Physics 228, Lecture 19
Thursday, April 7, 2005

Radioactivity, Nuclear Reactions. Ch. 43:3-4

Note: we will skip section 43:5

1 Radioactivity

We began our discussion of nuclei with the radiation that comes from unstable nuclei when they decay. We saw that there were three types. Alpha rays come from the ejection of a helium nucleus from a large nucleus. Beta rays are electrons emerging at very high energies, or the **antiparticles** of electrons, called **positrons**. Positrons are just like electrons except that they have a charge of $+e$ rather than $-e$. We will learn more about antiparticles in Ch. 44. Electrons and positrons are referred to in equations as e^- and e^+ respectively, where the superscript gives the charge, in fundamental units. Finally, gamma rays we now know to be photons of very high frequency and energy. There is no sharp division between X-rays and gamma rays, but somewhere around 100 KeV energy is a rough dividing line.

In each case, a particle is emitted from the nucleus with quite high energy. These particles often leave the sample and can be detected by various devices, in particular by a Geiger counter. Here we have several radioactive materials, each of which is giving off rays which produce clicks from the Geiger counter. The rays have differing capability to penetrate materials, depending both on which kind they are and on their energy.

Display sources, show counts, show absorbers.

The alpha particles have a charge $+2e$ and a heavy mass, at least compared to the two beta rays, which have charges of $\pm e$, and the gamma rays, which have no charge. So the kinds of radiation can be destinguished by whether they bend in a magnetic field, and in which direction and by how much. The alpha and beta rays are charged and interact much more strongly with matter and so do not penetrate nearly as far as gamma rays, which can pass through inches of lead. Alpha particles also have "strong interactions", and so they are the least penetrating.

When will a nucleus decay and give off one of these particles? Whether a nucleus can decay at all depends on whether there is a lower energy configuration for the constituents that make it up. Whether it decays quickly or slowly depends on how easily the transition to the more stable configuration can take place. This determines a **decay rate** or **decay constant** λ.

Let's do an experiment to see how decays proceed. We have here, in the small cylinder, some radioactive cesium, which undergoes beta decay

start barium demo

$$^{137}_{55}\mathrm{Cs} \longrightarrow {}^{137}_{56}\mathrm{Ba}^* + \mathrm{e}^- + \bar{\nu},$$

a process we will discuss in a few minutes. The star on the barium indicates the nucleus is not in its ground state, and it will decay by emitting a 0.662 MeV γ into its ground state. The cesium is decaying continuously, so barium builds up in the cylinder but can be "eluted" into the liquid pushed out by a syringe, into the little tray. We will place this barium, containing unstable nuclei, under a gamma ray counter and record the decays. In the meantime, I will continue the lecture

The decay rate λ gives the probability per unit time that any extant nucleus will decay. Notice that λ does not change as the number of nuclei remaining do. If there is a one-third chance that a nucleus will decay in the next second, but if after waiting 5 seconds it is still around, the chance it will decay in the following second is still only one in three. When the nucleus does decay, the number of extant nuclei of the type in question will decrease by one. Let $N(t)$ be the number of a certain kind of unstable nucleus in some sample at time t. What can we say about the function $N(t)$? Let's do it with these radioactive dice (just kidding). The dice each have a hole drilled through it, and if it lands on either of the two faces with the hole, the light will shine through it and we will declare it decayed, and remove it from the sample. The probability that any given die will die on any given toss is 1/3, as there are two fatal faces out of a total of 6. Let's see what happens.

do dice demo

Returning to a continuous process of decay, if there are $N(t)$ nuclei present, in time dt each has a probability of decaying of $\lambda\, dt$, so the expected number which will decay in that time is $\lambda N(t)\, dt$, and $N(t + dt) =$

$N(t) - N\lambda dt$, or

$$\frac{dN}{dt}(t) = \lim_{dt\to 0} \frac{N(t+dt) - N(t)}{dt} = -\lambda N(t),$$

which has as its only solution

$$N = N_0 e^{-\lambda t},$$

with N_0 an arbitrary initial number of nuclei present at time $t = 0$. If we detect the radiation coming from this sample, the **activity**, or the number of decays detected per second, is the rate at which the number of extant nuclei is decreasing,

$$R(t) = -\frac{dN}{dt}(t) = -N_0 \frac{d}{dt} e^{-\lambda t} = \lambda N_0 e^{-\lambda t}.$$

There are two units of radioactive activity in use. The standard is the **becquerel**[1], one decay per second. A more traditional unit is the Curie,

$$1\,\mathrm{Ci} \equiv 3.7 \times 10^{10}\ \mathrm{decays/s} = 3.7 \times 10^{10}\,\mathrm{Bq},$$

which is roughly the activity of a gram of pure radium.

