Physics 228, Lecture 7
Thursday, Feb. 10, 2005

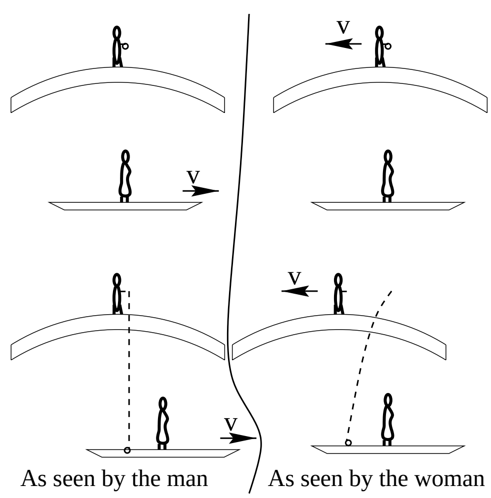
The Principle of Relativity. An expansion on Ch 37:1;

1 Galilean Relativity

Today we begin our descussion of what is generally called modern physics. Although there are many phenomena and several scales on which we discuss physics, the most dramatic breakthroughs of modern physics are two abrupt changes in our perception of reality — relativity and quantum mechanics.

We begin with relativity. It is part of our culture that Einstein proclaimed that everything is relative, and this has been used by postmodern professors of English to justify teaching rap music instead of Shakespeare. Of course Einstein never said "everything is relative". In fact, in his first breakthrough theory, the special theory of relativity, not much is relative that wasn't relative 300 years earlier. He simply showed that the relationship between these relative quantities is changed in a most surprizing way.

Relativity was invented, as far as I know, by Galileo, whose picture looks down on you as you enter this auditorium, and who flourished around 1600. Consider a man on a bridge and a woman in a boat floating, at constant velocity, under the bridge. The man on the bridge drops a stone at the moment he sees the woman beneath him. The stone does not hit her, however. According to the man on the bridge, the stone has travelled in a straight line, with



[Man's view of stone:]

$$x(t) = 0, \quad y(t) = H - \frac{1}{2}gt^2,$$

while the woman has moved at

[Man's view of woman:] $\qquad x(t) = vt, \quad y(t) = 0.$

"Damn", he says, "I should have realized that by the time the stone gets down to the river, at $t = \sqrt{2H/g}$, she will no longer be at $x = 0$".

Now this Neanderthal isn't very bright and certainly isn't very nice, but his physics is correct. However our lady in the boat would not describe things as he does. As it is a calm day and the stream straight and flowing at a constant rate, she is feeling in her prime, and she is perfectly entitled to consider herself at rest,

$$\text{Woman's view of herself:} \qquad x'(t) = y'(t) = 0.$$

She sees the man moving in the negative x direction with speed v, and when he drops the stone, it starts with an x component of velocity. Therefore it moves in a parabola:

$$\text{Woman's view of stone:} \qquad x'(t) = -vt, \quad y'(t) = H - \frac{1}{2}gt^2.$$

Her physics is also impeccable — in fact she is using the same physics from 123 as the man is. But they have different values of the positions, as a function of time, and of the velocities, of the two objects in question, her head and the stone. But they **do** agree on the consequences — she does not get knocked out.

Thus both observers agree on what might be called the physical events, but do not agree on the coordinates which describe these events. An event (in relativity) need not be anything of importance; it is merely a point in space at a particular time. The stone hitting the boat is an event. Its coordinates are $x = 0, y = 0, t = \sqrt{2H/g}$ according to the man and at $x' = -v\sqrt{2H/g}, y' = 0, t' = \sqrt{2H/g}$ according to the woman. More generally, the coordinates of any event as measured by the woman are given in terms of those of the man by

