GRADUATE QUANTUM MECHANICS: 502 Spring 2001

Assignment 4: due Monday, March 25th. This assignment will be treated as a mid-term and graded as equivalent to two homeworks.

Read: Sakurai 336-345.

1. Two atoms, A and B each with angular momentum $j = 1$ are combined into a bound-state of total angular momentum $\mathbf{j} = \mathbf{j_A} + \mathbf{j_B} = \mathbf{0}$. If the z-component of angular momentum of atom A is measured, what is the probability that $J_z^A = (\pm\hbar, 0)$?

2. A quantum mechanical state is known to be a simultaneous eigenstate of two Hermitean operators A and B which anticommute, i.e $\{A, B\} = AB + BA = 0$. What can you say about the eigenvalues of A and B for state $|\psi\rangle$? Illustrate your point using the parity operator Π and momentum operator.

3. A simple harmonic oscillator of angular frequency ω is acted apon by a time dependent *force*

$$F(t) = \frac{F_0}{\omega} \frac{\tau}{\tau^2 + t^2}$$

At $t = -\infty$ the oscillator is in its ground state. Using time-dependent perturbation theory, calculate the probability that the system is in the first excited state at $t = +\infty$.

4. The ground-state of a hydrogen atom $(n = 1, l = 0)$ is subjected to a time dependent potential as follows:

$$V(\mathbf{x}, t) = V_0 \cos(kz - \omega t).$$

Using time-dependent perturbation theory, obtain an expression for the transition rate at which the electron is emitted with momentum $\mathbf{p}$. Show, in particular, how you may compute the angular distribution of the ejected electron in terms of θ and ϕ defined with respect to the $z-$ axis. For the initial wavefunction, you may take

$$\psi_{n=1,l=0}(\mathbf{x}) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}.$$

If you find you have a normalization problem, you may find it useful to take the final state to have the form

$$\psi(\mathbf{x}) = \left(\frac{1}{L^{3/2}}\right) e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$$

with L very large, but you should be able to show that the observable effects are independent of L.