Three Lectures on Heavy Fermion Physics

Lecture III. SOME NEW DEVELOPMENTS

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Last time - Lecture II: Kondo Lattice.

Mott/Doniach hypothesis.

Strong Coupling Limit

Large N Approach

\[ \frac{J}{N} c_{j\alpha} s_{\alpha\beta} \equiv \vec{V}_j f_{j\beta} \]

Composite Fermion

Read & Newns 1983.
Reconstruction of the Fermi Surface and mass divergence

Three Lectures on HF Lecture III:

1. Topological Kondo Insulators

2. Composite pairing.

3. Hidden Order in URu2Si2
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Collaborators

Maxim Dzero            Kent State
Kai Sun                     Michigan
Victor Galitski          Maryland


arXiv.org > cond-mat > arXiv:1211.5104
Condensed Matter > Strongly Correlated Electrons

Discovery of the First Topological Kondo Insulator: Samarium Hexaboride
Steven Wolgast, Cagliyan Kurdak, Kai Sun, J. W. Allen, Dae-Jeong Kim, Zachary Fisk
(Submitted on 21 Nov 2012 (v1), last revised 27 Nov 2012 (this version, v2))
Topological insulators

Hasan and Kane (RMP 2009)
Qi and Zhang (RMP 2010)
Conventional band insulator: adiabatic continuation of the vacuum.
Topological insulator: adiabatically disconnected to vacuum.

Gap must close at interface between two different vacua.
Kondo insulators: theory

- Anderson model: Renormalized

\[
\hat{H} = \sum_{k, \alpha} \xi_k c_{k \alpha}^\dagger c_{k \alpha} + \sum_{k, j, \alpha} \left[ W^* c_{j \alpha}^\dagger \Phi_{\alpha} \Phi_{\alpha}^\dagger (k) c_{k \alpha} \right] + \sum_{j, \alpha} \left[ E_f (0) \sum_{k, \alpha} \left[ \Phi_{\alpha} \Phi_{\alpha}^\dagger (k) \right] n_{f, j} + U n_{f, j} \right]
\]

\[
\psi_{j \alpha} = \frac{1}{\sqrt{V}} \sum_{k} \Phi_{\alpha \sigma} (\hat{k}) e^{-i k \cdot x_j} c_{k \sigma}
\]

- Hybridization

Strong spin-orbit coupling is encoded in the hybridization.
Kondo insulators: theory

- Anderson model: Renormalized

\[ \hat{H} = \sum_{k, \sigma} \xi_k c_{k \sigma}^{\dagger} c_{k \sigma} + \sum_{k \alpha} \left[ V^* c_{k \sigma}^{\dagger} \Phi_{\sigma \alpha}(k) f_{k \alpha} + \text{h.c} \right] + \sum_{k \alpha} \left[ E_{f}^{(0)} f_{k \alpha}^{\dagger} f_{k \alpha} \right] \]

\[ \psi_{j \alpha} = \frac{1}{\sqrt{V}} \sum_{k} \Phi_{\alpha \sigma}(\hat{k}) e^{-i k \cdot x_j} c_{k \sigma} \]

Form factors:

\[ [\Phi_{\Gamma k}]_{\alpha \sigma} = \sum_{m} \langle \Gamma \alpha | jm \rangle \langle jm | k \sigma \rangle \]

Matrix element between Bloch and Wannier states

\[ \Phi_{\alpha \sigma}(k) = -1 \times \Phi_{\alpha \sigma}(-k) \]

Strong spin-orbit coupling is encoded in the hybridization.

