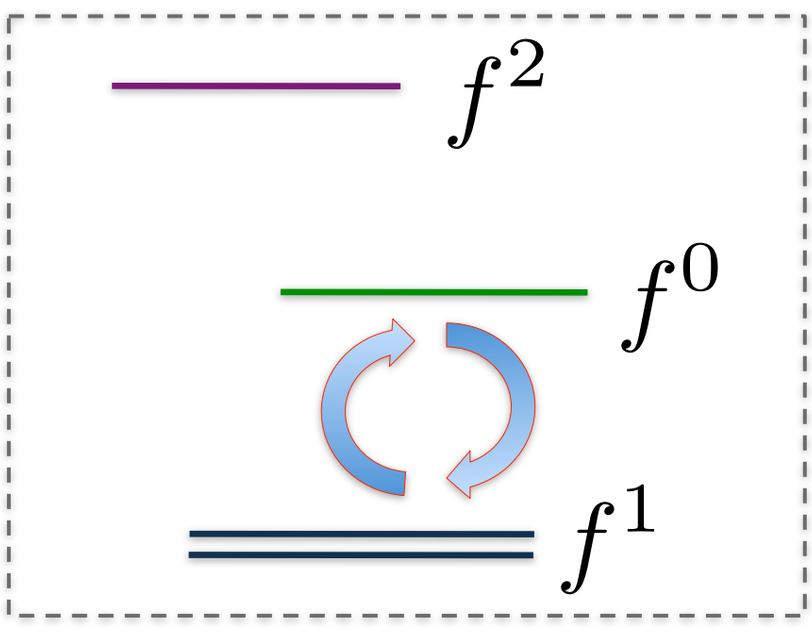
Kondo insulators: theory

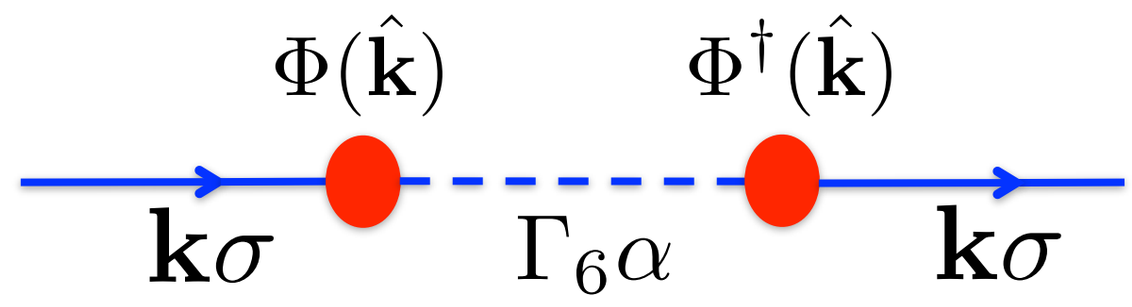
- Anderson model: Renormalized

$$\hat{H} = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}j\alpha} \left[V^* \psi^\dagger_{\mathbf{k}j\alpha} f_{j\alpha} + \text{h.c.} \right] + \sum_{j\alpha} \left[E_f^{(0)} \sum_{\mathbf{k}\alpha} n_{f\alpha} + \frac{U_f}{2} \sum_{\mathbf{k}\alpha, j\alpha} n_{f,j\alpha} n_{f,j\bar{\alpha}} \right]$$

$$\psi_{j\alpha} = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \underbrace{\Phi_{\alpha\sigma}(\hat{\mathbf{k}})}_{\text{f-electron form factor}} e^{-i\mathbf{k}\cdot\mathbf{x}_j} c_{\mathbf{k}\sigma}$$

- Hybridization





Strong spin-orbit coupling is encoded in the hybridization.

Kondo insulators: theory

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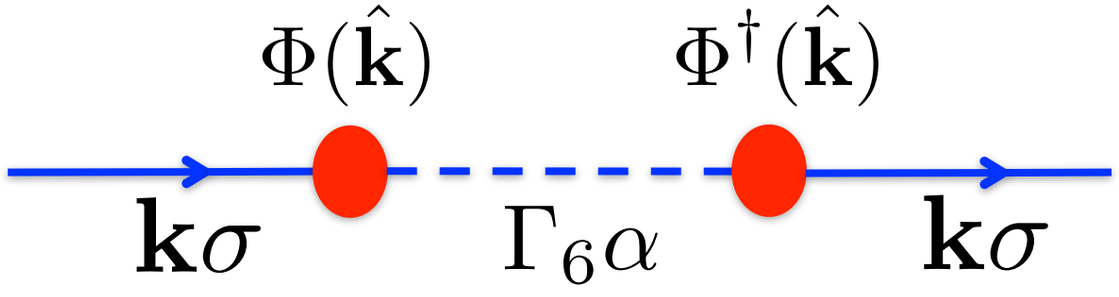
$$\psi_{j\alpha} = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \underbrace{\Phi_{\alpha\sigma}(\hat{\mathbf{k}})}_{\text{f-electron form factor}} e^{-i\mathbf{k}\cdot\mathbf{x}_j} c_{\mathbf{k}\sigma}$$

form factors:

$$[\Phi_{\Gamma\mathbf{k}}]_{\alpha\sigma} = \sum_m \langle \Gamma\alpha | jm \rangle \underbrace{\langle jm | \mathbf{k}\sigma \rangle}_{\text{Matrix element between Bloch and Wannier states}}$$

$$\Phi_{\alpha\sigma}(\mathbf{k}) = -1 \times \Phi_{\alpha\sigma}(-\mathbf{k})$$

ODD PARITY!

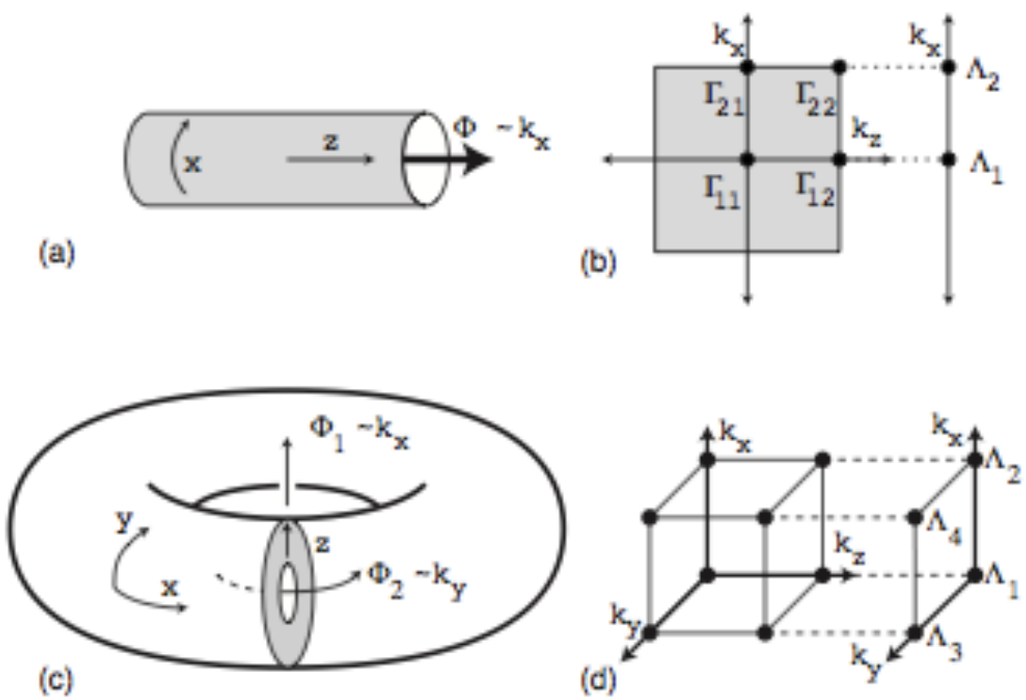


Strong spin-orbit coupling is encoded in the hybridization.

Topological insulators

L. Fu & C. Kane, “Topological Insulators with Inversion Symmetry”, PRB 76, 45302 (2007).

Response to a fictitious applied magnetic field



- 2D: Flux plays the role of the edge crystal momentum k_x
- 3D: two fluxes corresponding to two components of the surface crystal momentum

Z_2 invariants are computed from the **parity** of the occupied bands: change in time reversal polarization due to changes in bulk Hamiltonian

$$Z_2 = \prod_i \delta(\Gamma_i)$$

$$Z_2 = \prod_i \delta(\Gamma_i)$$

$$V_{\alpha\sigma}(\mathbf{k_m}) = V_{\alpha\sigma}(\mathbf{k_m} + \mathbf{G}) = -V_{\alpha\sigma}(-\mathbf{k_m}) = -V_{\alpha\sigma}(\mathbf{k_m}) = 0$$

Vanishes at high symmetry points

$$H_{mf}(\mathbf{k}_m) = \frac{1}{2}(\xi_{\mathbf{k}_m} + \varepsilon_f)\underline{1} + \frac{1}{2}(\xi_{\mathbf{k}_m} - \varepsilon_f)P$$

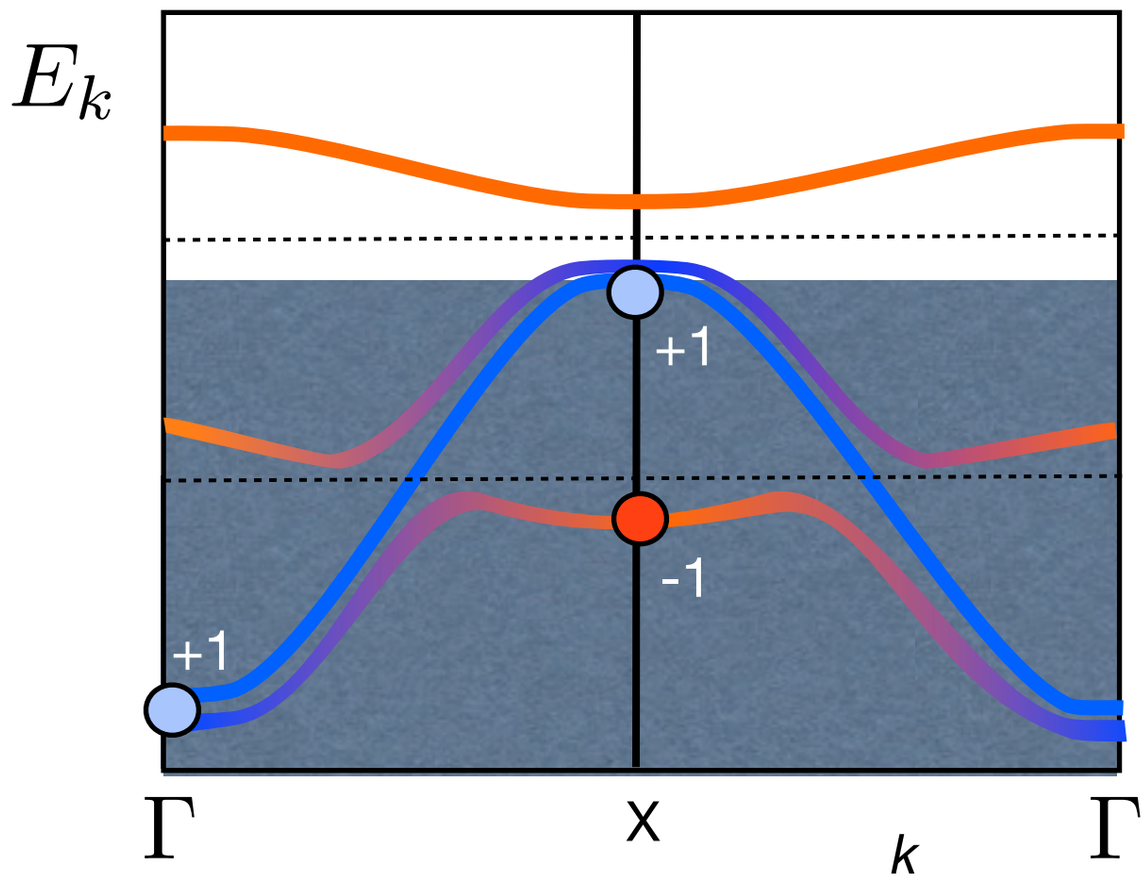
- Z_2 invariants are characterized by the parity eigenvalues:

$$\delta(\Gamma_m) = \mathrm{sgn}(E_f^* - \xi_{\mathbf{k_m}})$$

1D Kondo Insulator

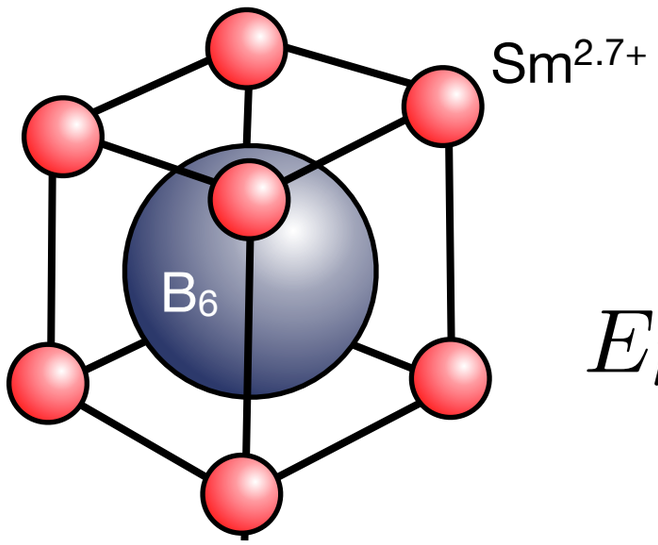
$$d^2 \quad \rightarrow f^1 d^1$$

$$\nu = +1 \quad \nu = -1$$

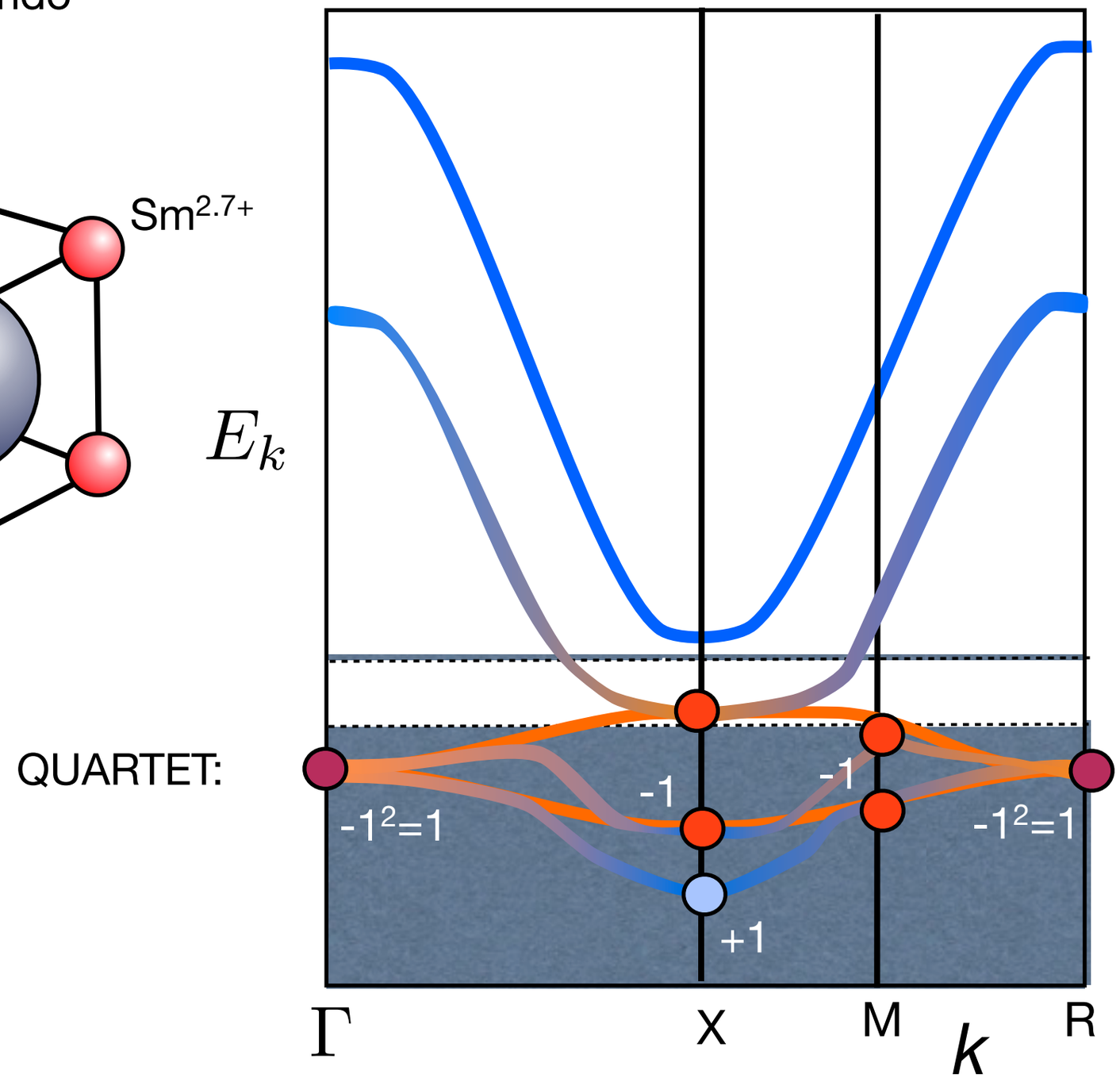


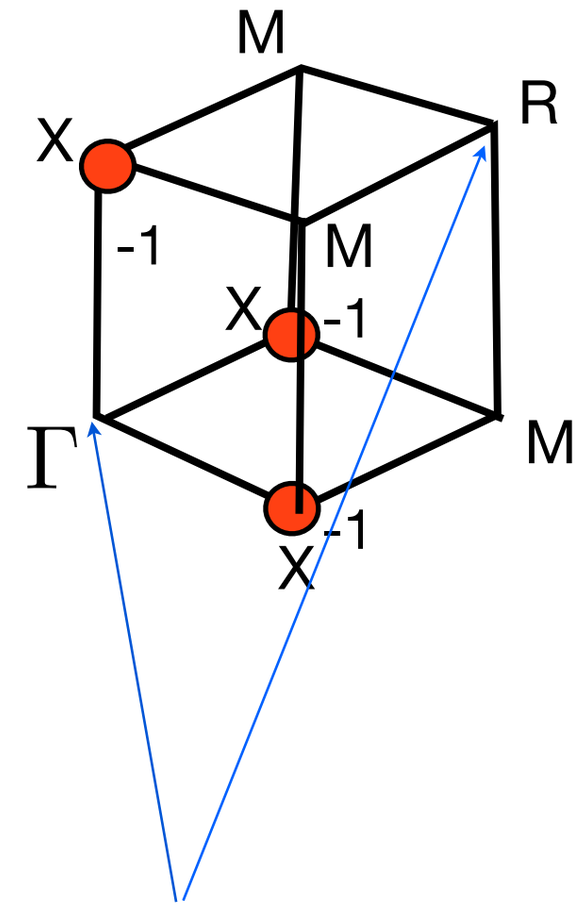
Alexandrov, Dzero and PC (2013)

3D Cubic Kondo Insulator



SmB_6





QUARTET:
topologically inert

$$d^0 f^6 \longrightarrow d^1 f^5$$

$$\nu = +1 \qquad \nu = -1$$

THREE DIRAC CONES ON SURFACE.

Alexandrov, Dzero and PC (2013)

arXiv.org > cond-mat > arXiv:1211.5104

Condensed Matter > Strongly Correlated Electrons

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(Submitted on 21 Nov 2012 (v1), last revised 27 Nov 2012 (this version, v2))

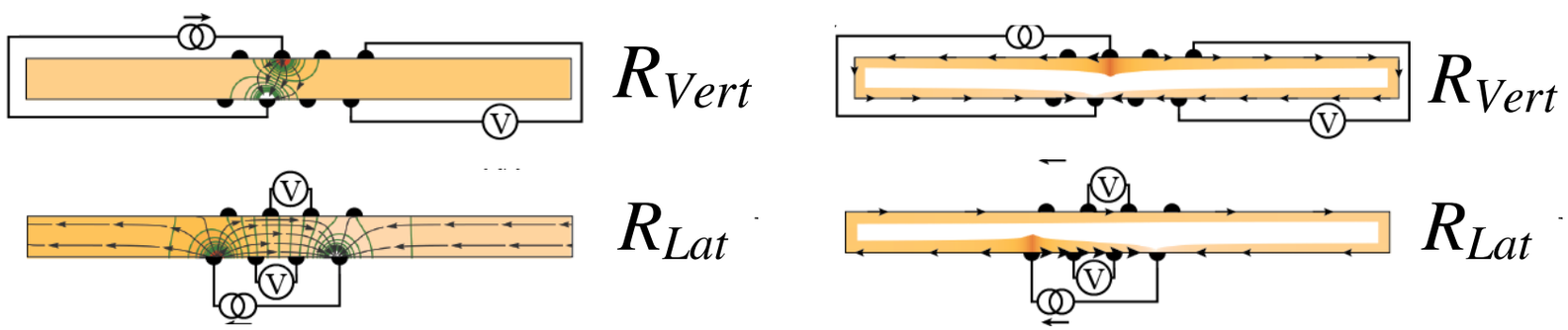
arXiv.org > cond-mat > arXiv:1211.5532

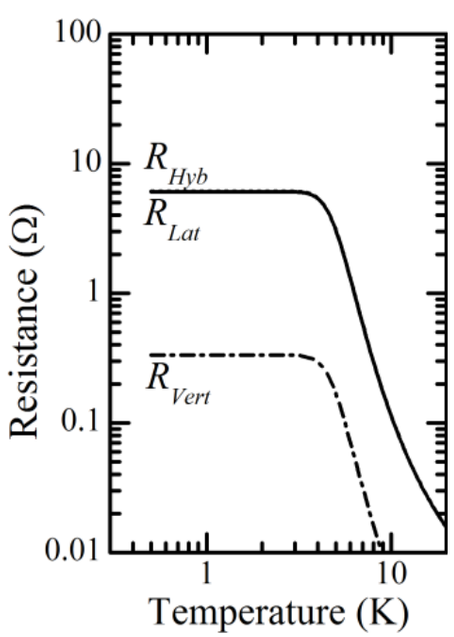
Condensed Matter > Strongly Correlated Electrons

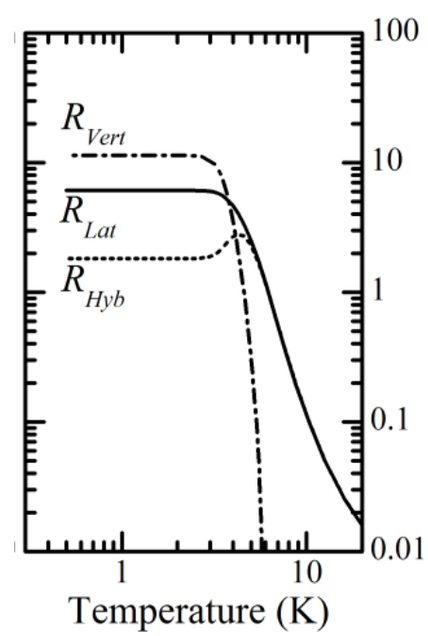
Hybridization, Correlation, and In-Gap States in the Kondo Insulator SmB6

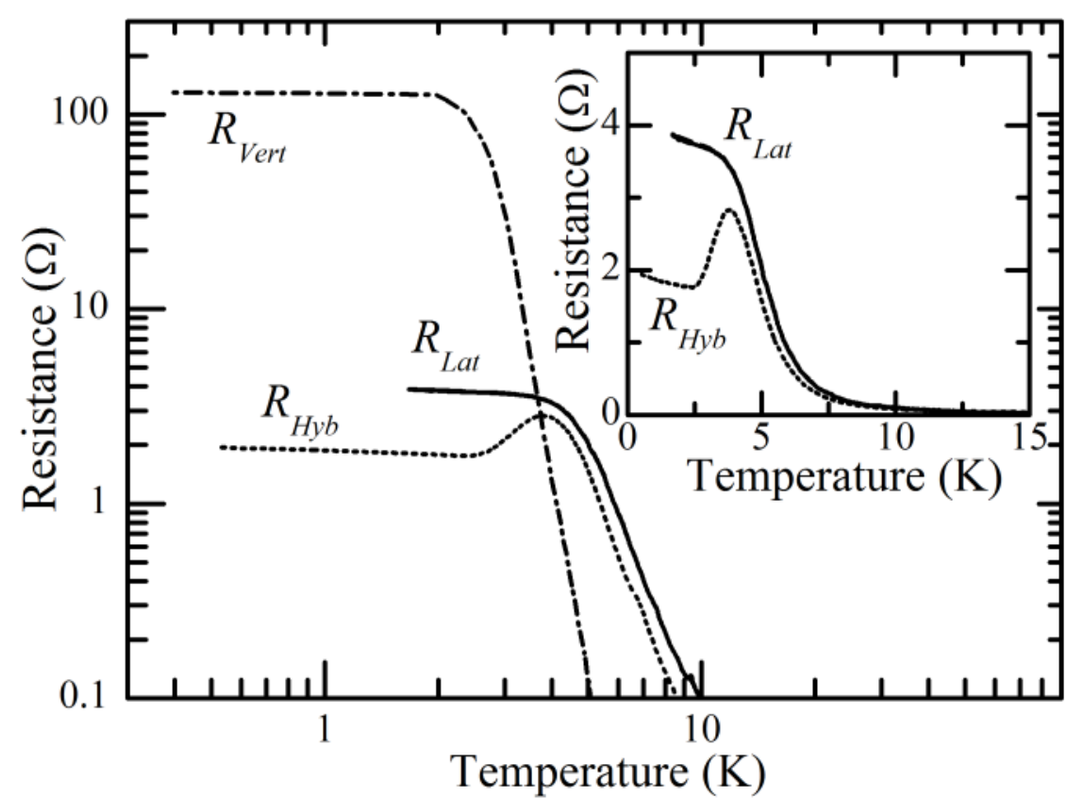
Xiaohang Zhang, N. P. Butch, P. Syers, S. Ziemak, Richard L. Greene, J. Paglione

(Submitted on 23 Nov 2012)









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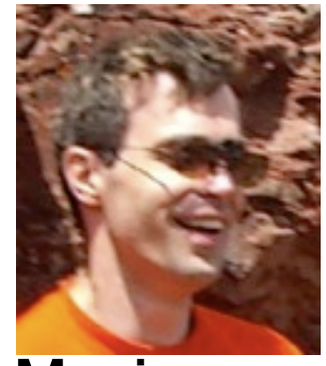
Rutgers
Center for Materials Theory

Collaborators


Rebecca


Maxim


Andriy


Alexei

Rebecca Flint	MIT
Maxim Dzero	Kent State
Andriy Nevidomskyy	Rice
Alexei Tsvelik	Brookhaven NL
Hai Young Kee	U. Toronto
Natan Andrei	Rutgers
Tzen Ong	Rutgers

PRB 60, 3605 (1999).

Nature Physics 4, 643 (2008).

PRL, 105, 246404 (2010)

PRB 84, 064514 (2011).

PRL, 108, 107201, (2012).

ICAM-I2CAM
Institute for Complex Adaptive Matter

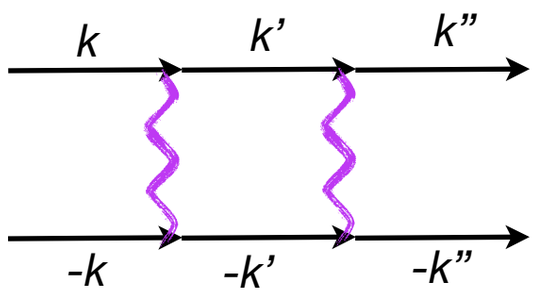
Rutgers
Center for Materials Theory

Culture war: glue vs fabric.

