12. THE UNCERTAINTY PRINCIPLE

Wave particle duality forces upon us a profound re-interpretation of the meaning of particles and the world around us. Waves have the property that the more precisely they are localized, the less precisely we know their wavelength or frequency. In 1923 a 21 year old student named Werner Heisenberg puzzled over this and almost failed his Ph.D. exam over his confusion. He barely passed his Ph.D, but went on to sharpen his ideas — he realized that if particles of

Matter move as waves with momentum given by the de Broglie relation $p = h/\lambda$, then the more precisely we determine their position, the more uncertain we become about their wavelength & momentum. This is the famous "Uncertainty Principle" which states that the product of the uncertainty in position & momentum can never be less than $h/4\pi$.

$$\Delta x \, \Delta p \geq \frac{h}{4\pi}$$

This is something you understand intuitively for sound.

A sudden "clap" or "pulse" of sound produces

a shock wave of well defined position & time —

but the shock wave contains a broad range of

frequencies & wavelengths. If an electron is a wave —

then it must suffer the same fate & our whole world

view is forced to change.....

39.2 Electron Diffraction

But despite the success of Bohr's atom — how do really know that electrons are waves? The answer to this question came from a remarkable experiment conducted by Clinton Davisson & Lesley Germer, working at the old Bell Labs in New York City, in 1927.

crystal.
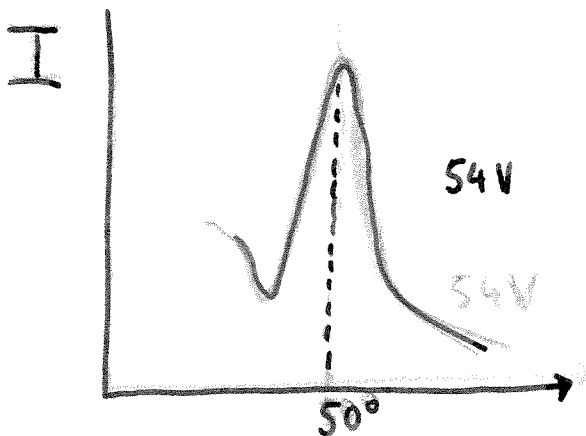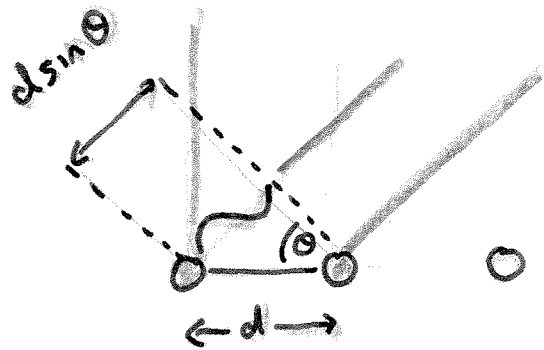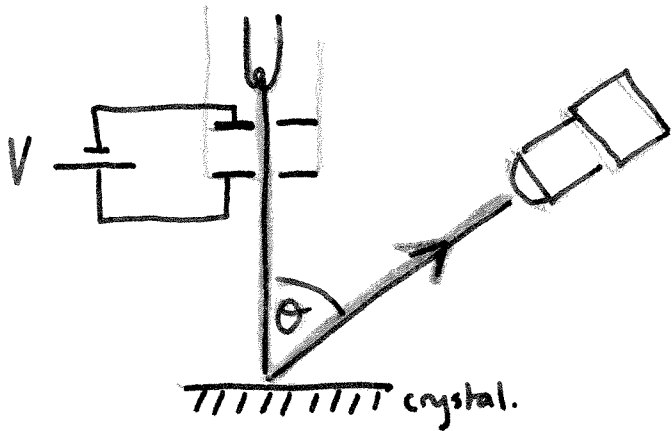
$$d\sin\theta = \lambda$$

constructive interference.