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

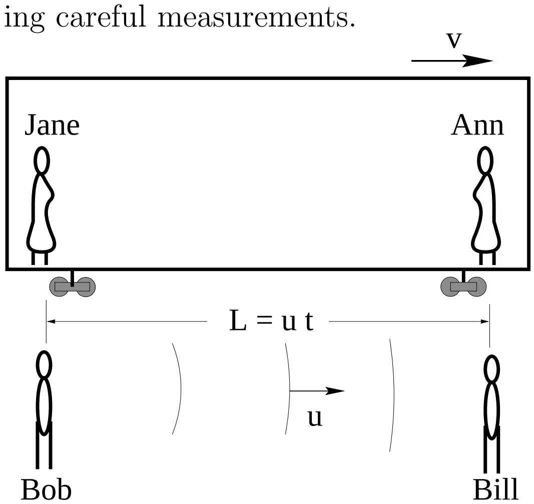
If we follow the motion of a point object, such as the stone, we have a path in space as a function of time. To find the velocities each ascribes to the object, which we will call u and u', we can differentiate

$$u'_x = \frac{dx'(t)}{dt'} = \frac{d}{dt'}(x - vt) = \frac{dx}{dt} - v = u_x - v$$

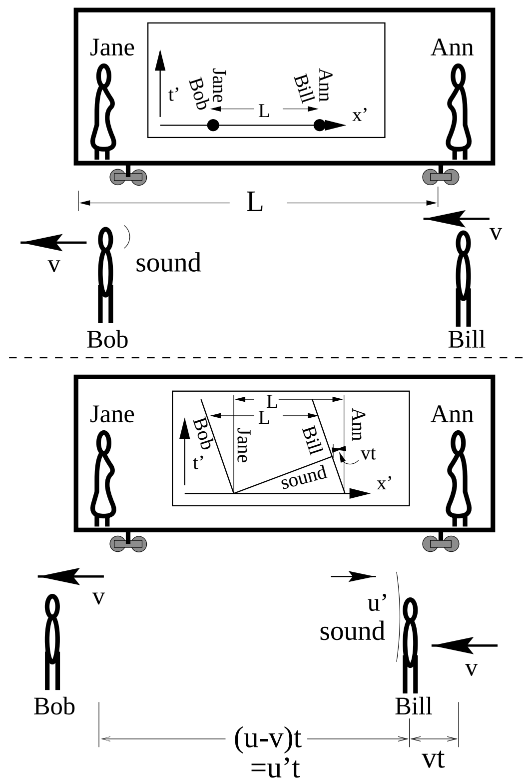
while $u'_y = u_y$ and $u'_z = u_z$. Differentiating again gives accelerations — and both observers agree on the accelerations, because v is a constant and does not contribute to the next derivative. Thus both can agree on the forces and the truth of Newton's second law, the basis of motion.

Galileo recognized the importance of this — that the laws of physics are the same whether formulated in the reference frame of the man or of the woman, even though the coordinates and velocities are described differently. Thus there is no way to tell who is moving and who is at rest. It is possible for Galileo and for us to maintain this view, that all "inertial reference frames" are equivalent in the sense that the laws of physics are the same and there is no way to consider one better than the other, because the laws under consideration involved accelerations, not velocities, and the accelerations are the same, so if the forces are unchanged, everything works for both observers.

Now consider another situation: Under suitably specified conditions, the speed of sound is $u = 333$ m/s. So if Bob and Bill are $L = 10$ m apart, and Bob shouts to Bill at $t = 0$, he will hear it at time $t = L/u = 30$ milliseconds later. Suppose at this moment, Anne and Jane are in a train car passing at a velocity v, also a distance L from each other. They are carrying meter-sticks and clocks with them, and making careful measurements.



As seen by the men



As seen by the women

Now how do the women describe this situation? They see Bill moving to the left at speed v. By his reaction they know that Bill heard the sound at $t = 30$ ms. During that time Bill has moved to the left a distance vt. The sound has thus not travelled a distance L but rather $L - vt$. To help us see this the ladies have drawn a diagram of positions as a function time (but drawn, as is the usual convention, with time running upwards on the blackboard) of the men and the sound wave. As the sound has only travelled a distance $L - vt$ in time t, its speed is $u' = (L/t) - v = u - v$. This is just as we discussed for the stone. But how do the women explain that this sound is not moving at 333 m/s, as we claimed the laws of physics determine? Do they admit their reference frame is not as good as that of the men? **No way!**

