First mid-term exam

- Monday, February 25, 2019 in lecture: 5:15-6:10 pm
- 12 multiple choice questions from Chapters 21-24, inclusive
- Exam review in Feb 22, 2019 recitations
- Pre-recitation – bring 2 page draft of your formula sheet
- Recitation activity: Review problems to solve collaboratively, referring to your formula sheet. The collaborative problems will be available online by Saturday Feb 23 morning.

All exams are closed-book, no calculators or other electronic devices allowed. All questions will be multiple choice. For the midterms, you may bring with you a single "formula sheet", one and only one 8.5 x 11 inch sheet of paper (OK to use both sides) on which you may hand write any formulae or diagrams or notes or problem solutions that might be helpful to you during the exam. Information on the sheets must be handwritten, no attachments are allowed. The numerical values of relevant constants will be provided to you. You should bring #2 pencils to the exams for the computer forms.

Types of questions: Like I-clickers, simple numbers, formulae
Study: Homework, I clickers, examples in textbook, collaborative+pre-REC
Old exams:

Free tutoring via MSLC: https://rlc.rutgers.edu/services/peer-tutoring
The nature of capacitors
• Any two conductors separated by an insulator (or a vacuum)

Capacitance measures ability to store charge.  \( Q = CV \Rightarrow C = \frac{Q}{V} \)

Capacitance only depends upon geometry
• Worked examples; Special: parallel plate

Capacitors connected in a network.
• In series  \( \frac{1}{C_{eq}} = \sum \frac{1}{C_i} \)
• In parallel  \( C_{eq} = \sum C_i \)

Energy stored in capacitor  \( U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{Q}{2V} \)

Energy density independent of geometry  \( u = \frac{\text{energy}}{\text{volume}} = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2 \)

Dielectrics: increase capacitance, increase mechanical stability
• Dielectric constant \( K \)
• Increase voltage that can be applied before dielectric breakdown

\[ C_{\text{parallel\ plate}} = \frac{\varepsilon_0 A}{d} \]
Lecture 9 – Chapter 25

Current and Resistance
Learning Goals for Chapter 25+26

- Meaning of electric current, and how charges move in a conductor.
- Use a simple model to understand the flow of current in metals.
- Calculate the resistance of a conductor from its dimensions and its resistivity or conductivity.
- Electromotive force (emf) makes it possible for current to flow in a circuit.
- Single loop circuits
- Calculate energy and power in circuits.
- Multiloop circuits
- RC Circuits with resistor and capacitor
Current: motion of charge from one region to another

- Motion of charged particles in a conductor of cross section area = \( A \)
- They move a distance \( \Delta x \) in time \( t \)
- Volume element \( A \Delta x \)

Analog: flow of (incompressible) fluid
- Increase cross section => \( v \) decreases
- \( A \ v = A \ (\Delta x/t) = \text{constant} \)
Current: motion of charge from one region to another

- Motion of charged particles in a conductor of cross section area \( A \),
- Volume element \( A \Delta x \)
- Total number of charge carriers: \( N = n A \Delta x \)
  - \( n = \) Number of mobile charge carriers/unit volume
- Charge in this volume element: \( \Delta Q = Nq = (nA \Delta x)q \)
  - \( q = \) charge of each charge carrier
- If charges moving at a speed \( v_d \) in time \( \Delta t \) they will travel distance \( \Delta x = v_d \Delta t \)

\[
\Delta Q = (nA \Delta x)q = nA(v_d \Delta t)q
\]

\[
\frac{\Delta Q}{\Delta t} \rightarrow \frac{dQ}{dt} = nAv_d |q| = I
\]
Defines current \( I \)
Current: motion of charge from one region to another

- Current $I$ through an area is the rate at which charge flows through the area:
  \[ I = \frac{dQ}{dt} \]

- Depends upon:
  - Concentration of moving charges: $n$
  - Charge per particle: $|q|$
  - Cross sectional area: $A$
  - Drift speed of the particles: $v_d$

- Current units: Ampere = Coulomb/sec

\[ I = \frac{dQ}{dt} = n|q|v_dA \]
Two copper wires of different diameter are joined end-to-end, and a current flows in the wire combination. When electrons move from the larger diameter wire into the smaller diameter wire

A. their drift speed increases.
B. their drift speed decreases.
C. their drift speed stays the same.
D. Not enough information is given to decide.
• In general, a conductor may contain several different kinds of moving charged particles.