54 V

54 V

50°

$$eV = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{1}{2}\frac{p^2}{m}$$

$$\Rightarrow \quad p = \sqrt{2meV}$$

$$\Rightarrow \quad \boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}}$$

de Broglie Wavelength of e^-

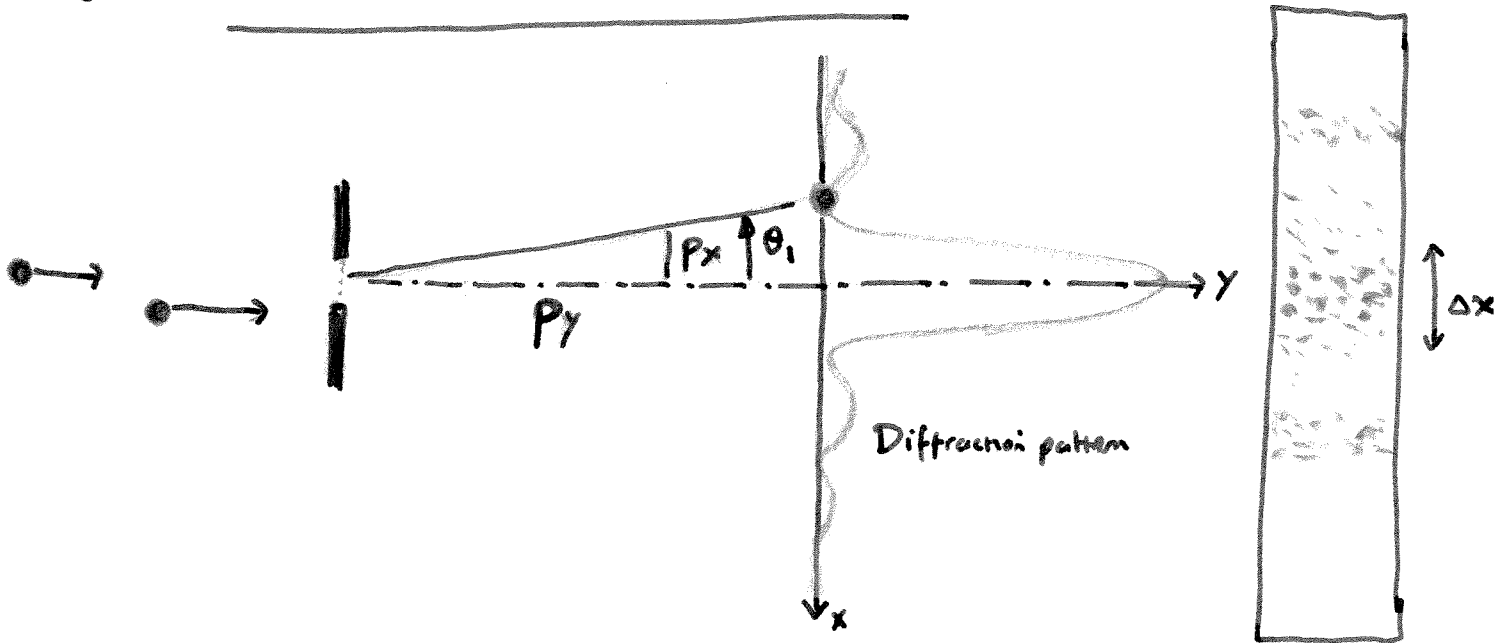$$d\sin\theta = \frac{h}{\sqrt{2meV}}$$

e.g $V = 54V$ what is λ?

$$\lambda = \frac{6.62\times10^{-34} \, Js}{\sqrt{2\times(9.1\times10^{-31})\times(1.6\times10^{-19})\times(54V)}} = 1.7\times10^{-10}m$$

what is d if the peak occurs at $50°$

$$d\sin\theta = \lambda \quad \Rightarrow \quad d = \frac{1.7\times10^{-10}m}{\sin50°} = 2.15\times10^{-10}m$$

39.3 PROBABILITY + UNCERTAINTY

Diffraction pattern

$$\theta_1 = \frac{\lambda}{a}$$

$$p_x = p_y \theta_1 = p_y \lambda/a$$

Uncertainty of momentum of e^- in central fringe $\qquad \Delta p_x \gtrsim p_y \left(\frac{\lambda}{a}\right)$