ODD PARITY!
Topological insulators

Response to a fictitious applied magnetic field

- 2D: Flux plays the role of the edge crystal momentum $k_x$
- 3D: two fluxes corresponding to two components of the surface crystal momentum

$Z_2$ invariants are computed from the parity of the occupied bands: change in time reversal polarization due to changes in bulk Hamiltonian

$$Z_2 = \prod_{i} \delta(\Gamma_i)$$
\[ Z_2 = \prod_i \delta(\Gamma_i) \]

\[ V_{\alpha\sigma}(k_m) = V_{\alpha\sigma}(k_m + G) = -V_{\alpha\sigma}(-k_m) = -V_{\alpha\sigma}(k_m) = 0 \]

Vanishes at high symmetry points

\[ H_{mf}(k_m) = \frac{1}{2}(\xi_{k_m} + \varepsilon_f)1 + \frac{1}{2}(\xi_{k_m} - \varepsilon_f)P \]

\[ Z_2 \text{ invariants are characterized by the parity eigenvalues:} \]

\[ \delta(\Gamma_m) = \text{sgn}(E_f^* - \xi_{k_m}) \]
1D Kondo Insulator

\[ d^2 \rightarrow f^1 d^1 \]

\[ \nu = +1 \quad \nu = -1 \]

Alexandrov, Dzero and PC (2013)
3D Cubic Kondo Insulator

$E_k$

QUARTET: topologically inert

THREE DIRAC CONES ON SURFACE.

$\nu = +1 \quad \nu = -1$

$d^0 f^6 \rightarrow d^1 f^5$

Alexandrov, Dzero and PC (2013)
independent contributions: bulk and surface resistivities can be suppressed or exaggerated depending on the position of the Arrhenius $\rho$-plot. Measured (solid grey line) vs inverse temperature. A linear model of $\rho$ is understood as a surface conductivity that buries the bulk conductivity at low temperatures. It plateaus at low temperatures. The activation energy is $3.47 \text{ meV}$.

In the conventional scenario, only a small fraction of the current path around the edges; thus, in this case, the voltage contacts are very close to the bulk, far away from the voltage contacts. At low temperatures, the current will continue to flow in this configuration by the thickness of the sample along the electrical contacts. Arrows indicate current through the sample vertically directly if the bulk is conductive, as shown in Fig. 3a, 3b, 3c, 3d. These configurations are illustrated in vertical measurement side and back side voltages are measured at similar potentials. The bulk in this case where the saturation conductivity is a surface conductor.

For this reason, such a configuration is accidental. The bulk in this scenario, thermal activated conductivity should be nearly identical to the bulk, far away from the voltage contacts. If the plateau is a bulk transport phenomenon, the resistance will be proportional to the temperature. If the current contacts, compared to the total resistance as a function of temperature. We performed Finite Element Analysis simulations dying the divergence between theoretical values and our sample's resistances as a function of temperature.
Three Lectures on HF Lecture III:

1. Topological Kondo Insulators

2. Composite pairing.

3. Hidden Order in URu2Si2
Collaborators

Rebecca Flint              MIT
Maxim Dzero               Kent State
Andriy Nevidomskyy    Rice

Alexei Tsvelik            Brookhaven NL
Hai Young Kee             U. Toronto
Natan Andrei              Rutgers

Tzen Ong                  Rutgers

PRB   60, 3605 (1999).
PRL, 105, 246404 (2010)
PRB   84, 064514 (2011).
Culture war: glue vs fabric.
Glue vs Fabric.

Spin fluctuations = pairing glue.

Glue

Fabric: spins **build** the pairs

HiTc: Anderson: RVB (1987)
Heavy Fermions: Andrei, PC (1989)

$R \ln W = \int_0^T dT' \frac{C'}{T'}$

“Hilbert Space Spectroscopy”
SPIN Hilbert space BUILDS the pairs.

How?
115 Mystery.
The Mystery of NpPd$_5$Al$_2$

How does the spin form the condensate?

4.5K Heavy Fermion S.C
NpAl$_5$Pd$_5$
Aoki et al 2007
Large first order jump in magnetization at $H_{c2}$.

Signals a release of the local moment from the condensate.

CeCoIn$_5$

$H_{c2}$

$H/\parallel[001]$

NpPd$_5$Al$_2$

Singlet condensate

$H/\parallel[100]$

1.8 K

0.2 $\mu_B$


T. Tayama et al., RPB 65, 180504R (2002)
A neutral magnetic moment cannot form a charged superconducting condensate.

**Paradox:**

How can a neutral magnetic moment form a charged superconducting condensate?

