# L13 Summary

- $|\Psi(x)|^2 =$ probability / unit volume (3D) / length (1D)

- The stationary states in 1D satisfy

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V(x)\,\Psi(x) = E\,\Psi(x)$$

- For a particle in a box

$$\boxed{\begin{gathered}\Psi(x) = \underbrace{\sqrt{\frac{2}{L}}}_{\text{NORMALIZATION}}\sin\left(k_n x\right), \quad k_n = \frac{n\pi}{L} = \frac{p_n}{\hbar} \\ E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{h^2}{8mL^2}\times n^2\end{gathered}}$$

- SPECIAL NOTE ON UNCERTAINTY PRINCIPLE

Young + Freedman drop the factor of 1/2 in the uncertainty principle, and write

$$\Delta x \Delta p \gtrsim \hbar$$

$$\Delta t \, \Delta E \gtrsim \hbar$$

We will use these formulae in Homework + exams.

- Misprint in Young + Freedman:-

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \qquad \text{eq.} \quad 38.22$$

Note that for a particle in a box

$$\Delta x = L/2 = \text{uncertainty in position}$$

$$p_n = n\left(\frac{h}{2L}\right) \Rightarrow \Delta p = \frac{h}{2L} = \text{uncertainty in momentum.}$$

$$\Delta x \, \Delta p = L/2 \times \frac{h}{2L} = h/4 \left( > \frac{h}{4\pi} \right)$$

So the product of the uncertainties is consistent with Heisenberg's uncertainty relation.

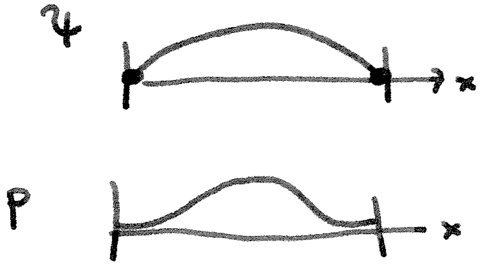
e.g. Electron in an atom sized box $L = 0.5\,\text{Å}$

$$\text{Ground state energy} = \frac{\hbar^2}{2m} \times \left(\frac{\pi}{L}\right)^2 = \frac{1}{2m}\left(1.05\times10^{-34}\right)^2 \times \left(\frac{\pi}{5\times10^{-11}}\right)^2$$

$$= 2.4\times10^{-19}\,\text{J}$$

$$\equiv \frac{2.4\times10^{-19}}{1.6\times10^{-19}}\,\text{eV}$$

$$= \underline{\underline{1.5\,\text{eV}}}$$




e.g. An electron is in the $n=4$ state of a particle in a box. Calculate the probability it can be found in the first 1/4 of the box.

$$P = \int_0^{L/4} dx\,|\psi(x)|^2 = \int_0^{L/4} dx \left(\sqrt{\frac{2}{L}}\,\sin\frac{4\pi x}{L}\right)^2$$

$$= 2\int_0^{L/4} \frac{dx}{L}\,\sin^2\!\left(\frac{4\pi}{L}x\right) = 2\int_0^{1/4} du\,\sin^2(4\pi u)$$

$$= 2\times\frac{1}{4}\times\frac{1}{2}$$

$$= \underline{\underline{\frac{1}{4}}}$$