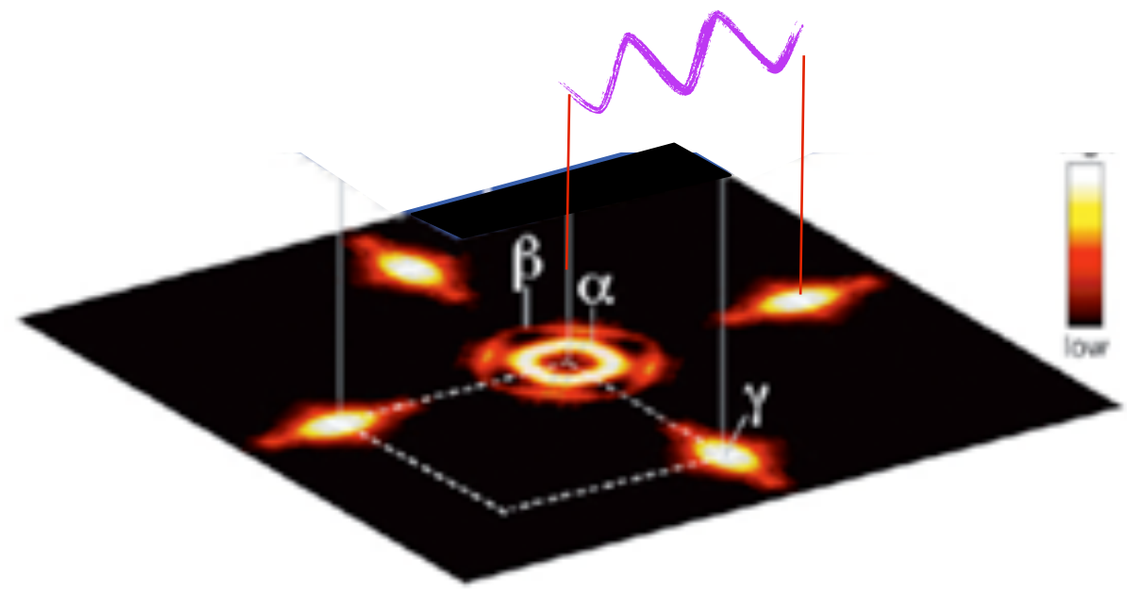
Glue vs Fabric.

Spin fluctuations = pairing glue.

Glue



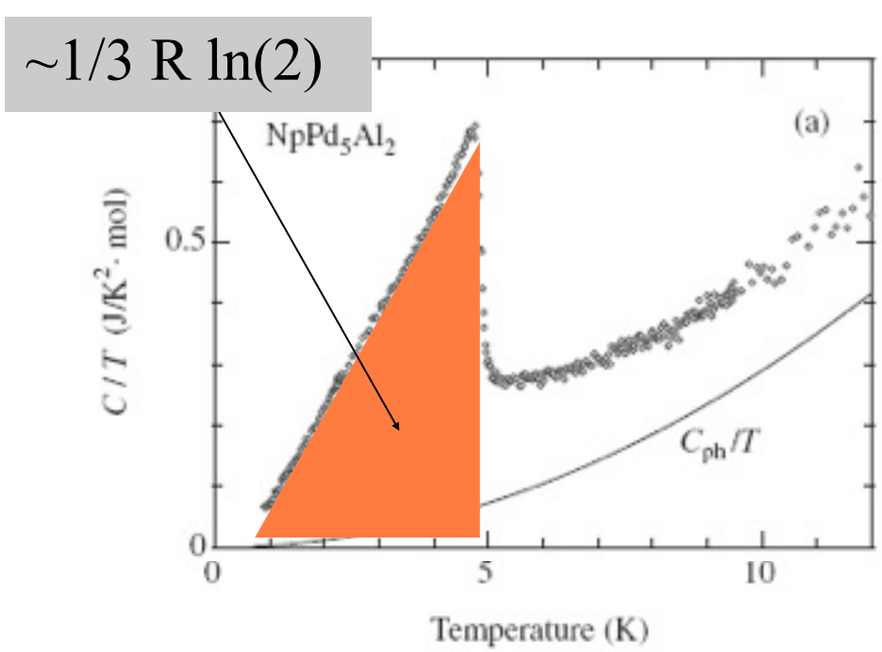
$(\pi,0) \rightarrow s^{\pm}$



Fabric: spins *build* the pairs

HiTc: Anderson: RVB (1987)

Heavy Fermions: Andrei, PC (1989)



$$R \ln W = \int_0^T dT' \frac{C'}{T'}$$

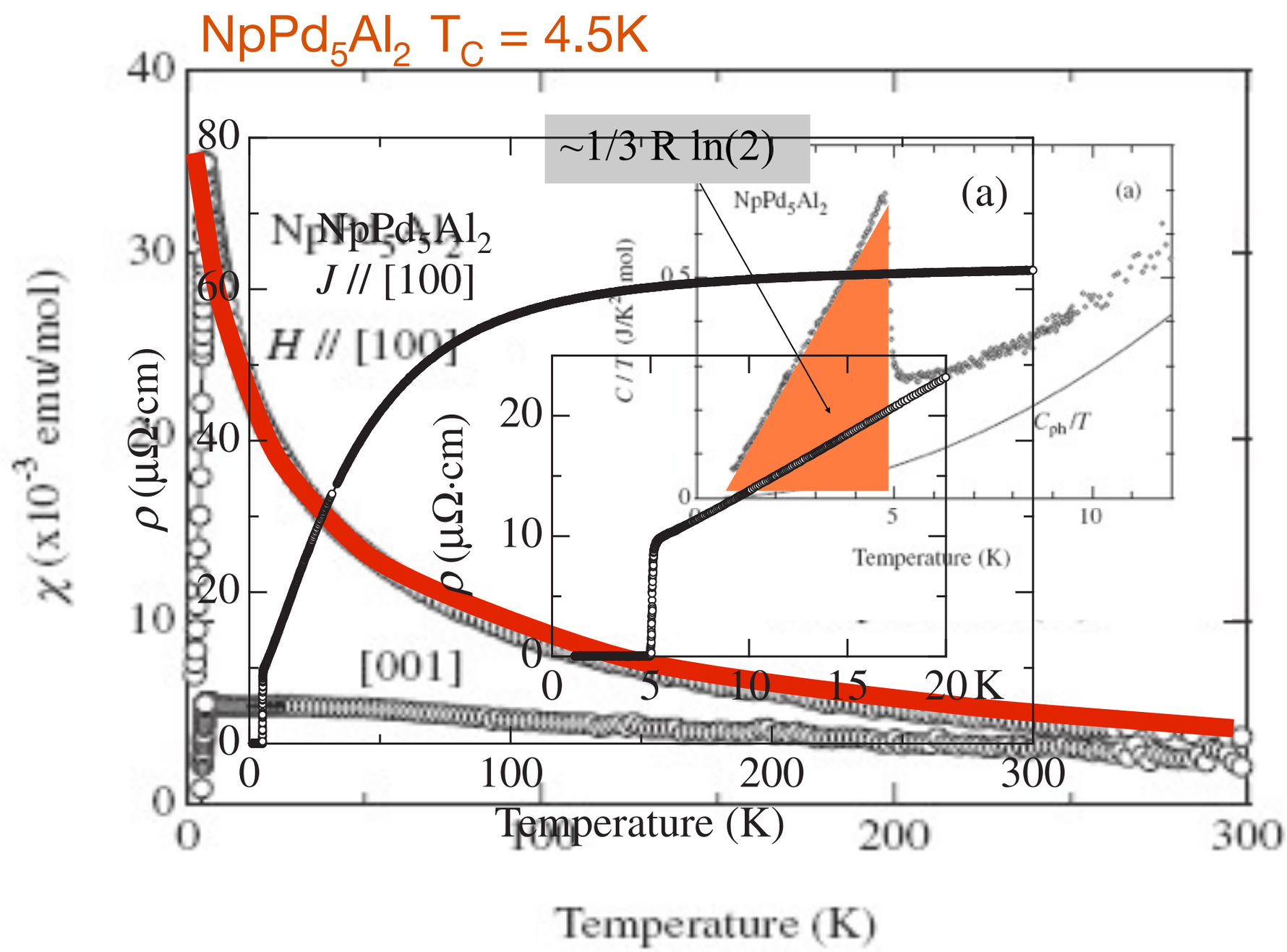
"Hilbert Space Spectroscopy"

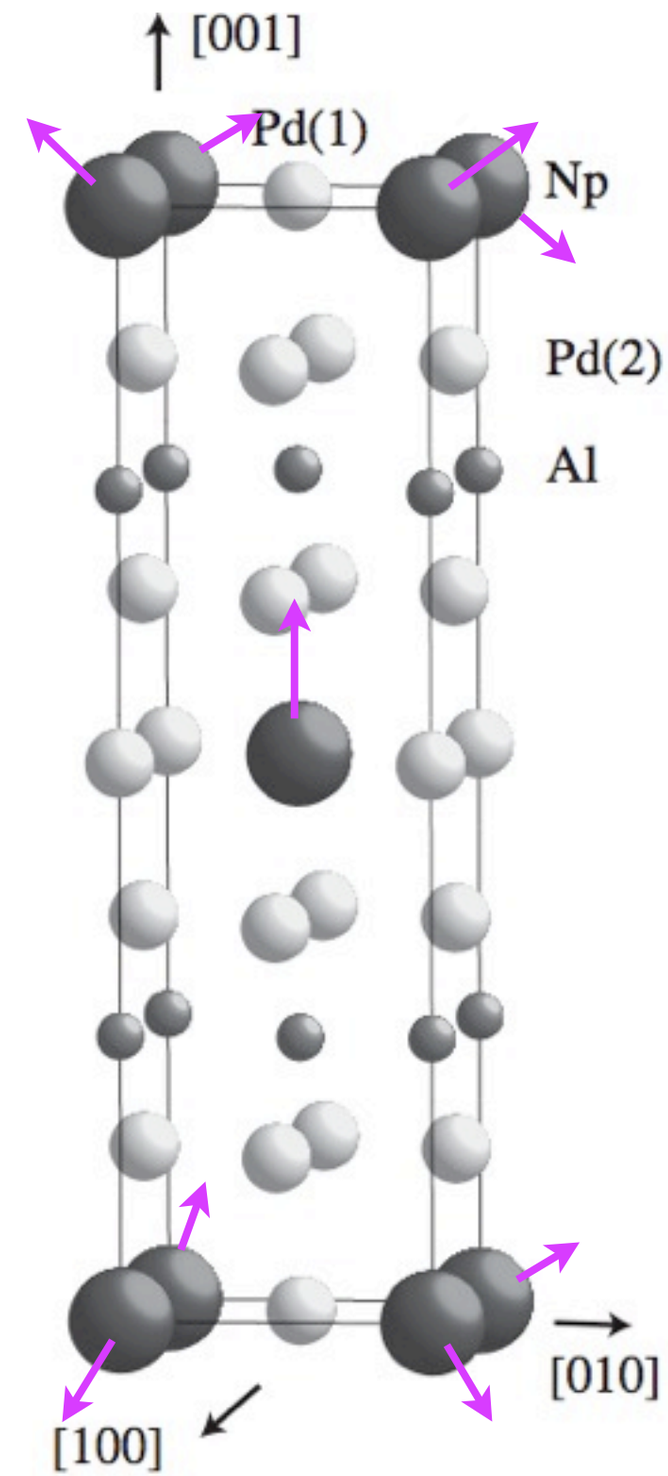
SPIN Hilbert space BUILDS the pairs.

How?

115 Mystery.

The Mystery of $NpPd_5Al_2$

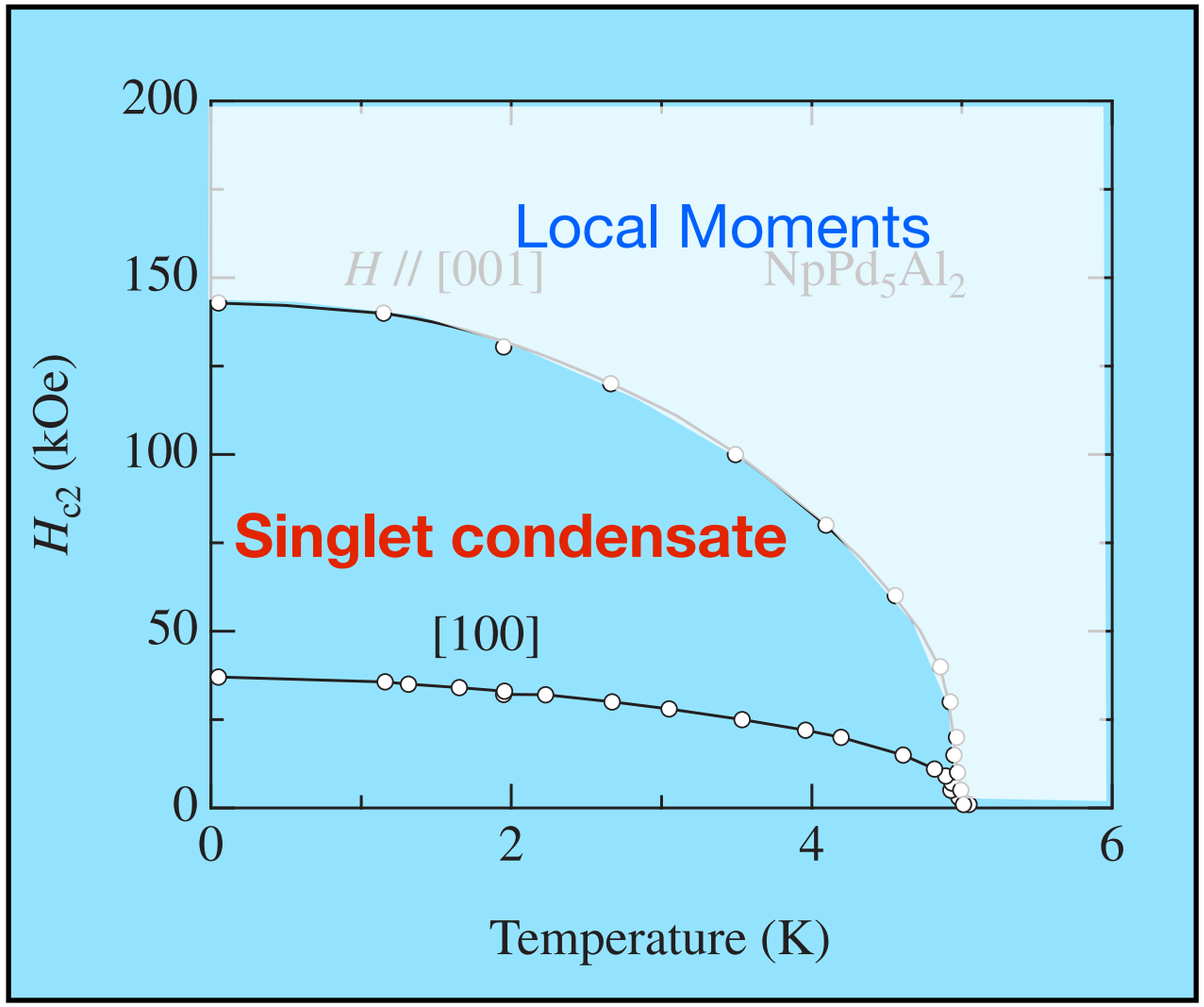




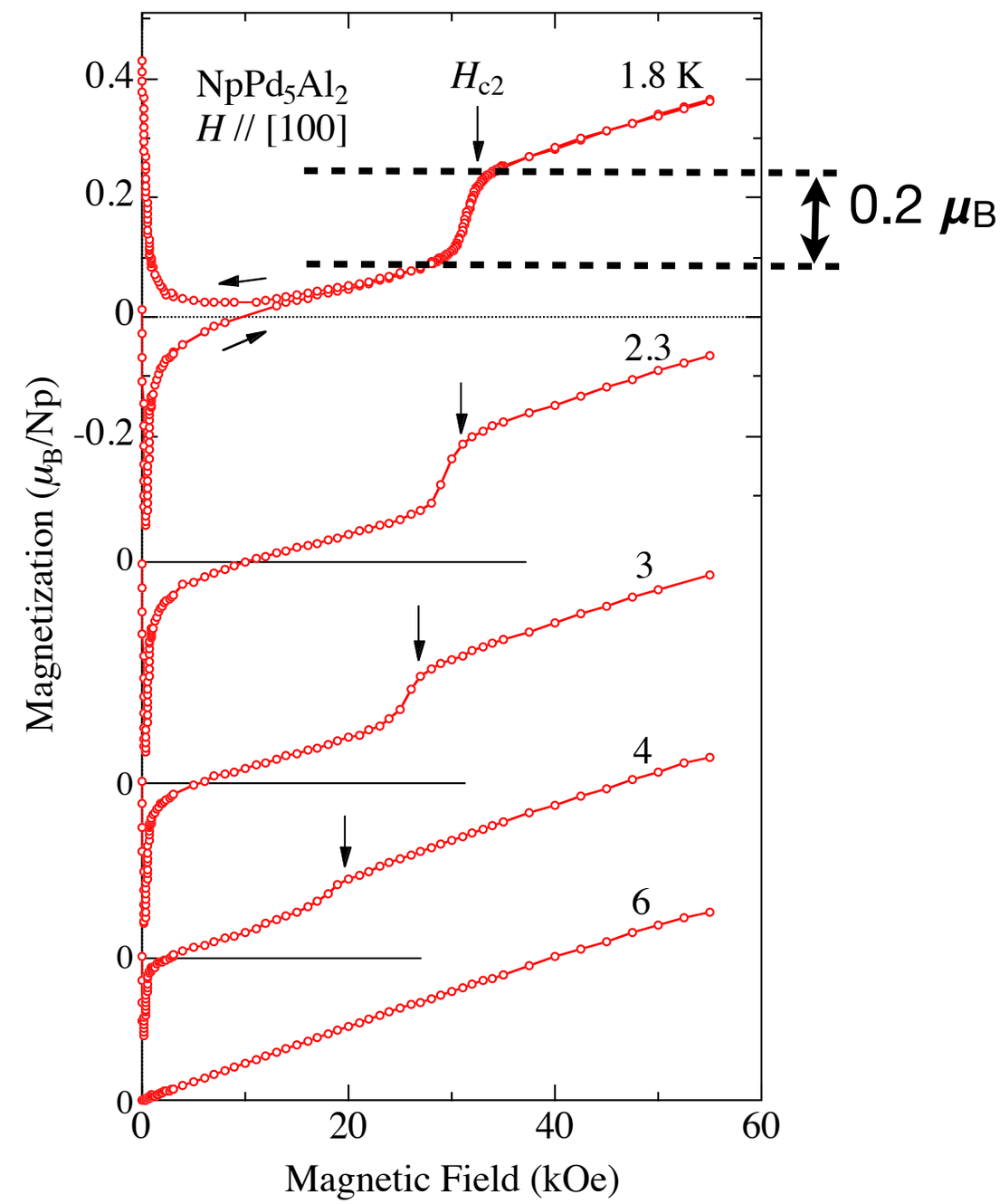
How does the spin form the condensate?

4.5K Heavy Fermion S.C
$NpAl_2Pd_5$
Aoki et al 2007

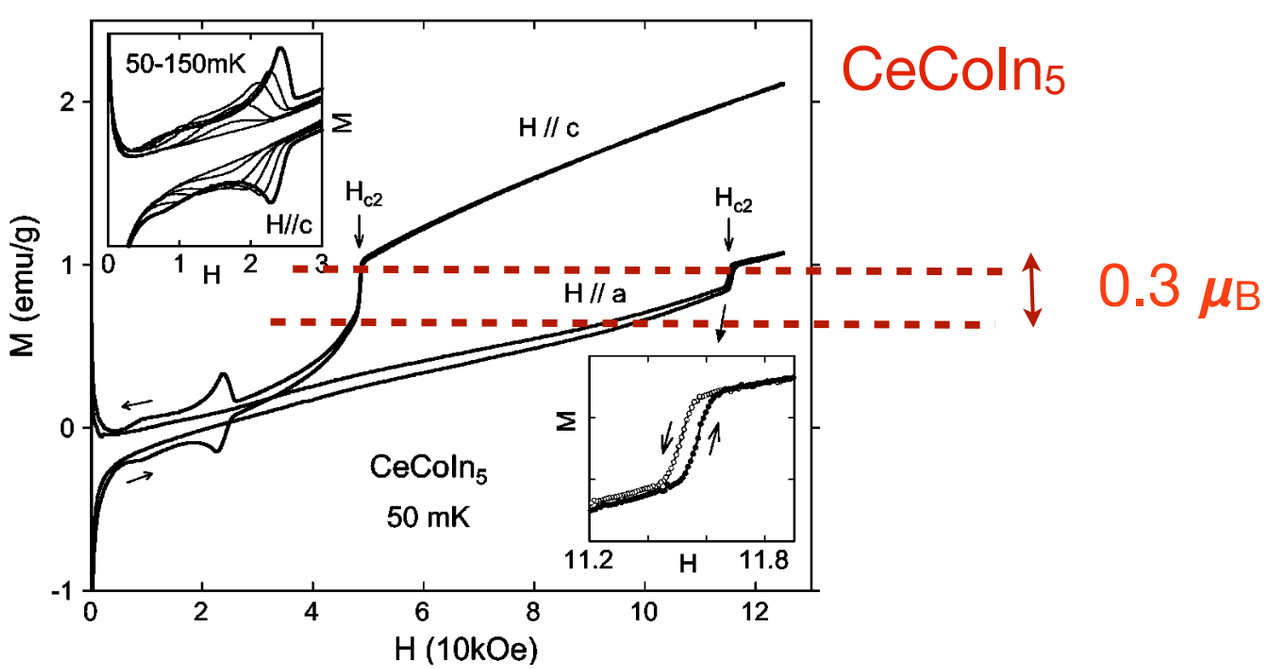
Large first order jump in magnetization at H_{c2}.



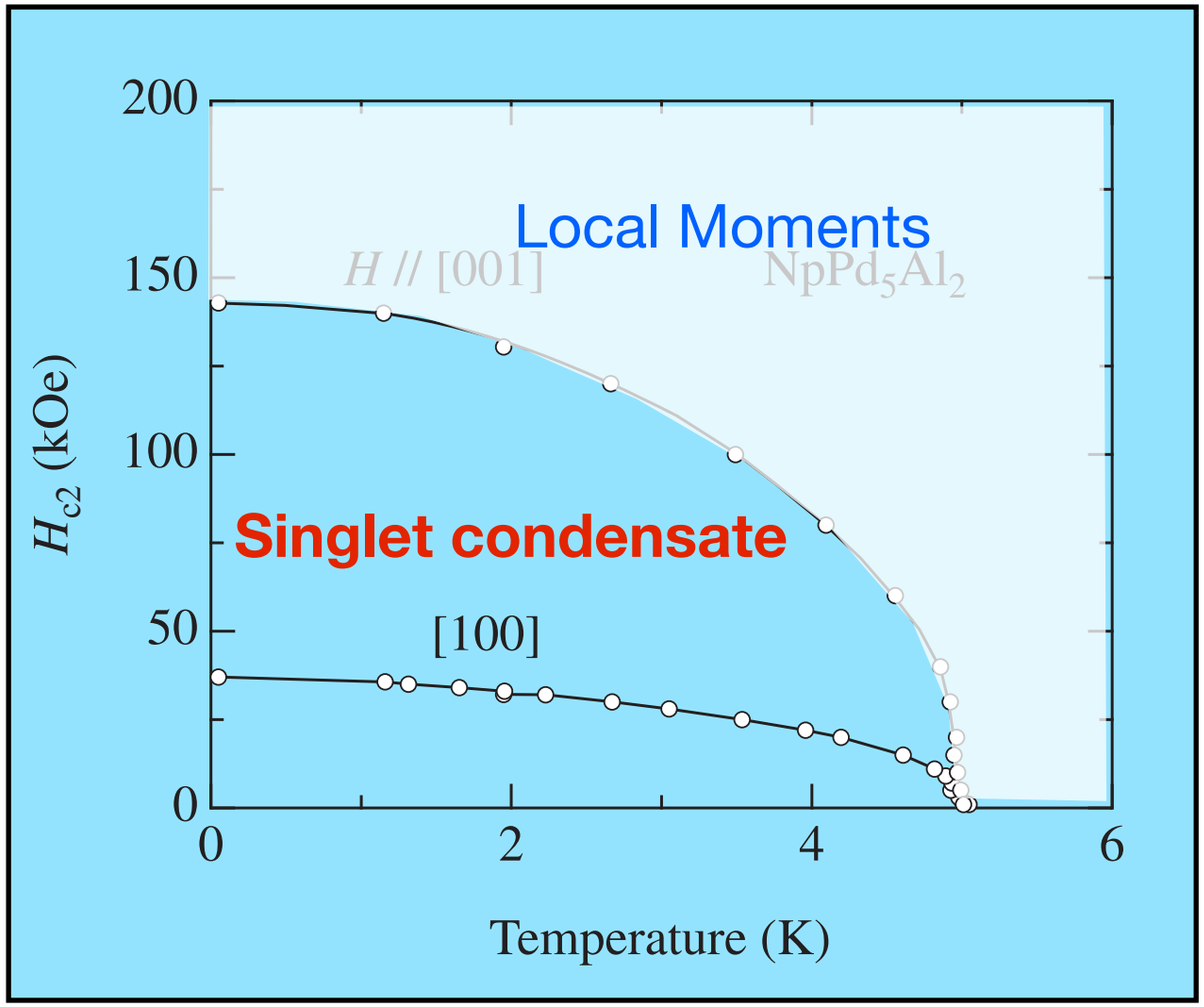
Signals a release of the local moment from the condensate.

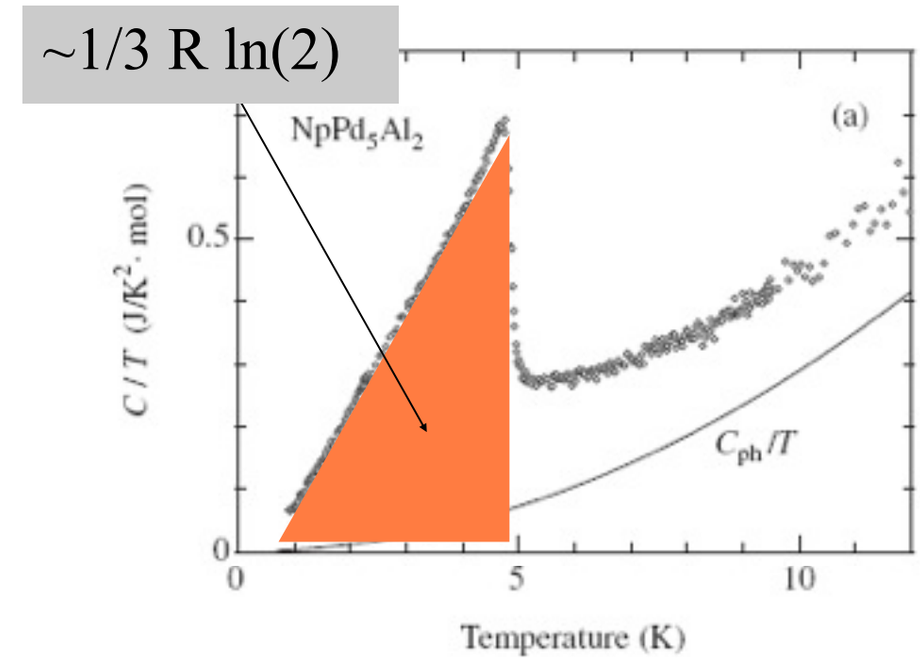


D. Aoki et al., J. Phys. Soc. Jpn. **76** (2007) 063701.



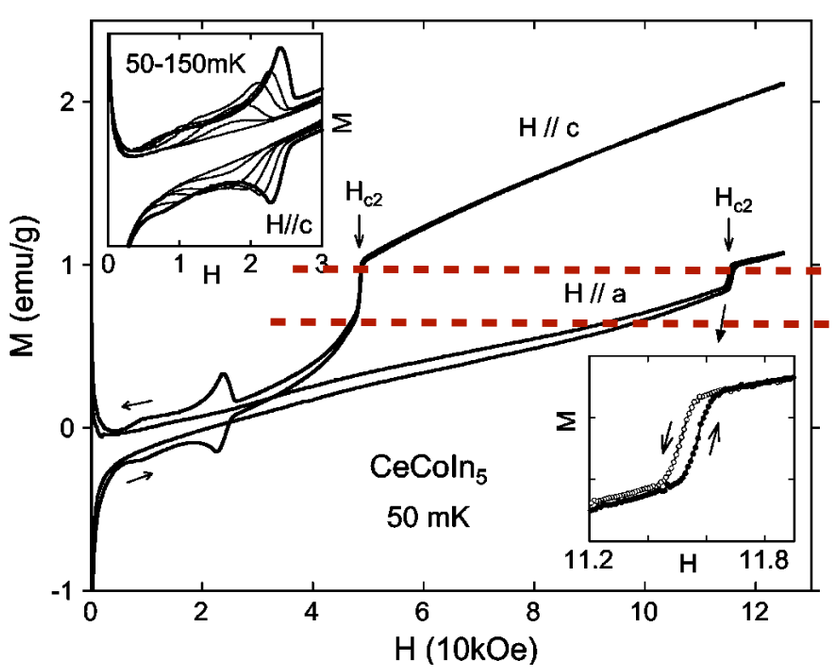
T. Tayama et al., RPB **65**, 180504R (2002)





Paradox:

How can a neutral magnetic moments form a charged superconducting condensate?



