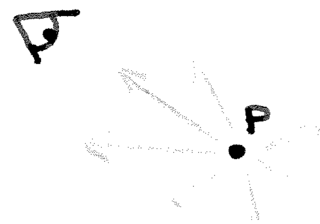
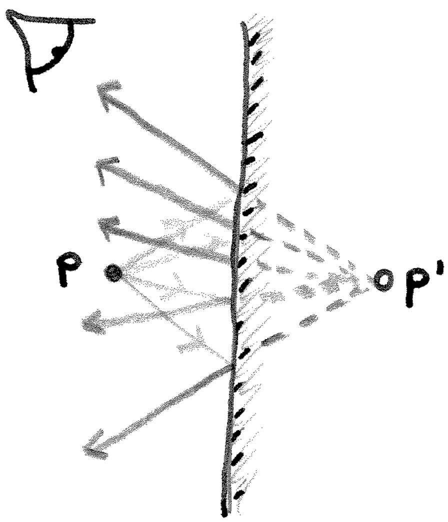
2. Images + Ray Optics

Everything we see – everything, depends on ray optics. From our reflection in the mirror to images of distant galaxies caught by the Hubble Space telescope – all depend on the way light from a distant <u>object</u> is manipulated so that it appears to come from an <u>image</u>. Understanding the relationship between the positions, sizes & orientations of the object & image is the topic of today's lecture, and we shall make extensive use of ray optics.

34.1 Object & image points

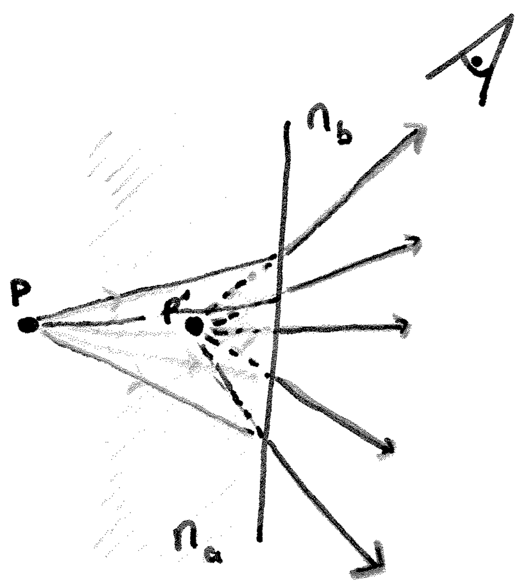
P — the position of a self luminous, point
OBJECT

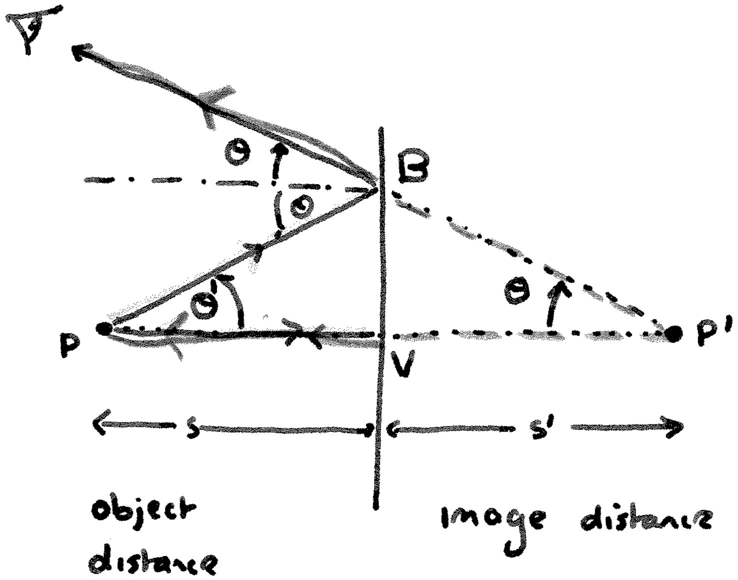




P' — apparent origin of light rays, the IMAGE point.

If rays do not actually pass through the image point P', it is said to be a VIRTUAL IMAGE



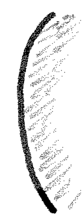


$\triangle$ PVB & P'VB are congruent

$\Rightarrow$ $s = s'$ for all points P & P'.

