16. SPIN + EXCLUSION PRINCIPLE

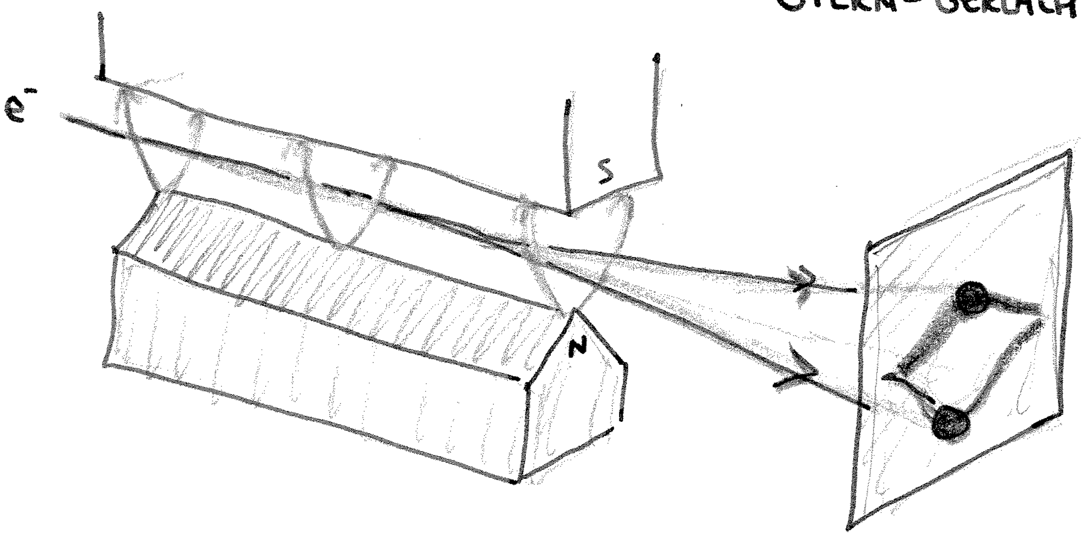
In lecture 15 we focussed on the Zeeman effect.

Today we will see that electrons behave like tiny spinning tops. When a magnetic field is applied - the spin can point "up" or "down" and has a z-component of angular momentum equal to $\pm \hbar/2$.

We'll go onto discuss the exclusion principle - which is the basis of chemistry & the periodic table. At the end of the lecture we shall discuss X-ray lines & lasers.

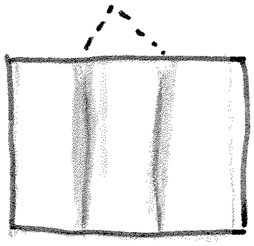
41.3 ELECTRON SPIN

- STERN-GERLACH (1922)



"up" $S_z = \frac{\hbar}{2}$

"down" $S_z = -\frac{\hbar}{2}$

- 

$3p \to 3s$

DOUBLET

WITHOUT FIELD!

Anomalous Zeeman effect.

These two experiments are a result of **ELECTRON SPIN**. Electrons behave as elementary "tops".

$$\boxed{S_z = m_s \hbar = \pm \frac{1}{2}\hbar}$$

COMPONENTS

- $$\boxed{S = \hbar\sqrt{\frac{1}{2}\left(1+\frac{1}{2}\right)} = \sqrt{\frac{3}{4}}\,\hbar}$$