T. Tayama et al., RPB **65**, 180504R (2002)

$CeCoIn_5$

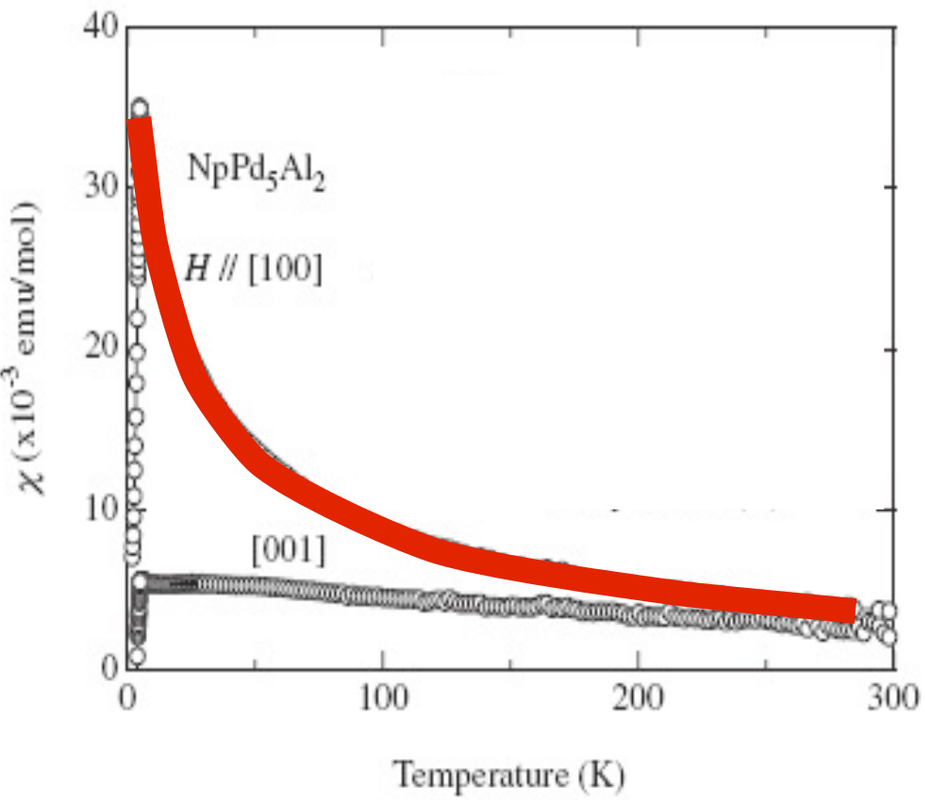
0.3 μ_B

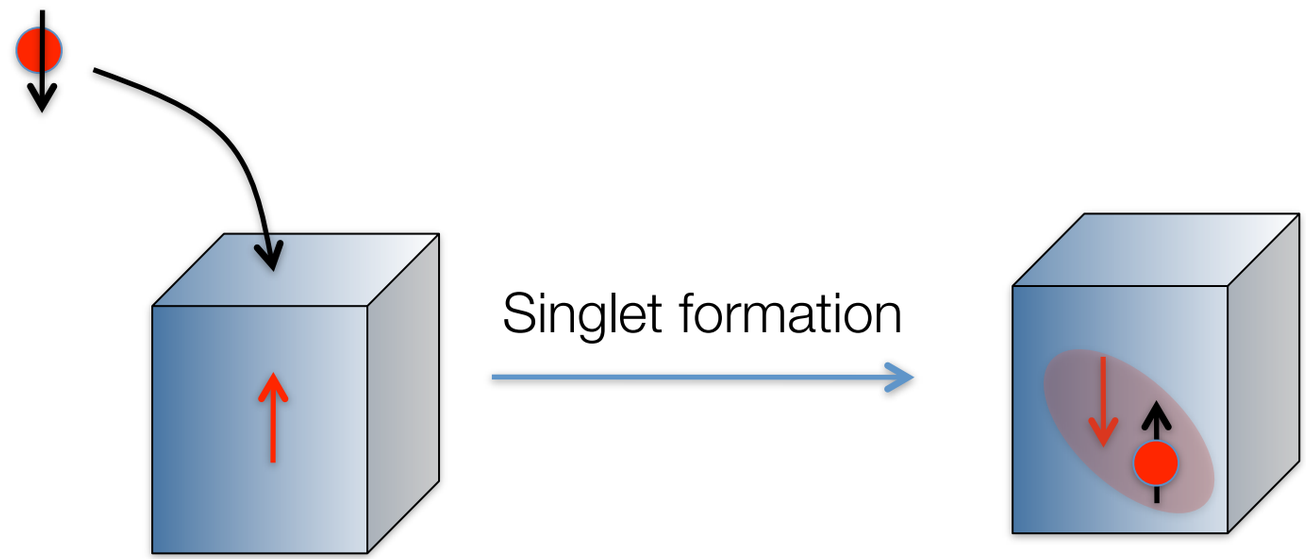
Charge = Condensate Hilbert Space

Composite pairing Hypothesis.

Cotunneling and composite fermions

$NpPd_5Al_2$ T_C = 4.5K

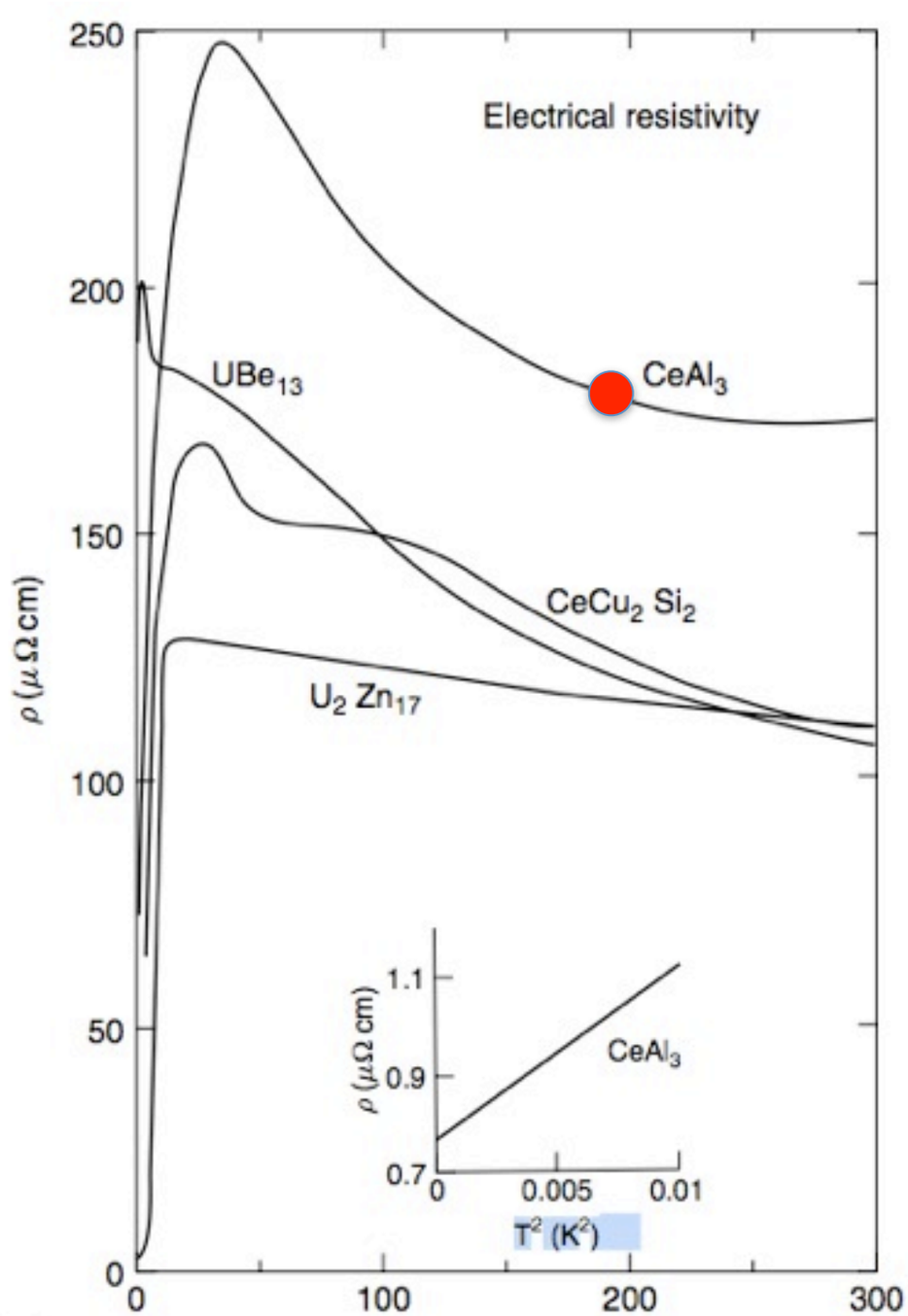


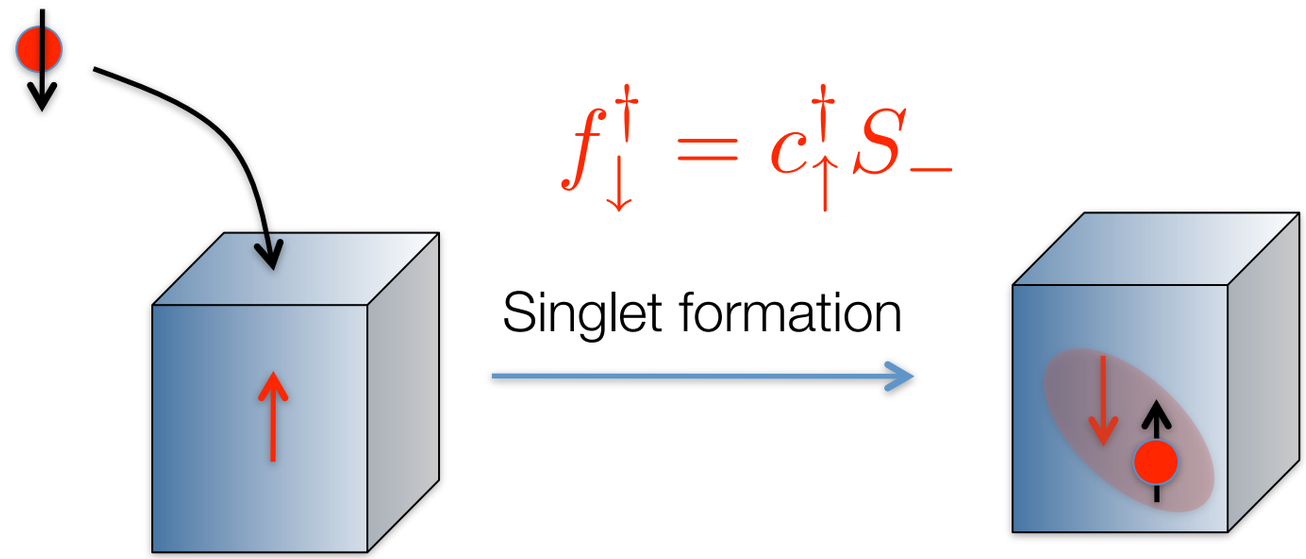


Heavy electron = (electron x spinflip)

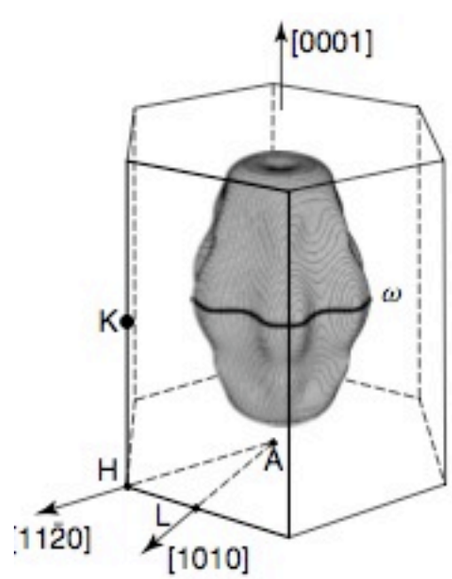
$$f_\downarrow^\dagger = c_\uparrow^\dagger S_-$$

Coherence and composite fermions

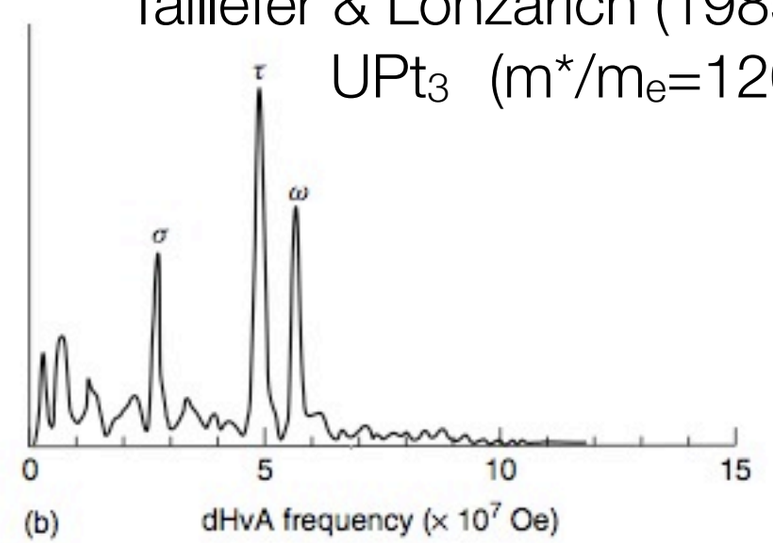




Heavy electron = (electron x spinflip)

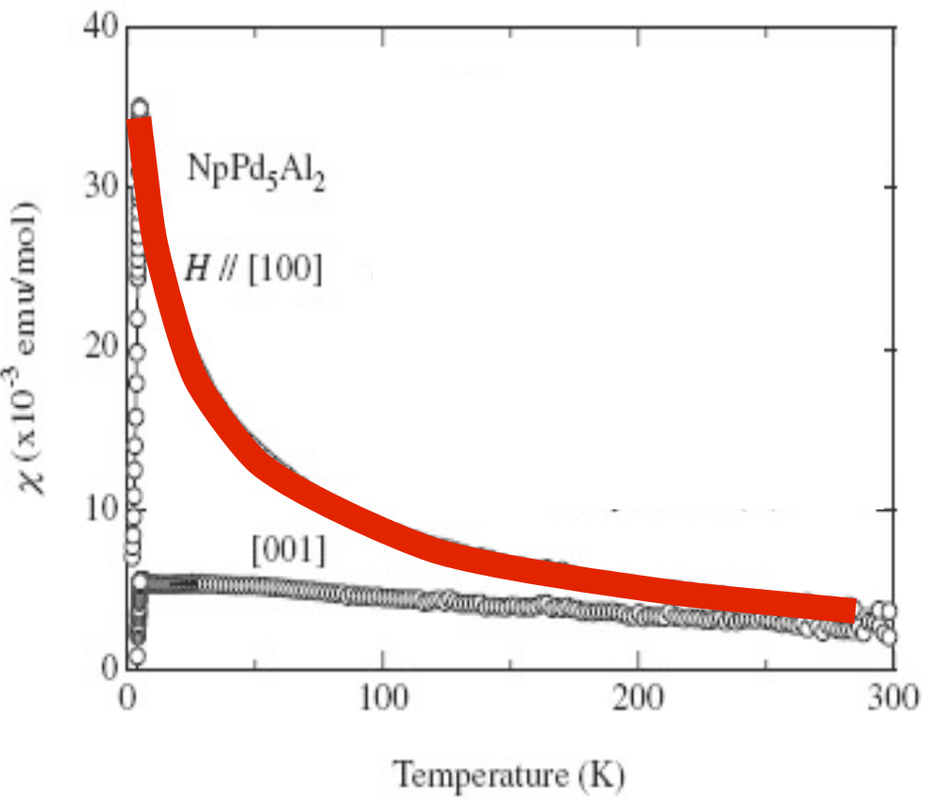


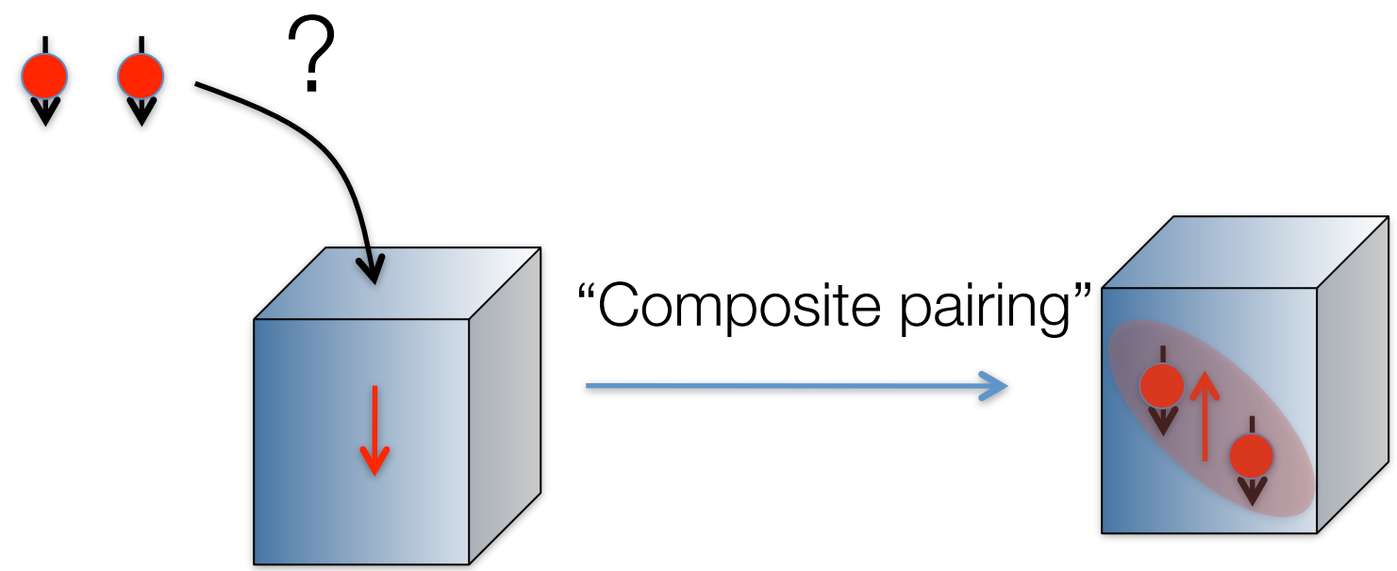
Taillefer & Lonzarich (1985)
UPt_3 (m^*/m_e=120)



Composite pairing

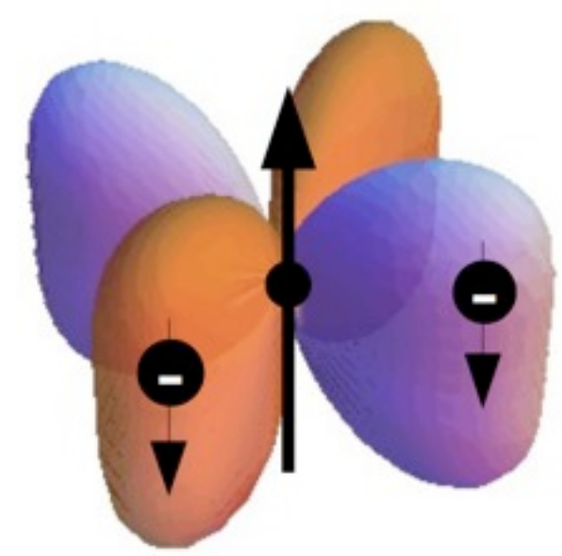
$NpPd_5Al_2$ T_C = 4.5K





Heavy Cooper pair = (pair x spinflip)

$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$



Abrahams, Balatsky, Scalapino, Schrieffer 1995

A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei PRB 60, 3605 (1999).

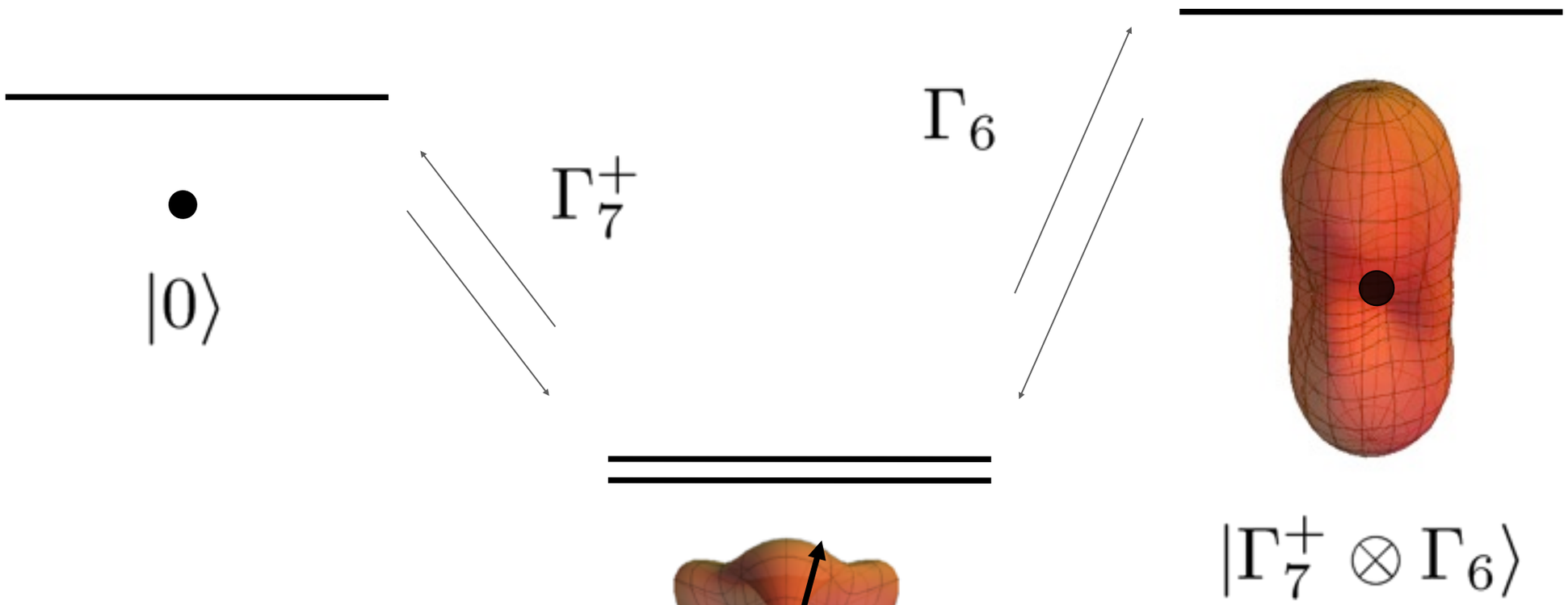
Flint, Dzero, PC, Nature Physics 4, 643 (2008).

Flint, PC, PRL, 105, 246404 (2010).

Flint, Nevidomskyy, PC, PRB 84, 064514 (2011).

The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j$$



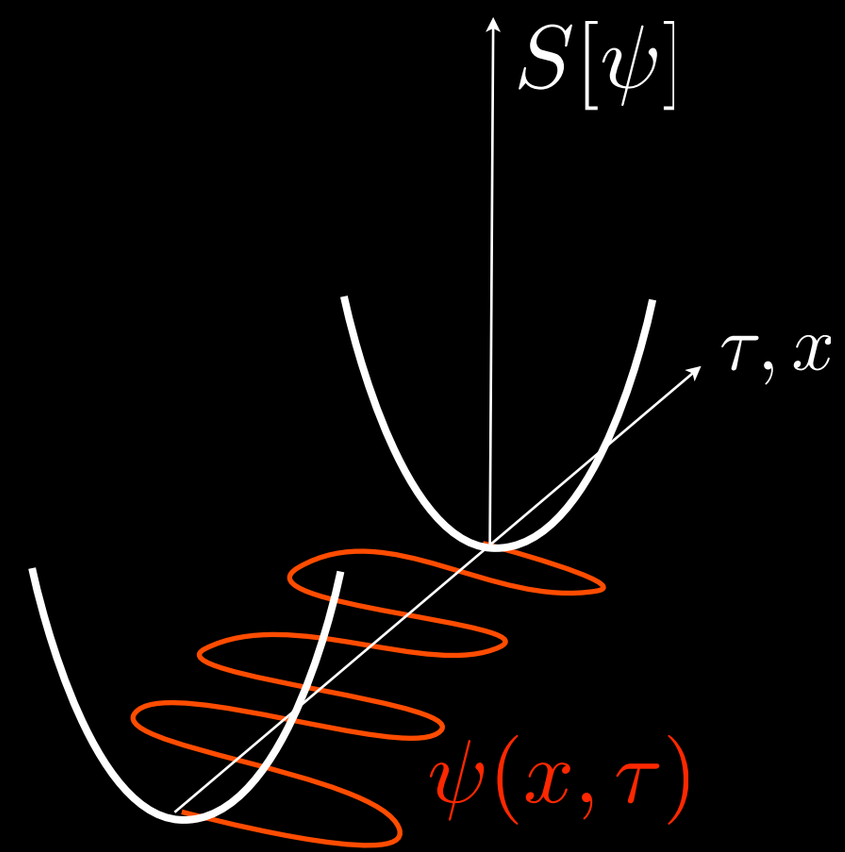
Wannier functions at site j:

$$\psi_{\Gamma j}^\dagger = \sum_k \Phi_{\Gamma k} e^{i\vec{k}\cdot\vec{R}_j} c_k$$

- Both $\begin{cases} f^0 + e^- \rightleftharpoons f^1 \\ f^1 \rightleftharpoons f^2 + h^+ \end{cases}$

$$Z = \int_{\text{history}} \mathcal{D}[\psi] e^{-S[\psi_\sigma]}$$

Feynman



Wild quantum fluctuations!

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \; c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Method: large N

Flint, Dzero Coleman Nature Physics '08
PRB 79, 014424(2009)

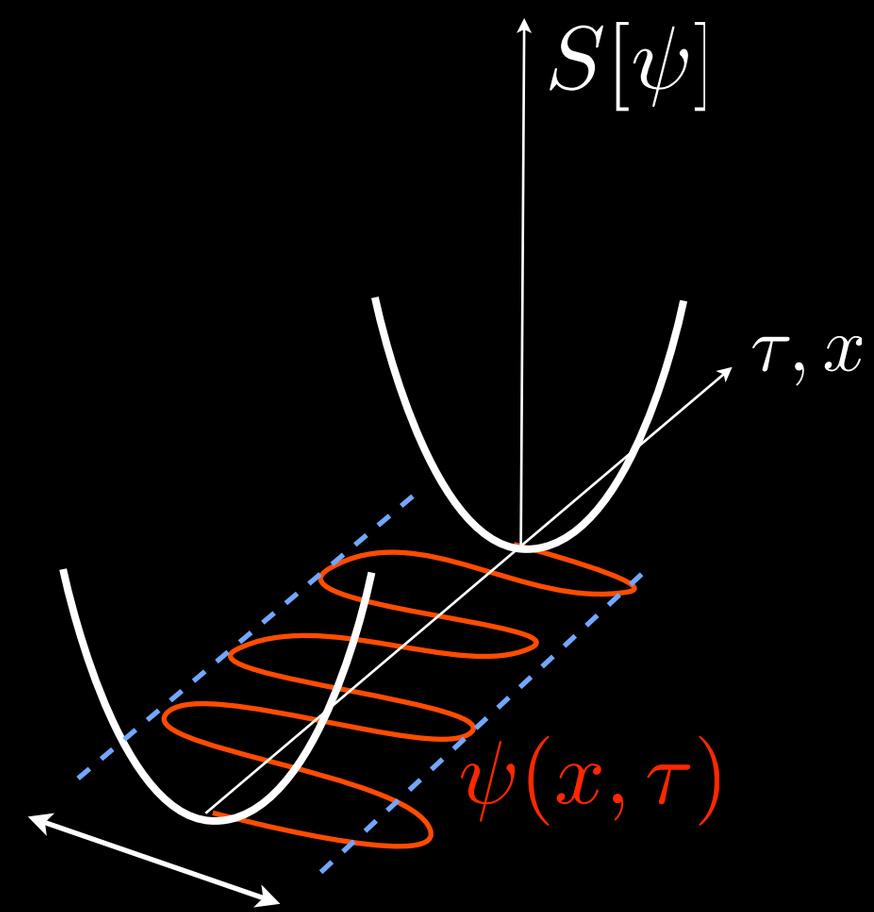
$$Z = \int_{\text{history}} \mathcal{D}[\psi] e^{-N S[\psi_\sigma]}$$

Feynman

$$= \int_{\text{history}} \mathcal{D}[\psi] e^{-\frac{S[\psi_\sigma]}{1/N}}$$

So how can we solve this model?