SIGN RULES


"concave"

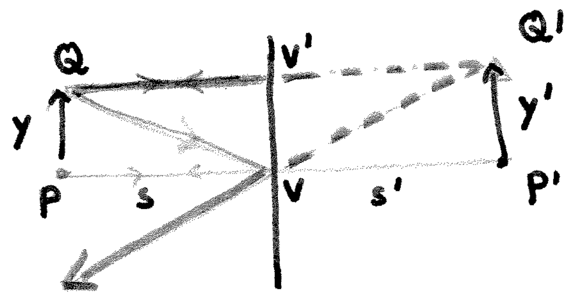

"convex"

Object	same side as incoming light	$s > 0$ (otherwise $s < 0$)
Image	same side as outgoing light	$s' > 0$ (otherwise $s' < 0$)
Radius of curvature	Center of curvature on same side as outgoing light	$R > 0$ (otherwise < 0)

e.g Plane mirror — virtual image on opposite side to outgoing light $s' < 0$

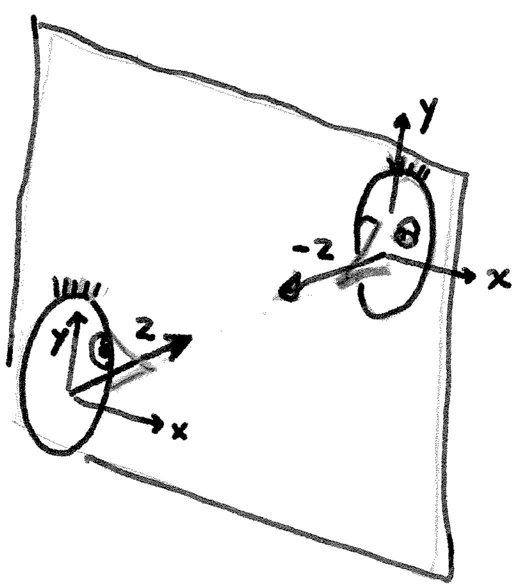
$$s' = -s$$

Magnification



$m = y'/y$

$m' > 0$ erect image.



$z \rightarrow -z$

$x = x'$

$y = y'$

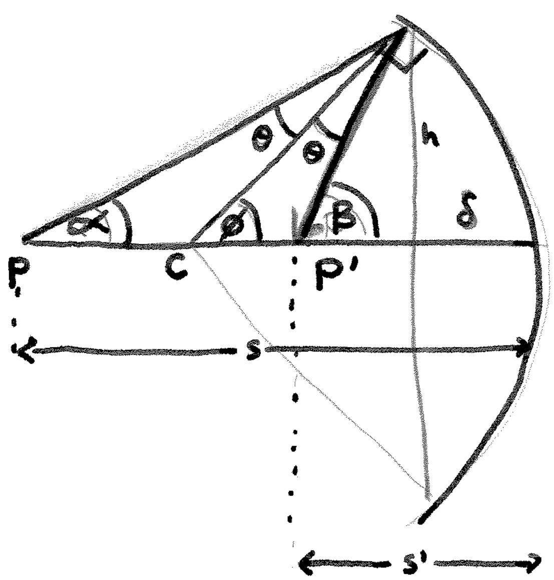
Mirror reverses back to front

34.2 Reflection @ Spherical Surface

$$\boxed{\begin{array}{l} \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R} \\[2em] m = -s'/s \end{array}}$$

object-image reln, spherical mirror

lateral magnification, spherical mirror.



$$\left.\begin{array}{l} \alpha + \theta = \phi \\ \phi + \theta = \beta \end{array}\right\}$$

exterior ∠ = sum of opp interior ∠s.

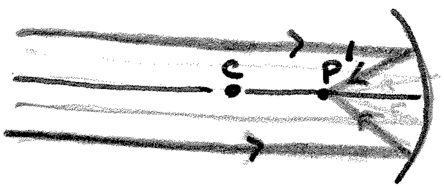
$$\Rightarrow \quad \underline{\alpha + \beta = 2\phi} \qquad (1)$$

$$\frac{h}{s-\delta} = \tan\alpha \qquad \frac{h}{R-\delta} = \tan\phi \qquad \frac{h}{s'-\delta} = \tan\beta$$

$$\alpha \approx \tan\alpha \quad \phi \simeq \tan\phi \quad \beta \simeq \tan\beta \quad s \gg \delta \quad s' \gg \delta \quad R \gg \delta$$