MAGNITUDE OF SPIN ANGULAR MOMENTUM

$$\boxed{\mu_z = -2.00\underbrace{232}\,\frac{e}{2m}\,S_z}$$

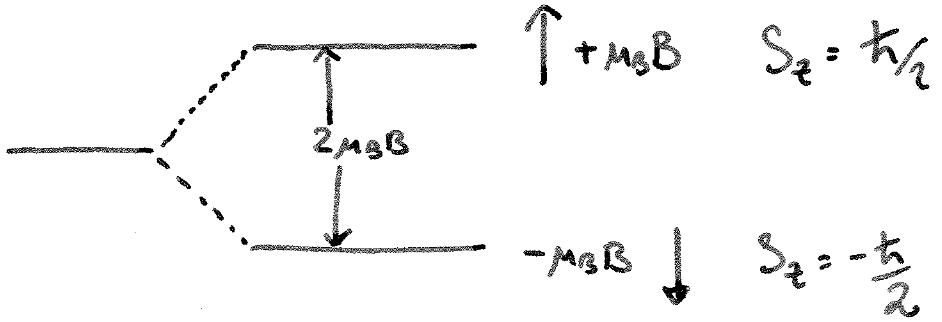
relativistic version of Schrödinger eqn
≡ DIRAC EQN

"Quantum Electrodynamics"
QED
2.0023193043737(82)

$$\mu_z \simeq -2\left(\frac{e}{2m}\right)m_s\hbar = -2\mu_B m_s = \mp\mu_B$$

Energy in a magnetic field

$$U = -\mu_z B = 2(\mu_B B)\,m_s = \pm(\mu_B B)$$



e.g. An electron absorbs a photon to flip from spin down to spin up in a magnetic field of 10 Tesla. What is the energy and frequency of the photon

$$\Delta E = 2\mu_B B = 2 \times 5.788 \times 10^{-5}\,\text{eV/T} \times 10$$

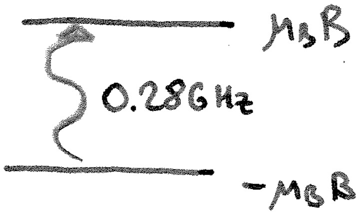
$$= 1.15 \times 10^{-3}\,\text{eV}$$

$$= 1.15\,\text{meV}$$

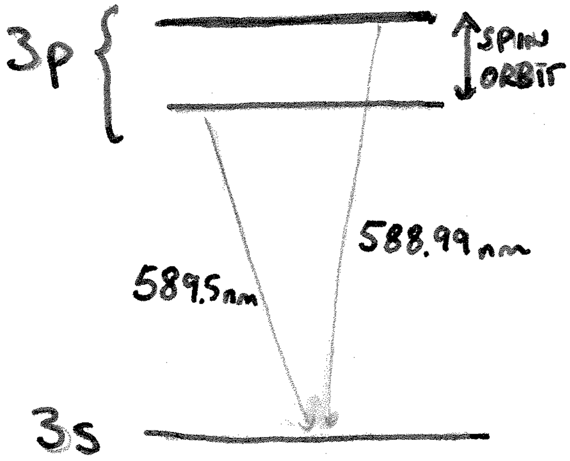
$$hf = \Delta E \Rightarrow f = \frac{1.15 \times 10^{-3}\,\text{eV}}{4.14 \times 10^{-15}\,\text{eV s}}$$

$$= 2.80 \times 10^{11}\,\text{Hz}$$

$$= 0.28\,\text{GHz}$$

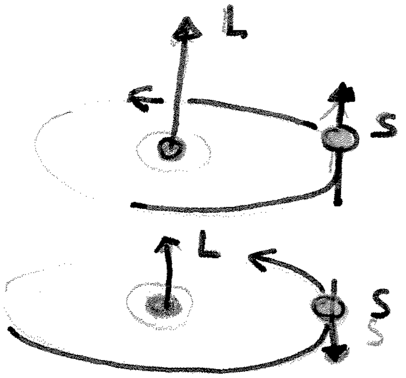


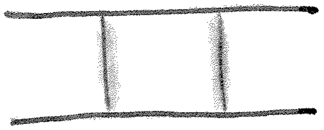
THIS IS THE BASIS OF SPIN RESONANCE



$J = L + S = 3/2$

$J = L - S = \frac{1}{2}$



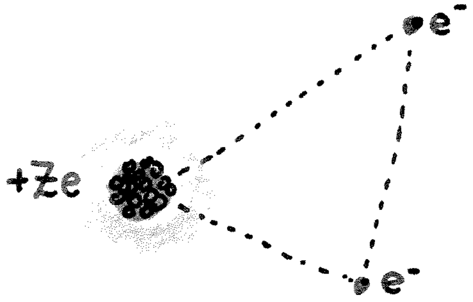


The Na D-LINE DOUBLET is a result of Spin-Orbit coupling between the $\ell=1$ orbital angular momentum & the $S=\frac{1}{2}$ spin angular momentum.