$\sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow$



$\frac{1}{N} \sim \hbar_{eff}$

Wild quantum fluctuations!

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \; c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

"Symplectic N" *(large N + time reversal symmetry)*

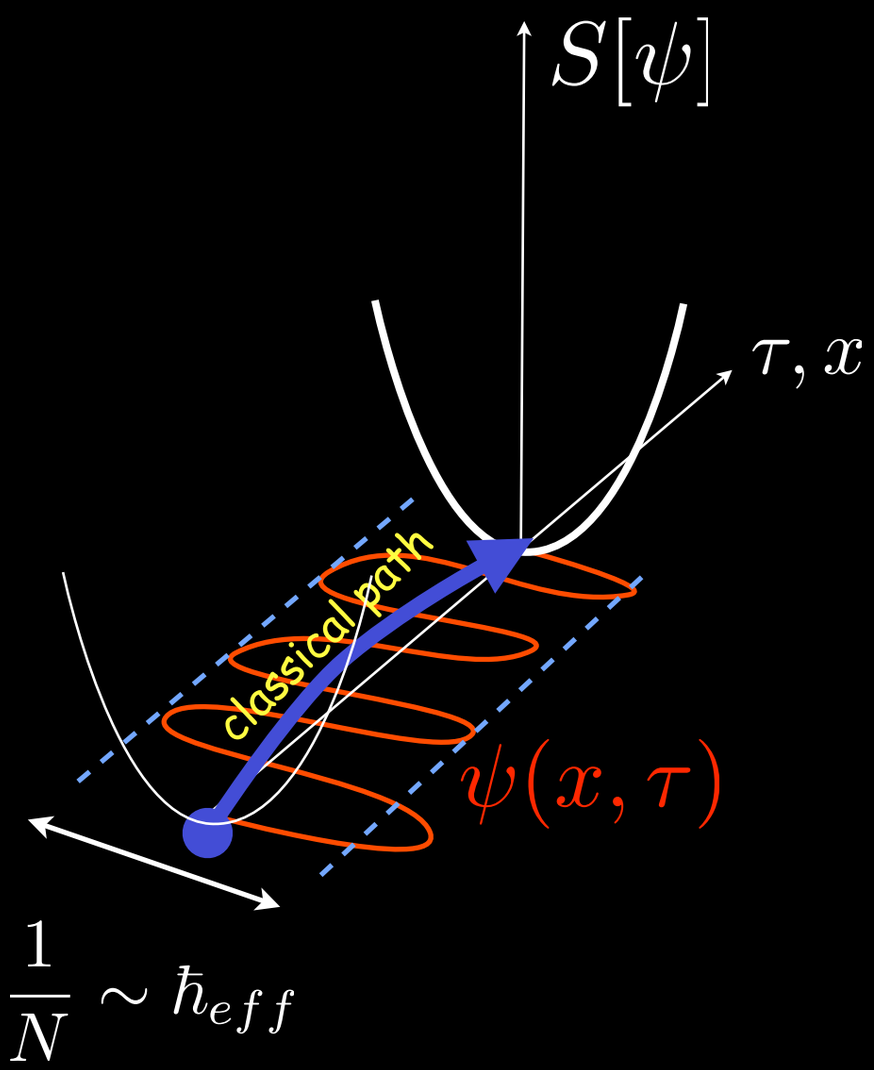
R. Flint, Dzero and PC '08

$N \to \infty$

$$Z = \int_{\text{history}} \mathcal{D}[\psi] e^{-N S[\psi_\sigma]}$$

Feynman

$$= \int_{\text{history}} \mathcal{D}[\psi] e^{-\frac{S[\psi_\sigma]}{1/N}}$$



$$\sigma \in \left(-\tfrac{1}{2}, \tfrac{1}{2}\right) \longrightarrow \left(-\tfrac{N}{2}, \tfrac{N}{2}\right)$$

$$S^{ba} = f_b^\dagger f_a - \mathrm{sgn}(a)\mathrm{sgn}(b) f_{-b}^\dagger f_{-a}$$

?

$$H = \sum_k \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi_{1a}^\dagger(j) \psi_{1b}(j) + J_2 \psi_{2a}^\dagger(j) \psi_{2b}(j) \right) S^{ba}(j)$$

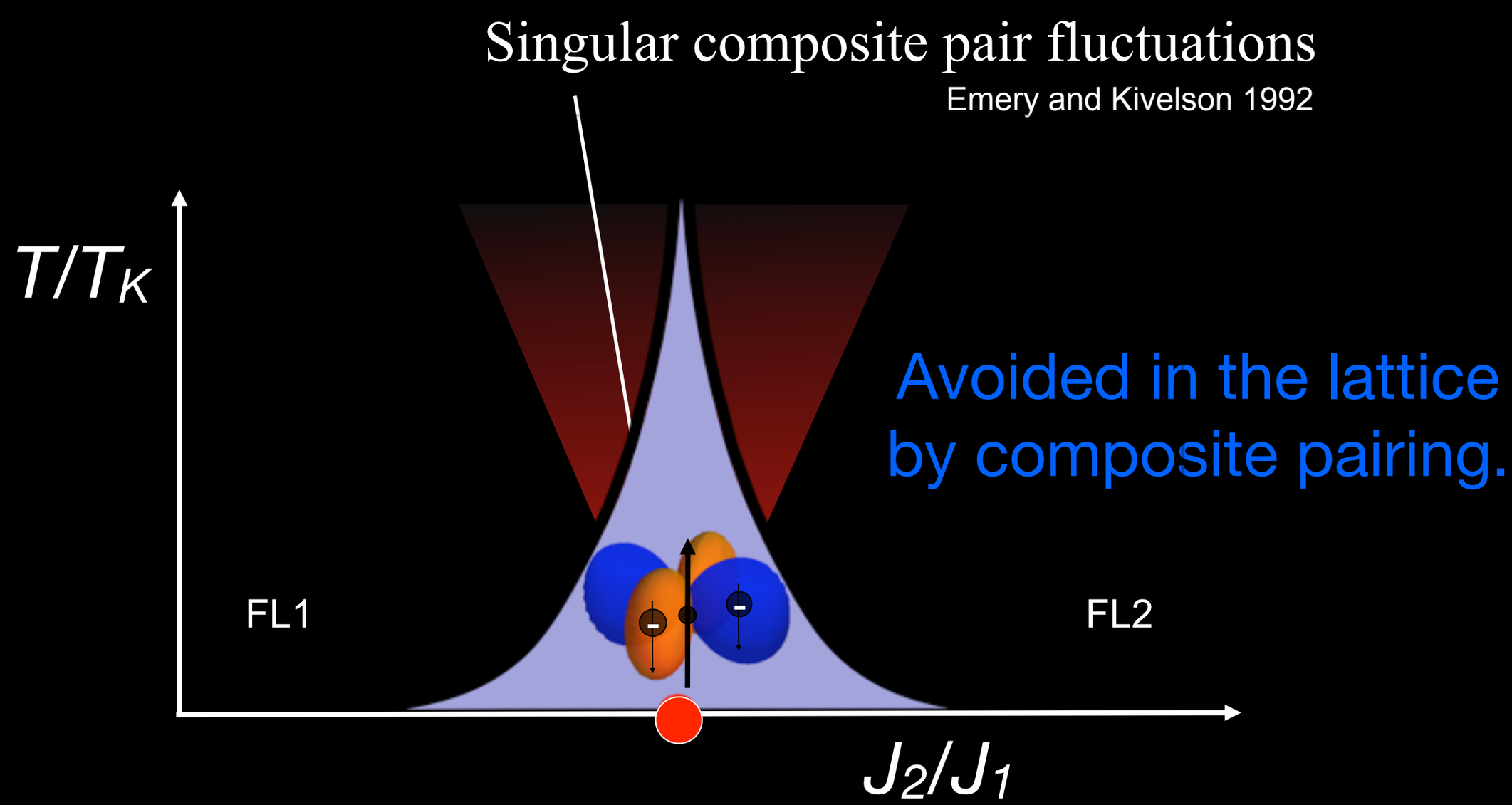
Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \; c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

Single FS, two channels.

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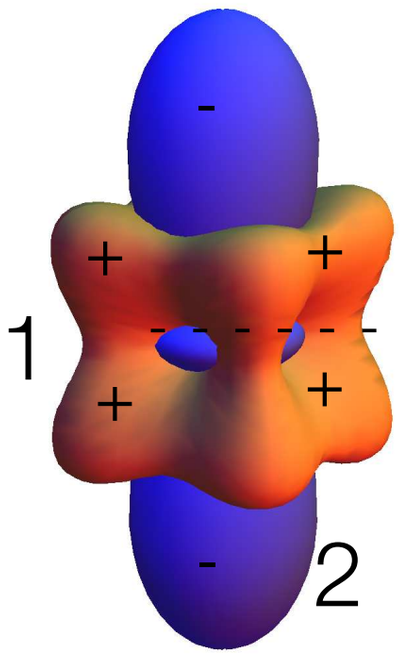
cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

4f/5f superconductors

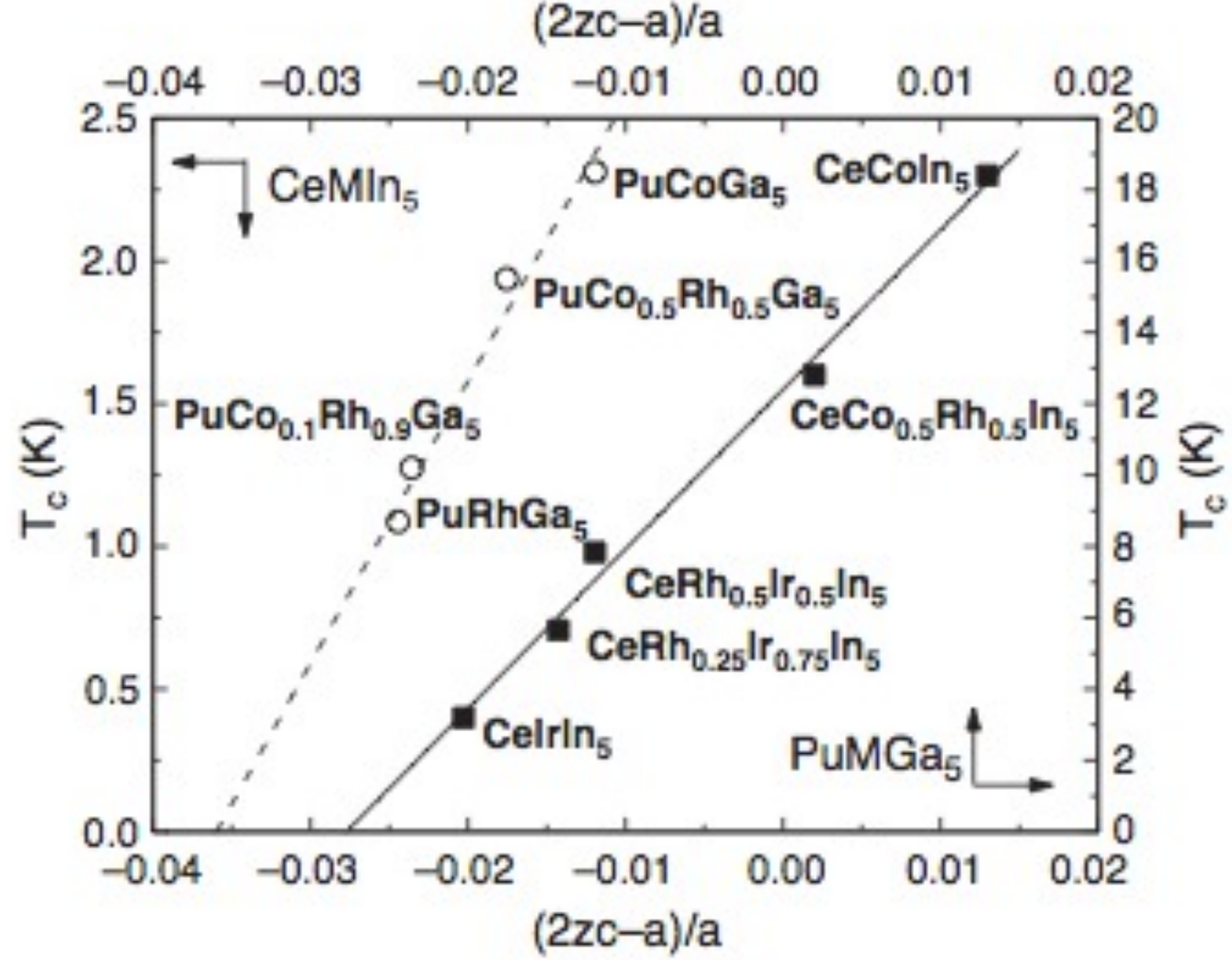


$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

$$Q_{zz} \propto \Psi_C^2$$

$$\Delta F \propto -Q_{zz} u_{tet}$$

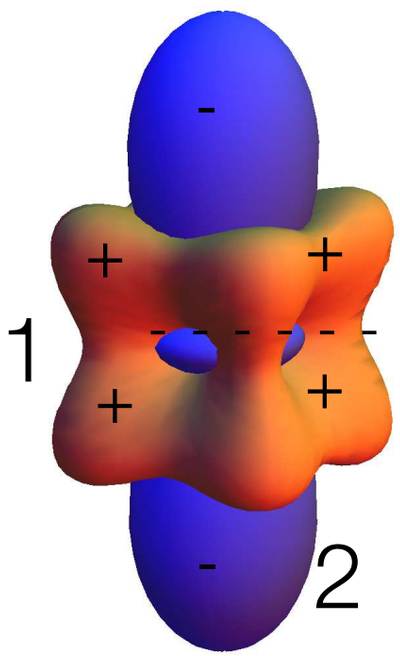
$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$$



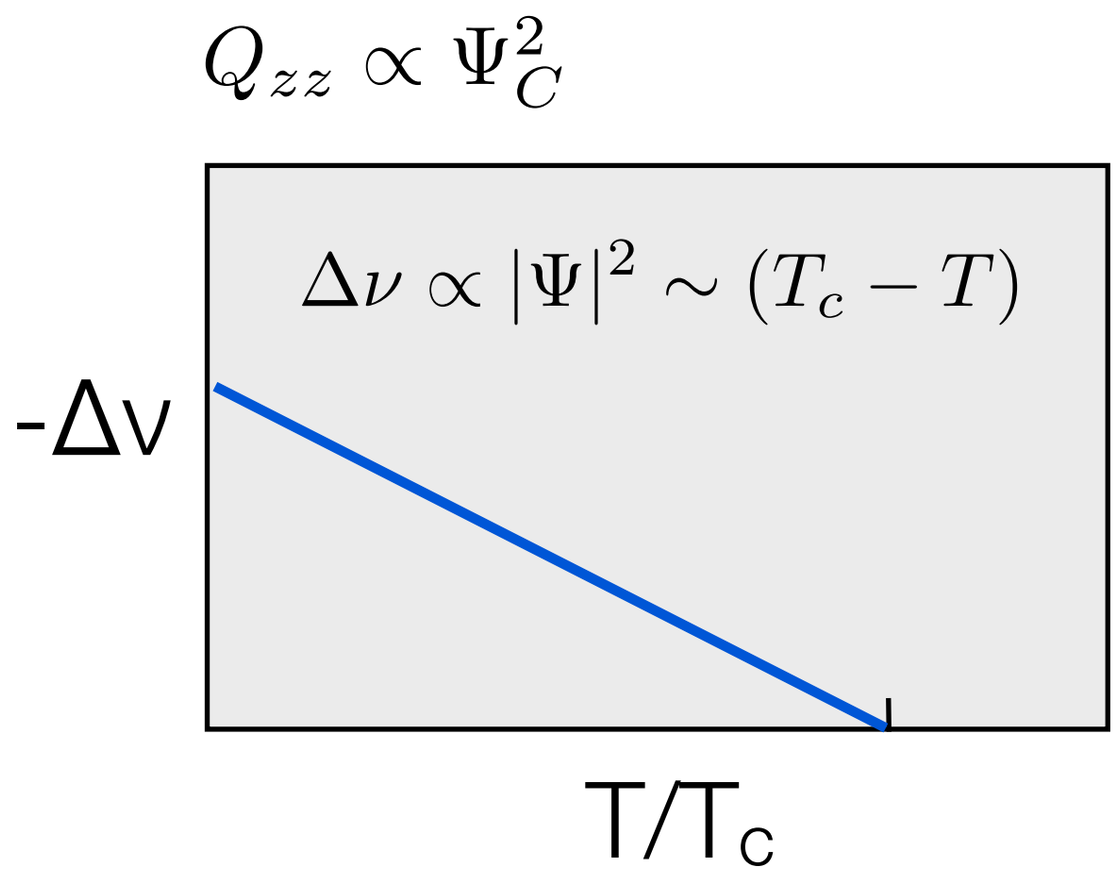
J.L. Sarrao et al. / Physica B 359– 361 (2005) 1144–1146

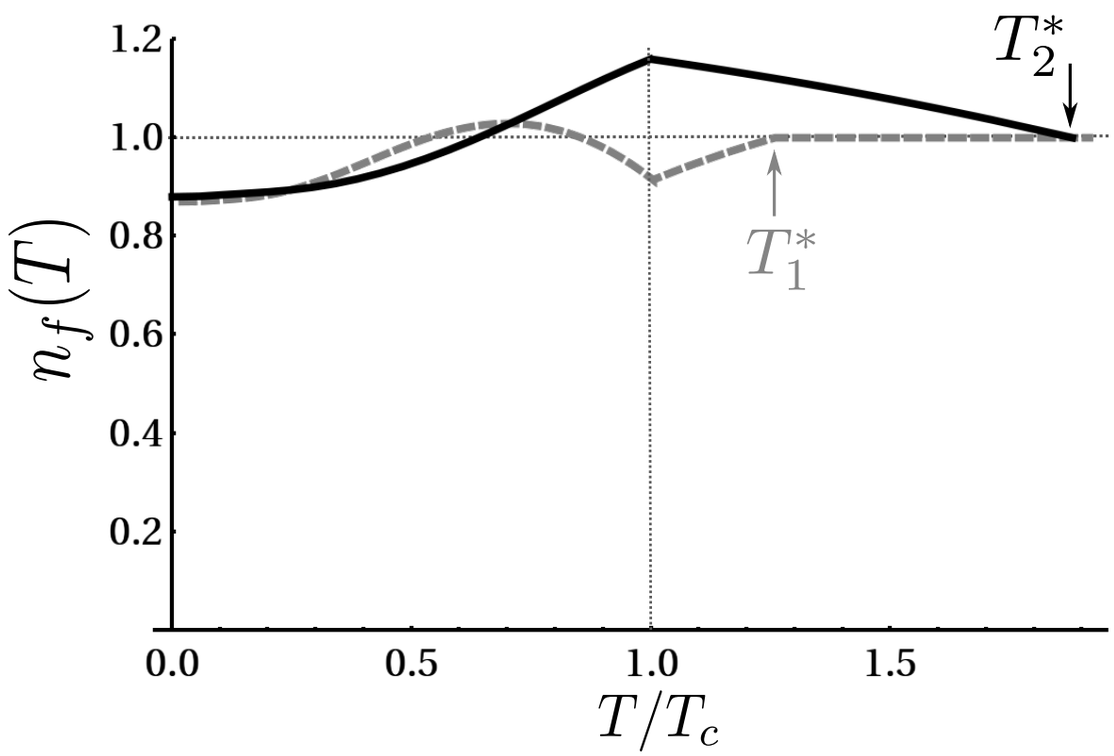
$$\Rightarrow T_c = T_{c2} + \lambda u_{tet}$$

Strain expected to enhance Tc

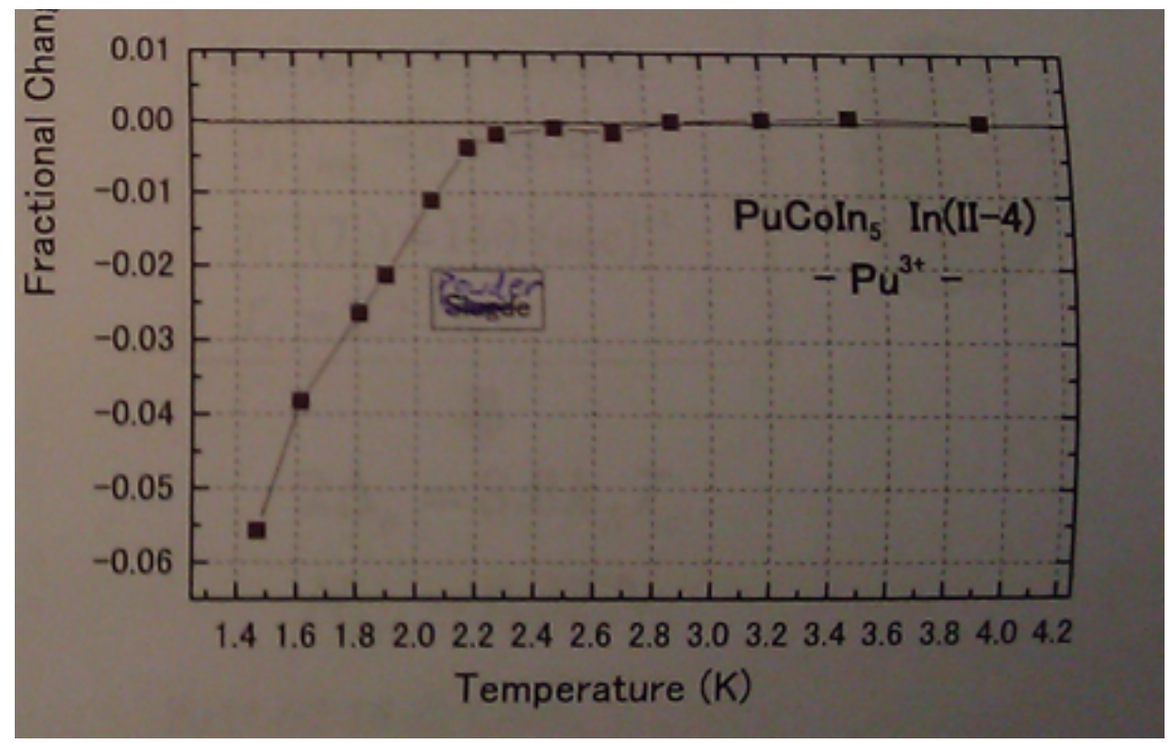


$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$





Flint et al, PRB 84, 064054, (2011)



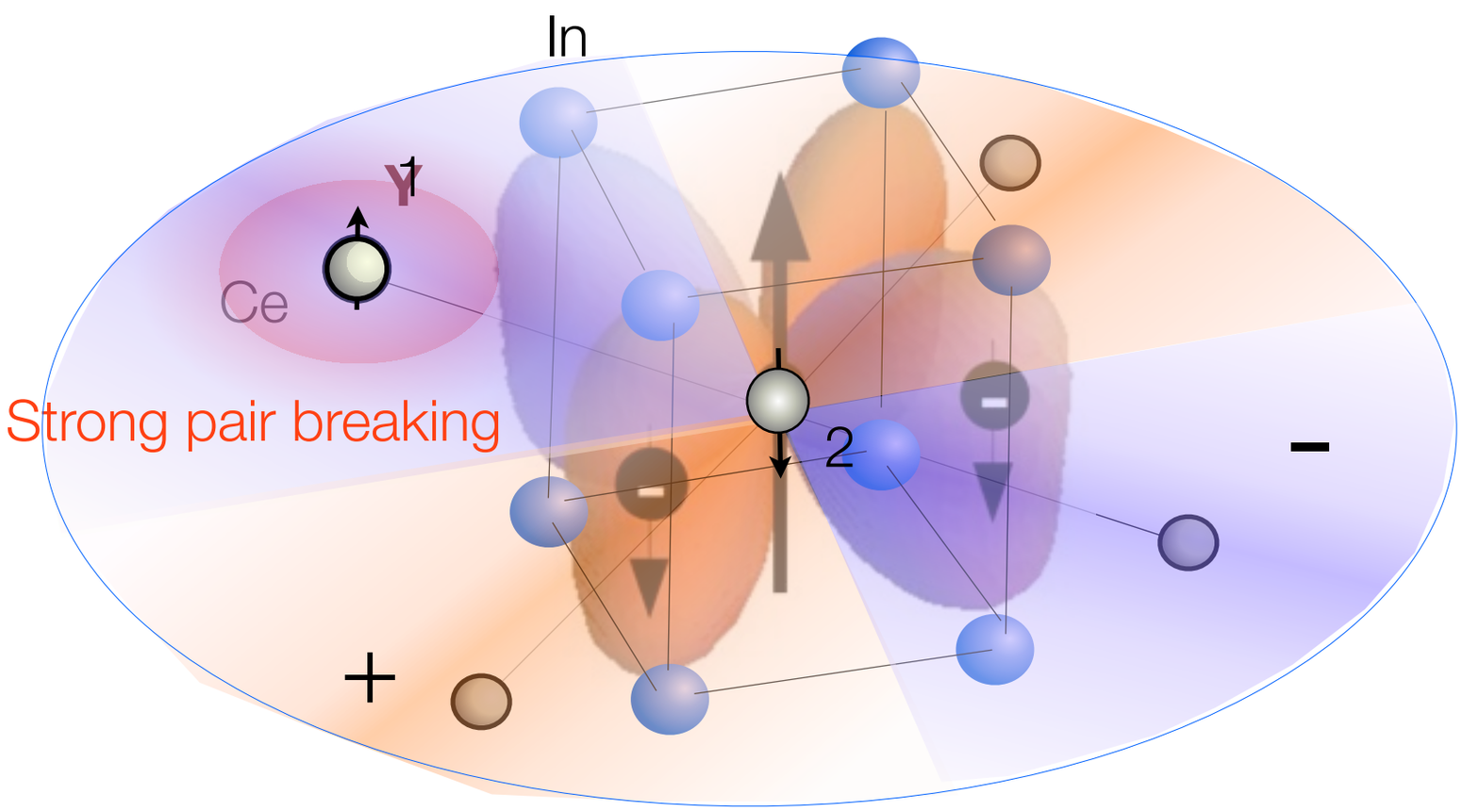
G. Koutroulakis and Yasuoko, Unpublished (2012)

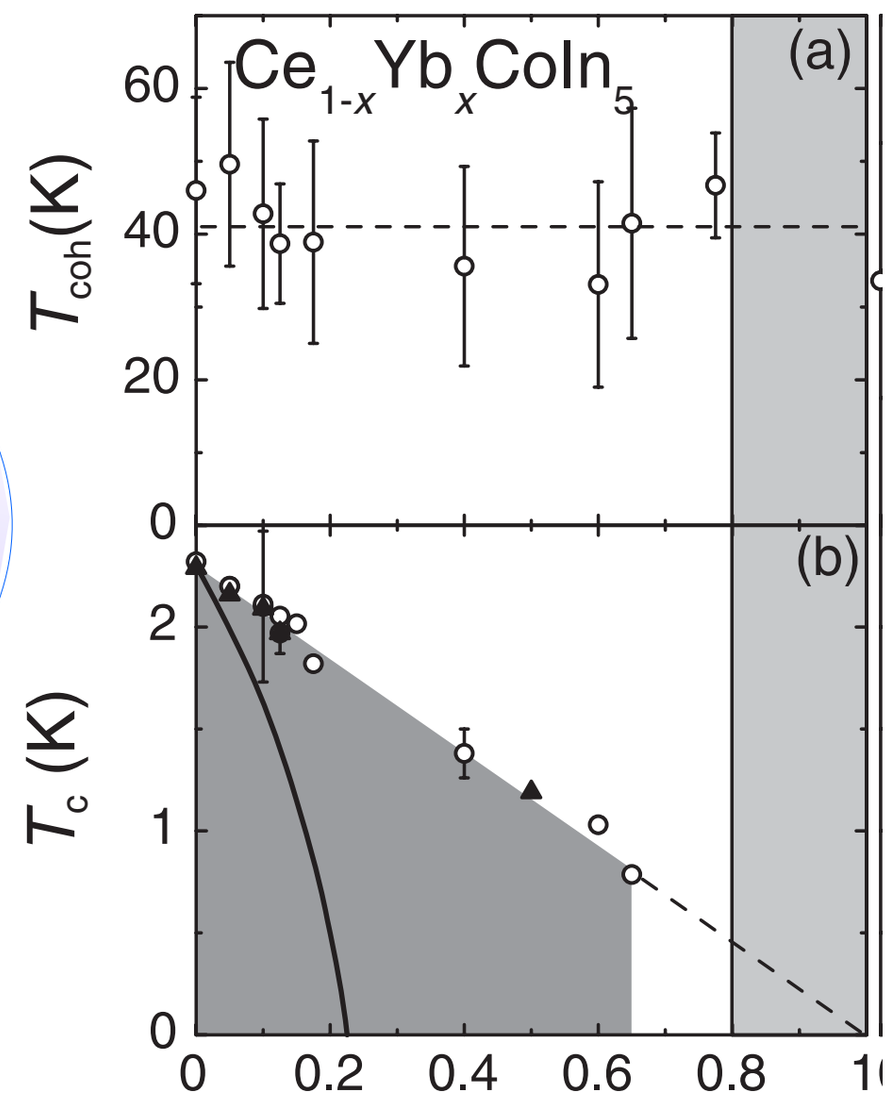
Real-space structure of pair

Extreme Resilience to doping on Ce site.

