GRADUATE QUANTUM MECHANICS: 501 Fall 2001

Assignment I. (Due Weds, Sept 12th.)

Read Sakurai, pages 1-36.

1. (Sakurai, Problem 4, ch 1.)Using the rules of bra-ket algebra, prove or evaluate the following:
   (a) $\mathrm{Tr}(XY) = \mathrm{Tr}(YX)$
   (b) $(XY)^\dagger = (Y^\dagger X^\dagger)$
   (c) $\exp[if(\hat{A})] =?$ in bra-ket form, where A is a Hermitian operator whose eigenvalues are known.
   (d) $\sum_a \psi_a^*(\mathbf{x})\psi_a(\mathbf{x})$, where $\psi_a(\mathbf{x}) = \langle \mathbf{x}|a\rangle$.

2. (Sakurai, Problem 11., ch 1.) A two-state system is characterized by the Hamiltonian
$$H = h_{11}|1\rangle\langle 1| + h_{22}|2\rangle\langle 2| + h_{12}[|1\rangle\langle 2| + |2\rangle\langle 1|] \tag{1}$$
where h_{11}, h_{22}, and h_{12} are real numbers with the dimensions of energy, and $|1\rangle$ and $|2\rangle$ are eigenstates of some observable *other than the Hamiltonian*. Find the energy eigenstates and the corresponding energy eigenvalues. Make sure tha your answer makes good sense for $h_{12} = 0$. (If you wish, make use of the result
$$(\vec{\sigma}\cdot\hat{\mathbf{n}})|\hat{\mathbf{n}};+\rangle = |\hat{\mathbf{n}};+\rangle \tag{2}$$
where $|\mathbf{n};+\rangle$ is given by
$$|\mathbf{n};+\rangle = \cos\frac{\beta}{2}|+\rangle + e^{i\alpha}\sin\frac{\beta}{2}|-\rangle \tag{3}$$
where β and α are the polar and azimuthal angles, respectively, which characterize $\hat{\mathbf{n}}$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the three Pauli matrices.)

3. Find the uncertainty relation for the operators $\hat{q}$ and $F(\hat{p})$, if $\hat{q}$ and $\hat{p}$ satisfy the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$.

4. This question concerns whether, according to the uncertainty principle, it is possible to measure the magnetic moment of a free electron. The magnetic field produced by a free electron depends on the motion of the particle as well as on its intrinsic magnetic moment. From electrodynamics, we know that the intensity of the magnetic field of a moving charge is of order
$$B_1 \sim \mu_o\frac{ev}{r^2} \tag{4}$$
where μ_0 is the permittivity of free space, v is the velocity of the charge and r is the distance from the measuring apparatus. The field intensity of a magnetic dipole of moment m is of order
$$B_2 \sim \mu_o\frac{m}{r^3}. \tag{5}$$
The magnetic moment of a free electron can be measured by measuring the field $B_1 + B_2$ it produces, provided that B_1 is negligibly small and the uncertainty in position Δr is small compared with r, i.e. $B_2 >> B_1$ and $\Delta r << r$. Can these two conditions be satisfied simultaneously? Please explain your answer carefully.
(*Hint*: Use the Uncertainty principle and the magnetic moment of the electron $m = \frac{e\hbar}{2m}$.)