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Physics 228, Review for Second Midterm

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1 Relativity

- Relativity is the idea that all reference frames moving at constant velocity with respect to each other are equivalent, that the laws of physics hold equally in each.

- The different frames describe the same events differently. In particular, if frame S' is moving in the x direction with velocity v , according to observers S , then the **Galilean transformation** gives

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

and $u'_x = u_x - v, \quad u'_y = u_y, \quad u'_z = u_z.$

- The Michelson Morley experiment shows either all distances parallel to the motion of a reference frame shrink by a factor of $\sqrt{1 - v^2/c^2}$ or light must travel at speed c in all directions in each reference frame, regardless of relative motions.

- **Einstein** postulated that **relativity is true** and **light travels at speed c in every inertial reference frame**, even though that is inconsistent with the Galilean transformations, which need to be replaced by the Lorentz transformations.

- An **event** happens at a particular point in space and at a particular time, but the coordinates which describe that point in space and time depend on the reference frame.

- The time interval Δt between two events in a reference frame (S) in which the two events take place at the same point is called the

proper time interval. In any other frame, moving with speed v with respect to S , the time interval between the events is longer by a factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \Delta t' = \gamma \Delta t.$$

- Lengths perpendicular to the relative motion of two reference frames appear the same, $L'_\perp = L_\perp$.
- The length of a physical object as measured in a reference frame (S) in which it is at rest is called the **proper length** of the object. If another frame S' is moving with velocity v in the direction of the length of the object, relative to S , the length of the object as measured by S' is contracted by a factor of γ ,

$$L' = L/\gamma = L\sqrt{1 - v^2/c^2}.$$

- **Simultaneity is relative:** two events which are simultaneous in one reference system need not be simultaneous in another. Even which event came first differs from one system to another.

- In the **twin paradox**, we see the importance of applying these rules only in **inertial** reference frames, which do not change their velocities.

- The **Lorentz transformation** describes the way to determine the coordinates of an event in one reference system given the coordinates of the event in another. If the S' coordinate system is moving in the $+x$ direction relative to S with velocity v ,

$$x' = \gamma x - v\gamma t, \quad y' = y, \quad z' = z, \quad t' = \gamma t - \frac{v\gamma}{c^2}x,$$

$$x = \gamma x' + v\gamma t, \quad t = \gamma t' + \frac{v\gamma}{c^2}x'.$$

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2 Quantum Mechanics

- And the velocities (for \vec{v} in the x direction) are related by

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}, \quad u'_y = \frac{u_y}{\gamma(1 - vu_x/c^2)}, \quad u'_z = \frac{u_z}{\gamma(1 - vu_x/c^2)}.$$

- A light signal emitted at frequency f_s is perceived by an observer to have a different frequency f_o given by

$$f_o = f_s \sqrt{\frac{c - u}{c + u}},$$

if the observer and source are moving away from each other with velocity u .

- The **momentum** of a particle of (rest) mass m and velocity \vec{u} is not $m\vec{u}$ but

$$\vec{p} = m\gamma\vec{u}, \quad \text{with } \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}.$$

- And the **energy** of such a particle (including the previously ignored “rest energy” mc^2) is

$$E = mc^2\gamma$$

- mass is **not** conserved, but the energy and momentum of an isolated system are. When mass is lost, the equivalent energy Δmc^2 appears in some other form, often in the form of kinetic energy.
- In particular, if an atomic nucleus of mass M_I decays into fragments with masses which sum up to M_F , the fragments will have kinetic energies which sum up to $Q = (M_I - M_F)c^2$. Because c^2 is a big number in everyday units, even a conversion of 1/20 of one percent of the mass of a kg of uranium, say, is a large amount of energy.

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- The **blackbody spectrum**, which gives the power per unit area per unit wavelength radiated by a black body, is given by

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)}.$$

This form cannot be understood classically, but was “explained” by an *ad-hoc* model of Planck in which he assumed that the energy of oscillators is quantized. This replaced the Rayleigh-Jeans law, $I(\lambda, T) = 2\pi ck_B T/\lambda^4$, although they agree for long wavelengths.

- The constant of nature h in this empirical law is known as **Planck’s constant**,

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}.$$

- The maximum of this intensity as a function of wavelength is at $\lambda_{\max} = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$. This is called **Wien’s displacement law**.
- In the **photoelectric effect**, when electrons are emitted from a metal by photoemission from light of frequency f , they emerge with a maximum kinetic energy of

$$K_{\max} = hf - \phi,$$

where ϕ depends on the metal and is called the **work function** of that metal. The intensity effects the rate at which electrons are emitted but not whether they are emitted nor what the maximum energy at which they are emitted is. This is explained by viewing light **as composed of quanta** with energy hf . Intense light has many quanta, not more energetic ones.

- The **Compton effect** describes the scattering of an electron by light. It can be understood as a scattering with a photon, at which the final

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photon has a longer wavelength than the incident photon, with

$$\Delta\lambda := \lambda' - \lambda_0 = \frac{hc}{m_e}(1 - \cos\theta),$$

where θ is the angle of scatter of the photon. This is understood as a collision of two particles, an electron and a photon.