18.6

41.4 MANY ELECTRONS + PAULI EXCLUSION PRINCIPLE

Complex atom



$$U(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

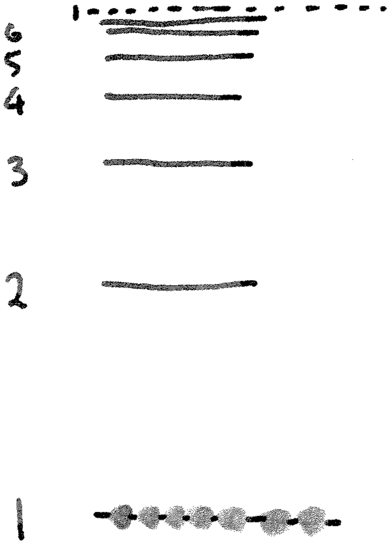
$$e^2 \longrightarrow Ze^2$$

$$E_n = -\frac{1}{(4\pi\epsilon_0)^2}\frac{mZ^2e^4}{2\hbar^2}\left(\frac{1}{n^2}\right)$$

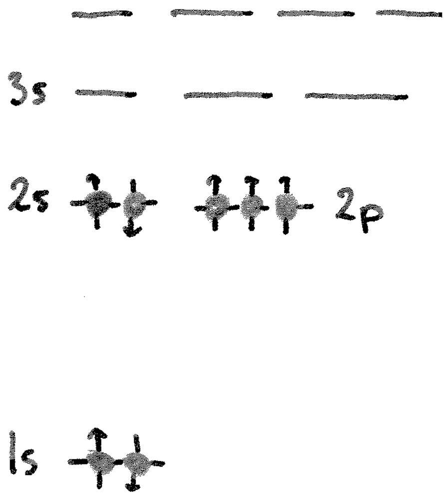
$$= -\frac{Z^2}{n^2}(13.6)\,eV$$

Each e^- — Four Quantum numbers

$\boxed{n, l, m_l, m_s.}$ $n \geq 1$, $0 \leq l \leq n-1$, $|m_l| \leq l$, $m_s = \pm\frac{1}{2}$



NO TWO ELECTRONS CAN OCCUPY THE SAME QUANTUM STATE



If all e^- in lowest state → ALL ELEMENTS ~SAME

No Chemistry.

O

$1s^2 2s^2 2p^3$

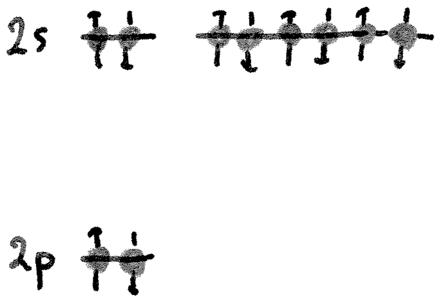
ELECTRON SHELLS

n	ℓ	m_ℓ	Notation	# states		SHELL
1	0	0	1s	2		K
2	0	0	2s	2	8	L
2	1	-1,0,1	2p	6		
3	0	0	3s	2	18	M
3	1	-1,0,1	3p	6		
3	2	-2,-1,0,1,2	3d	10		
4	0	0	4s	2	32	N
	1	-1,0,1	4p	6		
	2	-2,-1,0,1,2	4d	10		
	3	-3 —— +3	4f	14		