Charge = Condensate Hilbert Space
Composite pairing Hypothesis.
Cotunneling and composite fermions

\( \text{NpPd}_5\text{Al}_2 \ T_C = 4.5 \text{K} \)

Singlet formation

Heavy electron = (electron x spinflip)

\[ f_\uparrow^\dagger = c_\uparrow^\dagger S_- \]
Coherence and composite fermions

Heavy electron = (electron x spinflip)

Singlet formation

Tailléfer & Lonzarich (1985)

UPt$_3$ ($m^*/m_e = 120$)
Composite pairing

\[ NpPd_5Al_2 \ T_C = 4.5K \]

Heavy Cooper pair = (pair x spinflip)

\[ \Psi^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+ \]

Abrahams, Balatsky, Scalapino, Schrieffer 1995
A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei     PRB   60, 3605 (1999).
Flint, PC,                        PRL, 105, 246404 (2010).
Flint, Nevidomskyy, PC, PRB   84, 064514 (2011).
The Two Channel Kondo Model

\[ H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \tilde{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j \]

- Both

Wannier functions at site j:

\[ \psi_{\Gamma_j}^\dagger = \sum_k \Phi_{\Gamma k} e^{i\vec{k} \cdot \vec{R}_j} c_k \]

\[ |\Gamma_7^+\rangle \]

| \Gamma_7^+ \otimes \Gamma_6 \rangle

Both

\[ \{ f^0 + e^- \equiv f^1, \quad f^1 \equiv f^2 + h^+ \} \]
$Z = \int \mathcal{D}[\psi] e^{-S[\psi_{\sigma}]}$

Feynman

$H = \sum_k \epsilon_k c_k^{\dagger} c_k + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_2 \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$

$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot x_j}$

Wild quantum fluctuations!

Single FS, two channels.

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)
Method: large $N$

\[
Z = \int \mathcal{D}[\psi] e^{-NS[\psi]} \]

Feynman

\[
= \int \mathcal{D}[\psi] e^{-\frac{S[\psi]}{1/N}}
\]

So how can we solve this model?

\[
\sigma \in (-\frac{1}{2}, \frac{1}{2}) \rightarrow \frac{1}{N} \sim \hbar_{c.m.}
\]

\[
H = \sum_k \epsilon_k c_k^\dagger c_k + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi_{1a}^\dagger (j) \psi_{1b} (j) + J_2 \psi_{2a}^\dagger (j) \psi_{2b} (j) \right) S^{ba}(j)
\]

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Single FS, two channels.

\[
\psi_T (j) = \frac{1}{\sqrt{V}} \sum_k \gamma k c_k e^{ik \cdot x_j}
\]
"Symplectic N" (large $N +$ time reversal symmetry) 
R. Flint, Dzero and PC '08

\[ Z = \int D[\psi] e^{-NS[\psi_\sigma]} \]

Feynman
\[ = \int D[\psi] e^{-\frac{S[\psi_\sigma]}{1/N}} \]

\[ \sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right) \]

\[ S^{ba} = f^+_b f_a - \text{sgn}(a) \text{sgn}(b) f^+_b f_a \]

\[ H = \sum_k \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j)\right) S^{ba}(j) \]

Single FS, two channels.
\[ \psi_I(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{I\Gamma} c_k e^{i\mathbf{k} \cdot \mathbf{x}_j} \]
Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980

$H = \sum_k \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{N} \sum_{k,k'} \left( J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$

Singular composite pair fluctuations

Emery and Kivelson 1992

Avoided in the lattice by composite pairing.

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Single FS, two channels.