Magnetic pair: intercell

$$\Psi_M^\dagger = \Delta_d(1-2) f_\uparrow^\dagger(1) f_\downarrow^\dagger(2)$$





Composite pair**: intra-cell boson**

$$\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998)
Flint, Dzero, Coleman , Nat. Phys, (2008)

Lei Shu et al, PRL, (2011)

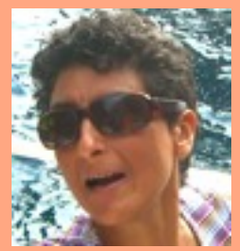
Three Lectures on HF Lecture III:

1. Topological Kondo Insulators

2. Composite pairing.

3. Hidden ("Hastatic?") Order in URu2Si2

Rebecca Flint
(MIT)

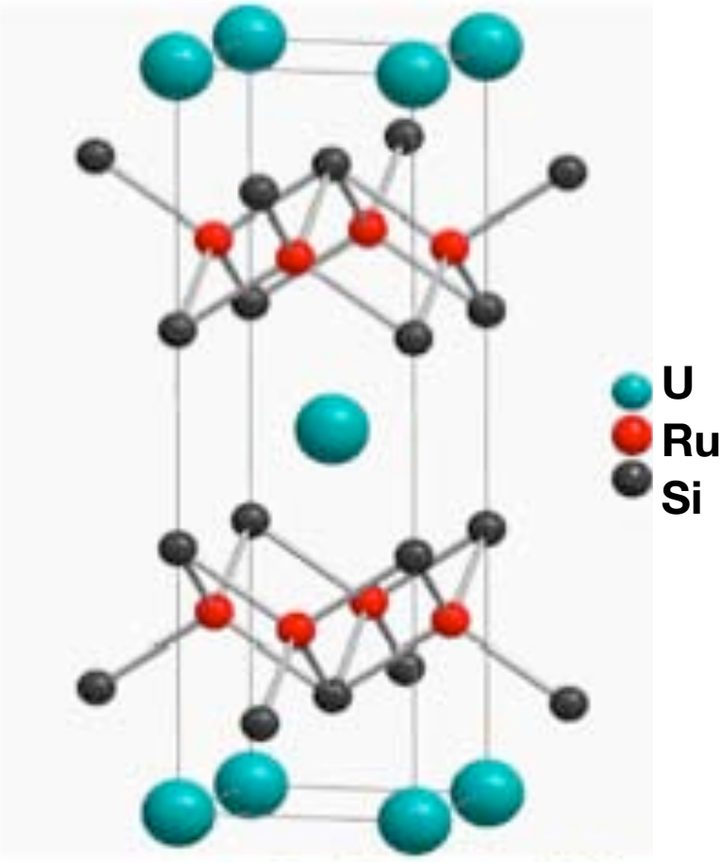


Premi Chandra
(Rutgers)

"Hastatic order in URu_2Si_2", Nature, in press.

Rutgers
Center for Materials Theory

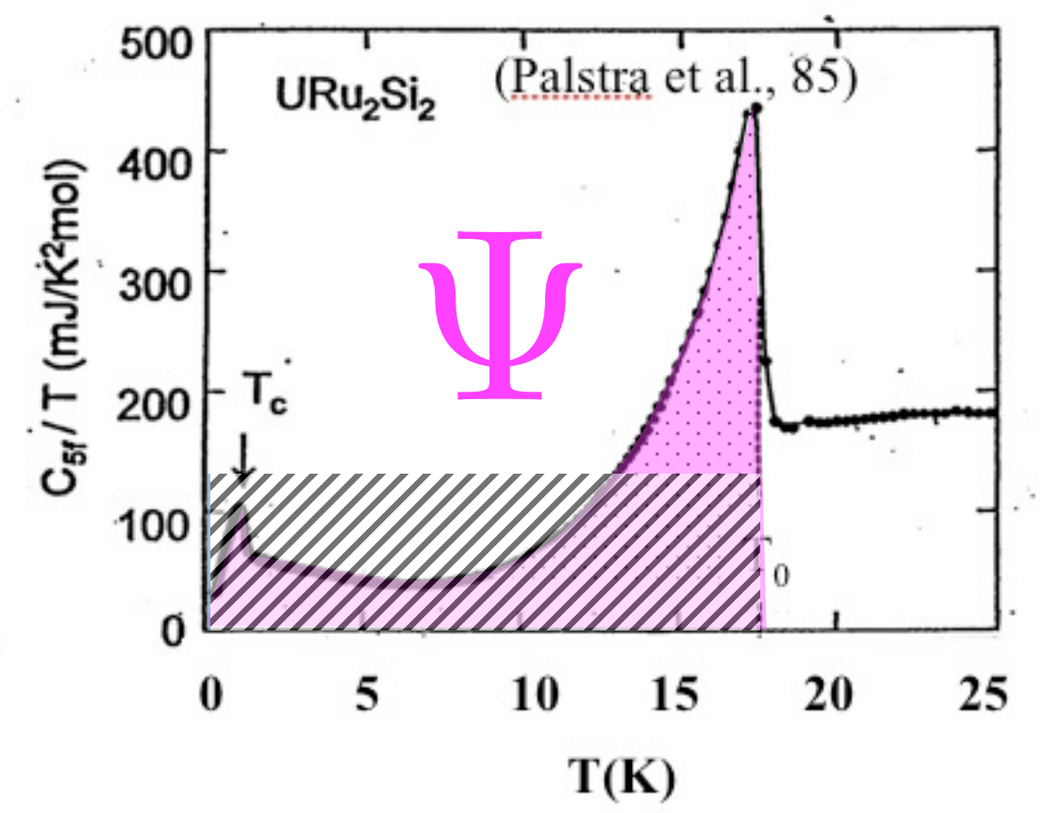
Hidden Order in URu_2Si_2



$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$

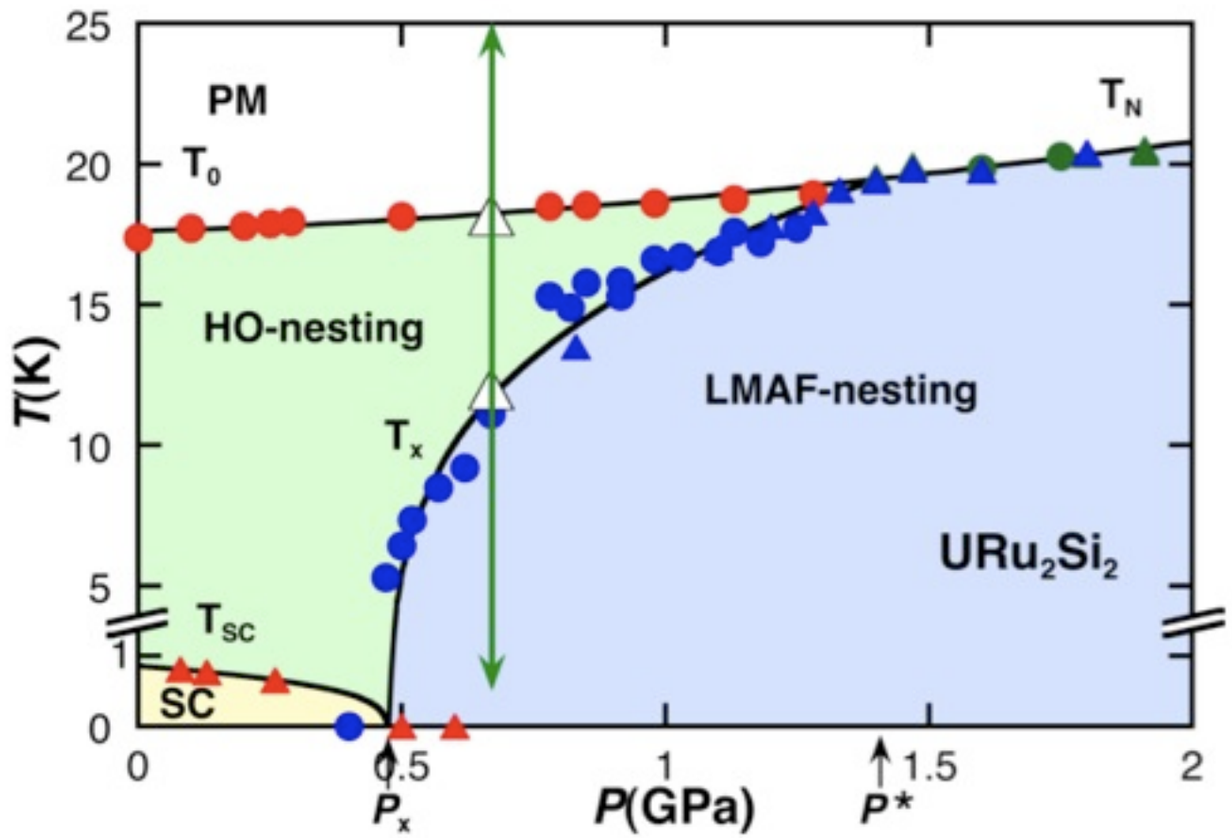
=0.14 x 17.5 K
=2.45 J/mol/K
=0.42 R ln 2

Large entropy of condensation.

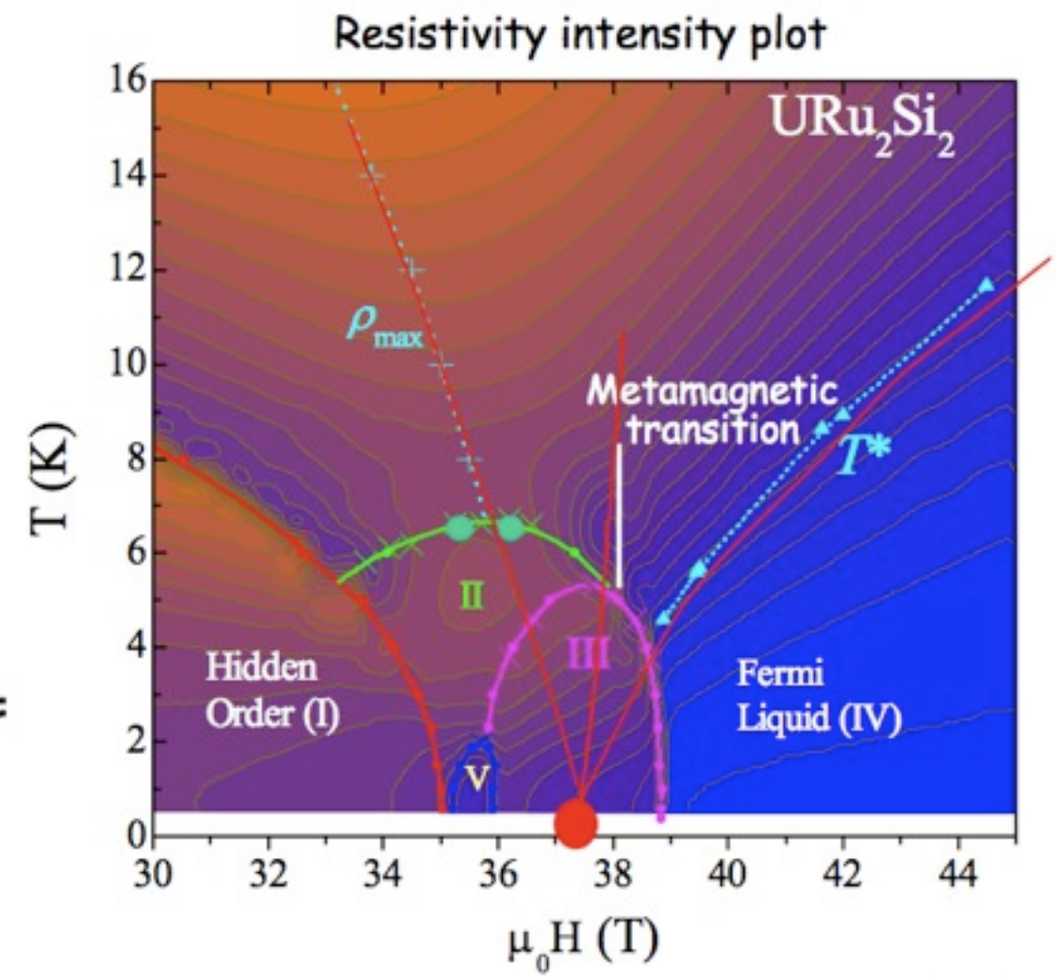


What is the nature of the hidden order?

High pressures, high fields



Villaume et al. (08)



Kim et al (03)

Ising order, present in LMAF, vanishes in the hidden order state. (NMR,MuSR).

25 Years of Theoretical Proposals

Local

Barzykin & Gorkov, ’93 (three-spin correlation)
Santini & Amoretti, ’94, Santini (’98) (Quadrupole order)
Amitsuka & Sakihabara (Γ_5, Quadrupolar doublet, ‘94)
Kasuya, ’97 (U dimerization)
Kiss and Fazekas ’04, (octupolar order)
Haule and Kotliar ’09 (hexa-decapolar)

Landau Theory

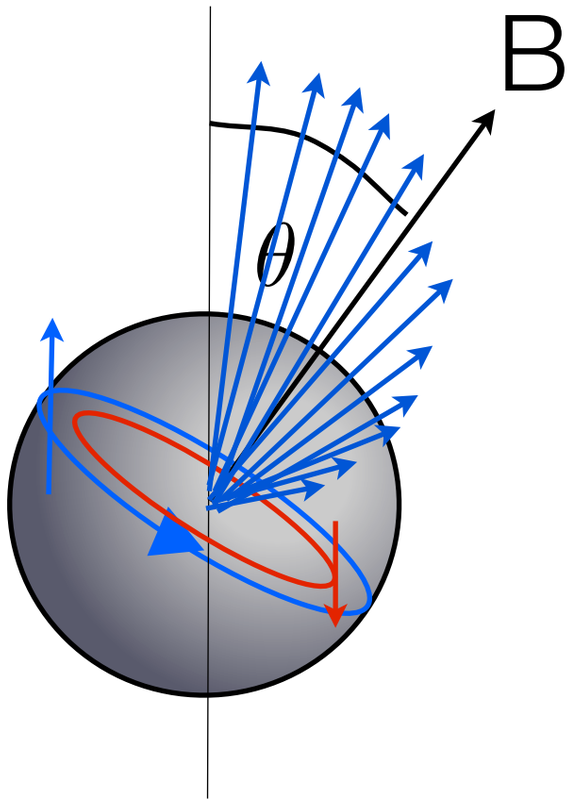
Shah et al. (’00) “Hidden Order”,
Ramirez et al, ’92 (quadrupolar SDW)

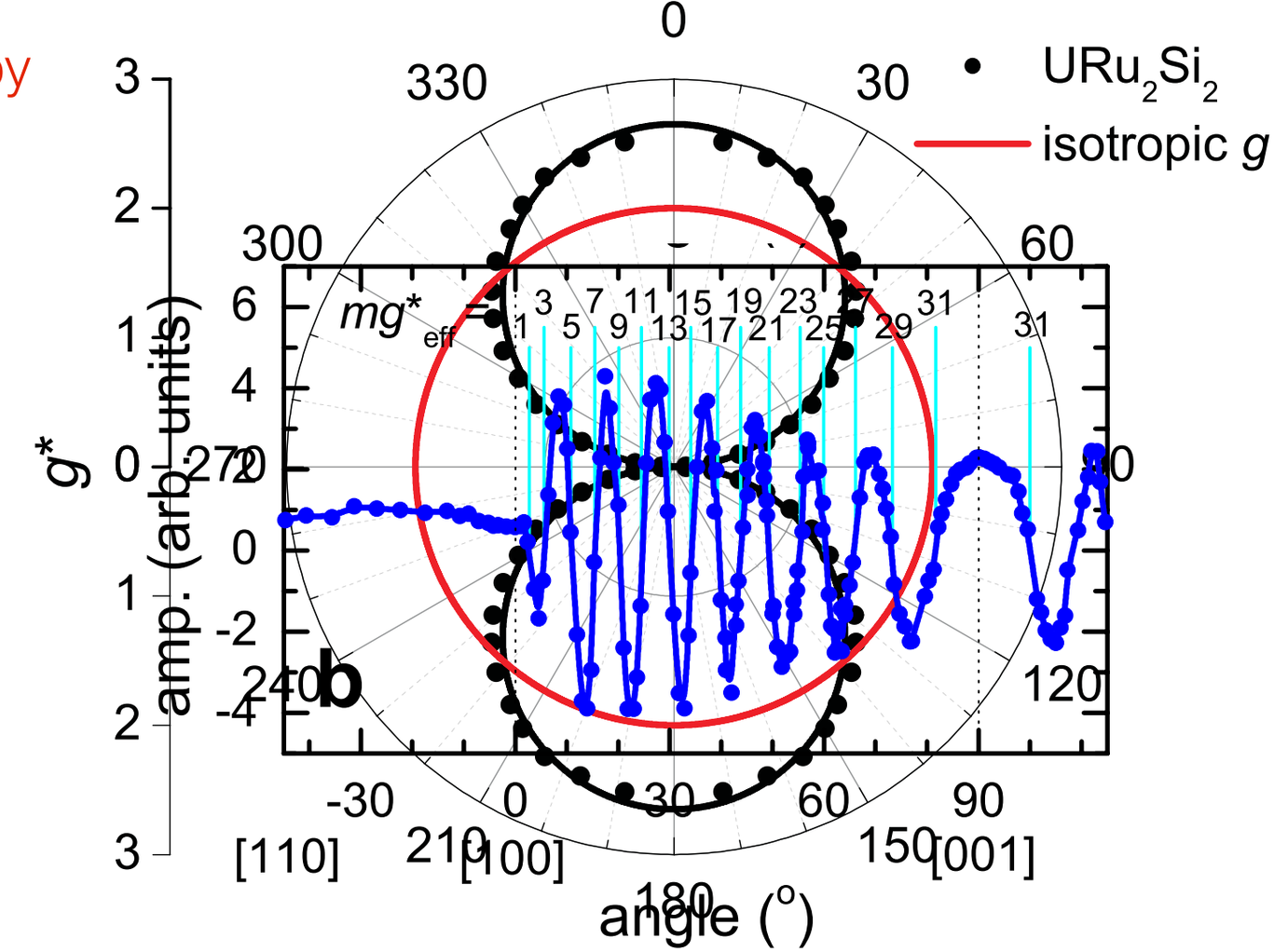
Itinerant

Ikeda and Ohashi ’98 (d-density wave)
Okuno and Miyake ’98 (composite)
Tripathi, Chandra, PC and Mydosh, ’02 (orbital afm)
Dori and Maki, ’03 (unconventional SDW)
Mineev and Zhitomirsky, ’04 (SDW)
Varma and Zhu, ’05 (spin-nematic)
Ezgar et al ’06 (Dynamic symmetry breaking)
Pepin et al ’10 (Spin liquid/Kondo Lattice)
Dubi and Balatsky, ’10 (Hybridization density wave)
Fujimoto, 2011 (spin-nematic)
Rau and Kee 2012 (Rank 5 pseudo-spin vector)

The Giant Ising Anisotropy.

Quantum Oscillations: Giant Ising Anisotropy





$$M \propto \cos\left[2\pi \frac{\text{Zeeman}}{\text{cyclotron}}\right]$$

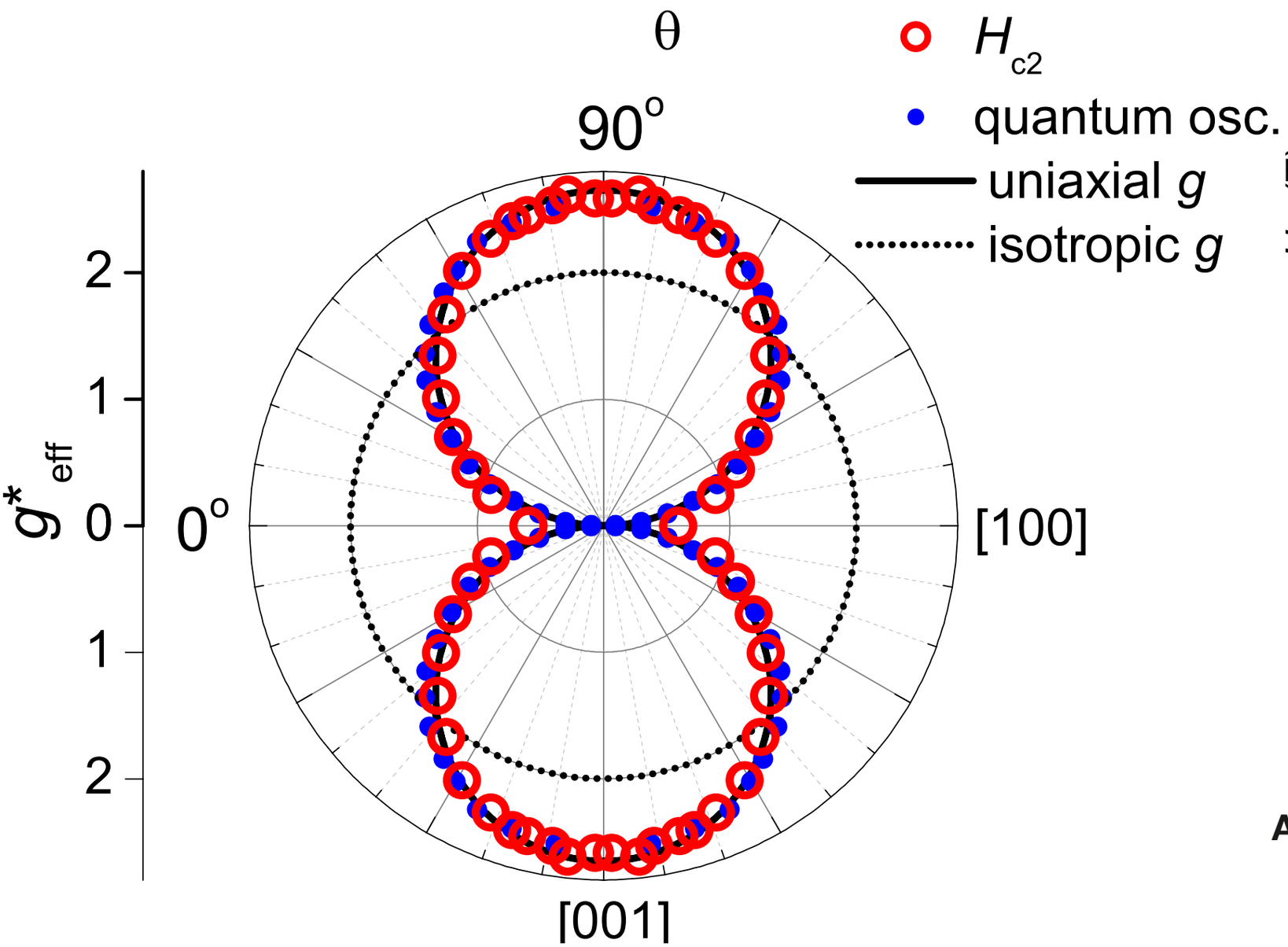
$$\frac{m^*}{m_e} g(\theta) = 2n + 1$$

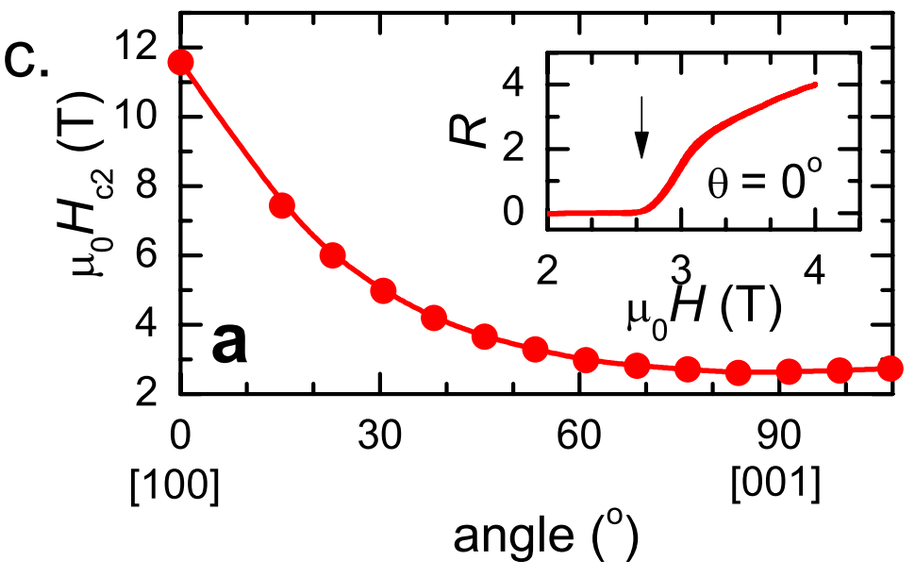
Spin Zero condition

M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011).
H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

17 spin zeros!

