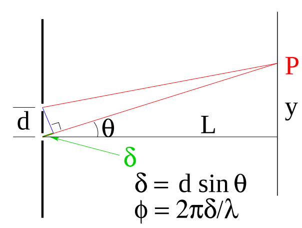
Physics 228, Lecture 4
Monday, Jan. 31, 2005

Interference. Ch 35:3-5, Multiple Thin Slits

1 Intensity of a Double Slit

Last time we discussed qualitatively the interference between the light coming from two slits, where we saw that in some positions there was constructive interference and in some destructive. We start today by asking quantitatively how the intensity varies, not only at the maxima and minima but also in between. This is not primarily to get the answer for the double slit, but to develop techniques for understanding multiple slits. A diffraction grating, for example, can be considered a multiple slit situation with perhaps 10000 slits. And for diffraction we will treat a wider slit as a collection of infinitely many infinitesimal slits. But let's start with just two.



We will assume the two slits are equal sized and equally illuminated, so the electric fields radiating from them will have equal amplitudes. And the angle from the two slits to our screen point P are almost the same, so the amplitudes of the electric fields at the point P, generated by each of the slits, will be the same, but they will not in general have the same phase. We can arbitarily set one field's phase to zero, so

$$E_1 = E_0 \cos \omega t, \quad E_2 = E_0 \cos(\omega t + \phi), \text{ with } \phi = 2\pi \frac{\delta}{\lambda}.$$

The electric field at P is the sum,

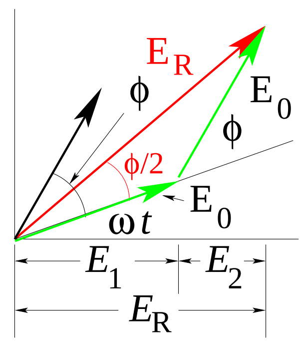
$$E_P = E_1 + E_2 = E_0[\cos \omega t + \cos(\omega t + \phi)].$$

We learned how to add oscillating terms with the same frequency but different phases when we dealt with RLC circuits. One way is to use trig addition formulas, like

$$\cos A + \cos B = 2 \cos\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right),$$

and another way is to draw phasors.

We draw the phasor for E_1 as a vector of length E_0 at an angle of ωt, recalling that it is really the projection on the x axis that tells us what the electric field is at any moment. The phasor for E_2 is also of length E_0 but is at an angle of $\omega t + \phi$. Both of these vectors are rotating, but the angle between them is fixed at ϕ. To add vectors we translate one of them so its tail rests on the head of the other, giving us the isosceles triangle, so each base angle is $\phi/2$, and the base has length $|E_R| = 2E_0 \cos(\phi/2)$. The electric field at P is the x projection of this,



$$E_P = 2E_0 \cos(\phi/2) \cos(\omega t + \phi/2).$$

The intensity of the light at P is proportional to time average of the square of this field,

$$I_P \propto 4E_0^2 \cos^2(\phi/2)\langle\cos^2(\omega t + \phi/2)\rangle = 2E_0^2 \cos^2(\phi/2).$$

Recalling that $\phi = 2\pi\delta/\lambda = 2\pi d \sin\theta/\lambda$, and expressing the intensity in terms of the maximum intensity (at $\theta = 0 = y$), we have

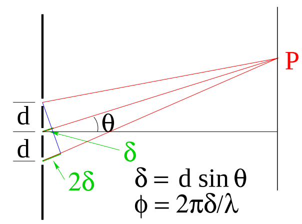
$$I = I_{\max} \cos^2 \left(\frac{\pi d \sin\theta}{\lambda}\right) = I_{\max} \cos^2 \left(\frac{\pi d}{\lambda L} y\right),$$

where the last expression assumes $y \ll L$.

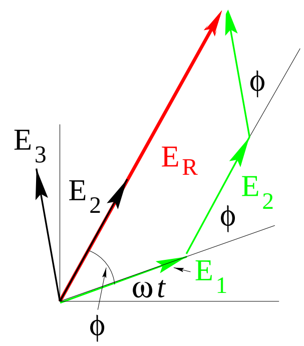
The cosine squared has maximum whenever the argument is an integer multiple of π, and zero when it is a multiple which is half an odd integer, verifying the results we obtained last time.