$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_k \gamma_{\Gamma k} c_k e^{i k \cdot \mathbf{x}_j}$
4f/5f superconductors

\[ \Psi^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S^+ \]

\[ Q_{zz} \propto \Psi^2_C \]

\[ \Delta F \propto -Q_{zz} u_{tet} \]

\[ \alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi^2_C \]

\[ \Rightarrow T_c = T_{c2} + \lambda u_{tet} \]

Strain expected to enhance \( T_c \)

\[ \Psi^\dagger = c_1^\dagger c_2^\dagger S^+ \]

\[ Q_{zz} \propto \Psi_C^2 \]

\[ \Delta \nu \propto |\Psi|^2 \sim (T_c - T) \]

Flint et al, PRB 84, 064054, (2011)

G. Koutroulakis and Yasuoko, Unpublished (2012)
Real-space structure of pair

Magnetic pair: intercell
\[ \Psi_M^+ = \Delta_d (1 - 2) f_{\uparrow}^+ (1) f_{\downarrow}(2) \]

Strong pair breaking

Composite pair: **intra-cell boson**
\[ \Psi_C^+ = c_{1\downarrow}^+ c_{2\downarrow}^+ S^+ \]

Abrahams, Balatsky, Scalapino, Schrieffer 1995
Andrei, Coleman, Kee & Tsvelik PRB (1998)
Flint, Dzero, Coleman, Nat. Phys, (2008)

Extreme Resilience to doping on Ce site.
Three Lectures on HF Lecture III:

1. Topological Kondo Insulators

2. Composite pairing.

3. Hidden ("Hastatic?") Order in URu2Si2

"Hastatic order in URu2Si2", Nature, in press.

Rebecca Flint (MIT)
Premi Chandra (Rutgers)
Hidden Order in URu$_2$Si$_2$

Large entropy of condensation.

\[ \Delta S = \int_0^{T_0} \frac{C_V}{T} dT \]

= 0.14 x 17.5 K
= 2.45 J/mol/K
= 0.42 R ln 2

What is the nature of the hidden order?
Ising order, present in LMAF, vanishes in the hidden order state. (NMR, MuSR).
25 Years of Theoretical Proposals

**Local**

Barzykin & Gorkov, ’93 (three-spin correlation)
Santini & Amoretti, ’94, Santini (’98) (Quadrupole order)
Amitsuka & Sakihabara (Γ₅, Quadrupolar doublet, ‘94)
Kasuya, ’97 (U dimerization)
Kiss and Fazekas ’04, (octupolar order)
Haule and Kotliar ’09 (hexa-decapolar)

**Landau Theory**

Shah et al. (’00) “Hidden Order”,
Ramirez et al, ’92 (quadrupolar SDW)
Ikeda and Ohashi ’98 (d-density wave)

**Itinerant**

Okuno and Miyake ’98 (composite)
Tripathi, Chandra, PC and Mydosh, ’02 (orbital afm)
Dori and Maki, ’03 (unconventional SDW)
Mineev and Zhitomirsky, ’04 (SDW)
Varma and Zhu, ’05 (spin-nematic)
Ezgar et al ’06 (Dynamic symmetry breaking)
Pepin et al ’10 (Spin liquid/Kondo Lattice)
Dubí and Balatsky, ’10 (Hybridization density wave)
Fujimoto, 2011 (spin-nematic)
Rau and Kee 2012 (Rank 5 pseudo-spin vector)
The Giant Ising Anisotropy.
Quantum Oscillations: Giant Ising Anisotropy

\[ M \propto \cos \left[ 2\pi \frac{\text{Zeeman}}{\text{cyclotron}} \right] \]

\[ \frac{m^*}{m_e} g(\theta) = 2n + 1 \]

Spin Zero condition

17 spin zeros!

Ising quasiparticle with giant Ising anisotropy > 30.
Pauli susceptibility anisotropy > 900
Giant anisotropy observed in Quantum Osc. & $H_{c2}$: Ising QP's pair condense.

Electrons hybridize with Ising 5f state to form Heavy Ising quasiparticles.
Hybridization = the order parameter?

