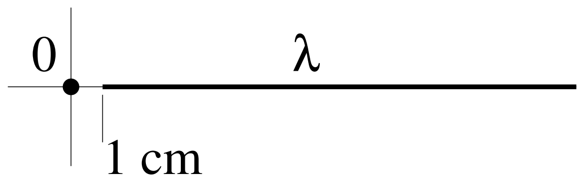
Solutions
Physics 227 - First Common Hour Exam
Profs. Shapiro and Schnetzer

1. A semi-infinite uniform line of charge that extends along the $+x$-axis from $x = +0.01$m to $x = +\infty$ establishes an electric field of magnitude 27.0N/C at the origin, pointing in the $-x$ direction. The linear charge density λ, of the line, in C/m, is:



Use the fact that the electric field due to a point charge is

$$\vec{E} = (kq/r^2)\hat{r}$$

and the principle of superposition

$$\vec{E}_{tot} \;=\; \vec{E}_1 \;+\; \vec{E}_2 \;+\; \ldots$$

Each element dx of the line produces an electric field at P that has magnitude

$$dE = kdq/x^2 \;\; \text{where} \;\; \mathrm{dq} = \lambda \mathrm{dx}$$

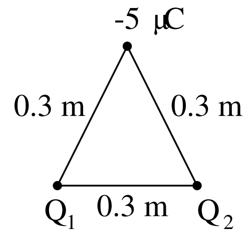
is the amount of charge in dx and x is the distance the element dx is away from P. The fields from all of the dx's point in the negative x-direction so there is only an x-component. We then simply add up these x-components to get the magnitude of E_{tot}. In other words we integrate over each element dx in the line

$$E \;=\; \int dE \;=\; \int_{0.01\ \mathrm{m}}^{\infty} k\lambda dx/x^2$$
$$= -k\lambda/x \mid_{0.01\ \mathrm{m}}^{\infty} \;=\; k\lambda/0.01\ \mathrm{m}$$

from this we get

$$\lambda \;=\; E(0.01\ \mathrm{m})/\mathrm{k}$$
$$= \;(27\ \mathrm{N/C})(0.01\ \mathrm{m})/(9\times 10^9\ \mathrm{N\cdot m^2/C^2})$$
$$= \;3.00\times 10^{-11}\ \mathrm{C/m}$$

2. Two charges, $Q_1 = +7.00\ \mu$C and $Q_2 = -5.00\ \mu$C are 0.300 m apart. How much work is done in bringing a third charge of $-5.00\ \mu$C to form the third vertex of an equilateral triangle, 0.300 m from each of the other charges?



The work we want is the work done by an external force is

$$W = \Delta U$$

where U is the potential energy. [Note that the work done by the electrical forces is $-\Delta U$.] Each pair of charges contributes a potential energy which is

$$U = kq_1q_2/r_{12}$$

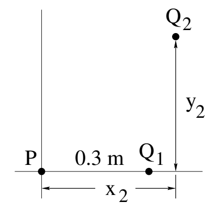
where r_{12} is the distance between the two charges. In the initial state, there is only the potential energy between the two charge Q_1 and Q_2. In the final state there is still this potential energy but there is also the potential energy between the third charge and Q_1 and between the third charge and Q_2. So,

$$W = \Delta U = U_f - U_i = kq_3Q_1/0.3\text{ m} + \text{kq}_3\text{Q}_2/0.3\text{ m}$$

$$= ((9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5.0\times10^{-6}\ \text{C})(7.0 - 5.0)\times10^{-6}\ \text{C})/(0.3\ \text{m}))$$

$$= -0.3\ \text{J}$$

3. Charge $Q_1 = +7.00\ \mu$C is located on the x-axis at $x =$ 0.300 m, and $Q_2 = -20.0\ \mu$C is located at $x_2 = 0.400$ m, $y_2 = 0.400$ m. What is **magnitude** of the total electric field at the origin, point P?



Use the fact that the electric field due to a point charge is

$$\vec{E} = (kq/r^2)\hat{r}$$

and the principle of superposition

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = -(kQ_1/(0.3\text{ m})^2)\hat{\text{i}}$$
$$\vec{E}_2 = -(kQ_2/((0.4\text{ m})^2 + (0.4\text{ m})^2))(\cos 45^o\hat{\text{i}} + \sin 45^o\hat{\text{j}})$$
$$E_{tot,x} = -k((Q_2/0.32\text{ m}^2)(1/\sqrt{2}) + (\text{Q}_1/0.09\text{ m}^2))$$

$$= (9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)((-20\times10^{-6}\text{C}/0.32\text{ m}^2)(1/\sqrt{2}) + (7.0\times10^{-6})/(0.09\text{ m}))^2$$
$$= 3.02 \times 10^5\text{ N/C}$$

$$E_{tot,y} = k(Q_2/0.32\text{ m}^2)(1/\sqrt{2})$$

$$= 3.98 \times 10^5\text{ N/C}$$

Magnitude of E_{tot} is

$$\sqrt{E_{tot,x}^2 + E_{tot,y}^2}$$

$$= 5.00 \times 10^5\text{ N/C}$$

4. Consider a uniformly charged, insulating sphere of radius $R = 5$ m. The total amount of charge in the sphere is $Q = 6.0$ nC. What is the total electrical flux through a spherical surface of radius $r = 3.0$ m that is concentric with the insulating sphere?