- Atoms radiate light only at certain characteristic frequencies. For hydrogen, the **Rydberg formula** gives

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad \text{with } R_H = 1.097\,373\,2 \times 10^7 \text{m}^{-1},$$

and n_i and $n_f < n_i$ are positive integers. The “lines” or wavelengths with $n_f = 2$ are called the **Balmer series**.

- Generally the spectrum of radiation from atoms can be understood by assuming the atom has certain discrete energy levels, and the transition gives off or absorbs a single photon with the extra energy.
- In the **Bohr model**, electrons can travel in orbits around the nucleus obeying classical laws except that only orbits with angular momentum $L = n\hbar$ are allowed, and radiation is not considered in the classical motion.
- For circular orbits, this predicts discrete orbits with

$$r = a_0 n^2, \quad v = \frac{k_e e^2}{n\hbar}, \quad E = \frac{E_0}{n^2},$$

where

$$a_0 = \frac{\hbar^2}{k_e m_e e^2} = 5.29 \times 10^{-11} \text{m}, \quad E_0 = -\frac{1}{2} \frac{k_e^2 m_e e^4}{\hbar^2} = -13.606 \text{eV}.$$

- Light is emitted or absorbed by atoms as they make a **transition** from one **energy level** to another, emitting or absorbing a photon with the energy required by energy conservation.

- For hydrogen, the Bohr model thus explains the Rydberg formula for the spectrum, including explaining the Rydberg constant R_H in terms of fundamental physical constants

- **Bohr’s correspondence principle** says that for large excitation levels, the quantum mechanical results should agree with classical expectations.

- According to de Broglie, particles also have associated frequencies and wavelengths given by

$$E = hf, \quad p = \frac{h}{\lambda}.$$

Whatever these represent, it is possible for a particle to interfere with itself, as demonstrated by electron scattering off a crystal by Davisson and Germer. This also explains the quantization of L in the Bohr model, as requiring that the electron interfere constructively with itself going around the orbit.

- Ordinary particles as well as photons exhibit interference, for example in a double slit experiment, as predicted by their **de Broglie wavelength** $\lambda = h/p$.
- The wave function that has this wavelength (and frequency), generally called ψ , is the **probability amplitude**, a complex number whose length squared, evaluated at some position and time, is proportional to the probability of finding the particle in a small volume around that position at that time.
- Because positions are uncertain to the extent the wave function is spread out, the **Heisenberg uncertainty principle** tells us that the position and velocity can never be simultaneously determined to

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an accuracy better than

$$\Delta x \Delta p \geq \frac{\hbar}{2}.$$

- A state which lasts a finite time Δt does not have a precise energy but has an energy uncertainty

$$\Delta E \Delta t \geq \frac{\hbar}{2}.$$

This also applies to any measurement of energy which is done within a time interval Δt . As a result, spectral lines have some small spread in the value of their wavelengths.

- Complex numbers are a form of two-dimensional vector with an additional multiplication rule (which is not a dot product or a cross product). The ordinary real numbers are one line in this two dimensional space, and negative numbers have square roots which are the “imaginary” numbers, lying on the other axis in this space.
- De Broglie’s concepts of wavelengths and frequencies for particles can be turned into a differential equation for the wave function, called the Schrödinger Equation.
- The time-independent Schrödinger Equation (in one dimension)

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2} [E - U(x)] \psi(x)$$

gives the wave function for a stationary state, that is, a state of definite energy.

- The probability density is given by $|\psi|^2$, and the expectation value for the position is

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi(x)|^2 dx.$$

- Localized waves cannot have a single wavelength, but are wave packets consisting of a superposition of amplitudes over a spread of wavelengths.
- a particle trapped in a box with infinitely strong walls has states labelled by a positive integer n with wave functions and energies

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \quad E_n = \frac{\hbar^2}{8mL^2} n^2.$$

3 Atomic Physics

- The states of a hydrogen atom or hydrogen-like ion with nuclear charge Ze are solutions of Schrödinger’s equation with a potential $U(r) = -k_e Z e^2 / r$. The wave function can be written as a function of r times a function of spherical angles.
- The solutions are indexed by
 - **principle quantum number** $n = 1, 2, 3, \dots$
 - **orbital quantum number** $\ell = 0, 1, \dots, n - 1$
 - **orbital magnetic quantum number** $m_\ell = -\ell, -\ell + 1, \dots, \ell - 1, \ell$

but the full state of the electron also needs to have the **spin magnetic quantum number** specified as $\pm \frac{1}{2}$.