Ne. $Z = 10$

Filled L shell – Chemically inert

$1s^2 2s^2 2p^6$

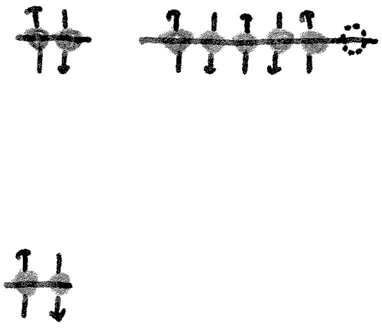


F $Z = 9$

$1s^2\ 2s^2\ 2p^5$

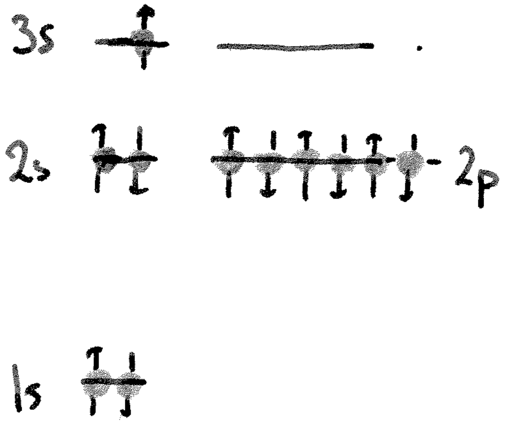
Strong Affinity to attract electrons + fill shell. Highly reactive

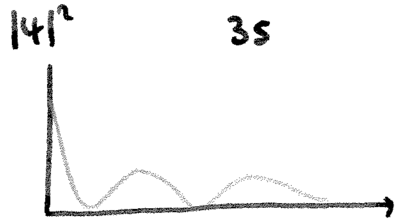
$F + e^- \longrightarrow F^-$ FILLED SHELL

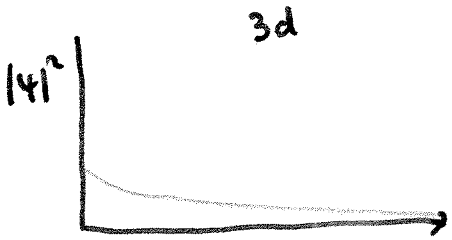


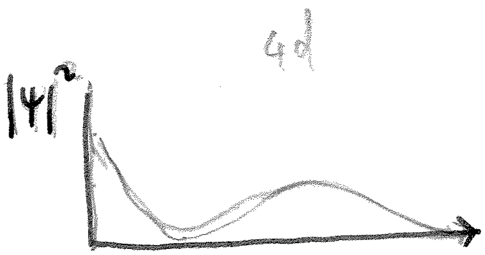
Na $Z = 11$

$1s^2 \quad 2s^2 \quad 2p^6 \quad 3s^1$









Filled shell + 1e^-

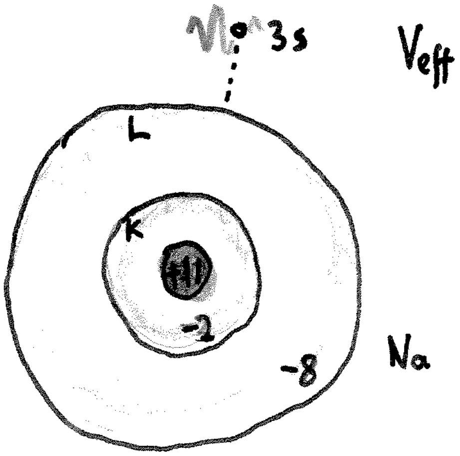
$$Z_{eff} = 11 - 10 = 1$$

$$E_n \approx -\frac{Z_{eff}^2}{n^2}(13.6)\ \text{eV}$$

		Z_{eff}
E_{3s}	$= -5.138\ \text{eV}$	1.84
E_{3p}	$= -3.035\ \text{eV}$	
E_{3d}	$= -1.521\ \text{eV}$	~1.0
E_{4s}	$= -1.947\ \text{eV}$	1.51