Pegor Aynajian et al, PNAS (2010)

Spectroscopy: H-gap in STM/Optics

Aynajian* et al, PNAS (2010)


* cf Session P1, Weds 8am, room A1
Spectroscopy H-gap in STM/Optics
* cf Session P1, Weds 8am, room A1

Aynajian* et al, PNAS (2010)

\[ \Psi_H \sim \left(1 - \frac{T}{T_C}\right)^{\frac{1}{2}} \]

Hybridization \textit{is} the order parameter.
Morr et al (10), Dubi and Balatsky (10)
Symmetry Implications of giant Ising anisotropy

**Giant Ising anisotropy** indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons $|k\sigma\rangle$ and **Ising 5f** doublet

$$|\alpha\rangle \equiv |\pm\rangle$$

$$\mathcal{H} = (|k\sigma\rangle V_{\sigma\alpha}(k)\langle\alpha| + \text{H.c})$$

4 fold symmetry: crystal mixes states differing by $4\hbar$

$$|+\rangle = \sum_n a_n |M - 4n\rangle$$

$$|-\rangle = \sum_n a_n |-M + 4n\rangle$$

$$|\Gamma,\pm\rangle = a |\pm 3\rangle + b |\mp 1\rangle$$

"$\Gamma_5$" non-Kramers doublet 5f²

$$\langle + | J_\pm | - \rangle = \sum_{n,n'} a_n a_{n'} \langle M - 4n | J_\pm | - M + 4n' \rangle = 0$$

$$M - 4n \neq -M + 4n' \pm 1$$

$$M \neq 2(n - n') \pm \frac{1}{2}$$

:. Integer spin $M$
**Symmetry Implications of giant Ising anisotropy**

**Giant Ising anisotropy** indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons \(|k\sigma\rangle\) and **Ising 5f** doublet

\[ H = (|k\sigma\rangle V_{\sigma\alpha}(k) \langle \alpha| + \text{H.c}) \]

\[ |\alpha\rangle \equiv |\pm\rangle \]

\[ 1/2 \text{ integer} \]

\[ 1/2 \text{ integer} \]

\[ |\Gamma, \pm\rangle = a|\pm3\rangle + b|\mp1\rangle \]

"\( \Gamma_5 \)" non-Kramers doublet 5f^2

Hybridization is a spinor.

Hybridization **IS** the OP.
Symmetry Implications of giant Ising anisotropy

**Giant Ising anisotropy** indicates fixed point Hamiltonian governing quasiparticles with a hybridization between electrons $|k\sigma\rangle$ and **Ising 5f** doublet

$$|\alpha\rangle \equiv |\pm\rangle$$

$$\mathcal{H} = (|k\sigma\rangle V_{\sigma\alpha}(k)\langle\alpha| + H.c)$$

**Kramers index $K$:** quantum no of double time reversal $\Theta \times \Theta = \Theta^2$. $\Theta^2$ is equivalent to a $2\pi$ rotation:

$$\Theta^2 |\psi\rangle = K |\psi\rangle = |\psi^{2\pi}\rangle$$

Half-integer spins change sign, integer spins do not.

$$K = (-1)^{2J}$$

Since the microscopic Hamiltonian must be Kramers-invariant,

$$V = -V^{2\pi}$$

The hybridization transforms under $\Theta$ as a $1/2$ integer spin. Unlike magnetism, it breaks single **and** double time reversal symmetry.

**It is thus a new kind of order parameter.**

Hybridization is a spinor.
“Hastatic Order”
“Hastatic” order.

In conventional heavy fermion materials a hybridization derives from virtual excitations between a Kramers doublet and an excited singlet. A uniform hybridization breaks no symmetry and develops as a cross-over.

**hasta:** spear (Latin)

![Diagram showing states and transitions](image)
“Hastatic” order.

But if the ground-state is a non-Kramer’s doublet, the Kondo effect occurs via an excited Kramer’s doublet.

\[ |5f^3, \sigma\rangle = \hat{\Psi}_\sigma^\dagger |0\rangle \]

\[ |5f^2, \alpha\rangle = \hat{\chi}_\alpha^\dagger |0\rangle \]

\[ |5f^3, \sigma\rangle \langle 5f^2, \alpha| = \hat{\Psi}_\sigma^\dagger \hat{\chi}_\alpha \]

hasta: spear (latin)

"Hastatic" order.

