Physics 228, Lecture 20
Monday, April 11, 2005

Decay Q Values, Fission. Ch. 43:6–7

Note: we are skipping section 43:5

Last time, in order to understand the nuclear transitions involved in our Barium experiment, I skipped over the calculations of energy release. We begin today by filling this in for alpha and beta decay, and for nuclear reactions.

1 Alpha Decay

In alpha decay,

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}. $$

Total energy and momentum are also conserved. If the parent nucleus is initially at rest, the alpha particle will fly off in one direction and the daughter nucleus will recoil in the other, at a slower speed but with equal but opposite momentum. Energy is also conserved, but we must be sure to include the rest-mass energy as well as kinetic energy. The initial energy is the rest mass of the parent X, which is M_Xc^2. The final energy includes both rest energy and kinetic energy for the daughter and for the alpha particle:

$$ M_Xc^2 = M_Yc^2 + K_Y + M_\alpha c^2 + K_\alpha, $$

so the total kinetic energy gain, which we call Q, is

$$ Q = K_Y + K_\alpha = M_Xc^2 - M_Yc^2 - M_\alpha c^2 = (M_X - M_Y - M_\alpha)c^2. $$

where

$$ Q = \text{net change in } \textit{kinetic} \text{ energy} $$

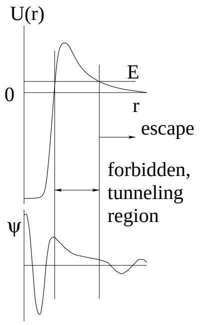
For a decay, Q is also called the *disintegration energy.*

For practical calculations we usually measure the masses in atomic mass units u, and the energy in Mev, so it is useful to state the speed of light squared as

$$ c^2 = 931.494 \text{ MeV/u}. $$

For the decay of radium the Q-value is 4.87 MeV.

How do we understand the alpha decay process, and why is it so slow? We spoke of two models of the nucleus, one of these the independant particle model in which nucleons move independantly. But for many purposes, especially for heavy nuclei, it is more useful to think of alpha particles moving independantly. Consider the potential which one alpha particle feels. If it is within the nucleus, there is a strong binding energy, so the potential is negative. But if it is outside the nucleus, it feels a Coulomb repulsion



$$U(r) = k_e \frac{2e \times (Z-2)e}{r}$$

If we set r to the sum of the radii of helium and radon,

$$\text{At } \; r \approx r_0(4^{1/3} + 222^{1/3}) = 7.6 \times 10^{-15} \text{ m}, \qquad U(r) = 32.6 \text{ MeV}.$$

This is much higher than the energy available in initial state, which has a total energy (relative to the separated final products at infinite separation, which we have taken to be 0, $U(\infty) = 0$) of 4.87 MeV. Classically, if the alpha particle starts inside, it does not have nearly enough energy to roll over the barrier. But by quantum mechanical tunneling, there is a very small probability each time it hits the barrier that it will penetrate and emerge out at 51 fm, where it can enjoy the infinite space now classically allowed for it.

Alpha decay is the primary method by which nuclei which are too heavy to be stable decay to lighter nuclei, though they can also undergo fission, as we will discuss later.

2 Beta Decay

We also discussed β^- and β^+ decay:

$$^A_Z\mathrm{X} \longrightarrow {}^{\;\;A}_{Z+1}\mathrm{Y} + \mathrm{e}^- + \bar{\nu}_e$$

or

$$^A_Z\mathrm{X} \longrightarrow {}^{\;\;A}_{Z-1}\mathrm{Y} + \mathrm{e}^+ + \nu_e.$$

and electron capture.

$$^A_Z\mathrm{X} + \mathrm{e}^- \longrightarrow {}^{\;\;A}_{Z-1}\mathrm{Y} + \nu_e.$$

In these reactions, the total energy is conserved, with some rest mass energy converted to kinetic energy. Just as for alpha decay, that energy is called the Q value. For β decay,

$$Q = K_Y + K_e + K_\nu = (m_X - m_Y - m_e)c^2,$$

where the m_X and m_Y are the masses of the nuclei. I used a lower case m here rather than the capital M which I used in alpha decay. Why? The masses given in the tables, for example Table 43.2, are the masses of the neutral atom, including the Z electrons. The difference of these masses, M_X, from the nuclear mass m_X is primarily the mass of the Z electrons[1],

$$\text{For } {}^A_Z\text{X}: \quad M_X \approx m_X + Zm_e$$

so for alpha decay,

$$\begin{aligned} \text{For} \quad & {}^A_Z\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\text{He}, \\ Q \quad = \quad & (m_X - m_Y - m_\alpha)c^2 \\ & \approx (M_X - Zm_e - [M_Y - (Z-2)m_e] - [M_\alpha - 2m_e])c^2 \\ & \quad = (M_X - M_Y - M_\alpha)c^2. \end{aligned}$$