Ising *quasiparticle* with giant Ising anisotropy > 30.
Pauli susceptibility anisotropy > 900





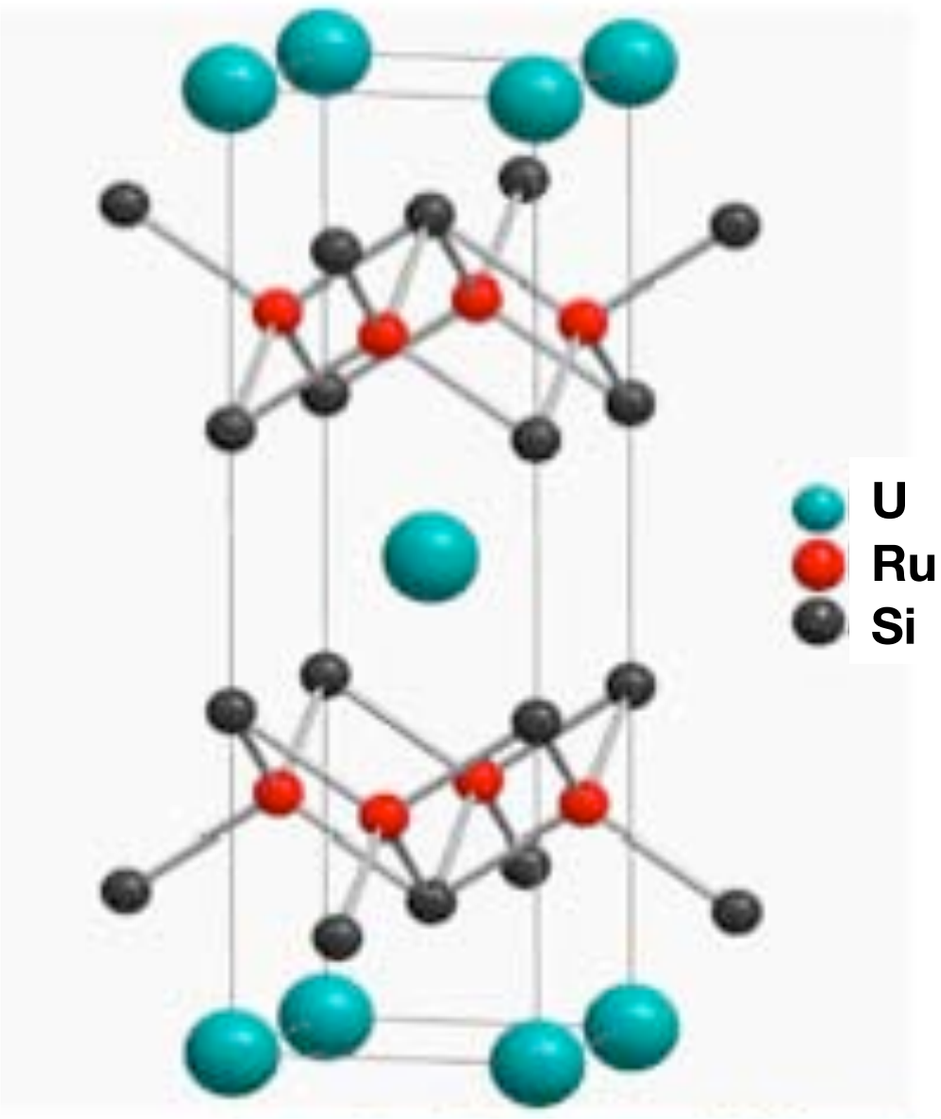
$$\mu_0 H_{\mathrm{p}} = \frac{2\Delta}{\sqrt{2}\,\mu_{\mathrm{B}} g^*_{\mathrm{eff}}}$$

Altarawneh et al., PRL 108, 066407 (2012)

Giant anisotropy observed in Quantum Osc. & H_{c2}:
Ising QP's pair condense.

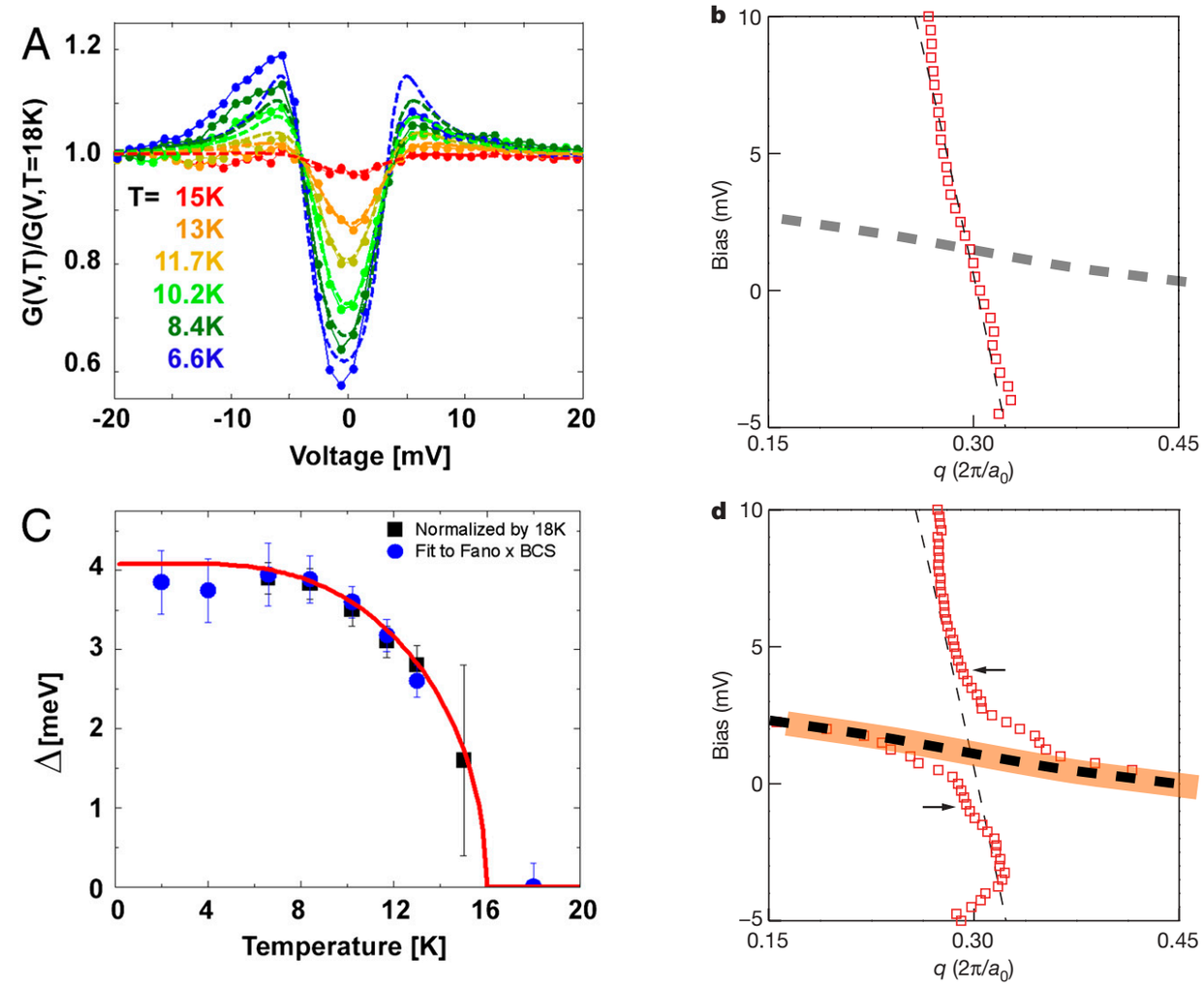
Electrons hybridize with Ising 5f state to form Heavy Ising quasiparticles.

Hybridization = the order parameter ?

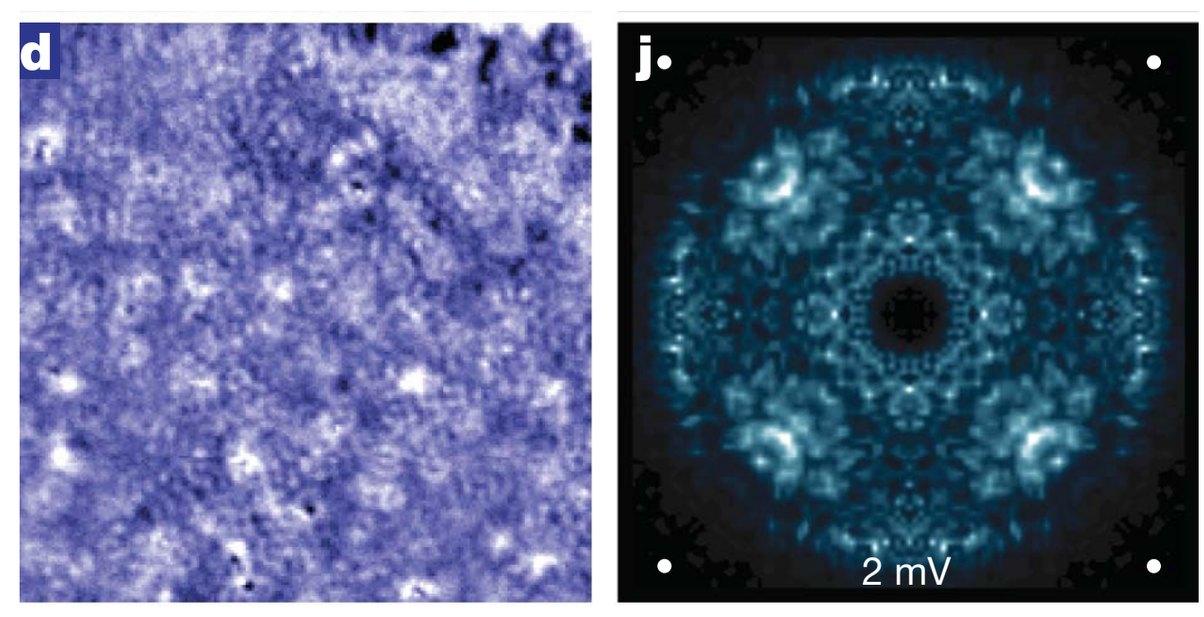


URu_2Si_2

Pegor Aynajian et al, PNAS (2010)



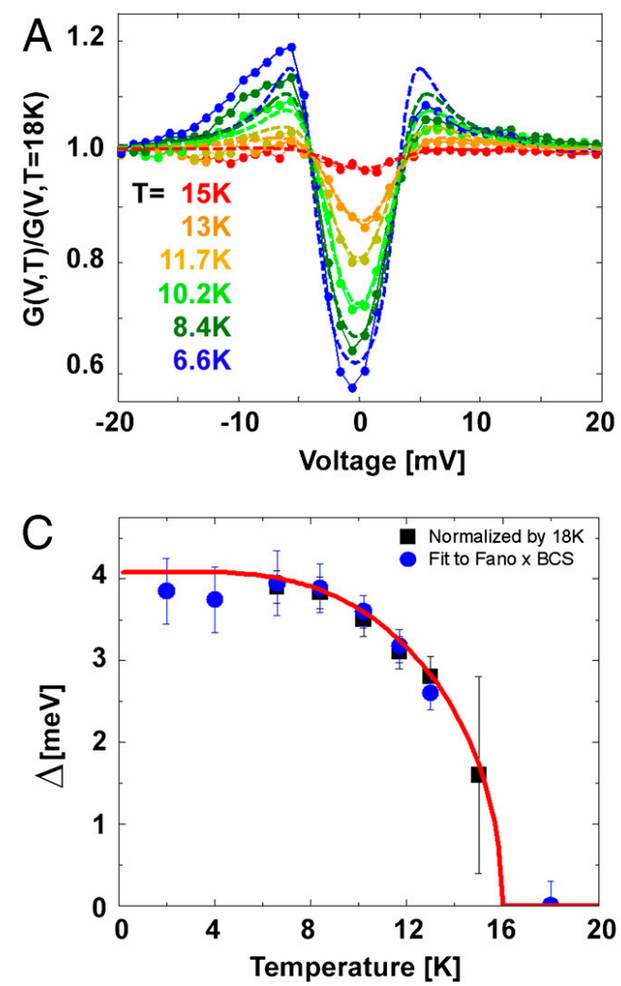
A. R. Schmidt et al., Nature (2010).



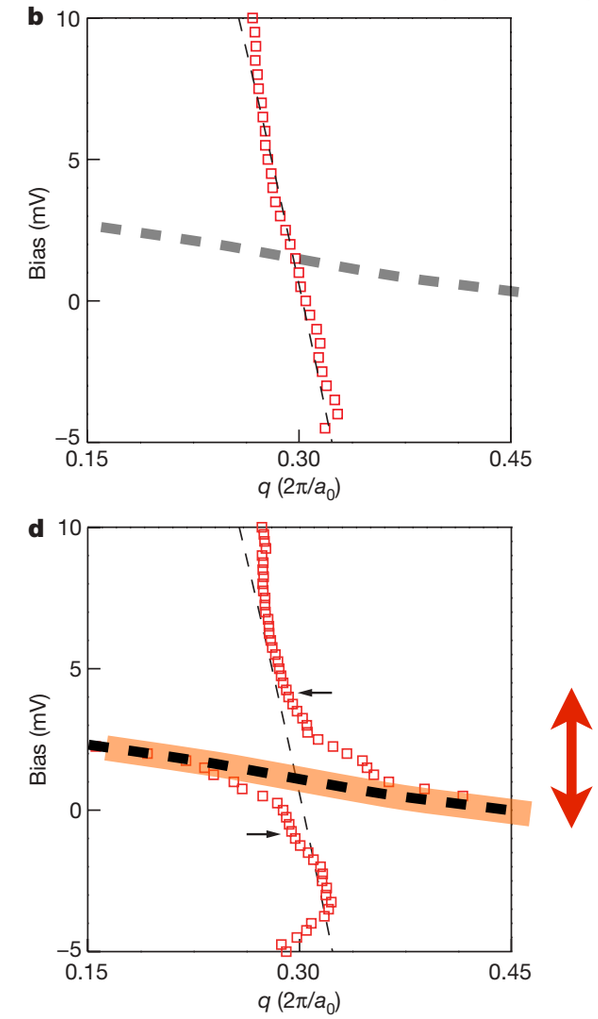
Spectroscopy: H-gap in STM/Optics

* cf Session P1, Weds 8am, room A1

Aynajian* et al, PNAS (2010)



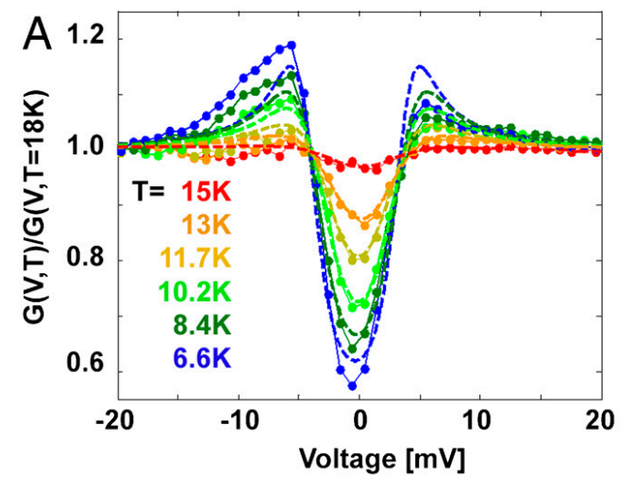
Schmidt*, Hamidian et al, Nature (2010)

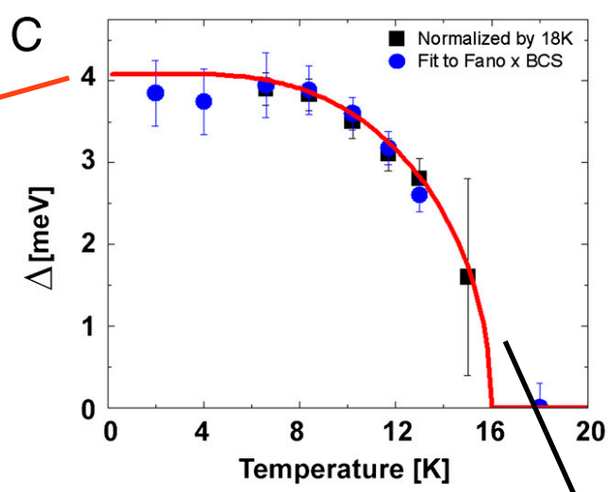


Spectroscopy
H-gap in STM/Optics

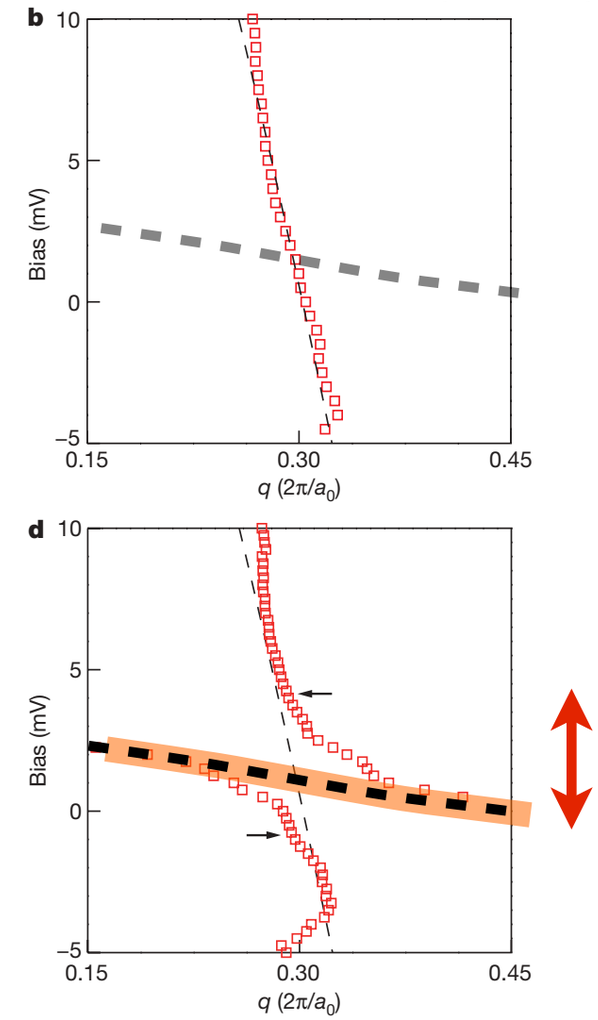
* cf Session P1, Weds 8am, room A1

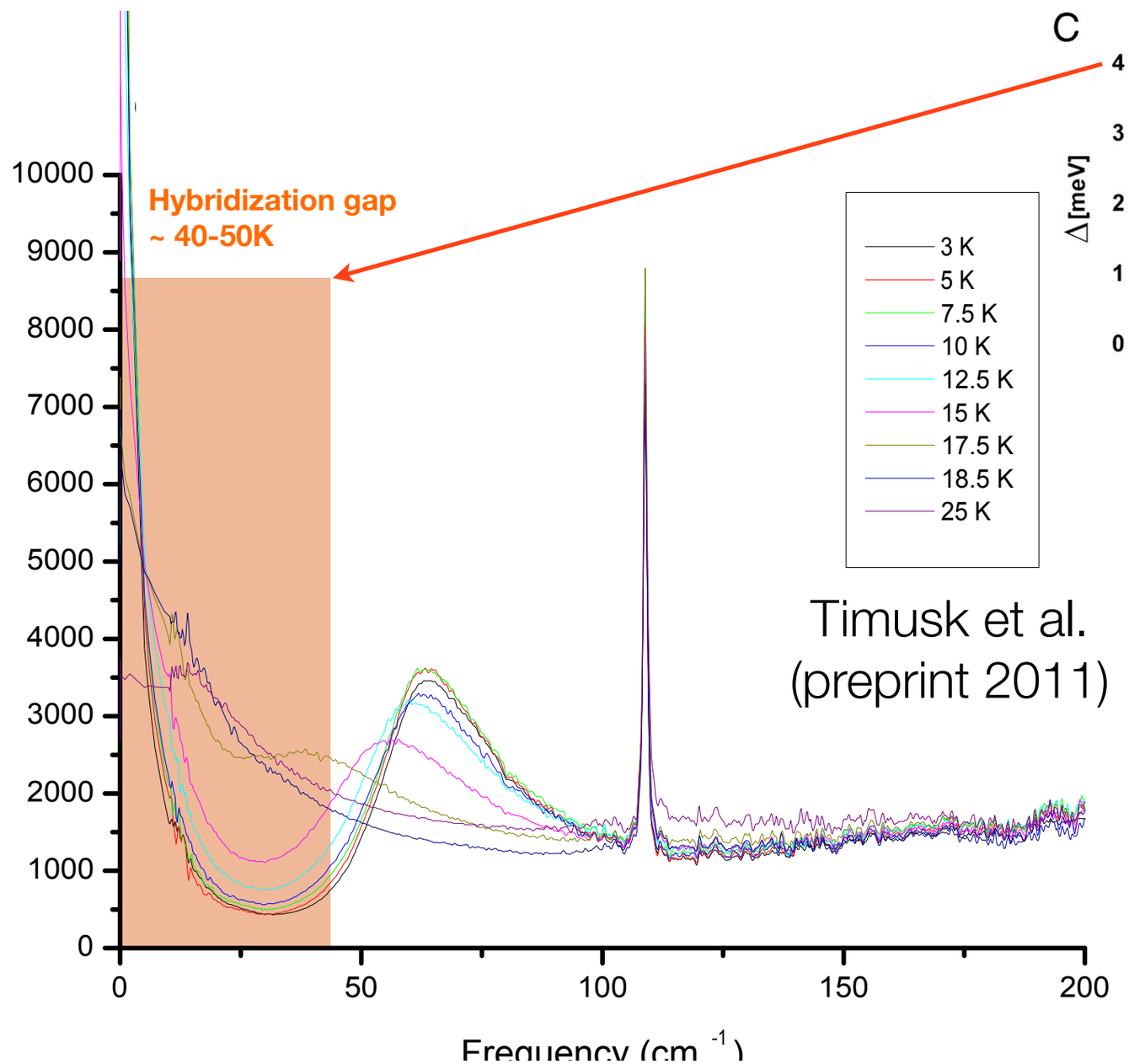
Aynajian* et al, PNAS (2010)





Schmidt*, Hamidian et al, Nature (2010)





Timusk et al. (preprint 2011)

$$\Psi_H \sim \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$$

Hybridization *is* the order parameter.

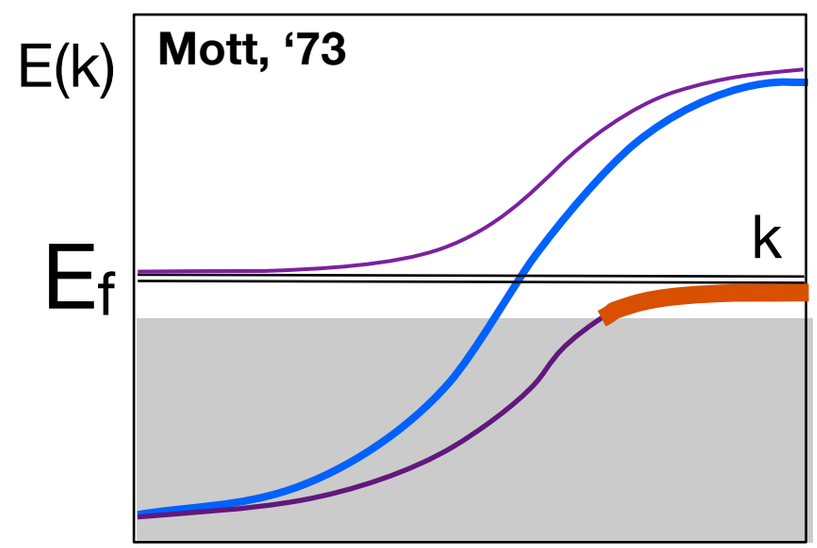
Morr et al (10), Dubi and Balatsky(10)

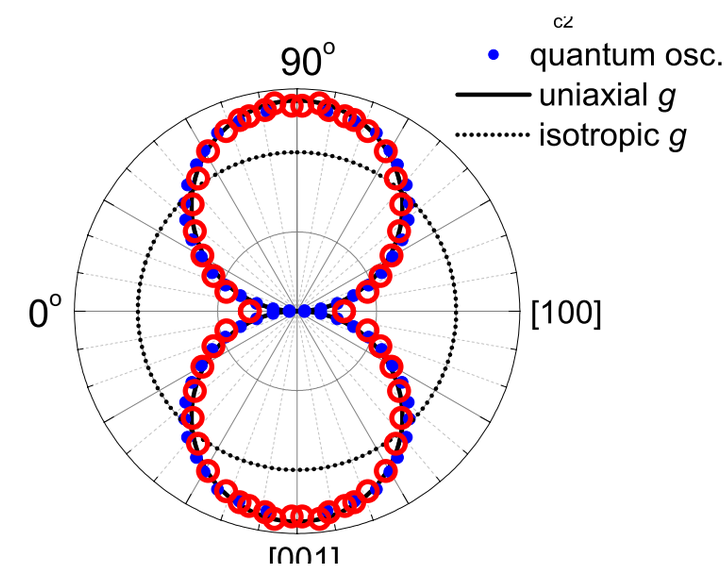
Symmetry Implications of giant Ising anisotropy

Giant Ising anisotropy indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons $|\mathbf{k}\sigma\rangle$ and **Ising 5f** doublet

$$|\alpha\rangle \equiv |\pm\rangle$$

$$\mathcal{H} = (|\mathbf{k}\sigma\rangle V_{\sigma\alpha}(\mathbf{k})\langle\alpha| + \mathrm{H.c})$$





Must vanish, =0

4 fold symmetry: crystal mixes states differing by 4n

$$\langle +|J_\pm|-\rangle = \sum_{n,n'} a_n a_{n'} \langle M-4n|J_\pm|-M+4n'\rangle = 0$$

$$|+\rangle = \sum_n a_n |M-4n\rangle$$

$$|-\rangle = \sum_n a_n |-M+4n\rangle$$

$$M - 4n \neq -M + 4n' \pm 1$$

$$\boxed{M \neq 2(n-n') \pm \tfrac{1}{2}}$$

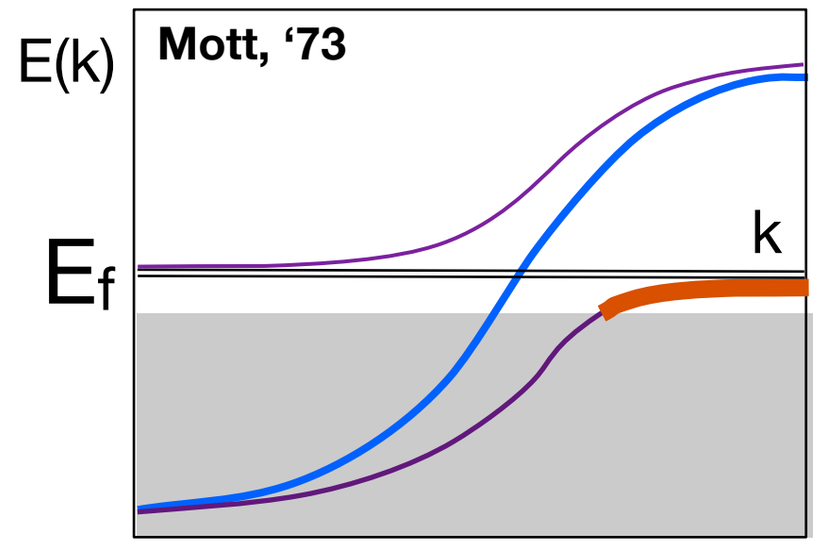
$$|\Gamma,\pm\rangle = a|\pm 3\rangle + b|\mp 1\rangle$$

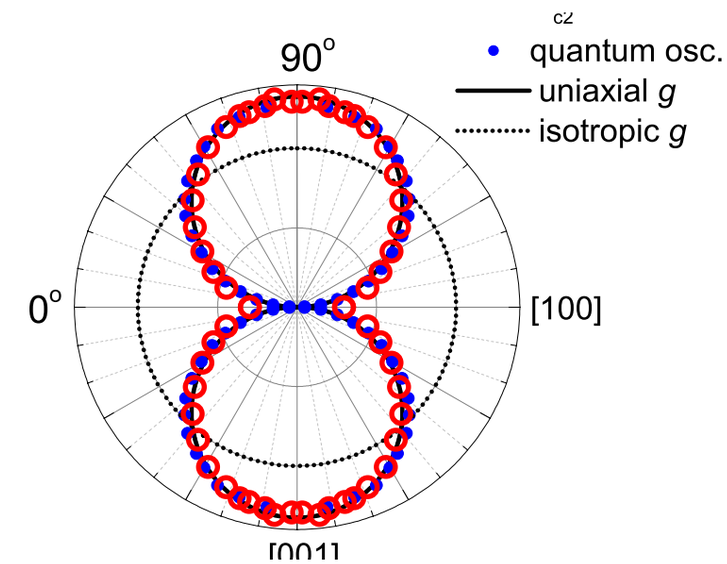
"Γ_5" non-Kramers doublet $5f^2$

$\therefore$ Integer spin M

Symmetry Implications of giant Ising anisotropy

Giant Ising anisotropy indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons $|\mathbf{k}\sigma\rangle$ and **Ising 5f** doublet





$$|\alpha\rangle \equiv |\pm\rangle$$

$$\mathcal{H} = (|\mathbf{k}\sigma\rangle V_{\sigma\alpha}(\mathbf{k})\langle\alpha| + \mathrm{H.c})$$

Integer

1/2 integer

1/2 integer

$$|\Gamma,\pm\rangle = a|\pm 3\rangle + b|\mp 1\rangle$$

"Γ_5" non-Kramers doublet $5f^2$

Hybridization is a spinor.