3s penetrates into filled shell

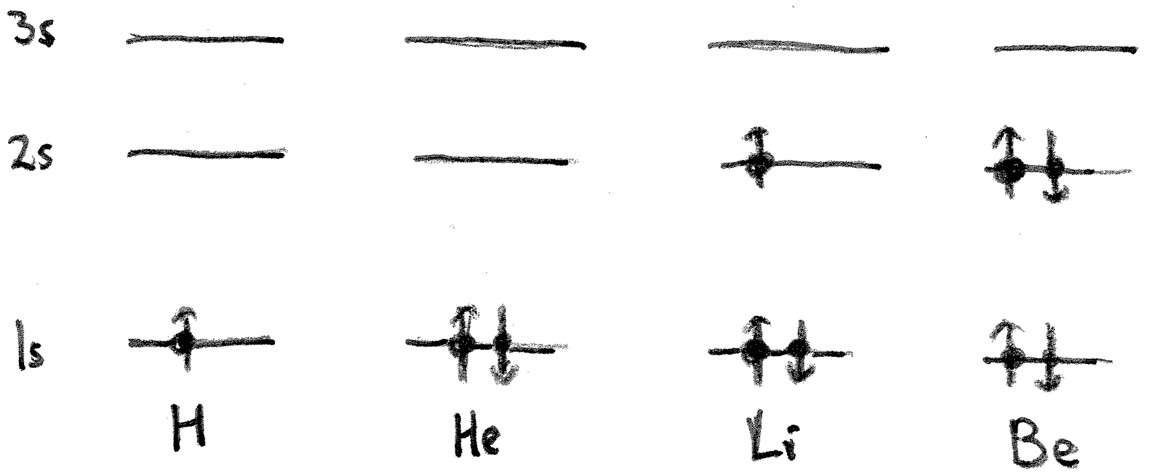
$\Rightarrow$ higher Z_{eff}



$$V_{eff} = -\frac{e^2 Z_{eff}}{4\pi\epsilon_0 r}$$

$11 - 2 - 8 = Z_{eff} = 1$

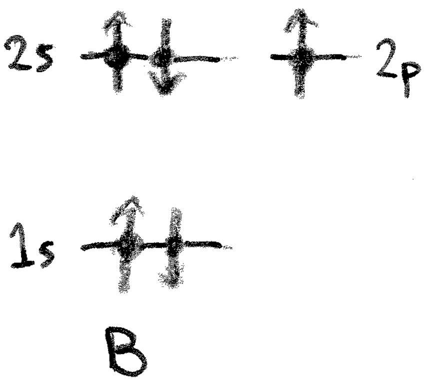
This is the basis of the Periodic Table.