So we didn't make any mistake in alpha decay if we ignored the mass of the cloud of electrons which surround the decaying nucleus. In beta decay, however, we need to be more careful. For β^- decay,

$${}^A_Z\text{X} \longrightarrow {}_{Z+1}^{\;\;A}\text{Y} + \text{e}^- + \bar\nu_e$$

the Q value is

$$\begin{aligned} Q &= (m_X - m_Y - m_e)c^2 = (M_X - Zm_e - [M_Y - (Z+1)m_e] - m_e)c^2 \\ &= (M_X - M_Y)c^2 \end{aligned}$$

while for β^+ decay,

$${}^A_Z\text{X} \longrightarrow {}_{Z-1}^{\;\;A}\text{Y} + \text{e}^+ + \nu_e.$$

we have

$$\begin{aligned} Q &= (m_X - m_Y - m_{e^+})c^2 = (M_X - Zm_e - [M_Y - (Z-1)m_e] - m_e)c^2 \\ &= (M_X - M_Y - 2m_e)c^2 \end{aligned}$$

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[1] The difference is not entirely due to the rest mass of the electrons — there is binding energy for the atom which subtracts from the total mass. This mostly cancels between the parent and the daughter terms, and we ignore it.

3 Nuclear Reactions

In decay processes all the energy comes from the initial rest mass, but in nuclear reactions we have a collision in which there is kinetic energy in the initial as well as final states. The total energy, kinetic plus rest-mass, is conserved, so if we define the Q value, or *reaction energy*, as **the increase in kinetic energy**, we can also evaluate it in terms of the masses:

$$Q := K_Y + K_b - K_a - K_X = (M_X + M_a - M_Y - M_b)c^2.$$

This value can be positive or negative. In the reaction

$${}^1_1\mathrm{H} + {}^7_3\mathrm{Li} \longrightarrow {}^4_2\mathrm{He} + {}^4_2\mathrm{He},$$

the helium nuclei have more binding energy, and the process is *exothermic*, that is, there is more kinetic energy in the final state than in the initial. The reaction

$${}^1_1\mathrm{H} + {}^7_3\mathrm{Li} \longrightarrow {}^7_4\mathrm{Be} + {}^1_0\mathrm{n}$$

is *endothermic*, requiring more kinetic energy in the initial beam than is left afterwards, because beryllium 7 has more mass than Lithium 7 and the neutron has more mass than the proton.

Notice that in both these reactions, the total atomic mass A is conserved, and so is the number of protons. So far as we know, conservation of baryon number (a generalization of atomic mass or nucleon number) is always true, and conservation of charge is always true. In strong interactions like the two given, that implies the number of protons is conserved, but in weak interactions, like those in beta decay, the change can be changed by the production of electrons, and the number of protons may change.

4 Interactions with Neutrons

We have just discussed nuclear reactions, in which two nuclei, collided together, can rearrange their constituents and emerge as different nuclei. One particularly important class of such reactions or variant perhaps, is what happens when a neutron collides with a heavy nucleus.

Unlike charged nuclei, a neutron can approach a charged nucleus closely without overcoming any coulomb barrier, and neutrons will pass through material, even through atoms, without any interaction most of the time. But if the neutron comes up to the nucleus, the short range strong interactions can cause the neutron to be scattered, or they can be captured by the nucleus, a process called **neutron capture**. This is the more likely to occur the slower the neutron is moving. Even though the neutron captured has almost no kinetic energy, the strong interaction attractive potential energy means there is excess energy in the combined nuclear configuration which results. The nucleus formed is therefore generally in an excited energy state, indicated by a star on its chemical symbol. It will usually give off the excess energy by relaxing to the ground state for the compound nucleus and emitting a gamma ray,

$${}^1_0n + {}^A_ZX \longrightarrow {}^{A+1}_{\ \ Z}X^* \longrightarrow {}^{A+1}_{\ \ Z}X + \gamma,$$

where X can be the nucleus of any element. Even in its ground state, the nucleus ${}^{A+1}_{\ \ Z}X$ may have too many neutrons to be stable, and may decay by beta emission or nuclear fission.

Fast neutrons seldom undergo capture; their interactions are nearly always scattering. For some materials, called **moderators**, most interactions are elastic scattering of the neutron from the moderator nucleus. Although the collisions are elastic, if the average kinetic energy of the moderator nucleus is less than the neutron kinetic energy, some of the neutron's kinetic energy will be transferred to the moderator, slowing down the neutrons. Moderators are used to slow down the neutrons in order to enhance the nuclear capture cross section by other nuclei. For this purpose, it is better if the moderator is a light nucleus, because then the recoil energy from a collision is greater, removing more energy from the neutron.

If the moderator does not capture any of the neutrons, the energy will redistribute itself until the neutrons are in thermal equilibrium with their surroundings, so on the average

$$\frac{1}{2}m_nv^2 = \frac{3}{2}k_BT \Longrightarrow v = 2800\text{m/s at T} = 300\text{ K}, \quad \text{K} \approx 0.04\,\text{eV}.$$

A good moderator can slow neutrons to room temperature in a millisecond.