Use Gauss's Law.

$$\text{The total flux through a closed surface} = \mathrm{Q_{inside}}/\epsilon_0$$

To find the flux through a spherical surface of radius 3.0 m, we need to find how much charge is in a sphere of radius 3.0 m. Since the 5.0 m sphere is uniformly charged, the charge inside the 3.0 m radius sphere is equal to the charge on the 5.0 m radius sphere times the ratio of the volumes. In other words,

$$Q_{inside} Sthethe = Q(3.0\text{ m})^3/(5.0\text{ m})^3$$

the flux then is

$$\begin{aligned} Q_{inside}/\epsilon_0 &= Q(3.0\text{ m})^3/(5.0\text{ m})^3/\epsilon_0 \\ &= 4\pi k Q(3.0\text{ m})^3/(5.0\text{ m})^3 \\ &= 4\pi(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{-9}\text{ C})(3.0\text{ m})^3/(5.0\text{ m})^3 \\ &= 146\text{ N}\cdot\text{m}^2/\text{C} \end{aligned}$$

16. An infinitely long, straight string has a uniform linear charge density of λ expressed in C/m. A sphere of radius R has its center at a point on the string. What is the electric flux through the sphere?

a) $\lambda R/2\epsilon_0$
b) λ/ϵ_0
c) $\lambda R/2\pi\epsilon_0$
d) $\lambda/2\pi R\epsilon_0$
e) $2\lambda R/\epsilon_0$

Gauss's Law: The total flux through a closed surface is

$$= Q_{inside}/\epsilon_0$$

$$\begin{aligned} Q_{inside} &= \lambda\times\ \text{(length of string that is inside sphere)} \\ &= \lambda 2R \end{aligned}$$

So, flux is

$$\lambda 2R/\epsilon_0$$

17. An infinitely large, horizontal plane carries a uniform charge density of $+10\mu\text{C/m}^2$. What is the vertical component of the electric field a distance of 0.20m above the plane?

a) (-2.8×10^6) N/C
b) $(+1.13 \times 10^6)$ N/C
c) $(+5.65 \times 10^5)$ N/C
d) (-5.65×10^5) N/C
e) (-1.13×10^6) N/C

The electric filed due to a large plane of charge is perpendicular to the plane and has a magnitude of

$$\sigma/(2\epsilon_0)$$

independent of position for points whose distance from the plane is small compared to the size of the plane. So, the answer is

$$\begin{aligned} E &= \sigma/(2\epsilon_0) = 2\pi k\sigma \\ &= 2\pi(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10 \times 10^{-6} \text{ C/m}^2) \\ &= 5.65 \times 10^5 \text{ N/C} \end{aligned}$$

18. A metal sphere centered at the origin has a radius R and a net charge Q. The electric field at the point $x = 5R$ is E_o. The sphere is replaced by a different metal sphere centered at the origin with radius $2R$ and a net charge Q'. The field at $x = 5R$ is still E_o. Therefore, we can deduce:

a) Q′ = Q b) Q′ = 4Q c) Q′ = $\frac{1}{2}$Q d) Q′ = 2Q
e) Q′ = $\frac{1}{4}$Q

At a distance x from a conducting charged sphere the electric field has a magnitude of

$$E = kQ/x^2$$

As long as x is greater than the radius of the charge sphere the electric field does not depend upon the radius of the sphere. So for the two cases, if the electric field is the same, than

$$Q' = Q$$

19. A +1-μC point charge experiences a force F due to a +2-μC point charge that is 2-cm away. If the +2-μC charged is removed and replaced by a +4-μC charge that is 4-cm away, the 1-μC charge now feels a force F' where:

a) $F' = \frac{1}{4}\,F$ b) $F' = 4F$ c) $F' = F$ d) $F' = 2F$
e) $F' = \frac{1}{2}\,F$

The magnitude of the force between two point charges is given by

$$F = kq_1q_2/r^2$$

So, if the value of one of the charge is doubled and the separation of the two charges is increased by a factor of two then the resulting force between the charges is

$$F' = 2F/2^2 = F/2$$

20. A +2μC charge is located at the point (x, y) = (5cm, 0cm) and a +1μC charge is at the origin. The +2μC charge is moved to the point (3cm, 4cm). The force on the +1μC charge:

a) has changed in magnitude only.
b) has changed in neither magnitude nor direction.
c) has changed in direction only.
d) has changed sign.
e) has changed in both magnitude and direction.