Uncertainty of position $\qquad \Delta x \gtrsim a$

$$\boxed{\Delta x \, \Delta p_x \gtrsim p_y \lambda = h}$$

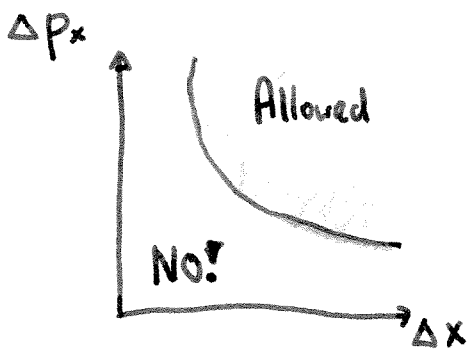If we reduce the slit $\Delta x \downarrow$ but $\Delta p_x \uparrow$
If we reduce $\Delta p_x \downarrow$ we increase $\Delta x \uparrow$

Heisenberg Uncertainty Principle $\quad \Delta X \Delta p_x \gtrsim \dfrac{\hbar}{2}$

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J}_s$$

(I include a factor of two which is dropped in Young + Freedman.

$$\Delta x \Delta p_x \gtrsim \hbar/2$$
$$\Delta y \Delta p_y \gtrsim \hbar/2$$
$$\Delta z \Delta p_z \gtrsim \hbar/2$$

UNCERTAINTY IN ENERGY

$$\Delta f \gtrsim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \gtrsim \hbar/2$$

$$\boxed{\Delta E \Delta t \gtrsim \hbar/2}$$

e.g   Electron confined in a box of side $1\overset{\circ}{A}$.

Estimate its K.E.

$$\Delta p_x \gtrsim \frac{\hbar}{2 \Delta X} \quad = \quad \frac{1.0 \times 10^{-34}}{2 \times 10^{-10}} \quad = \quad 5 \times 10^{-25} \text{ m/s}$$

$$\text{K.E} \sim \frac{\Delta p_x^2 + \Delta p_y^2 + \Delta p_z^2}{2 m_e} \quad = \quad \frac{3 \times (25 \times 10^{-50})}{2 \times (9.1 \times 10^{-31} \text{kg})} = 4.1 \times 10^{-19}$$
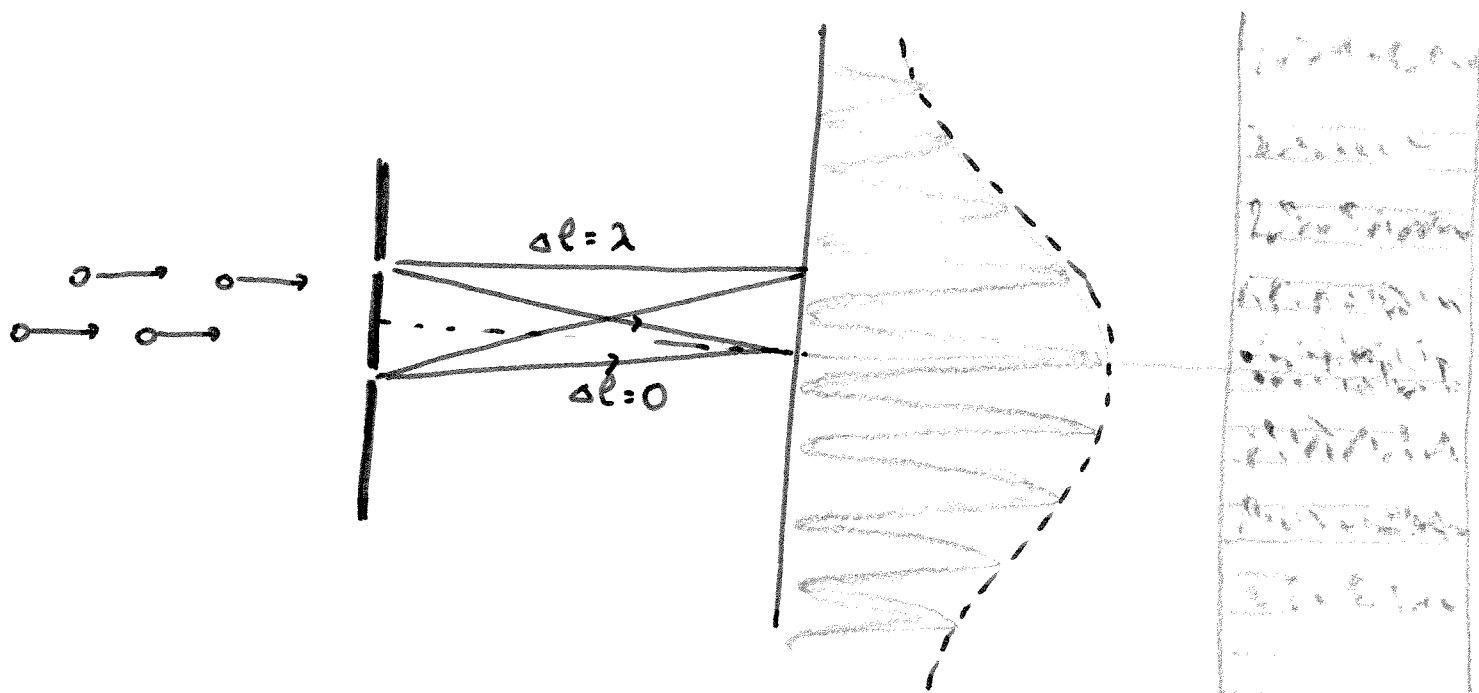
$$\text{K.E} \sim \frac{4.1 \times 10^{-19}}{1.6 \times 10^{-19}} \simeq 2.5 \text{ eV}$$

