GRADUATE QUANTUM MECHANICS: 502 Spring 2001

Assignment 5: due Thurs April 5th.

Read: Sakurai 357-377.

1. (Sakurai, Problem 36, Ch. 5) Derive an expression for the density of free particle states in two dimensions, that is, the two dimensional analog of

$$\rho(E)dEd\Omega = \left(\frac{L}{2\pi}\right)^2 \left(\frac{mk}{\hbar^2}\right) dEd\omega, \qquad \left(\mathbf{k} \equiv \frac{\mathbf{p}}{\hbar},\ E = \frac{p^2}{2m}\right) \tag{1}$$

   Your answer should be written as a function of k (or E) times $dEd\phi$, where ϕ is the polar andlge that characterizes the momentum direction in two directions.

2. (Sakurai, Problem 38, Ch 5) This is an excercise on the photo-electric effect, repeated for a new wavefunction.

3. (Sakurai, Problem 1, Chapter 6)

   (a) N identical spin $\frac{1}{2}$ particles are subjected to a one dimensional simple harmonic oscillator potential. What is the ground-state energy? What is the Fermi energy?

   (b) What are the ground-state and Fermi energies if we ignore the mutual interactions and assume N to be very large?

4. List three profound ways in which the properties of your environment depend on the Pauli Exclusion principle. In each case, describe how the world would be different if electrons were bosons.

5. (Sakurai, Problem 4, Chapter 6.) Three spin 0 particles are situated at the corners of an equilateral triangle. Let us define the z-axis to go through the center and in the direction normal to the plane of the triangle. The whole system is free to rotate about the z-axis. Using statistics considerations, obtain restrictions on the magnetic quantum numbers corresponding to J_z.