The magnitude of the force between two point charges is given by

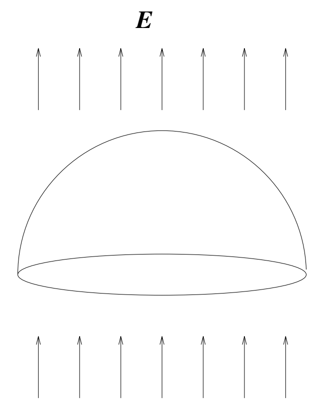
$$F = kq_1q_2/r^2$$

The separation between the two charges doesn't change. Initially it is 5 cm and in the final state it is

$$\sqrt{((3\text{ cm})^2 + (4\text{ cm})^2} = 5\text{ cm}$$

So, the magnitude of the force between the two charges doesn't change. The direction of the force is along the line connecting the two charges (attractive if the two charges are of opposite sign and repulsive if the charges are of same sign). Since the line connecting the charges changes, the direction will change.

21. A hemispherical surface (half of a spherical surface) of radius R is located in a uniform electric field $\mathbf{E}$ that is parallel to the axis of the hemisphere. What is the magnitude of the electric flux through the hemisphere surface?



a) 0
b) $4\pi R^2 E/3$
c) $2\pi R^2 E$
d) $\pi R^2 E$
e) $4\pi R^2 E$

The flux through a surface is given by

$$\text{flux} = \int \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{A}}$$

where $d\vec{A}$ is perpendicular to the surface. We could integrate over the hemispherical surface but we would have to take into account the fact that the angle between $\vec{E}$ and $d\vec{A}$ is changing. It's easier to recognize that the flux through the hemispherical surface is equal to but opposite in sign to the flux through the surface defined by the circular opening of the hemisphere. On this surface, $\vec{E}$ and $d\vec{A}$ are parallel to each other at every point so then the integral becomes trivial. It is just the constant value of E times the area of the circle.

$$\text{flux} = \pi \mathrm{R}^2 \mathrm{E}$$

22. The globe of a van de Graaff generator has a radius of 0.1m and is charged to a potential of $-(1 \times 10^4)$V relative to infinity. An electron escapes from the globe with an initial velocity of 0 m/s. What will be the velocity of the electron when the electron is very far away from the globe?

a) (1.5×10^7) m/s
b) (3×10^{-8}) m/s
c) (3.5×10^{15}) m/s
d) (6×10^7) m/s
e) (3×10^8) m/s

The initial potential energy of the electron is

$$U = q_e V = (-1.6 \times 10^{-19}\text{C}) * (-1 \times 10^4\text{V})$$
$$= 1.6 \times 10^{-15}\text{J}$$

When the electron is very far away its potential energy will be zero. By conservation of energy

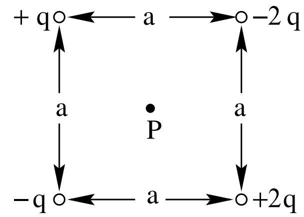
$$\Delta K = K_f - K_i = -\Delta U = U_f - U_i$$
$$= 1.6 \times 10^{-15}\text{J}$$

$K_i = 0$ because $v_i = 0$. So,

$$m_e v_f^2/2 = 1.6 \times 10^{-15}\text{J}$$

$$v_f = \sqrt{(2) * (1.6 \times 10^{-15}\text{J}}/(9.11 \times 10^{-31}\text{ kg})$$
$$= 6 \times 10^7 \text{ m/s}$$

23. Taking V = 0 at infinity, what is the electrostatic potential, V, at the point P which is at the center of the square formed by the charges as shown.



a) 0
b) $(6kq)/(\sqrt{2}\ a)$
c) $(4kq)/(\sqrt{2}\ a)$
d) $(3kq)/(\sqrt{2}\ a)$
e) $(kq)/(\sqrt{2}\ a)$

The potential due to a point charge is

$$V = kq/r$$

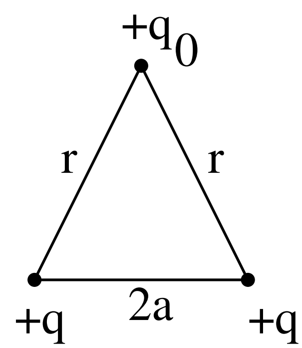
where r is the distance from the charge. When there are several point charges the total potential is the sum of the potentials due to each of the individual charges.