Quasiparticles acquire the Ising anisotropy of the non-Kramers doublet.

\[ \Psi = \begin{pmatrix} \langle \Psi_{\uparrow} \rangle \\ \langle \Psi_{\downarrow} \rangle \end{pmatrix} \]

\[ |5f^3,\sigma\rangle = \hat{\Psi}_\sigma^\dagger |0\rangle \]

\[ |5f^2,\alpha\rangle \]

\[ \langle 5f^3,\sigma|5f^2,\alpha \rangle = \hat{\Psi}_\alpha^\dagger \hat{\psi}_\sigma \hat{\chi}_\alpha \]

("Magnetic Higgs Boson")

hasta: spear (Latin)
What’s a Spinor ??

“No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the square root of geometry and, just as understanding the square root of -1 took centuries, the same might be true of spinors.

Think about going for a walk on a Mobius strip

M. Atiyah
“Hastatic Order”
Landau Theory of Hastatic Order. 1.

\[ \Psi = \begin{pmatrix} \Psi_{\uparrow} \\ \Psi_{\downarrow} \end{pmatrix} \]

\[ f[T, P] = \alpha(T_c - T)|\Psi|^2 + \beta|\Psi|^4 - \gamma(\Psi^\dagger \sigma_z \Psi)^2 \]

\[ \gamma = \delta(P - P_c) \]

AFM

\( \Psi_A \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \Psi_B \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)

Large f-moment

HO

\( \Psi_A \sim \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}, \Psi_B \sim \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix} \)

No f-moment: large Ising fluctuations
Landau Theory of Hstatic Order. I.

\[
\Psi = \begin{pmatrix} \Psi_{\uparrow} \\ \Psi_{\downarrow} \end{pmatrix}
\]

\[
f[T, P] = \alpha(T^c - T)|\Psi|^2 + \beta|\Psi|^4 - \gamma(\Psi^\dagger \sigma_z \Psi)^2
\]

\[
\gamma = \delta(P - P^c)
\]

Gap to Longitudinal Spin Fluctuations.

\[
\Delta \propto \sqrt{\delta|\Psi|^2} \sim |\Psi|\sqrt{P_c(T) - P}
\]
Landau Theory of Hastatic Order. II. Field dependence.

\[ \Psi = \begin{pmatrix} \Psi_{\uparrow} \\ \Psi_{\downarrow} \end{pmatrix} \]

\[ f[\{T, B_z\}] = \alpha(T_c - T) - \eta_z(B \cos \theta)^2 \]

\[ M = \chi_1 B + \frac{\chi_3}{3!} B^3 \]

\[ \Delta \chi_3 = -\frac{d^4 f}{dB^4} = \frac{6}{\beta} (\eta_z \cos \theta)^4 \]

Hastatic order only couples to \( B_z \)

Anisotropy in excess of 1000 predicted.

Ramirez et al '92
Microscopic theory of Hastic Order
\[ |\Gamma_7^\dagger, \sigma\rangle \equiv \Psi_{\sigma}^\dagger |0\rangle \]

\[ |\pm\rangle \equiv \chi_{\pm}^\dagger |0\rangle \]

\[ 5f^2 \equiv 5f^1 + e^- \quad \psi^\dagger_{\Gamma\sigma}(j) = \sum_k \left[ \Phi^\dagger_{\Gamma}(k) \right]_{\sigma\tau} c^\dagger_{\tau} e^{-ik \cdot R_j} \]

\[ H_{WF}(j) = W_6 \psi^\dagger_{\Gamma_6} (j) \Gamma^\dagger \chi_{\pm} \Gamma j_{\pm} \pm \Gamma \psi^\dagger_{\Gamma_7} (j) \Gamma^\dagger \chi_{\pm} \Gamma j_{\pm} \pm \Gamma \]

