GRADUATE QUANTUM MECHANICS: 502 Spring 2002

Assignment 3: due Weds, Mar 6th.

Read: Sakurai 285-310.

1. (Sakurai, problem 4, ch 5) Consider an isotropic harmonic oscillator in *two* dimensions. The Hamiltonian is given by

$$H_o = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2). \tag{1}$$

   (a) What are the energies of the three lowest lying states? Are there any degeneracies?

   (b) We now apply a perturbation

$$V = \delta m\omega^2 xy, \tag{2}$$

   where δ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenket and the corresponding energy to first order (that is the unperturbed energy obtained in (a) plus the first order energy shift) for each of the three lowest lying states.

   (c) Solve the $H_0 + V$ problem *exactly*. Compare with the perturbation results obtained in (b).

2. Sakurai Chapter 5, problem 9. A p-orbital characterized by $|n, l = 1, m = \pm 1, 0\rangle$ (ignore spin) is subjected to a potential

$$V = \lambda(x^2 - y^2) \tag{3}$$

   (a) Obtain the "correct" zeroth energy eigenstates that diagonalize the perturbation. You need not evaluate the energy shifts in detail, but show that the original threefold degeneracy is now completely lifted.

   (b) Because V is invariant under time-reversal and because there is no longer any degeneracy, we expect each of the energy eigenstates obtained in (a) to go into itself (up to a phase factor or sign) under time reversal. Check this point explicitly.

3. Sakurai, ch 5, Problem 17 Suppose the Hamiltonian of a rigid rotator in a magnetic field perpendicular to the axis is of the form

$$A\mathbf{L}^2 + BL_z + CL_y. \tag{4}$$

   If terms quadratic in the field are negelected. Assuming $B >> C$, use perturbation theory to lowest non-vainishing order to get approximate energy eigenvalues.