$$V_{tot} = V_1 + V_2 + \ldots$$

So,

$$V_{tot} = kq/(a/\sqrt{2}) - k2q/(a/\sqrt{2}) - kq/(a/\sqrt{2}) + k2q/(a/\sqrt{2}) = 0$$

24. Three positive charges are located as shown. The magnitude of the net force on the charge q_0 is given by



a) $2kq_0qa/r^3$
b) $2kq^2/4a^2$
c) $2kq_0q\sqrt{r^2-a^2}/r^3$
d) $2kq_0q/r^2 + kq^2/4a^2$
e) $2kq_0q/r^2$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$
$$= (kq_0q/r^2)((a/r)\hat{\imath}+(\sqrt{r^2-a^2}/r)\hat{\jmath})+(kq_0q/r^2)(-(a/r)\hat{\imath}+(\sqrt{r^2-a^2}/r)\hat{\jmath})$$
$$= (2kq_0q/r^2)(\sqrt{r^2-a^2}/r)\hat{\jmath})$$

So the magnitude of the force is

$$2kq_0q\sqrt{r^2-a^2}/r^3$$

25. A charge of -3.0 nC lies on the x-axis at $x = +6$ cm, and another equal charge of -3.0 nC is at $x = -6$ cm. What is the electric potential at the origin, assuming that it is zero at infinity?

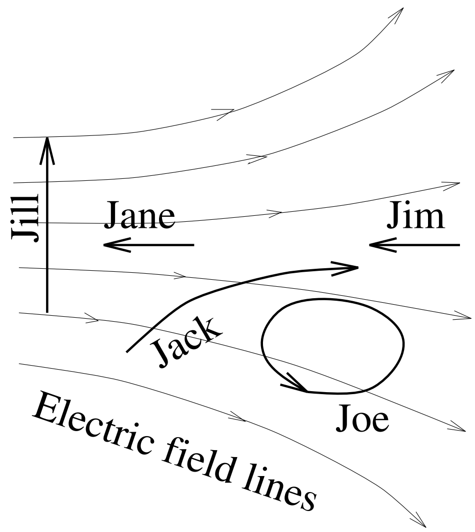
a) -900 V
b) Zero
c) 450 V
d) 900 V
e) -450 V

$$V = k(-3.0\times10^{-9}\text{ C})/(0.06\text{ m}) + \text{k}(-3.0\times10^{-9}\text{ C})/(0.06\text{ m})$$
$$= -900\text{ V}$$

Notice that the sign of q matters but the sign of r doesn't.

26. Jack, Jill, Joe, Jane, and Jim are assigned the tasks of moving equal positive charges through an electric field, each along his or her assigned path. In each case the charge is at rest at the beginning. Jim and Jane have paths of exactly equal lengths, shorter than each of the others. Who must do the most positive work?

a) Jack
b) Joe
c) Jim
d) Jane
e) Jill



$$\text{Work} = -\int \vec{\mathrm{F}} \cdot \vec{\mathrm{dr}} = -\int \mathrm{Q}\vec{\mathrm{E}} \cdot \vec{\mathrm{dr}}$$

- Jack does negative work since $\vec{E} \cdot \vec{dr}$ is positive over all of Jack's path.
- Jill does no work since $\vec{E}$ is perpendicular to $\vec{dr}$ and $\vec{E} \cdot \vec{dr}$ is therefore zero.
- Joe does zero work since $\int \vec{F} \cdot \vec{dr}$ is zero over a closed path.
- Jane and Jim both do positive work since $\vec{E}$ and $\vec{dr}$ are in opposite directions and $\vec{dr}$ and $\vec{E} \cdot \vec{dr}$ is therefore negative. The path lengths of Jane and Jim are the same but Jane does more work because the magnitude of the electric field is greater (notice the field lines are closer together) in the region where she moves the charge.

27. A solid conducting sphere of radius R_s is centered at the origin. It is inside a hollow conducting sphere, also centered at the origin, of inner radius R_i and outer radius R_o where $R_o > R_i > R_s$. The net charge on the solid sphere is –5μC and the net charge on the hollow sphere is +3μC. What is the charge on the inner surface of the hollow sphere?

a) It depends on the relative values of R_o, R_i, and R_s
b) Not enough information given.
c) +5 μC
d) –3 μC
e) +2 μC

The electric field within the hollow sphere must be zero since it is a conductor. By Gauss's Law that means that the net charge within a spherical surface of radius r such that $R_i < r < R_o$ must be zero. So the charge on the inner surface of the hollow sphere must exactly cancel the charge on solid sphere. That is, the charge on the inner surface of the hollow sphere is

$$= +5\ \mu\text{C}$$