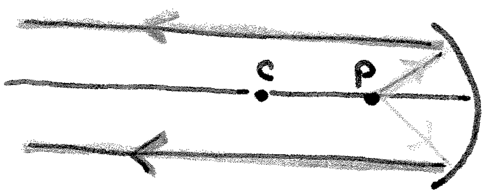
$$\alpha + \beta = \frac{h}{s} + \frac{h}{s'} = \frac{2h}{R} \Rightarrow \boxed{\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}}$$

The object-image relation is an approximate formula, valid for small angles. All rays from $P \longrightarrow P'$.



$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \qquad s' = \frac{R}{2}$$

$$f = \frac{R}{2} = \text{focal length.}$$



$$\frac{1}{s} + \frac{1}{\infty} = \frac{2}{R} \qquad s' = R/2$$

image at infinity — object at focal point

object at infinity — image at focal point.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \boxed{f = R/2}$$

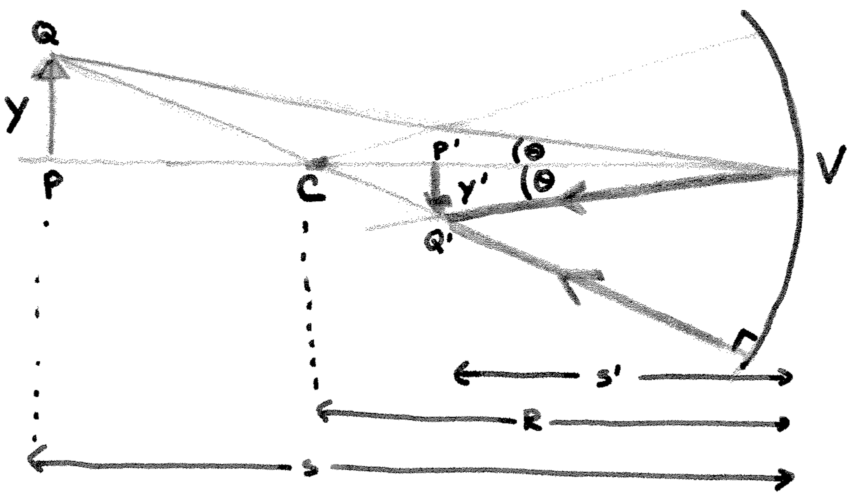
Larger angles — P' moves inward towards mirror $\Rightarrow$ SPHERICAL ABERRATION.

n.b.

- Solar furnaces : place furnace or generator at F.P of large mirror

- Initial problems + Hubble space Telescope in 1993 were caused by spherical aberration.

Image of a concave spherical mirror



$$m = \frac{y'}{y}$$

PVQ & P'VQ' are similar $\Rightarrow$ $\dfrac{y}{s} = -\dfrac{y'}{s'}$ $\quad (y' < 0)$

$$\boxed{m = \frac{y'}{y} = -\frac{s'}{s}}$$

lateral magnification, spherical mirror.

e.g 34.1

concave mirror forms image of filament which is 10cm from mirror at a distance of 3 m.

a) What is radius of curvature?

b) What is the height of the image if the filament is 5mm high?

a) $s = 10\text{cm}, \quad s' = 300\text{cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{10} + \frac{1}{300} = 0.1033 = \frac{2}{R}$$

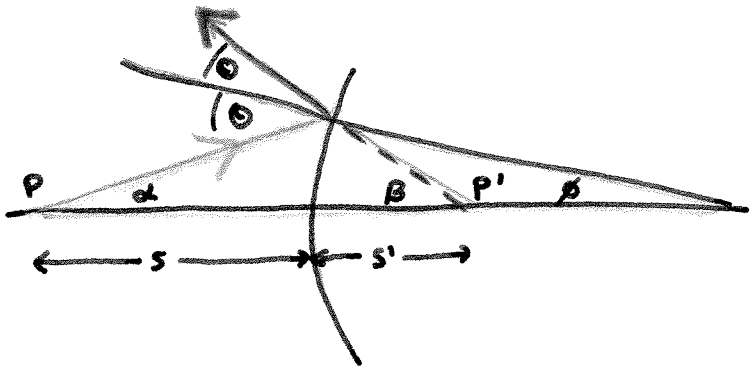
$$R = \frac{2}{0.1033} = \underline{19.4\text{ cm}}$$