- All states with a given n are called a **shell**, and all of those having a given ℓ are called a **subshell**, and are given letters s, p, d, f,
- The energy of a hydrogen-like state depends only on the principle quantum number

$$E_n = -13.606 \frac{Z^2}{n^2} \text{ eV}.$$

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- The hydrogen wave functions have a decaying exponential e^{-r/na_0} , possibly multiplied by polynomials in r (for $n > \ell + 1$) and by polynomials in (x, y, z) of order ℓ . The probability to lie within a range of values of r is proportional to $r^2|\psi|^2$ and peaks at a nonzero value, a_0 for the ground state 1s.

- The orbital angular momentum of an electron is quantized:

$$\begin{aligned} L_z &= m_\ell \hbar, & -\ell \leq m_\ell \leq \ell, \\ |L| &= \sqrt{\ell(\ell+1)} \hbar. \end{aligned}$$

- The radial wave functions have zeros at some values of r if $n > \ell + 1$, giving certain values for which the probability density vanishes. Angular momentum states other than the s have an angular dependence such that there are angles at which the wave function vanishes, but s states are spherically symmetric.
- Nonzero angular momentum gives rise to a magnetic moment and an energy in a magnetic field:

$$\vec{\mu} = \frac{e}{2m_e} (\vec{L} + g\vec{S}) \quad \text{and} \quad U = -\frac{e\hbar B}{2m_e} (m_\ell + gm_s) = -(m_\ell + gm_s)\mu_B B,$$

where $g \approx 2$ but more accurately $g = 2.00231930439$, where μ_B is called the **Bohr magneton**:

$$\mu_B = \frac{e\hbar}{2m_e} = 5.788 \times 10^{-5} \text{ eV/T}.$$

- This gives the **Zeeman effect**, a splitting of the spectral lines of an atom in a magnetic field.
- Electrons have an intrinsic **spin** in addition to the orbital angular momentum. $S_z = \hbar m_s$, $m_s = \pm \frac{1}{2}$.

- In an inhomogeneous magnetic field, Stern and Gerlach showed discrete bending paths, verifying that L_z can take on only $2\ell + 1$ discrete values and spin only two discrete values.
- Atoms are built up to form the Periodic Table by adding electrons according to the **Pauli exclusion principle**:
 - No two electrons can occupy the same quantum state.
- Each electron looks for the unoccupied state of lowest $n + \ell$, and among those the subshell of lowest n .
- within a subshell, **Hund's rule** applies:
 - In filling orbitals of equal energy, the electrons try to keep their spins unpaired.
- “noble” or “inert” gases are those with full subshells for which the next electron would go into a higher n state.
- Though most energy levels are not calculable, we can estimate
 - for an electron beyond the filled shells, $n > n_{\text{shell}}$, $E_n = -13.6 \frac{Z_{\text{eff}}^2}{n^2}$ eV.
 - For the innermost shell, or for the $n = 2$ state if only one $n = 1$ orbital is filled,

$$E_L = -\frac{13.6}{2^2} (Z - 1)^2 \text{ eV}, \quad E_K = -13.6 (Z - 1)^2 \text{ eV},$$
 so the X-ray will have an energy of approximately $10.2(Z - 1)^2$ eV. This $L \rightarrow K$ transition produces an X-ray known as the K_α line.
- In general, transitions between states by emission or absorption of one photon means $f = \Delta E/h$.

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- deexcitation can be **spontaneous** or it can be **induced** by light of the right energy.
- A **laser** (for visible light) or a maser (for microwaves) works by a chain reaction of induced deexcitations.
- **fluorescence** and **phosphorescence** occur when light excites an atom which then deexcites in multiple steps.

[We are skipping section 9, for which you are not responsible.]

4 Molecules and Solids

- ionic, covalent, Van der Waals and hydrogen bonds
- electronic, translational, rotational, and vibrational energies.
- Rotation energy states $E_J = \frac{\hbar^2}{2I}J(J+1)$, with allowed transitions $J \leftrightarrow J-1$ giving $\Delta E = \hbar^2 J/I$, where the moment of inertia $I = \mu r^2$, $\mu = m_1 m_2 / (m_1 + m_2)$.
- Vibrational energies $E_v = (v + \frac{1}{2})\hbar f$, $f = \sqrt{k/\mu}$, for integer $v \geq 0$, with allowed transitions $v \rightarrow v \pm 1$ having $\Delta E = \hbar f$. independent of v .
- Transitions can combine a change of ± 1 for J and ± 1 for v .
- Crystalline solids have a repeating lattice structure of several different arrangements. Ionic and covalent bonds generally give insulators.
- Another way of understanding the electronic states is by bands of electronic energy levels which come from the individual atomic states overlapping.

- Conductivity requires electrons to move from the states they occupy in the absence of an applied field to nearby states to cause an imbalance in current. This can happen only if there are available empty states close by in energy. If there are partially filled bands or filled bands overlapping unfilled bands, this will happen and we have a conducting metal. If the highest energy filled band is separated from an empty band by an energy gap of an eV or more, these transitions are very improbable and we have an insulator. In semiconductors there is a gap but it is smaller, so that there is a nonnegligible probability that electrons can make the jump and conduct. This may be enhanced by doping with acceptors or donors of extra electrons.