What the women explain is that the speed of sound is 333 m/s **relative**

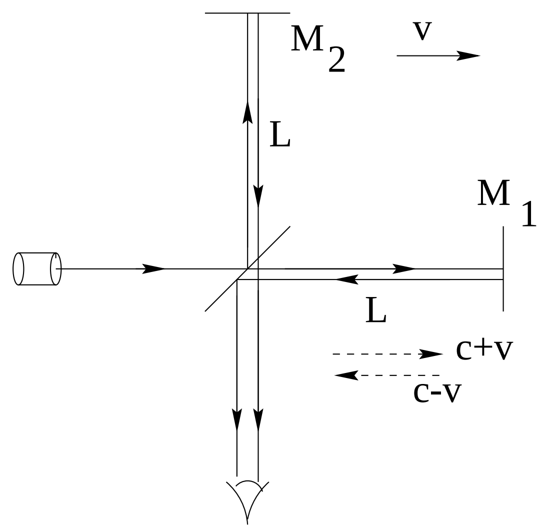
to the air in which the sound travels. If Jane shouted to Ann inside their enclosed car, the sound would travel at 333 m/s, as the air is at rest, but the air outside the car is blowing along at speed roughly v, to the left, and relative to that wind the speed of sound is fixed by the physical laws.

Now, instead of a sound wave, let us ask about what happens if Bob sends Bill a pulse of light. We have learned that Maxwell's equations tell us light and other electromagnetic waves travel at a speed c specified by the laws of physics. With respect to what? One possibility is, with respect to the source. That doesn't seem likely, because we understood the velocity of the wave from the equations of the electric and magnetic fields themselves, without any reference to how the wave was produced. Another possibility is that the velocity is with respect to the medium in which it travels. But light does not need any detectible medium — it travels through the best vacuum we can produce on Earth with no problem, and in fact also through cosmological space, which we believe to have much less matter than the best vacuum we can produce on Earth. If there is a medium in which it travels, it is ethereal indeed, so we will call it the **ether**.

But if the ether has a reference frame in which it is at rest, and other reference frames need to relate light's motion to this ethereal one, all inertial reference frames are not equal. How undemocratic! But physics needs to search for the truth, not what is politically correct. If the speed of light is determined with respect to the ether, we should be able to determine our velocity relative to the ether. How can we do that?

2 Michelson-Morley Experiment

One way we have seen to make very sensitive measurements of light travel time is the Michelson interferometer. If the two arms are of equal length, and if the light moves at velocity c with respect to the interferometer, the interference will be constructive. But suppose the light moves at c with respect to the ether, assumed at rest with respect to a reference frame which is itself moving at velocity v in the direction of one arm, rather than with respect to the reference frame, call it Mick's, in which the apparatus is at rest. How long does that take, and how many wavelengths will it be, and how does that compare to the wave that goes in the perpendicular direction?



If the light moves at speed c with respect to the ether, and the ether is moving to the right at speed v with respect to the equipment, then Galileo's calculations, or common sense, would tell us that the speed of the light is $c+v$ with respect to the mirror on the way to M_1 and $c-v$ on the way back. The time it takes to travel is therefore

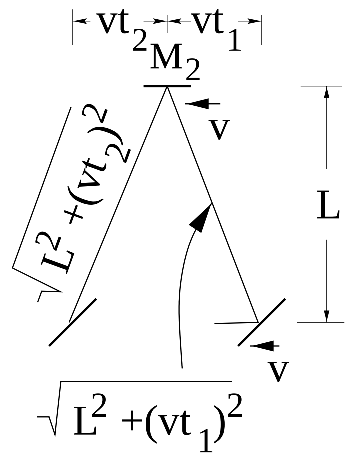
$$\text{Time to get to } M_1: \qquad \frac{L}{c+v}$$

$$\text{Time to return from } M_1: \qquad \frac{L}{c-v}$$

Total time:

$$t_L = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2L}{c}\frac{1}{1-v^2/c^2}$$