Hybridization IS the OP.

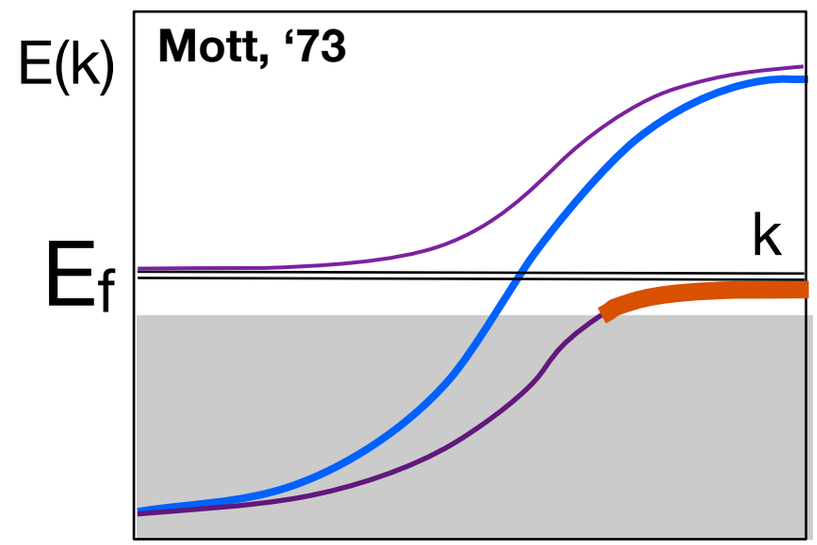
Symmetry Implications of giant Ising anisotropy

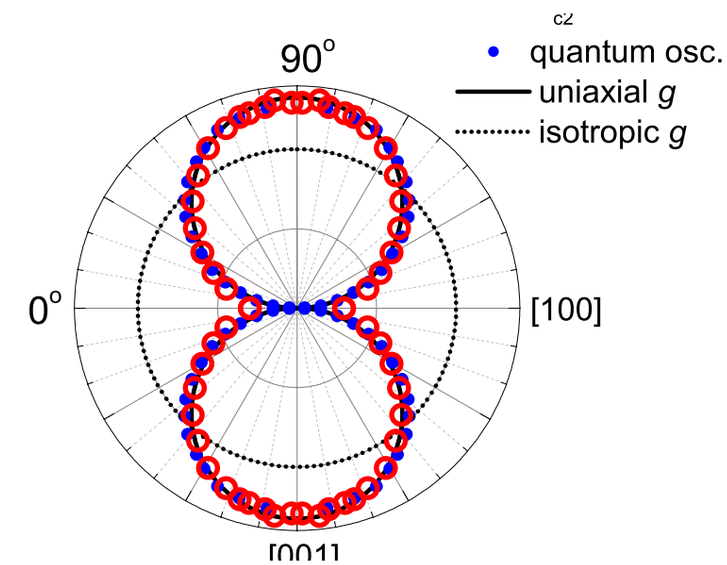
Giant Ising anisotropy indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons $|\mathbf{k}\sigma\rangle$ and **Ising 5f** doublet

$$|\alpha\rangle \equiv |\pm\rangle$$

$$\mathcal{H} = (|\mathbf{k}\sigma\rangle V_{\sigma\alpha}(\mathbf{k})\langle\alpha| + \mathrm{H.c})$$

-1 -1 +1





Kramers index K: quantum no of ***double*** time reversal θxθ = θ^2.

θ^2 is equivalent to a 2π rotation: $$\Theta^2|\psi\rangle = K|\psi\rangle = |\psi^{2\pi}\rangle$$

Half-integer spins change sign, integer spins do not.

$$K = (-1)^{2J}$$

Since the microscopic Hamiltonian must be Kramers-invariant, $$V = -V^{2\pi}$$

The hybridization transforms under θ as a 1/2 integer spin. Unlike magnetism, it breaks single **and** double time reversal symmetry.

It is thus a new kind of order parameter.

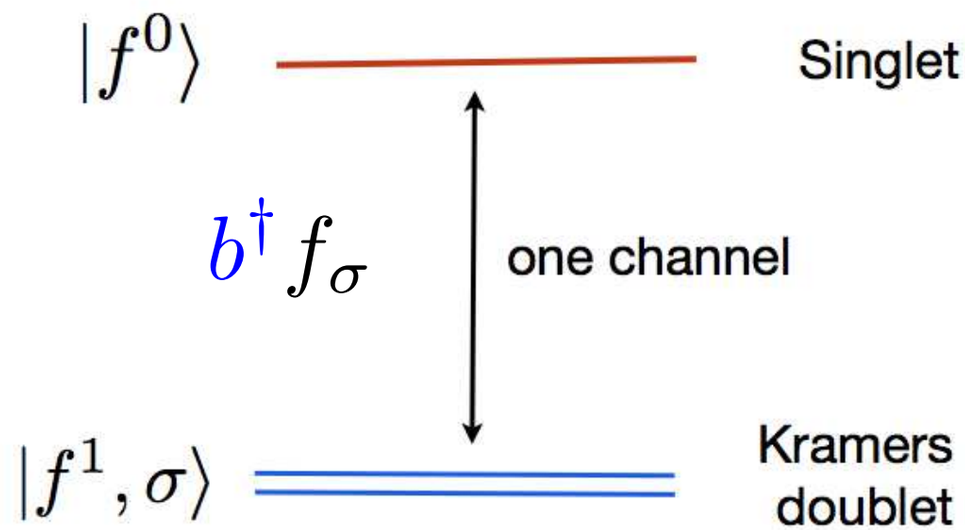
Hybridization is a spinor.

Hybridization IS the OP.

“Hastatic Order”

"Hastatic" order.

In conventional heavy fermion materials a hybridization derives from virtual excitations between a Kramers doublet and an excited singlet. A uniform hybridization breaks no symmetry and develops as a cross-over.

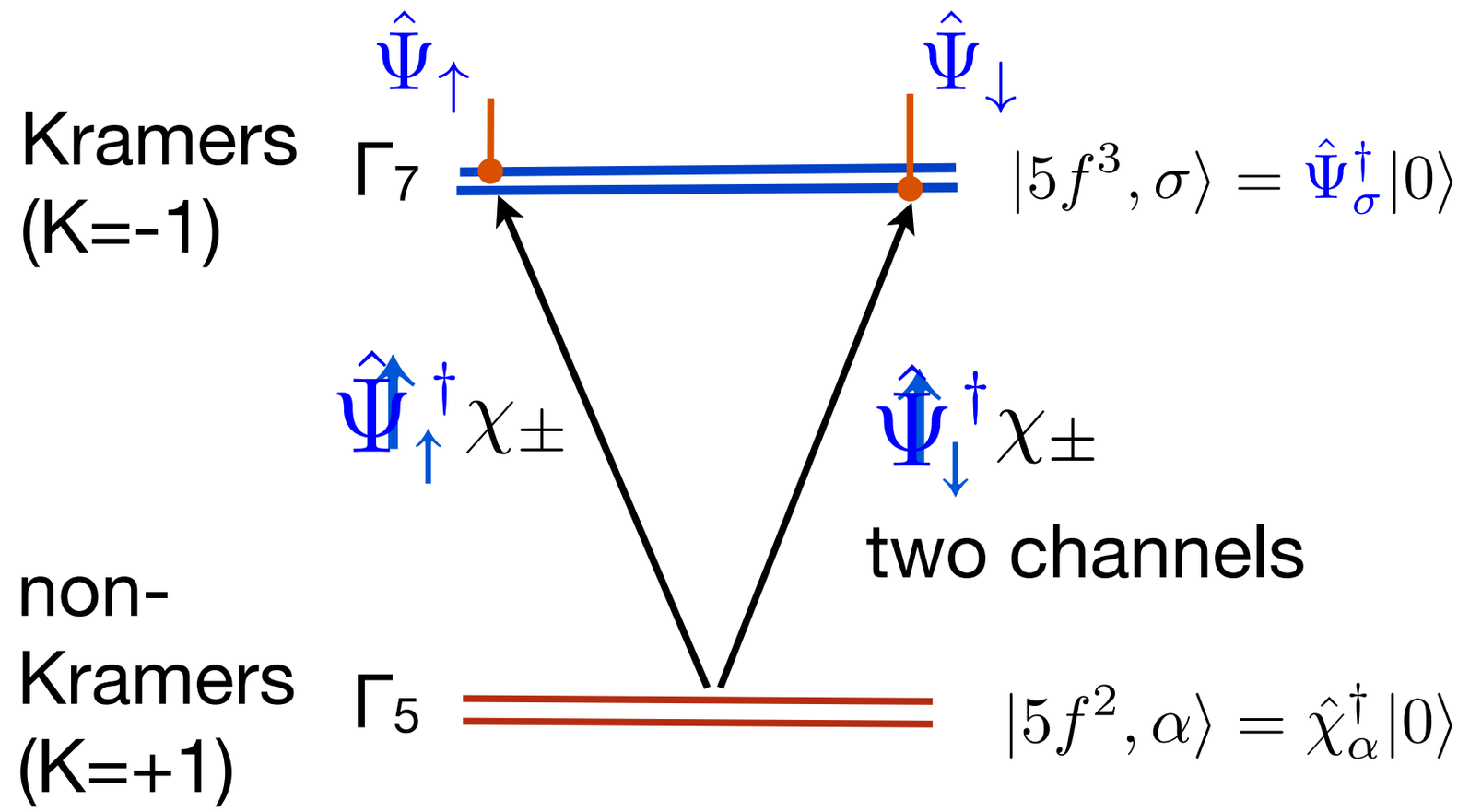


hasta: spear (latin)

"Hastatic" order.

H. Amitsuka and T. Sakakibara, *J. Phys. Soc. Japan* **63**, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.



$$|5f^3,\sigma\rangle\langle 5f^2,\alpha| = \hat{\Psi}^\dagger_\sigma\hat{\chi}_\alpha$$

hasta: spear (latin)

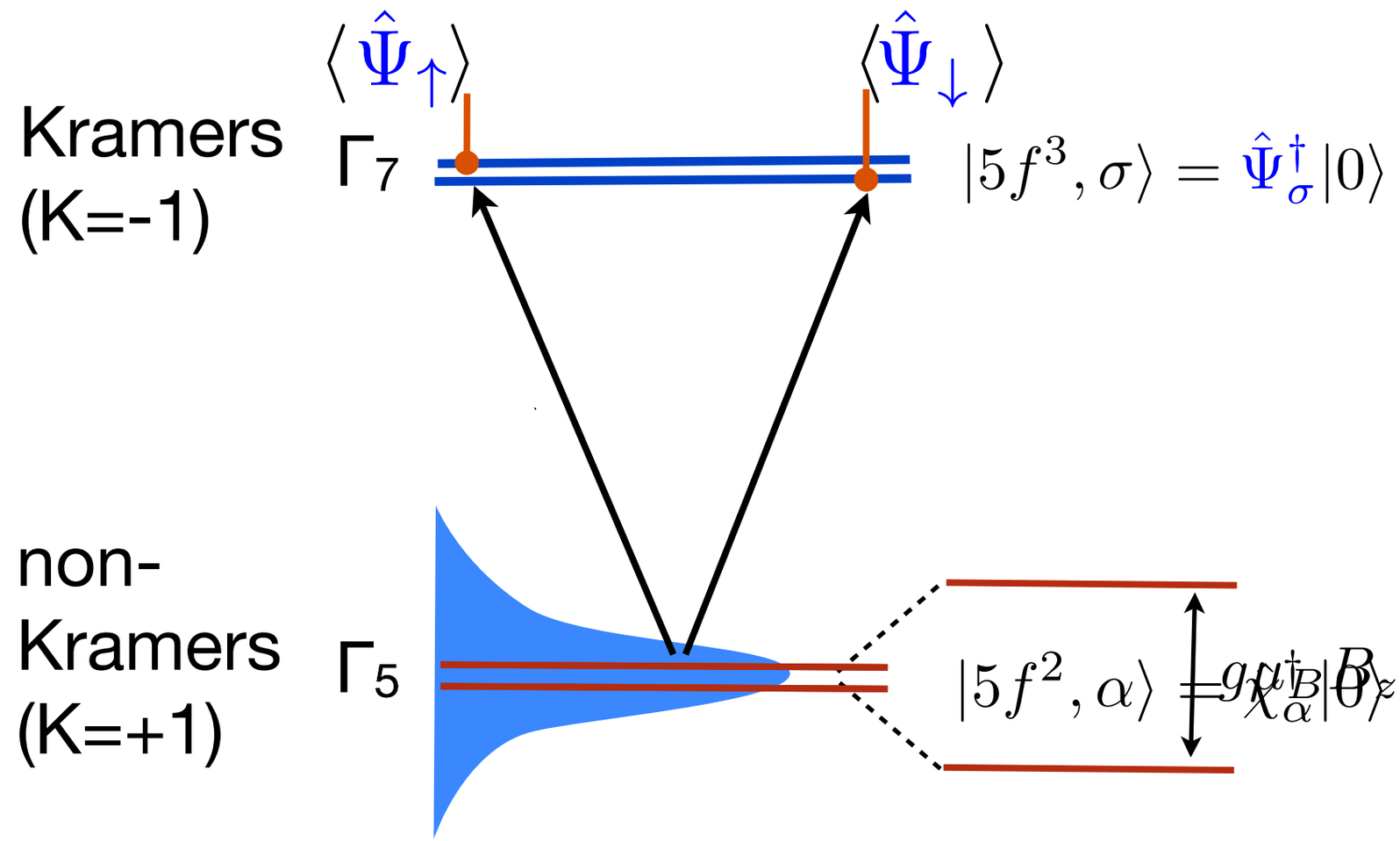
"Hastatic" order.

Quasiparticles acquire the Ising anisotropy of the non-Kramers doublet.

("Magnetic Higgs Boson")

$$\Psi = \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix}$$

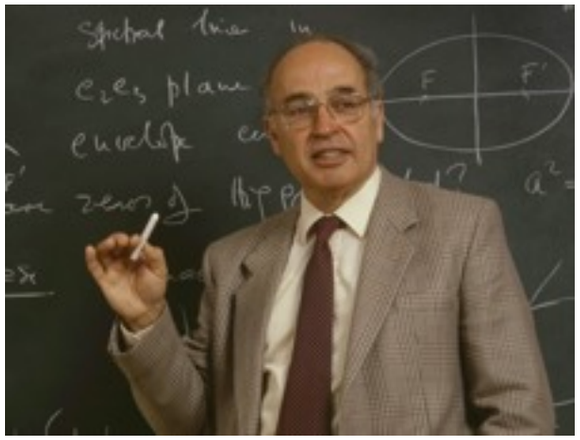
hasta: spear (latin)



$$|5f^3, \sigma\rangle\langle 5f^2, \alpha| = \hat{\Psi}^\dagger_\sigma \langle \hat{\Psi}^\dagger_\sigma \rangle \hat{\chi}_\alpha$$

What's a Spinor ??

"No one fully understands spinors.
Their algebra is formally understood but
their general significance is mysterious.
In some sense they describe the square root
of geometry and, just as understanding the
square root of -1 took centuries, the same
might be true of spinors.



M. Atiyah

Think about going for a walk
on a Mobius strip

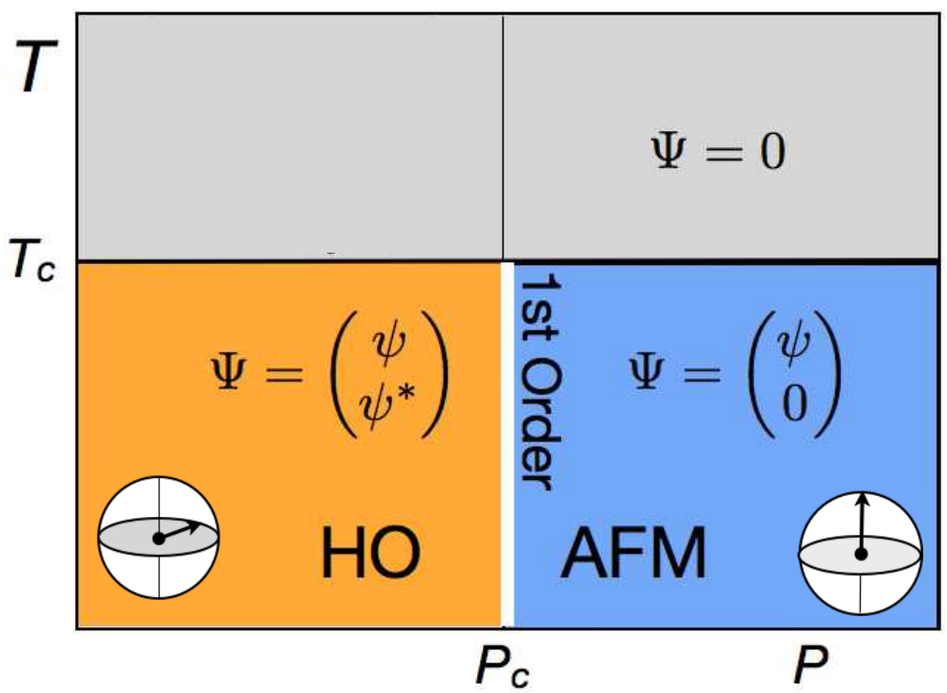
“Hastatic Order”

Landau Theory of Hastatic Order. I.

$$\Psi = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix}$$

$$f[T,P] = \alpha(T_c - T)|\Psi|^2 + \beta|\Psi|^4 - \gamma(\Psi^\dagger \sigma_z \Psi)^2$$

$$\gamma = \delta(P - P_c)$$



AFM

$$\Psi_A \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \Psi_B \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Large f-moment

HO

$$\Psi_A \sim \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}, \Psi_B \sim \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}$$

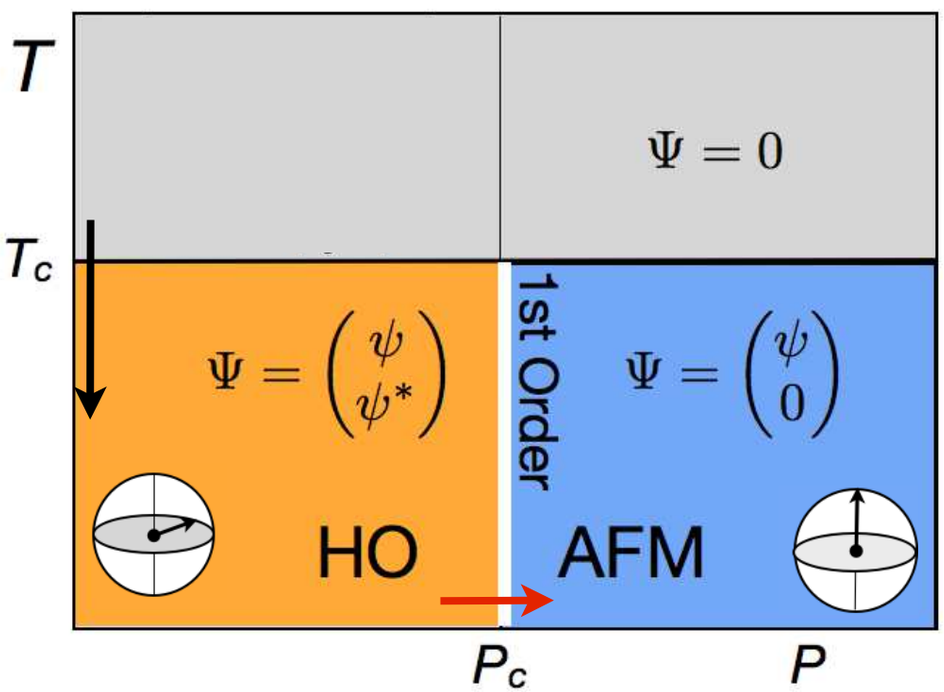
No f-moment: large Ising fluctuations

Landau Theory of Hastatic Order. I.

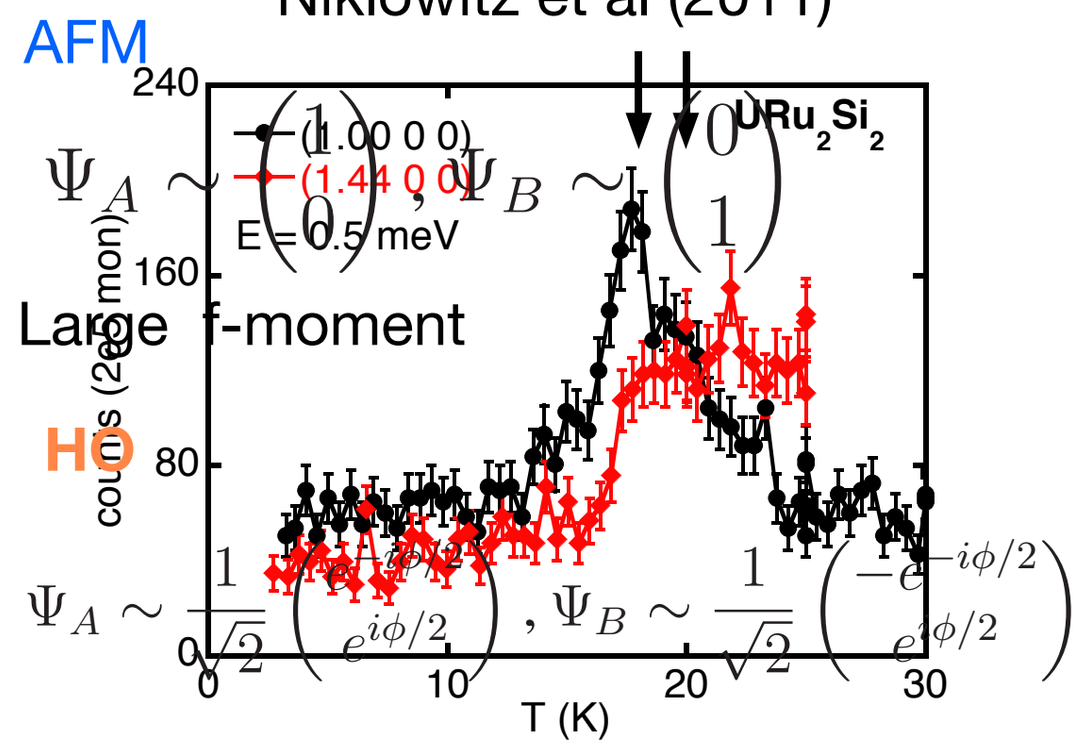
$$\Psi = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix}$$

$$f[T,P] = \alpha(T_c - T)|\Psi|^2 + \beta|\Psi|^4 - \gamma(\Psi^\dagger \sigma_z \Psi)^2$$

$$\gamma = \delta(P - P_c)$$



Niklowitz et al (2011)



No f-moment: large Ising fluctuations

Gap to Longitudinal Spin Fluctuations.

$$\Delta \propto \sqrt{\delta|\Psi|^2} \sim |\Psi|\sqrt{P_c(T) - P}$$

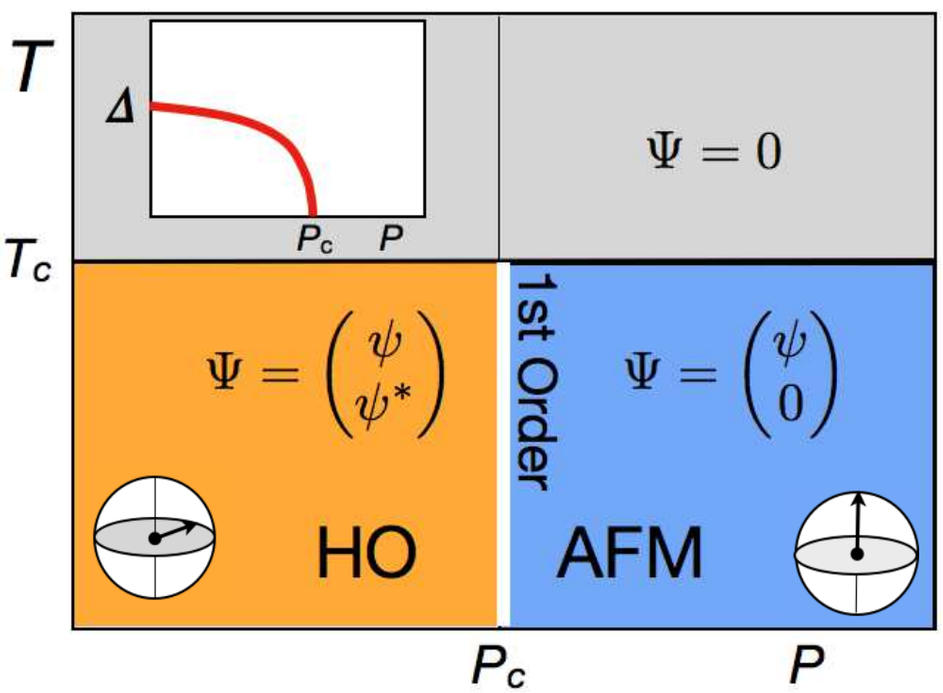
Landau Theory of Hastatic Order. II. Field dependence.

Hastatic order only couples to B_z

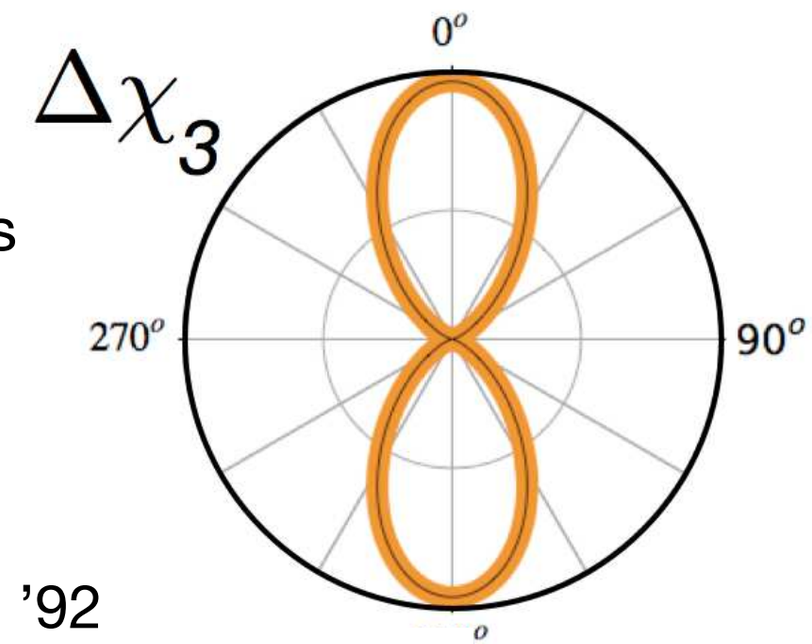
$$\Psi = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix}$$

$$f[T,P,B_z] = \alpha(T_c - T)|\Psi|^2 - \eta_z B_z^2 |\Psi|^2 + \beta|\Psi|^4 + \beta|\Psi|^4 (\Psi^\dagger \sigma_z \Psi)^2$$

$$\gamma = \delta(P - P_c)$$



Anisotropy in excess of 1000 predicted.