How long does it take for half of a sample to decay? That time is called the **half-life** $T_{1/2}$. From our formula for $N(t)$, we get half left when

$$N(T_{1/2}) = \frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}, \quad \text{so} \quad 2 = e^{\lambda T_{1/2}},$$

or

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

S&B 44.13,
Y&F 43.9
4"× 6"

How long will it take for half of what remains to decay? Another half-life. What fraction of the original sample is left after m half-lives? It is 2^{-m}, whether m is an integer or not.

Notice that this calculation gives the expected value of how many nuclei will be left. As a statistical process, there will be fluctuations about this expected number.

[1] Antoine Henri Becquerel discovered radioactivity as a spontaneous emission from uranium which could penetrate paper opaque to visible light and expose photographic plates. This was in 1896.

One application of our calculation is in the dating of fossils. The isotope ${}^{14}_{6}C$ is radioactive with a half-life of 5,730 years. There wouldn't be any of this stuff around except that cosmic ray neutrons are constantly bombarding the atmosphere, participating in a reaction with ordinary nitrogen (which is 80% of the air)

$${}^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}p.$$

This maintains a very small level of ${}^{14}_{6}C$ in the atmosphere, 1 atom out of every million million carbon atoms. Living things are constantly exchanging carbon with the atmosphere, so they also have one ${}^{14}_{6}C$ for every 10^{12} carbons. While that is a small percentage, you have roughly 10^{27} carbon atoms, so there are plenty of ${}^{14}_{6}C$[2]. Once a living thing dies, its remains cease exchanging carbon atoms with the atmosphere, so the carbon 14 which decays is not replaced (it turns into nitrogen), and the fraction of carbon 14/carbon 12 will fall according to the exponential. How old is a fossil that has a ratio of ${}^{14}_{6}C/{}^{12}_{6}C$ that is 1/32 of the atmospheric ratio, or 3.1×10^{-14}?

2 Decay Processes

Now that we know how to describe a decay process in which each nucleus has a certain finite chance of decaying per unit time, we will discuss what some of these processes are, why they occur, and some of their consequences. Then we will also discuss other nuclear reactions.

The three principal decay processes we will discuss are α, β, and γ decay.

Alpha decay occurs when a heavy nucleus emits an α particle, which is just the nucleus of a Helium atom. The helium nucleus has two protons and two neutrons, so what remains after it is subtracted from the parent nucleus is a daughter nucleus with two fewer protons and four fewer nucleons all together:

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}.$$

We saw last time that the binding energy per nucleon drops off for heavy nuclei, so it may well be advantageous for a heavy nucleus to expell the two protons and two neutrons in a package which has a pretty good binding energy per nucleon. The semiempirical mass formula can be used to predict whether this should release energy or not.

[2]Even at this small concentration and with this fairly long halflife, this still gives about 13 decays/minute/gram of carbon.

Notice that we are implicitly assuming that the nucleons themselves are conserved. In particular, the total number, called the baryon number, remains A after the decay, even though they are separated into the daughter nucleus Y, with $A-4$, and the helium nucleus with 4. The number of protons is also conserved in alpha decay, so the daughter is left with two fewer than the parent has, as the helium has run off with two. The separate conservation of the number of neutrons and of the number of protons is a feature of the **strong interactions**, and also of the **electromagnetic interactions**.

An example of alpha decay is the decay of radium in the earth, which leaves a daughter of radon gas, which seeps into your basement to give you cancer. The decay

$${}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

has a half-life of 1,600 years. The radon has about four days to seep into your house and get to you before it decays.

Total energy and momentum are also conserved. This determines the kinetic energy of the decay products, called the Q value. But in order to discuss the three decays today, we will put off Q values until next time.

Beta Decay

S&B 44.3, Y&F 43.4
stable nuclei
4 1/2" × 6 1/4"

In looking at the line of stability we see there are regions where a nucleus might not be too heavy to be stable but might have too many or too few protons. As we will learn when we discuss particle physics in Chapter 44, protons and neutrons are not separately conserved, though their total seems to be (right now - more on this later). The decays

$$\begin{aligned} n &\longrightarrow p + e^- + \bar{\nu}_e, \\ p &\longrightarrow n + e^+ + \nu_e \end{aligned}$$

can occur by the **weak interactions**, provided there is enough energy available. The neutrino ν_e and antineutrino $\bar{\nu}_e$ are massless particles, somewhat like the photon, while the electron and its antiparticle, the positron, have a rest energy of 0.511 MeV. So if the rest energy of a parent nucleus ${}^{A}_{Z}\mathrm{X}$ is more than .511 MeV more that that of a possible daughter ${}^{A}_{Z\pm1}\mathrm{Y}$, there will be enough energy to allow the decay of one of its constituent nucleons, either

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + \mathrm{e}^- + \bar{\nu}_e$$

or

$$ {}^A_Z\mathrm{X} \longrightarrow {}_{Z-1}^{\;A}\mathrm{Y} + \mathrm{e}^+ + \nu_e. $$

For historical reasons the electrons and positrons emerging from such decays are often called negative and positive beta rays.

