GRADUATE QUANTUM MECHANICS: 502 Spring 2002

Assignment 6: due Weds, April 24th 19th. Read: Sakurai 379-421.

1. (Sakurai Chapter 7, Problem 1) The Lippmann-Schwinger formalism can be applied to a one-dimensional transmission-reflection problem with a finite range potential $V(x) \neq 0$ for $0 < |x| < a$ only.

   (a) Suppose we have an incident wave coming from the left $\langle x|\phi\rangle = e^{ikx}/\sqrt{2\pi}$. Howe must we hand the singular $1/(E - H_o)$ operator if we are to have a transmitted wave only for $x > a$ and a reflected wave and the original wave for $x < -a$? Is the $E \to E + i\delta$ prescription still correct? Obtain an expression for the appropriate Green's function and write and integral equation for $\langle x|\psi^+\rangle$.

   (b) Consider the special case of an attractive delta-function potential

   $$V = -\left(\frac{\gamma\hbar^2}{2m}\right)\delta(x), \qquad (\gamma > 0), \tag{1}$$

   Solve the integral equation to obtain the transmission and reflection amplitudes.

   (c) The one dimensional delta-function potential with $\gamma > 0$ admits one and only one bound-state. Show that the transmission and reflection amplitudes you computed have bound-state poles at the expected positions when k is regarded as a complex variable.

2. (Sakurai Chapter 7, Problem 3) Consider a potential

   $$V = \begin{cases} 0, & (r > R) \\ V_o, & (r < R) \end{cases} \tag{2}$$

   where V_o may be positive or negative.

   (a) Use the method of partial waves, show that for $V_o << E = \hbar^2k^2/2m$ and $kR << 1$, the differential scattering cross-section is isotropic, and that the total cross-section is given by

   $$\sigma_{tot} = \left(\frac{16\pi}{9}\right)\frac{m^2V_o^2R^6}{\hbar^4} \tag{3}$$

   (b) Show that when the energy is raised slightly, the angular distribution can be written

   $$\frac{d\sigma}{d\Omega} = A + B\cos\theta \tag{4}$$

   Obtain an approximate expression for B/A.

   (c) Would you obtain the same result using Born Scattering Theory?