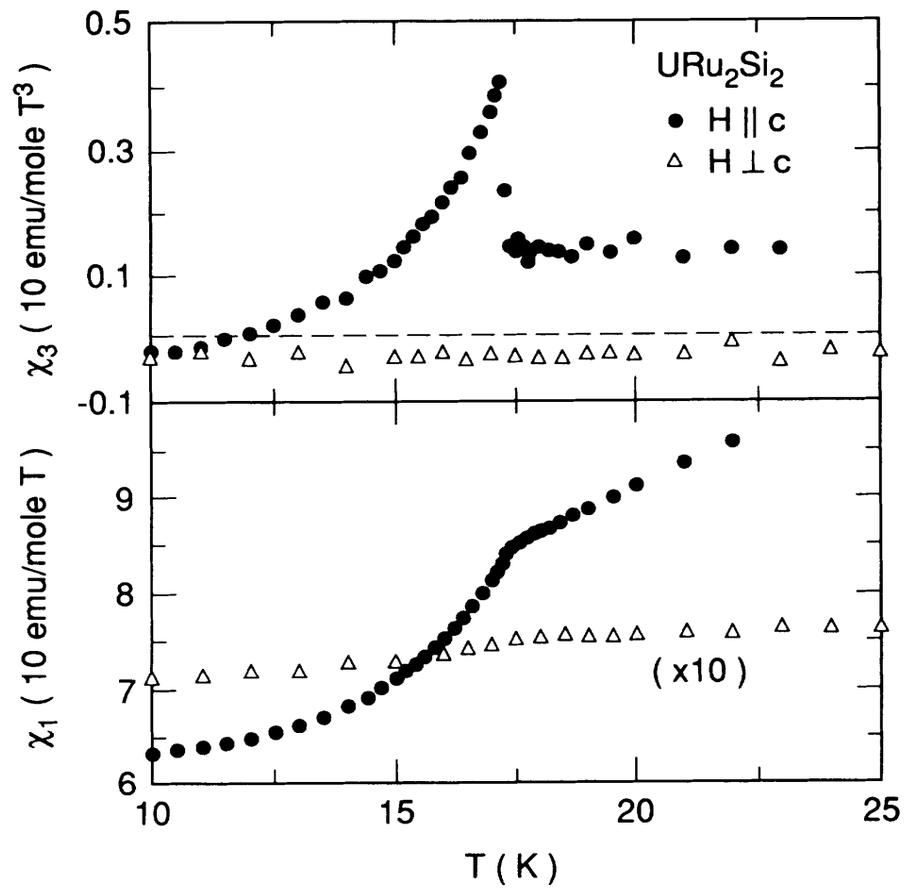


$$f = -\frac{1}{4\beta}\left[\alpha(T_c - T) - \eta_z (B\cos\theta)^2\right]^2$$

$$M = \chi_1 B + \frac{\chi_3}{3!} B^3$$

$$\Delta\chi_3 = -\frac{d^4 f}{dB^4} = \frac{6}{\beta}(\eta_z \cos\theta)^4$$

Ramirez et al '92



Microscopic theory of Hastatic Order

Schwinger/Slave boson

$$|\Gamma_7^+\sigma\rangle\langle\Gamma_5\alpha| = \hat{\Psi}^\dagger_\sigma \chi_\alpha$$

$$|\Gamma_7^+, \sigma\rangle \equiv \Psi^\dagger_\sigma |0\rangle$$

E_b

$$|\pm\rangle \equiv \chi^\dagger_\pm |0\rangle$$

$$5f^2 \rightleftharpoons 5f^1 + e^- \qquad \psi^\dagger_{\Gamma\sigma}(j) = \sum_{\mathbf{k}} \left[\Phi^\dagger_\Gamma(\mathbf{k})\right]_{\sigma\tau} c^\dagger_{\mathbf{k}\tau} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{R}_j}$$

$$H_{VF}(j) = \mathcal{V}_6 \psi^\dagger_{\Gamma_6^\pm}(j)\Psi^\dagger_{j\pm}\langle\Gamma_5^\pm| + \mathcal{V}_7 \psi^\dagger_{\Gamma_7^\mp}(j)\Psi^\dagger_{j\mp}\langle\Gamma_5^\pm| + \mathrm{H.c.}$$

$$\langle\Psi^\dagger_j\rangle = |\Psi| \begin{pmatrix} \mathrm{e}^{i(\mathbf{Q}\cdot\mathbf{R}_j+\phi)/2} \\ \mathrm{e}^{-i(\mathbf{Q}\cdot\mathbf{R}_j+\phi)/2} \end{pmatrix}, \qquad (\phi = \pi/4).$$

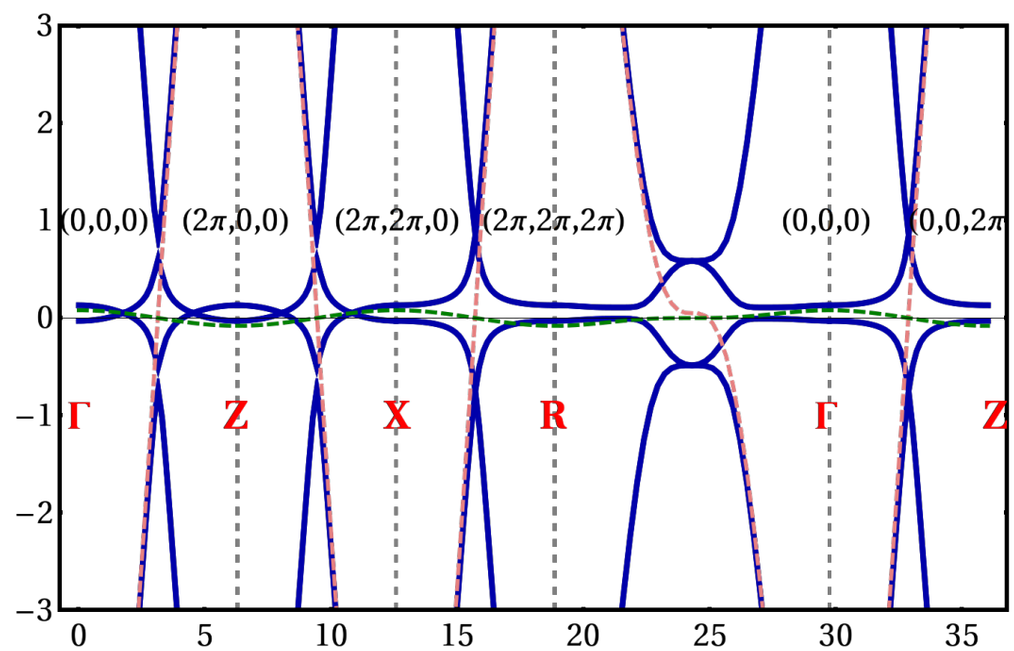
$$H_{VF} = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \mathcal{V}_6(\mathbf{k}) \chi_{\mathbf{k}} + c^\dagger_{\mathbf{k}} \mathcal{V}_7(\mathbf{k}) \chi_{\mathbf{k}+\mathbf{Q}} + \mathrm{h.c.}$$

$$H = \sum_{\mathbf{k}} \left(c^\dagger_{\mathbf{k}}, c^\dagger_{\mathbf{k+Q}}, \chi^\dagger_{\mathbf{k}}, \chi^\dagger_{\mathbf{k+Q}} \right) \overbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & 0 & \mathcal{V}_6(\mathbf{k}) & \mathcal{V}_7(\mathbf{k}) \\ 0 & \epsilon_{\mathbf{k+Q}} & -\mathcal{V}_7(\mathbf{k}) & -\mathcal{V}_6(\mathbf{k}) \\ \mathcal{V}_6^\dagger(\mathbf{k}) & -\mathcal{V}_7^\dagger(\mathbf{k}) & \lambda_{\mathbf{k}} & 0 \\ \mathcal{V}_7^\dagger(\mathbf{k}) & -\mathcal{V}_6^\dagger(\mathbf{k}) & 0 & \lambda_{\mathbf{k+Q}} \end{pmatrix}}^{\mathcal{H}_{\alpha\beta}(\mathbf{k})} \begin{pmatrix} c_{\mathbf{k}} \\ c_{\mathbf{k+Q}} \\ \chi_{\mathbf{k}} \\ \chi_{\mathbf{k+Q}} \end{pmatrix}$$

$$5f^2 \rightleftharpoons 5$$

$$H_{VF}(j) =$$

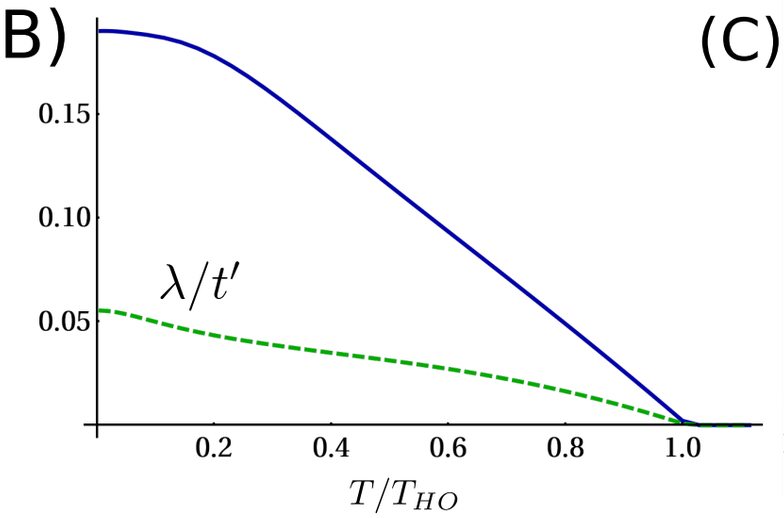
(A)



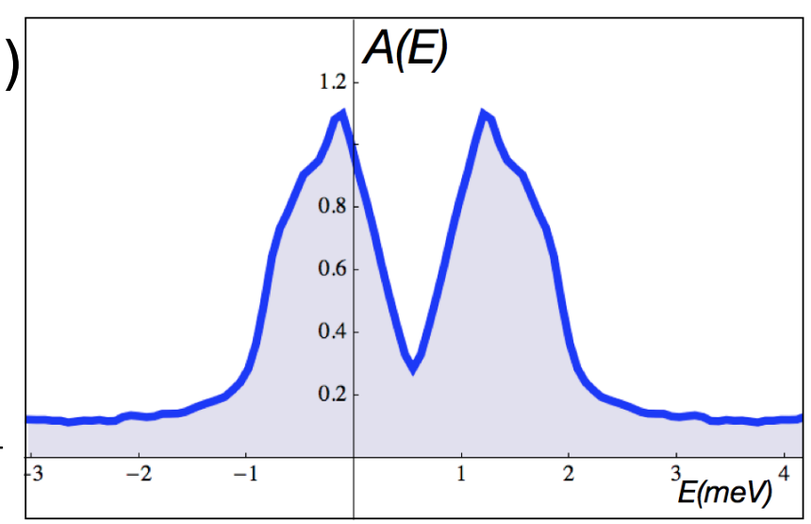
Ψ

$\langle \Psi_j^\dagger \rangle$

(B)

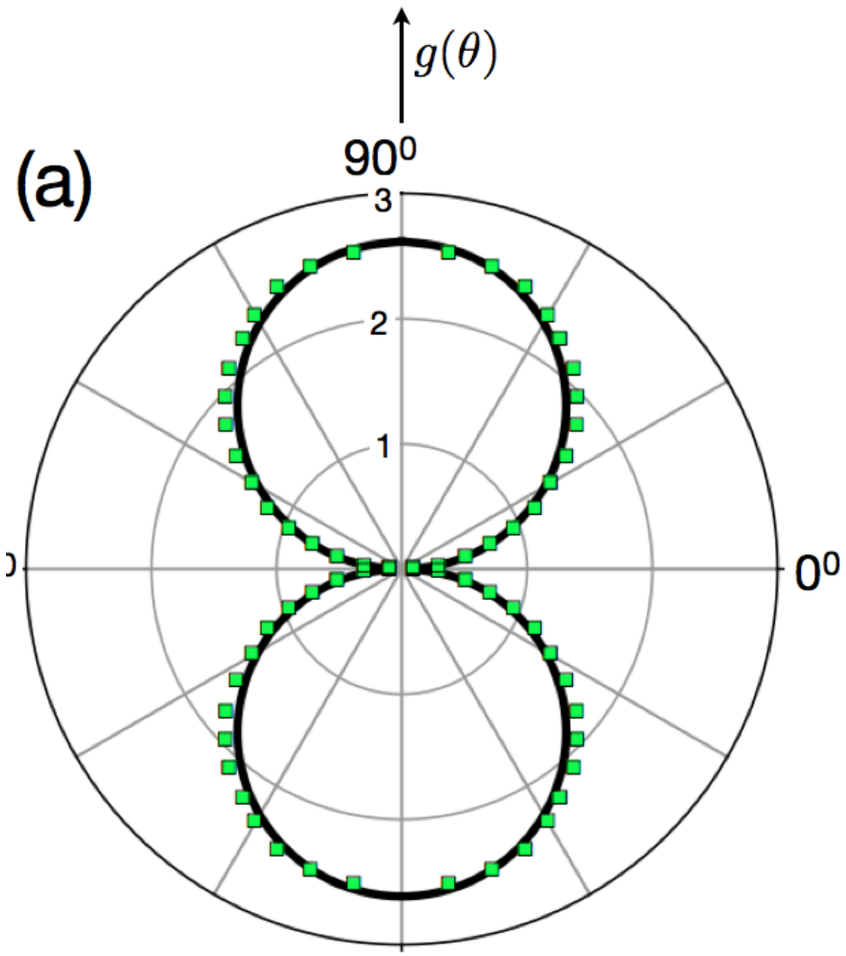


(C)

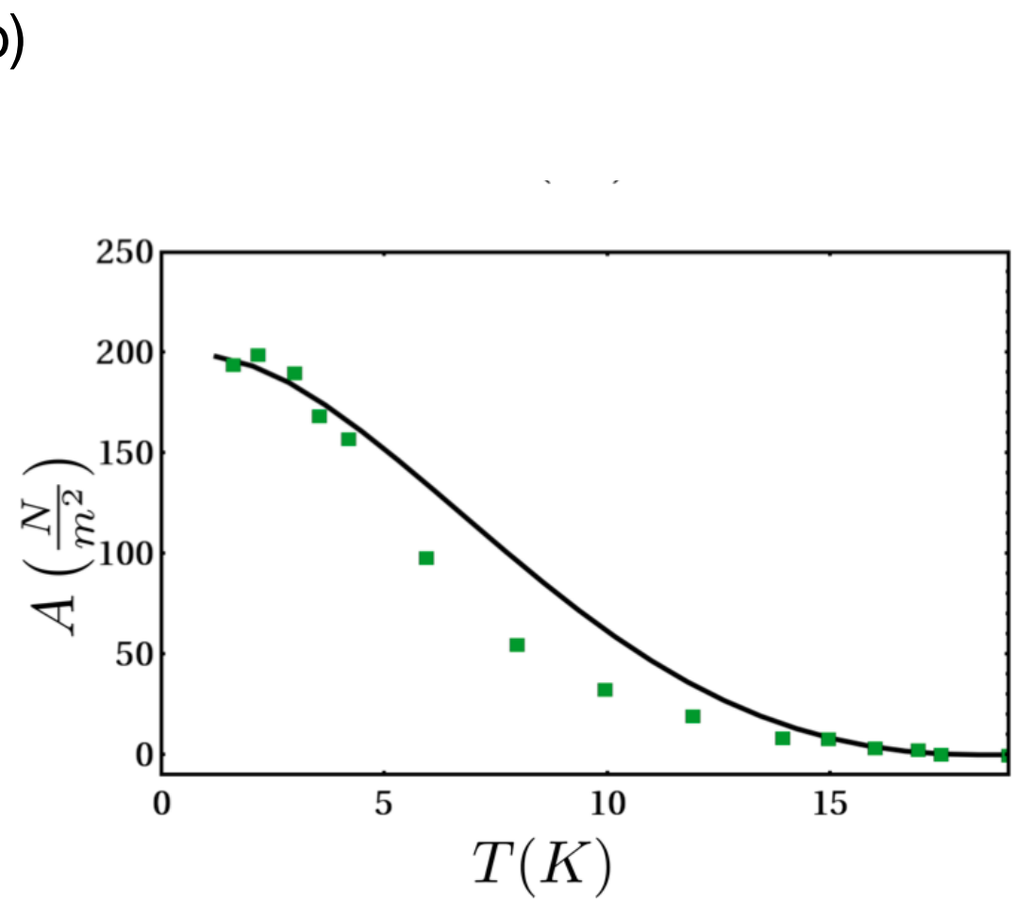


$$H_{VF} = \sum_{\mathbf{k}}$$

Consistency with experiment.



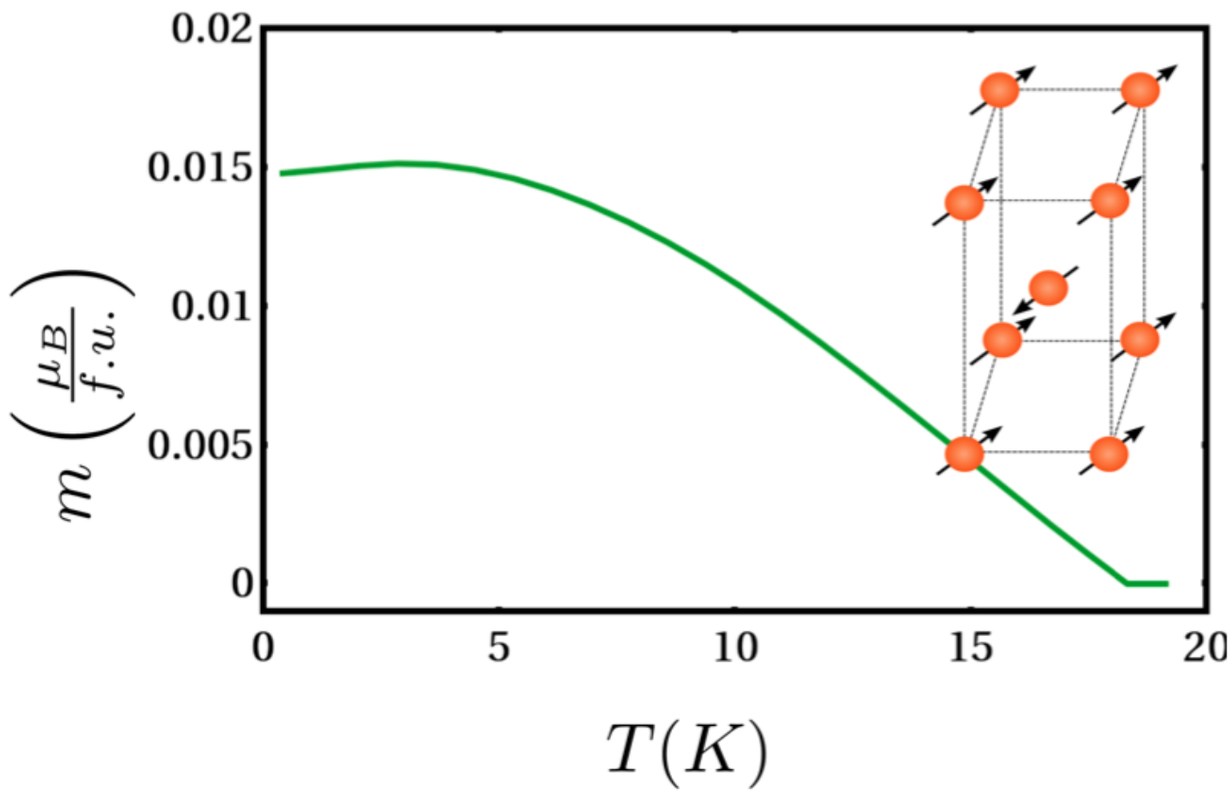
Giant Ising Anisotropy



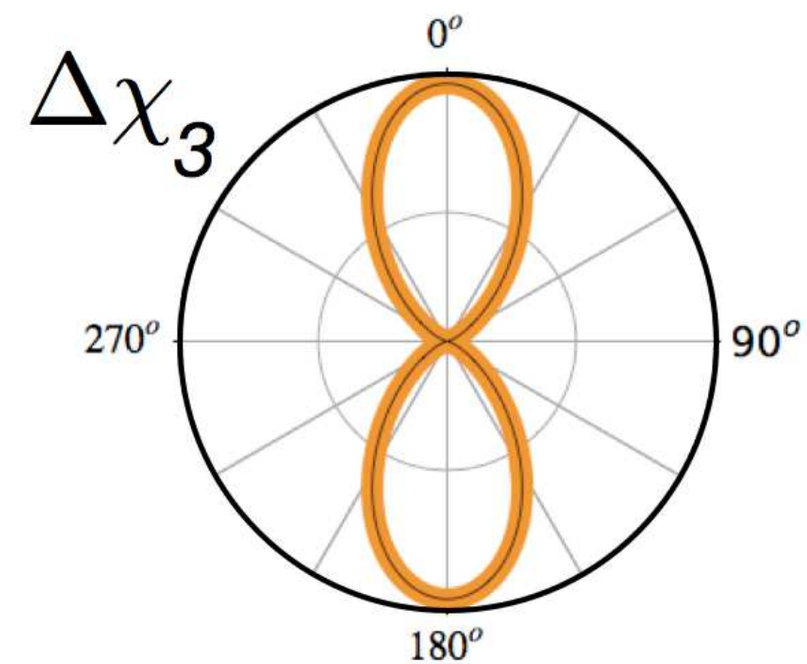
Nematicity: anisotropic X_{xy} consistent with observed susceptibility anomaly.

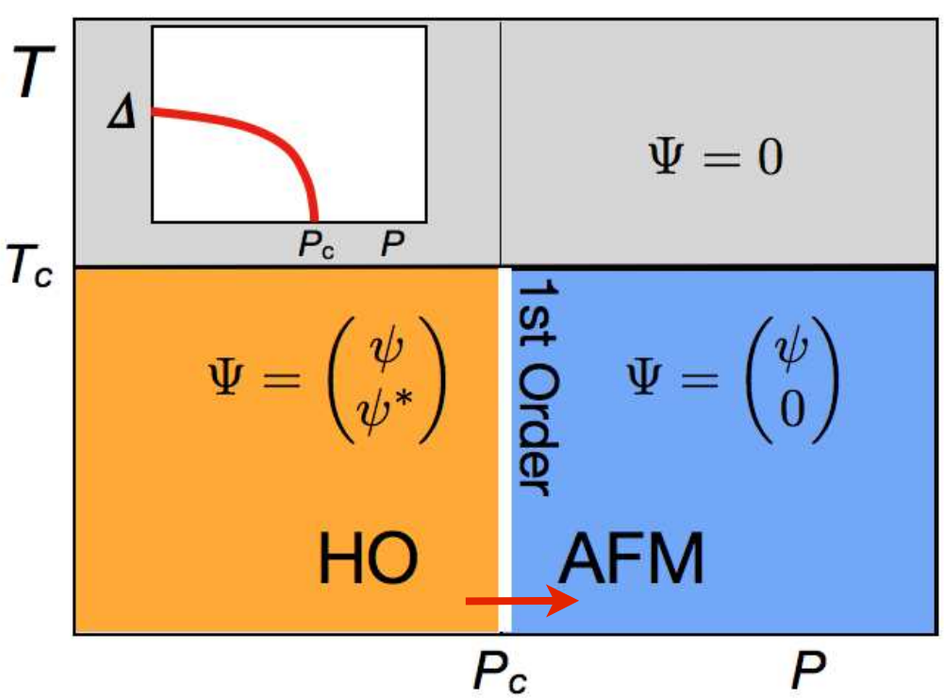
Predictions.

(a) Transverse moment in conduction sea m~ O(TK/D)~ 0.01μ_B.



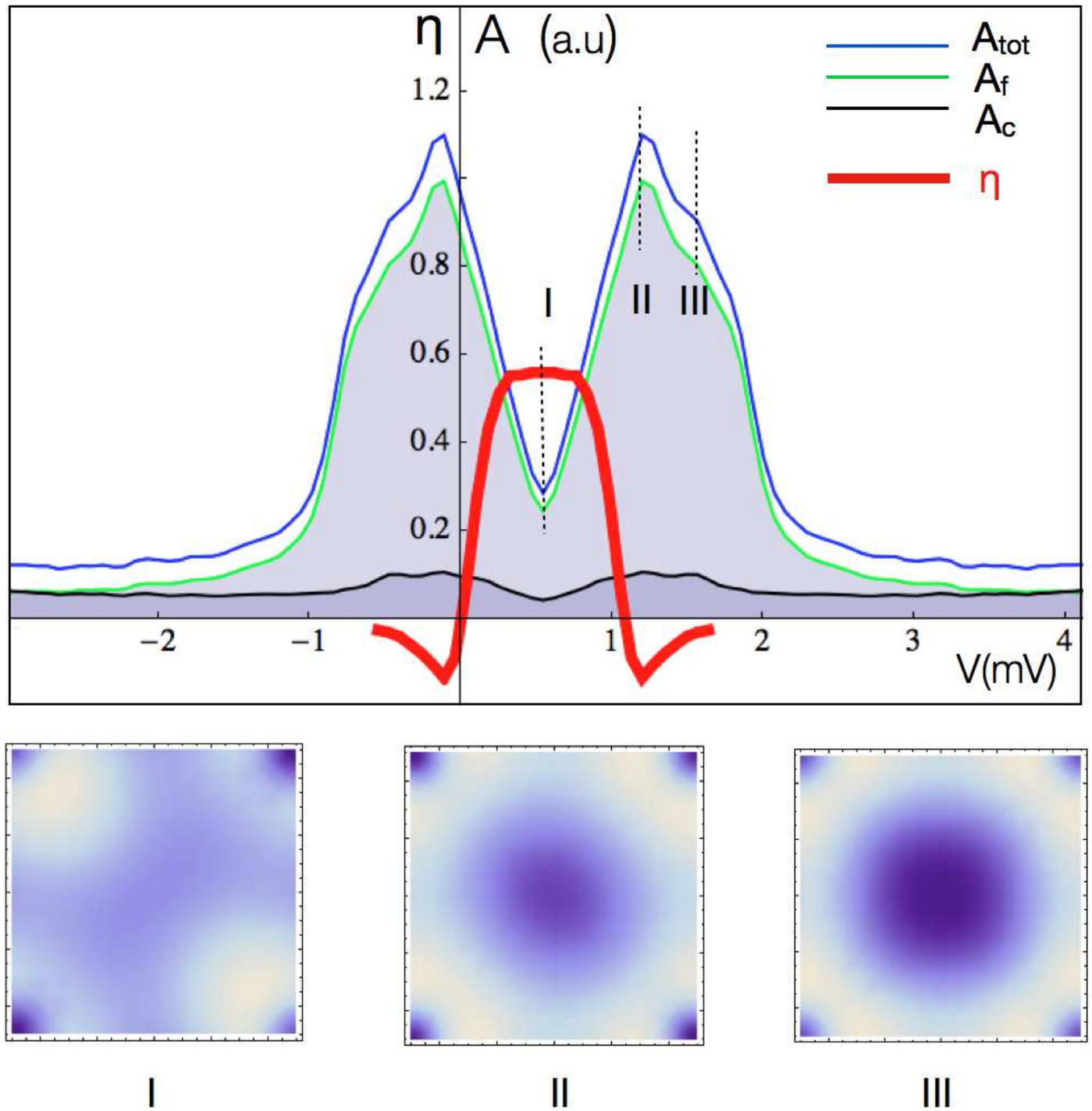
(b) Giant non-linear susceptibility anomaly.





(c) Collapse of gap to Ising fluctuations at 1st order transition line.

(d) Resonant Nematicity in STM



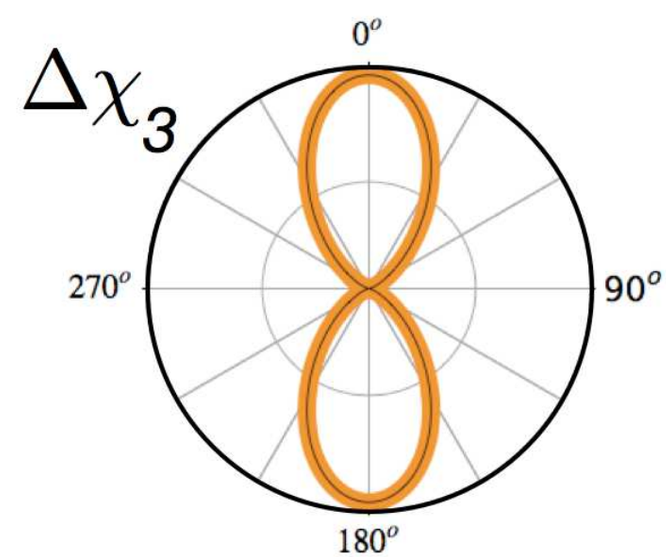
Conclusions about Hidden Order in URu2Si2.

•Giant quasiparticle Ising anisotropy indicates the development of a fixed point Hamiltonian with hybridization between half integer electrons and integer spin $5f^2$ U doublet.

•Hybridization mixes half-integer and integer spin states: must have half-integer character - a spinor.

•Unlike magnetism, hybridization breaks single and double time-reversal symmetry: a new kind of “Hastatic Order”.

•Broader implication: a new kind of broken symmetry where OP transforms under DOUBLE GROUP (S = 1/2) representation



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Another Definition of "Hastatic"

urban DICTIONARY

1. hastatic **7** up, **5** down

Happy. A word invented by Deena Nicole Cortese (Jersey Shore)

Hastatic is when you're like super happy and, like... really happy.