GRADUATE QUANTUM MECHANICS: 502 Spring 2002

Assignment 1. (Due Weds Feb 20th)

Read Sakurai, section 3.9-3.10

1. Find the Clebsch Gordon Coefficients for the $\frac{1}{2} \otimes \frac{3}{2} = 2 \oplus 1$ combination of angular momentum states.
2. Sakurai, problem 3.18.
3. Sakurai, problem 3.24.
4. Sakurai, 3.28 (Modified). The quadrupole moment operator has matrix elements given by

$$\langle jm|Q_{ab}|jm\rangle = e\langle jm|x_a x_b - \frac{1}{3}r^2\delta_{ab}|jm\rangle$$

where $x_a(a = 1, 3)$ are the position co-ordinates of a particle and $r^2 = x_1^2 + x_2^2 + x_3^2$. $|jm\rangle$ refers to a state of definite j and azimuthal quantum number m. The expectation value

$$Q = e\langle j, m = j|(3z^2 - r^2)|j, m = j\rangle$$

is known as the quadrupole moment.

   (a) Use the Wigner Eckart Theorem to relate the matrix elements of the quadrupole moment operator to the matrix elements of the operator

$$J_a J_b - \frac{1}{3}J^2\delta_{ab}.$$

   Write the constant of proportionality in terms of the quadrupole moment Q.

   (b) Evaluate the $2j + 1 \times 2j + 1$ matrix

$$e\langle jm|(x^2 - y^2)|jm'\rangle$$

   for the case where $j = 1$.