Now consider the light moving along the transverse arm. This will be easier to calculate if we do so from the point of view of an observer named Esther who is at rest with respect to the ether. From Esther's point of view, every piece of Mick's Michelson interferometer is moving to the left at speed v. Let's suppose the light leaves the half-silvered mirror at $t = 0$ and $x = 0$, and bounces off M_2 at time t_1, and then takes an additional time t_2 to return to the half-silvered mirror. During the first part of the trip, according to Esther, the mirror moves to the left by vt_1, so the light moves on the hypotenuse of a right triangle with sides vt_1 and L. As it is travelling at speed c, we have



$$L^2 + (vt_1)^2 = (ct_1)^2,$$

The same applies to the trip back — the half-silvered mirror is also moving, and again the full speed of the light is along a hypotenuse,

$$L^2 + (vt_2)^2 = (ct_2)^2.$$

These equations have the same solution,

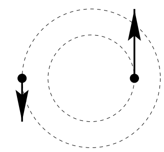
$$t_1 = t_2 = \frac{L}{\sqrt{c^2 - v^2}} = \frac{L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

and the total time for the transverse trip is

$$t_T = t_1 + t_2 = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}} \neq t_L = \frac{2L}{c}\frac{1}{1 - v^2/c^2}$$

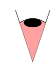
These two times are not the same, so the phases of the two beams will differ when they recombine at the half-silvered mirror, and we will be able to check, by means of the interference pattern, how much out of phase they are and therefore what the speed of the Earth through the ether is.

Michelson attempted to measure this speed in 1881 and then, in 1887, did a more conclusive experiment with Morley, and they found **nothing**. To the accuracy of their measurements, the Earth was at rest. How could this be?



1) A geocentric view would have been pretty generally accepted before 1600, but it is hard to maintain that the Earth, even the center of the Earth, is a nonaccelerating reference frame because it circles the sun, and the acceleration is enough to change the velocity of the Earth, during 6 months, by more than enough to have been detected.

2) is ruled out not only by the fact that Maxwell's equations have been very thouroughly checked, but also because we can tell that the differential delay in receiving light from binary stars is inconsistent with observations.



3) would imply a viscous drag that would eventually slow down the planets. It is also inconsistent with an effect called the aberation of starlight, by which the position of stars moves during the course of a year, due to the Earth's motion, beyond what you might expect from parallax

4) This idea is called the Lorentz-FitzGerald contraction. It is completely ad-hoc, but it does give an explanation of the Michelson-Morley results. But then we need to explain what force the ether can exert that always compresses all objects by the same ratio.

5) Why be chicken. Let's try a radical approach. We will see that it explains both the Michelson-Morley result, the Lorentz-Fitzgerald contraction, the atom bomb, and a whole lot of other things.

3 Einstein's Relativity Postulates

The radical approach to the observation that our reference frame seems to preferred by having the laws of physics referred to it is to claim, contrary to common sense, that all inertial reference frames are preferred, as given by Einstein's two postulates:

1. The laws of physics, all of them, are the same in all inertial frames of reference

2. The speed of light in vacuum is a fixed number c, independant of the velocity of the emittor or absorber.

We have already seen that this is inconsistent with our Newtonian concepts of how the coordinates describing events in one reference frame are related to those of another. In particular, if I shine light at you but you are moving towards me at half the speed of light, the light is not approaching at 3c/2, as you might expect, but only at c. This will take some careful thinking to work out.

But that will have to wait until next week.

4 Summary

- Relativity is the idea that all reference frames moving at constant velocity with respect to each other are equivalent, that the laws of physics hold equally in each.

- The different frames describe the same events differently. In particular, if frame S' is moving in the x direction with velocity v, according to observers S, then

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

$$\text{and} \quad u'_x = u_x - v, \quad u'_y = u_y, \quad u'_z = u_z$$

- The Michelson Morley experiment shows either all distances parallel to the motion of a reference frame shrink by a factor of $\sqrt{1 - v^2/c^2}$ or light must travel at speed c in all directions in each reference frame, regardless of relative motions

- Einstein postulated that relativity is true and light travels as speed c is each reference frame, even though that contradicts the second item.