2 Phasor analysis of multiple slits

Suppose we have more than two slits, evenly spaced. At a point P far away on the screen, each successive slit will be a distance $\delta = d \sin\theta$ further away than the previous, so the contribution of that slit will lag by an angle $\phi = 2\pi\delta/\lambda$. So if we take the longest path to have phase zero,



$$E_1 = E_0 \cos\omega t \quad (1)$$
$$E_2 = E_0 \cos(\omega t + \phi) \quad (2)$$
$$E_3 = E_0 \cos(\omega t + 2\phi) \quad (3)$$



The phasors are thus three equal length vectors with angles ϕ and 2ϕ between them. For two slits, to get cancelation we needed $\phi = 180°$, with the two slits out of phase.

Lets see what happens with three slits. We see that we get cancellation when $\phi = 120°$ and again at $\phi = 240°$, with a small maximum at 180°.

Show 3 slit slideshow. Pause at 40, 120, 180, 240°, then to 360.

Notice that the big maximum has $E = NE_0$, where N is the number of slits and E_0 is the electric field due to a single slit. Thus the intensity at the maximum will be proportional to N^2. Lets try this with a bigger N, say $N = 9$

Show 9 slit slideshow. Don't pause, but go slow

3 Phase change on reflection

One elegant way to do a double slit experiment without slits, in principle[1], is to take a point source of light and hold in near a mirror, shining on a distant

[1] Apparently it is hard to do this in practice, and we don't have the apparatus

screen. Each point on the screen receives some light directly from the source and some from the image of the source in the mirror, which looks just like a second source, and is coherent because the light really comes from the same source. So we would expect to see an interference pattern on the screen, with a bright spot at the mirror, because that point is equidistant from the source and its image, and fringes away from that spot. That is almost exactly what we do see, but with bright and dark reversed!

The reason is that reflection can change the phase of the light. That some change must take place can be seen if we consider an electromagnetic wave reflected perpendicular to a surface. As the direction of the wave is that of $\vec{E} \times \vec{B}$, if the direction is reversed one of $\vec{E}$ or $\vec{B}$ must be. We will not go into the details, but instead just use this rule

> **An electromagnetic wave undergoes a 180° phase shift when it is reflected by a higher n medium than the one in which the wave is travelling. A conductor counts as a higher n medium.**

So the wave reflected in the mirror has its phase changed by 180° which is what made points we expected to have constructive interference actually have destructive, and vice-versa.

The same reversal of phase applies to waves on a string as well. If a wave on a string hits a point at which it is unable to move, a reflected wave with the opposite sign returns, with the same amplitude. If the wave is on a tense string that changes to a heavier string (without a bulky knot) the reflected wave also has its phase reversed, but its amplitude is diminished and some of the wave is transmitted onto the heavier string. If the wave comes the other way, there is again both reflection and transmission, but in this case both the transmitted wave and the reflected wave have the same sign at the transition point as the incident wave, there is no phase shift upon encountering a less dense medium.

4 Thin film interference

Now I am going to show you light reflected off a soap bubble. The soap is transparent and colorless, the light beam is white, but we see a colorful display. Why?

Show reflection from new soap film.

Consider light incident on a thin piece of transparent material. Some of the light will be reflected from the surface entering the material, and some will pass through. Of the light which passes into the material, some will be reflected from the second surface, come back to the first surface and be transmitted. These two beams will interfere with each other, and whether that is constructive or destructive depends on the phase difference between the two paths.

The ray which is reflected off the front surface undergoes, according to what we have just learned, a phase shift of 180°, or $\phi_1 = \pi$. The other beam has a phase shift due to the extra path length that it travels. Let's assume the light is perpendicular to the material, so the extra distance the light has travelled is $2t$, where t is the thickness of the material. The phase change from this extra distance is

S&BV5 Fig 37.16
Two reflections
from thin film
2 7/8"× 3 1/2"

$$\phi_2 = 2\pi \times \frac{2t}{\lambda_n},$$

where λ_n is the wavelength in the material, shorter than the wavelength λ of the light in vacuum by a factor of n. So the phase difference between the two paths is

$$\Delta\phi = \pi\left(\frac{4tn}{\lambda} - 1\right) = 2\pi \begin{cases} m & \text{for constructive interference} \\ m + \frac{1}{2} & \text{for destructive interference} \end{cases},$$

where m means any integer. Dividing out the 2π and adding $\frac{1}{2}$, we have

$$2tn = \begin{cases} (m + \frac{1}{2})\lambda & \text{for constructive interference} \\ m\lambda & \text{for destructive interference} \end{cases}.$$

Soap bubbles provide a good example of the bright colors thin films can display. The colors are due to the fact that the different colors have different wavelengths, so some constructively interfere and are bright, while others are destructively interfering and are not present in the light your eye receives.