b) $\frac{y}{s} = -\frac{y'}{s'}$ $\quad y' = my = -\left(\frac{s'}{s}\right)y = -\left(\frac{300}{10}\right) \times 5\text{mm}$

$$= -30 \times 5$$

$$= \underline{-150\text{mm}}$$

Convex Mirrors

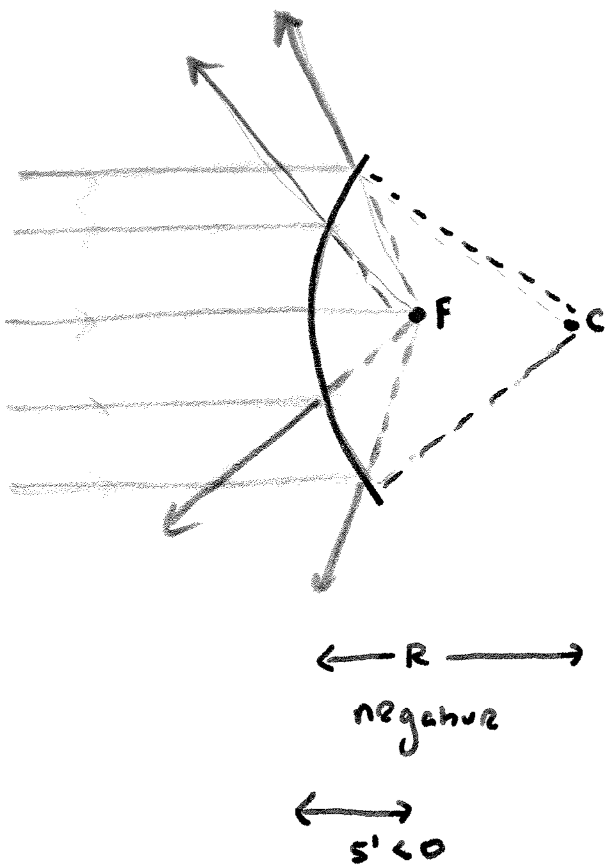


$$\boxed{\begin{array}{c} \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \\ \\ m = y'/y = -\frac{s'}{s} \end{array}}$$

BUT NOW $R < 0$

$\beta = \phi + \theta$

$\theta = \phi + \alpha$

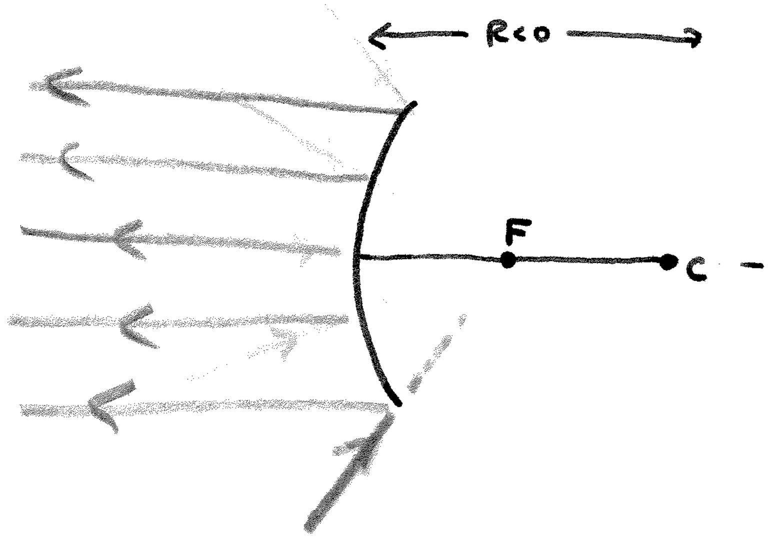


$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R}$$

$$s' = \frac{-|R|}{2}$$

$$\frac{1}{s} + \frac{1}{\infty} = \frac{2}{R}$$

$$s = \frac{R}{2} = -\frac{|R|}{2}.$$

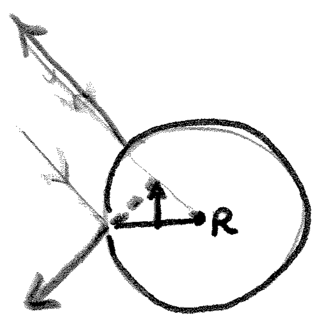


e.g Santa Claus reflected in an ornament

$d_{ornament}$: 7.2cm

$h_{Santa} = 1.6m$, standing $s = 0.75m$ from ornament

How high is the image of Santa?