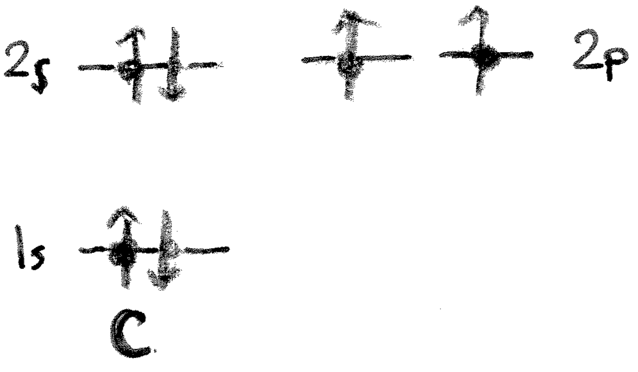
$1s$ $1s^2$ $1s^2 2s$ $1s^2 2s^2$

- FILLED SHELL.
- CHEMICALLY INERT

- Loosely bound $2s$ e^-.
- $\Delta E = 5.4 eV$
- Alkali metal

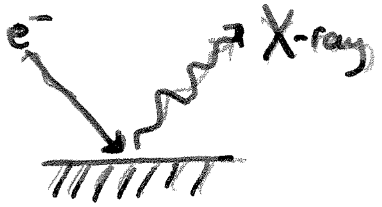


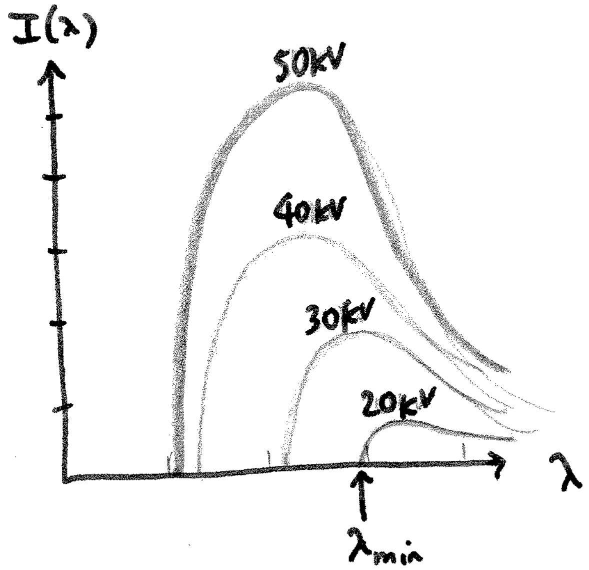
$1s^2 2s^2 2p$



$1s^2 2s^2 2p^2$

41.5 X-Ray Spectra



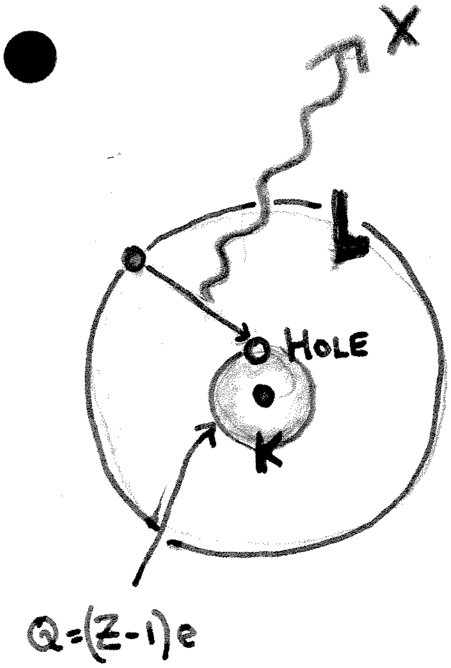
- 

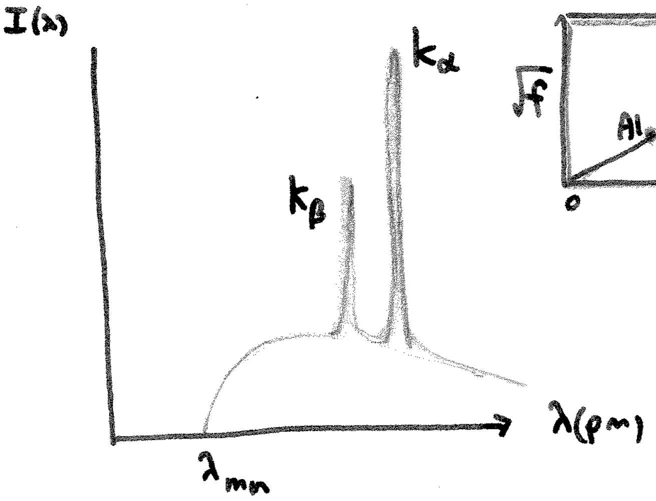
$$\lambda_{min} = \frac{hc}{eV_{AC}} = \frac{1240\ \text{nm-eV}}{E(\text{eV})}$$