If I halve the dimensions of the box in each direction, what happens to the kinetic energy?

$\Delta p \rightarrow 2 \Delta p$

$\text{K.E} \rightarrow 4 \text{ K.E}$.

Two Slits

$\Delta \ell = \lambda$

$\Delta \ell = 0$

- If I block one slit ⟹ no diffraction pattern.

- If I determine which slit the e^- goes through ⟹ destroy pattern.

In order to preserve the diffraction pattern, I must preserve the uncertainty about which slit the electron passes through. A FUNDAMENTAL LIMITATION.

39.4   Electron microscope

$$\Delta x \sim \lambda = \frac{h}{\sqrt{2meV}} \qquad \text{resolving power.}$$

e.g $\Delta x \sim 0.1 \overset{\circ}{A}$ what voltage?

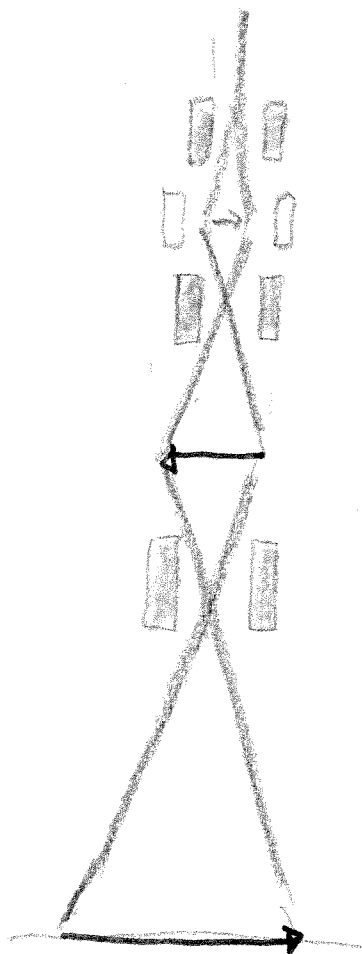$$(\Delta x)^2 = \frac{h^2}{2meV} \qquad V = \frac{h^2}{2m \Delta x^2} \left(\frac{1}{e}\right)$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 (9.1 \times 10^{-31}) \times (0.1 \times 10^{-10})^2} \frac{1}{1.6 \times 10^{-19}}$$

$$= 15 \times 10^3 \ V.$$

$$= 15 \ kV$$

e^- have a tunable short

wavelength $\Rightarrow$ extra resolution.

T.E.M.