5 Nuclear Fission

From the semiempirical mass formula, we have seen that the binding energy per nucleon is highest for nuclei of intermediate mass, so that the nucleons in a heavy nucleus might be more stable divided into two smaller nuclei. This process is called fission. For example, from Fig. 43.2, we see that a nucleus with $A \approx 200$ has about 7.5 MeV of binding energy per nucleon, but if it split into two nuclei of $A \approx 100$ it might have as much as 8.3 MeV per nucleon[2]. So it is energetically favorable for the heavy nucleus to split into two pieces, possibly two equal pieces. But to do so, the pieces must pass through a state in which they have lost some of their strong interaction binding (the two pieces have more surface than the one ball) but are still near enough to have a huge coulomb energy. For example, if each piece has $Z = 40$, $A = 100$, $r = A^{1/3} r_0 = 5.6$ fm, the coulomb potential when they are just touching is

S&B 44.8,
Y&F 43.2
5 1/4" × 5"

$$U = k_e \frac{Z^2 e^2}{2r} = 9 \times 10^9 \frac{(40 \times 1.6 \times 10^{-19})^2}{2 \times 5.6 \times 10^{-15}} \mathrm{J} = 206 \text{ MeV},$$

which is still considerably more than the energy the big nucleus has relative to the two fragments, $\approx 200 \times 0.8$ MeV. So to split, the system must tunnel through a forbidden region, just as for alpha decay.

Even a little bit of extra energy makes tunnelling a lot more likely, so when a heavy nucleus captures a neutron and gets the ≈ 7.5 MeV extra energy, instead of γ decay into the ground state the extra energy may be enough to cause fission.

When the nucleus splits up into two pieces, the pieces will have more neutrons than ideal. For example, the more stable parents with $A = 200$ have about 80 protons and 120 neutrons. But if it splits in two, each piece has 40 protons and 60 neutrons, while from Fig. 43.4 we see the stable nuclei with $Z = 40$ only have about 50 neutrons. So some neutrons may be spit off, and instead of two halves, we get two big nuclei and a few neutrons. Actually, the split is not usually into two equal pieces. A typical reaction is

S&B 44.3,
Y&F 43.4
4 1/2"× 6 1/8"

$$\mathrm{n} + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{236}_{92}\mathrm{U}^* \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,\mathrm{n} + 201 \text{ MeV}.$$

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[2] This exaggerates the advantage, because the most stable nuclei with $A = 100$ will have more protons and fewer neutrons that the two halves of the heavy nucleus, and so the halves are not be quite as well bound as we are assuming. Nonetheless, there is still energy to be gained from splitting.

Many other combinations of the protons and neutrons into two big clumps and a few excess neutrons are possible. There are excess neutrons because the lighter daughters don't want to have as big a percentage neutron excess as the parent did, because they don't suffer as much from the Coulomb repulsion.

Fission is interesting because it is a qualitatively different process than the emission of small pieces as we might have in alpha or beta decay. But it is also interesting because of those neutrons in the final state. ${}^{235}_{92}U$ by itself decays with a half-life of 700 million years, so it is pretty stable. But one neutron can trigger one fission which makes about three neutrons which might make three more fissions which makes about nine neutrons which could make nine more fissions which ... . You get the idea — we can have a chain reaction in which the fissioning nuclei cause others to fission. This is what happens in a atomic bomb.

S&B 45.3,
Y&F 43.15
chain reaction
10" × 5 3/4"

In a more moderate form, where the reactions maintain themselves but don't run haywire, we have a nuclear reactor.

The crucial parameter which determines whether we have a chain reaction and whether we have control or an explosion is the **reproduction constant** K, the average number of neutrons from each fission event that cause a new fission event. Clearly if $K < 1$ the reactions will die out, if $K > 1$ the fission rate will blow up exponentially. $K = 1$ is the condition for a reactor to be **critical**.

The reproduction constant is the product of two factors, the number of neutrons emitted per fission, and the fraction of those emitted which cause a new fission. For uranium the average number emitted is 2.5, so to be critical, 40% of those must cause another nucleus to fission. The neutrons released in the fission have energies about 2 MeV, with small probability of being captured by another uranium nucleus to cause fisssion before escaping from the pile. The neutron's efficiency at producing fission is very strongly dependant on how fast it is moving — slow "thermal" neutrons are much more likely to be absorbed and cause the fission than faster neutrons. For this reason, a nuclear reactor requires moderators, which are often carbon or heavy water, to slow down the neutrons. The K value can then be controlled by adjusting the moderator, or more likely by control rods containing absorbers such as cadmium, which absorb neutrons and thereby lower the K value.

We have discussed three types of nuclear decay processes, alpha, beta, and gamma emission, and various nuclear reactions, including spontaneous and neutron-induced fission. Next time we will discuss nuclear fusion.