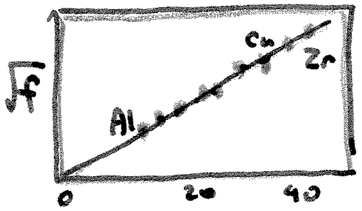
e.g 50kV

$$\lambda_{min} = \frac{1240}{50\times10^{3}} = 2.48\times10^{-2}\ \text{nm}$$

$$= 24.8\ \text{pm}$$

- 





$$E_i = -\frac{(Z-1)^2(13.6)}{2^2} = -(Z-1)^2(3.4\text{eV})$$

$$E_f = -\frac{(Z-1)^2(13.6)}{1^2} = -(Z-1)^2(13.6\text{eV})$$

- $E_{K\alpha} \approx (Z-1)^2\ 10.2\text{eV}$

$$\boxed{f = \frac{E}{h} = \frac{(Z-1)^2\ 10.2}{4.136\times10^{-15}\ \text{eV-s}} = 2.47\times10^{15}\ \text{Hz}\ (Z-1)^2}$$

MOSELEY 1913.

16.12

38.6 LASER

The conventional emission of light by an atom is called "spontaneous emission", and in this case, the direction & phase of the emitted light are random.

STIMULATED EMISSION is the emission of a photon in response to the arrival of a photon of matching frequency. In this process, the emitted photon has precisely the same frequency, phase & direction as the incoming photon. This "light amplification" effect is the basis of the operation of a laser.

POPULATION INVERSION

Under conventional thermal conditions, there is negligible excitation of atoms into their excited states. According to Boltzmann's distribution

$$n_g = \text{\# atoms in ground state} = Ae^{-E_g/k_BT}$$
$$n_{ex} = \text{\# atoms in excited state} = Ae^{-E_{ex}/k_BT}$$

$$\frac{n_{ex}}{n_g} = e^{-(E_{ex}-E_g)/k_BT}$$

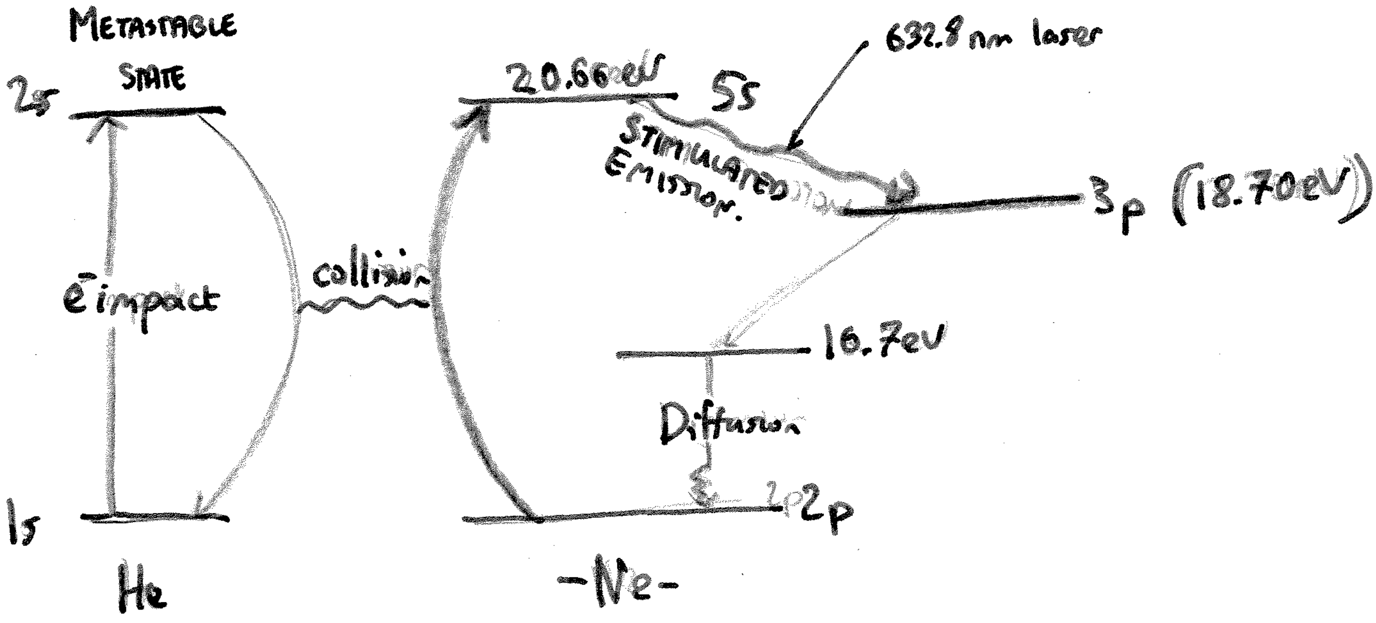
e.g if $E_{ex}-E_g = 2eV = 3.2\times10^{-19}J \equiv 620nm$ photon (visible)