$$|R| = \frac{7.2}{2} = 3.6 cm$$

$$f = \frac{R}{2} = -1.8 cm$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{-1}{1.8}$$

$$\frac{1}{75} + \frac{1}{s'} = \frac{-1}{1.8} \Rightarrow \frac{1}{s'} = \frac{-1}{1.8} - \frac{1}{75}$$

$$s' = -1.758 cm$$

$$m = -\frac{s'}{s} = \frac{1.758}{75} = +0.0234$$

$$y' = ym = 0.0234 \times 1.6 m = 0.0374m$$

$$y' = 3.74 cm$$

virtual & upright.

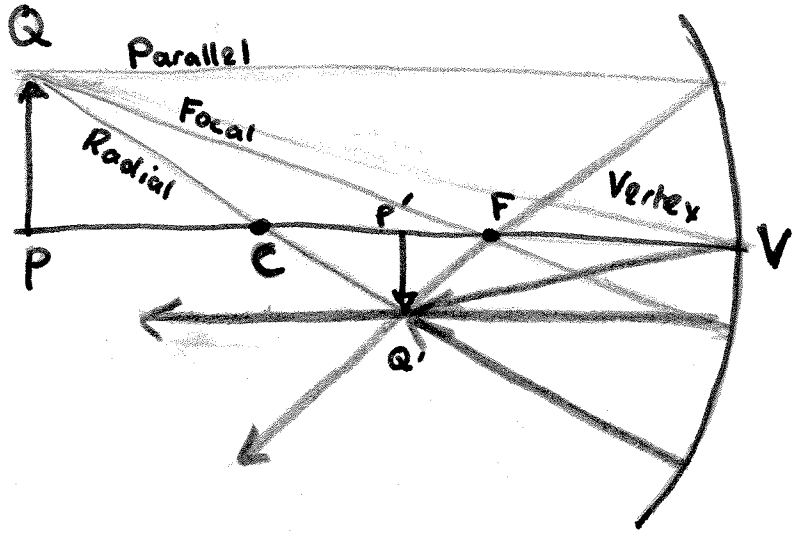
Graphical methods (mirrors) "PaFo RaVe"

1. Parallel ray to axis → passes through focal point F on reflection.

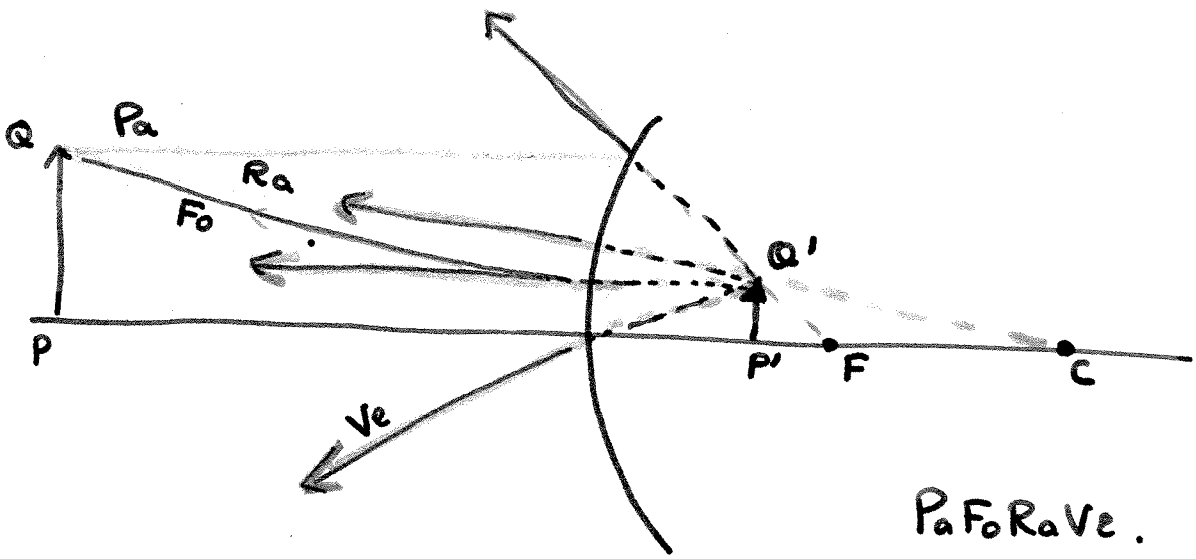
2. Focal ray, passing through focal point → is parallel on reflection.