\[ \langle \Psi^\dagger_j \rangle = |\Psi| \begin{pmatrix} e^{i(Q \cdot R_j + \phi)/2} \\ e^{-i(Q \cdot R_j + \phi)/2} \end{pmatrix}, \quad (\phi = \pi/4). \]

\[ H_{VF} = \sum_k c^\dagger_{k} V_6(k) \chi_{k} + c^\dagger_{k} V_7(k) \chi_{k+Q} + \text{h.c.} \]
where the hybridization form factors is staggered and the observed Ising anisotropy enables us to set some of th

\[ H = \sum_k \left( \hat{c}_k^\dagger, \hat{c}_{k+Q}^\dagger, \chi_k^\dagger, \chi_{k+Q}^\dagger \right) \]

\[ \mathcal{H}_{\alpha\beta}(k) \]

\[
\begin{pmatrix}
\epsilon_k & 0 & \nu_6(k) & \nu_7(k) \\
0 & \epsilon_{k+Q} & -\nu_7(k) & -\nu_6(k) \\
\nu_6(k) & -\nu_7(k) & \lambda_k & 0 \\
\nu_7(k) & -\nu_6(k) & 0 & \lambda_{k+Q}
\end{pmatrix}
\]

\[
\begin{pmatrix}
c_k \\
c_{k+Q} \\
\chi_k \\
\chi_{k+Q}
\end{pmatrix}
\]

\[ 5f^2 \Rightarrow 5 \]

\[ H_{VF}(j) = \Psi \]

\[ \langle \Psi_j^\dagger \rangle \]

\[ H_{VF} = \sum_k \]
Consistency with experiment.
Figure 3: Magnetic response of nematic order. (a) Polar plot of calculated g-factor, \( g(\theta) \) averaged over the Fermi surface, as a function of magnetic field angle \( \theta \) (see SOM for details), compared with results of Altarawneh et al. (20), overlaid in green. (b) As a consequence of the broken time-reversal symmetry, we predict a staggered conduction electron moment that onsets at the HO transition with a linear \( T_c - T \) temperature dependence (staggering pattern shown in inset). The magnitude of this moment is governed by \( T_K/D \sim 0.01\mu_B/U \). (c) We have calculated the tetragonal symmetry breaking component of the uniform susceptibility, \( \chi_{xy}(T) \). To compare our results to Okazaki et al. (18) (overlaid as green squares), we have plotted the two-fold oscillation amplitude of the magnetic torque, \( A \) (in black), where \( A \cos 2\phi \equiv \tau_2 = -\mu_0 H^2 V \cos 2\phi \chi_{xy}(T) \). \( A \) goes as \((T_c - T)^2\) just below the HO transition. For details of our calculation, including parameter choices, please see the supporting online material (32).
Predictions.
(a) Transverse moment in conduction sea
\[ m \sim O(TK/D) \sim 0.01 \mu_B. \]

(b) Giant non-linear susceptibility anomaly.

(c) Collapse of gap to Ising fluctuations at 1st order transition line.
Figure 4: Density of states and resonant nematicity predicted by theory. Upper pane: density of states as a function of energy predicted by model calculation (blue line), showing conduction electron components. Red line, voltage dependence of nematicity $\eta(V)$ in model calculation of scanning tunneling spectrum. Lower panels: spatial dependence of density of states for selected bias voltages in model calculation of scanning tunneling spectrum, showing the resonant character of the nematicity.
Conclusions about Hidden Order in URu2Si2.

- Giant quasiparticle Ising anisotropy indicates the development of a fixed point Hamiltonian with hybridization between half integer electrons and integer spin 5f\(^2\) U doublet.

- Hybridization mixes half-integer and integer spin states: must have half-integer character - a spinor.

- Unlike magnetism, hybridization breaks single and double time-reversal symmetry: a new kind of “Hastatic Order”.

- Broader implication: a new kind of broken symmetry where OP transforms under DOUBLE GROUP (S = 1/2) representation.
Another Definition of “Hastatic”

1. hastatic

Happy. A word invented by Deena Nicole Cortese (Jersey Shore)

Hastatic is when you're like super happy and, like... really happy.