Notice that while the number of protons changes in these reactions, both the baryon number (that is, the total number of protons and neutrons) and the total charge are conserved.

One important example of beta decay is the decay of ${}^{14}_{6}\mathrm{C}$, which we discussed earlier, and which has important applications in archeology for dating fossils.

One other reaction is closely related to β^+ decay, which is called electron capture. Instead of emitting a positron the nucleus can absorb an electron, usually from the K shell:

$$ {}^A_Z\mathrm{X} + \mathrm{e}^- \longrightarrow {}_{Z-1}^{\;A}\mathrm{Y} + \nu_e. $$

This reaction does not require as much excess energy in the parent nucleus, because an electron rest mass is eaten up rather than produced.

Gamma Decay

In beta decay the nucleus suddenly finds itself with a proton where there used to be a neutron, or vice versa. Thinking in terms of the independant particle model, this new proton is approximately in the state the neutron had been in, but this need not be the lowest energy state that a neutron is free to occupy. So it is not surprising that often a beta decay will be followed by a transition in which the nucleus goes to a lower energy state, giving off its excess energy in the form of a photon, or gamma ray. This is an interaction with the electromagnetic field, so it is an electromagnetic, rather than a weak or strong interaction. A nucleus is sometimes shown as being in an excited state with a star, so for example,

$$ \begin{aligned} \text{The beta decay:} \qquad & {}^{12}_{5}\mathrm{B} \longrightarrow {}^{12}_{6}\mathrm{C}^* + e^- + \bar\nu \\ \text{is followed by:} \qquad & {}^{12}_{6}\mathrm{C}^* \longrightarrow {}^{12}_{6}\mathrm{C} + \gamma \end{aligned} $$

and the beta decay

$$ {}^{137}_{55}\mathrm{Cs} \longrightarrow {}^{137}_{56}\mathrm{Ba}^* + \mathrm{e}^- + \bar\nu, $$

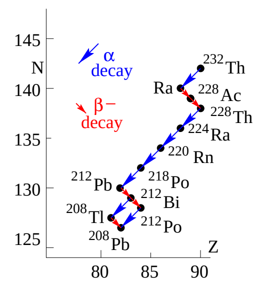
is followed by

$$^{137}_{56}\mathrm{Ba}^* \longrightarrow {}^{137}_{56}\mathrm{Ba} + \gamma.$$

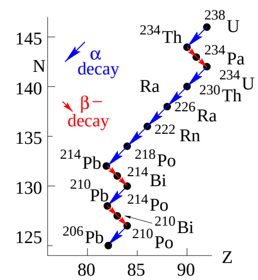
In fact, that is the experiment we set going and forgot about. Let's go back and look.

Discuss barium experiment.

There are other sequences of decays which often follow each other. For the nuclei which are too heavy to be completely stable but are long-lived, there is a large surplus of neutrons over protons. When the nucleus decays by alpha emission, the daughter is a somewhat lighter nucleus with the same excess as the parent. But for a lighter nucleus, a lesser excess would be better. So the daughters of alpha decays often will β^- decay to convert one of those neutrons to a proton.



Thorium series



Uranium series

3 Nuclear Reactions

Decay processes are nuclear transformations which occur spontaneously, but it is also possible for nuclei to be transformed when they collide. Scientists can accelerate nuclei in beams and shoot them at the nuclei of stationary atoms, or even collide two beams at high energies. Often, but not always, a small particle such as a proton or alpha is the projectile. If a particle a hits a nucleus X, and if what emerges is particle b and nucleus Y, we write this reaction

$$\mathrm{a} + \mathrm{X} \longrightarrow \mathrm{Y} + \mathrm{b}.$$

Nuclear physicists also write it as $\mathrm{X(a,b)Y}$, but we won't use that arcane notation.

Summary

We have discussed three types of nuclear decay processes, alpha, beta, and gamma emission, and two specific forms of nuclear reactions, fission and fusion, although there are many other nuclear reactions.