Other examples are thin films of oil floating on water, and the feathers of a peacock.

If we wait long enough, the soap film gets very thin. Once the thickness becomes less than $1/4\ \lambda$ for all visible light, all frequencies begin to interfere destructively, and the reflection is dark. This happens at the top of the soap bubble, but because we are viewing the relection through a lens which inverts the image, we see the dark area growing from the bottom.

Another thin-film effect is known as Newton's rings. If a convex spherical piece of glass is placed on a flat piece of glass, near the point of contact there will be a thin film of air. Even though the film now has a lower index of refraction than the surrounding medium (which in this case is glass) the conditions for constructive and destructive interference are the same. Here, though, the thickness is not constant, so one gets curves of constructive interference whereever the thickness is the right constant value. If the glass is truly spherical, we will get bright and dark circles, but if it is distorted even by a fraction of a wavelength, the fringes will be visibly distorted. This is one way lens grinders can check their work.

We have here a spherical lens of large R resting on a plane glass surface. We view Newton's rings by transmitted light rather than reflected light, as discussed in the book. For us, the interference is between light which is never reflected and light which is reflected twice off going from air into glass. So the extra phases cancel, and we have a bright spot where the glass surfaces touch.

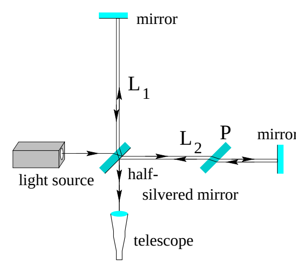
Demo Newton's rings on overhead.

A technology has developed using properties of thin films. A simple interface between two transparent materials must reflect a certain percentage of the light, depending on the relative indices of refraction, but if we put a thin film which produces destructive interference in the reflected light, more of the light will be transmitted. This is used to increase the efficiency of solar cells and high quality lenses in cameras.

5 Michelson Interferometer

The fact that two rays of light will interfere with an effect which depends on the difference of the distances travelled on a scale of wavelengths of light provides an extremely accurate way of measuring distances and wavelengths.

This is the principle by which a Michelson interometer works. A light beam hits a partially-silvered mirror, which reflects some of the light upwards and lets some of it pass to the right. The deflected beam travels a distance L_1 to a mirror and is then reflected back, and some of it passes through the half-silvered mirror to the telescope. The other part of the light goes a distance L_2 to another mirror, is reflected back and this time reflected by the half-silvered mirror.



Thus we have two beams interfering as they enter the telescope. To make sure the situation is symmetrical between the two beams, we place a plate of glass P in the second beam to compensate for the fact that the first beam (the one initially reflected by the half-silvered mirror) had to pass through the glass there twice. Thus if $L_1 = L_2$, the two beams will be in phase and will interfere **constructively**. But if one length is just 1/4 of a wavelength longer than the other, considering the trip to the mirror and back we see that the two beams will be totally out of phase (180° out of phase) and will interfere **destructively**.

We will see that this apparatus played an important historical role in motivating Einstein's theory of special relativity, but it is also very useful as an optical instrument. L_1 and L_2 can differ by some macroscopic distance, and for monochromatic light we can easily tell what the remainder is when we divide that distance by half the wavelength of the light. When using two different wavelengths, this provides a very accurate way of measuring the ratio of the wavelengths.

6 Summary

- The intensity received at an angle θ from a double slit varies as

$$I = I_{\text{max}} \cos^2 \left(\frac{\pi d \sin\theta}{\lambda} \right).$$

- The intensity for double or multiple slits is effectively calculated by using phasors.

- Light reflected from a higher n material or a conductor undergoes an extra 180° phase shift.

- Thin film interference comes from interference between the light reflected from the front and back surfaces. If the film is surrounded by material of lower n, there is constructive interference for $2nt = (m+\frac{1}{2})\lambda$ and destructive for $2nt = m\lambda$ and destructive for

- A Michelson interferometer slits a beam into two and recombines it, in a way that provides very accurate measurements of relative distances and of wavelengths.