3. Radial ray through center of curvature C, bounces back along radial path through C

4. Vertex ray, passing through vertex, is reflected forming equal angles with the optic axis.



PaFoRaVe

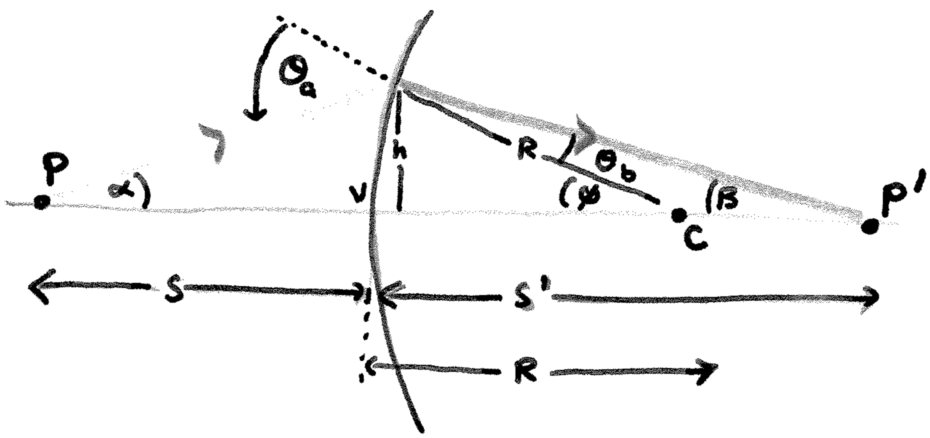


PaFoRaVe.

34.3 Refraction at a Spherical Surface.

$$\boxed{\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}}$$

object-image relation
spherical refracting surface.



- $\left.\begin{aligned} \theta_a &= \alpha + \phi \\ \phi &= \theta_b + \beta \end{aligned}\right\}$ exterior $\angle$ = sum of opposite interior $\angle$s.

- $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \quad n_a\theta_a \approx n_b\theta_b \quad \Rightarrow n_a(\alpha + \phi) = n_b(\phi - \beta)$

- $\tan\alpha = \dfrac{h}{s+\delta} \qquad \tan\phi = \dfrac{h}{R-\delta} \qquad \tan\beta = \dfrac{h}{s'-\delta}$

$$\Rightarrow \quad \alpha \approx \frac{h}{s} \qquad \phi \approx \frac{h}{R} \qquad \beta \approx \frac{h}{s'} \qquad \Rightarrow \quad n_a\left(\frac{h}{s} + \frac{h}{R}\right) = n_b\left(\frac{h}{R} - \frac{h}{s'}\right)$$

1

Snells law becomes

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$n_a \left(\frac{y}{s}\right) = n_b \left(\frac{-y'}{s'}\right)$$

$$\Rightarrow \quad m = \frac{y'}{y} = - \frac{(s'/n_b)}{(s/n_a)}$$

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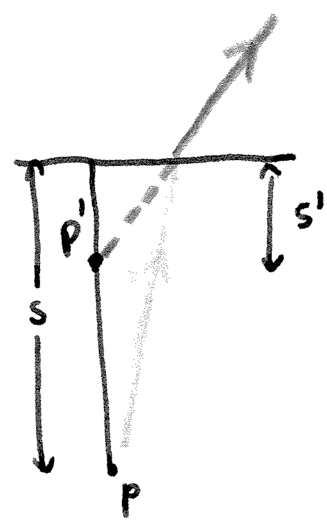
e.g PLANE SURFACE $\quad R = \infty$



$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

$$s' = - s\left(\frac{n_b}{n_a}\right)$$

Swimming pool $\quad s' = -s\left(\frac{n_{AIR}}{n_{WAT}}\right) = -\frac{s}{1.33}$