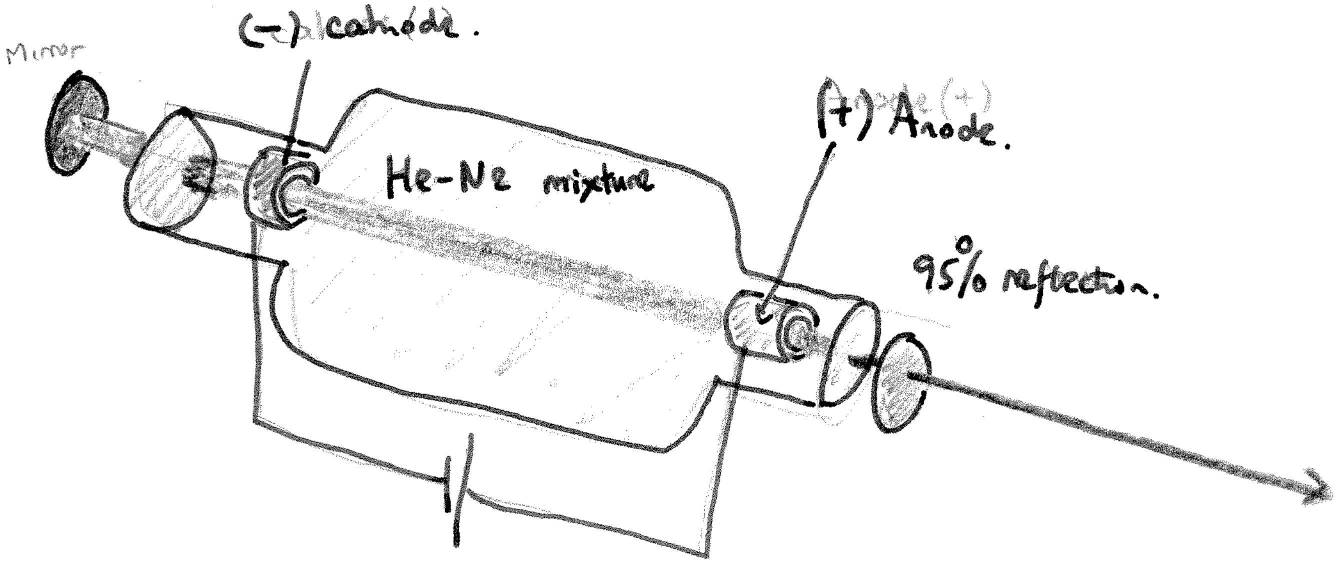
at $T = 3000K$

$$\frac{E_{ex}-E_g}{k_BT} = \frac{3.2\times10^{-19}J}{1.38\times10^{-23}\times(3000K)} = 7.73$$

$$e^{-(E_{ex}-E_g)/k_BT} = e^{-7.73} = 4.4\times10^{-4}.$$

To obtain a significant "population inversion", one must pump the atom into an excited state. e.g He-Ne laser.





- semiconductor laser (p-n)
- CO_2
- chemical laser
- Maser - microwaves.