Example

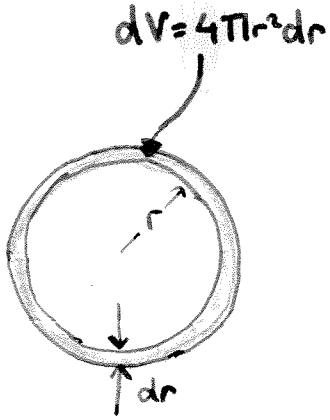
The ground-state wavefunction of Hydrogen is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

a_0 = Bohr radius $= 5 \times 10^{-11}$ m.

a) Sketch the radial probability distribution function $P(r)$

b) Calculate the probability that the electron is more than one Bohr radius from the atom.

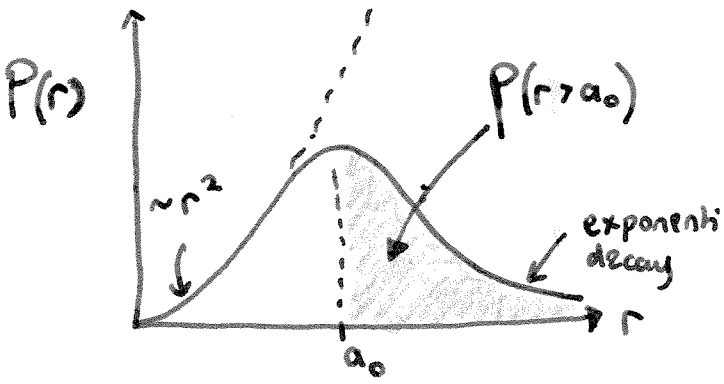


a) $P(r)\,dr$ = probability inside shell between r & $r+dr$

$$= |\psi(r)|^2 dV$$

$$= \left(\frac{1}{\pi a_0^3} e^{-2r/a_0}\right) 4\pi r^2 dr$$

$$P(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$$



b) $P(r > a_0)$ = area under curve with

$r > a_0$

$$= \int_{a_0}^{\infty} dr\, P(r)$$

$$= 4 \int_{a_0}^{\infty} dr\, \frac{r^2}{a_0^3} e^{-2r/a_0}$$

$x = r/a_0 \qquad \frac{dr}{a_0} = dx \qquad \frac{r^2}{a_0^2} = x^2$

$$P(r > a_0) = 4 \int_1^{\infty} dx\, x^2 e^{-2x} = 4 \left[\cdot \left(\frac{-x^2}{2} - \frac{x}{2} - \frac{1}{4} \right) e^{-2x} \right]$$

$$= 5 e^{-2} = 0.677$$