• An example is current flow in an ionic solution.

• In the “pickle battery” (sodium chloride solution), current can be carried by both positive sodium ions and negative chlorine ions.

• The total current $I$ is found by adding up the currents due to each kind of charged particle.
• A current can be produced by positive or negative charge flow.
• *Conventional current* is treated as a flow of positive charges.
• In a metallic conductor, the moving charges are electrons — but the current still points in the direction positive charges would flow.

\[
I = \frac{dQ}{dt} = n |q| v_d A
\]
Drift velocity of electrons in conductors

• Electrons in a conductor are free to move
  • Colliding at intervals with the stationary positive ions.

• Apply electric field (ΔV)
  ➢ Random motion + drift
  ➢ Net displacement/Δt = v_d
  ➢ Drift velocity v_d

Drift velocity is average velocity of the charge carriers
Calculating drift velocity: warning – it is slow!

- Example: copper wire
  - $A=1 \text{ mm}^2$  \hspace{0.5cm} $\text{Current } I=1 \text{ A}$  \hspace{0.5cm} $\rho(\text{Cu})=9 \times 10^3 \text{ kg/m}$

- First: need number of charge carriers

$$n = \frac{\text{charge carriers}}{\text{unit volume}} = \left( \frac{1 \text{charge carrier}}{\text{atom}} \right) \left( \frac{\text{atoms}}{\text{volume}} \right)$$

$$= \left( \frac{1 \text{charge carrier}}{\text{atom}} \right) \left( \frac{\text{atoms}}{\text{mole}} \right) \left( \frac{\text{mole}}{\text{kg}} \right) \left( \frac{\text{kg}}{\text{volume}} \right)$$
Calculating drift velocity: warning – it is slow!

- Example: copper wire
  - A=1 mm$^2$  Current $I=1$ A  $\rho$(Cu)=$9 \times 10^3$ kg/m

- First: need number of charge carriers

$$n = \frac{\text{charge carriers}}{\text{unit volume}} = \left( \frac{1\text{charge carrier}}{\text{atom}} \right) \left( \frac{\text{atoms}}{\text{volume}} \right)$$

$$= \left( \frac{1\text{charge carrier}}{\text{atom}} \right) \left( \frac{\text{atoms}}{\text{mole}} \right) \left( \frac{\text{mole}}{\text{kg}} \right) \left( \frac{\text{kg}}{\text{volume}} \right)$$

$$= \left( 1 \right) \left( \frac{N_{\text{Avogadro}}}{\text{amu(Cu)}} \right) \left( \frac{1}{\rho(\text{Cu})} \right)$$

$$= \left( \frac{1}{\text{atom}} \right) \left( 6 \times 10^{23} \frac{\text{atoms}}{\text{mol}} \right) \left( \frac{1}{64 \times 10^{-3} \text{kg/mol}} \right) \left( 9 \times 10^3 \frac{\text{kg}}{\text{m}^3} \right)$$

$$n = 8.4 \times 10^{28} \frac{\text{charge carriers}}{\text{m}^3}$$

A very large number
Calculating drift velocity

• Example: copper wire
  • \( A = 1 \text{ mm}^2 = 10^{-6} \text{ m} \)    \( \text{Current } I = 1 \text{ A } \)    \( q = 1.6 \times 10^{-19} \text{ C} \)

• Number of charge carriers
  \( n = 8.4 \times 10^{28} \text{ charge carriers/m} \)

• Drift velocity is related to the current
  \[ I = n |q| v_d A \Rightarrow v_d = \frac{I}{nqA} \]
Calculating drift velocity

- Example: copper wire
  - \( A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2 \)  \( \text{Current } I = 1 \text{ A} \) \( \text{q} = 1.6 \times 10^{-19} \text{ C} \)
- First: need number of charge carriers
  \[ n = 8.4 \times 10^{28} \text{ charge carriers/m} \]
- Drift velocity is related to the current

\[
I = n |q| v_d A \implies v_d = \frac{I}{nqA}
\]

\[
v_d = \frac{1.0 \text{ A}}{8.4 \times 10^{28} \text{ /m} \times (1.6 \times 10^{-19} \text{ C}) \times (1 \times 10^{-6} \text{ m}^2)} \]

\[ v_d = 7.4 \times 10^{-5} \text{ m/s} \approx 10^{-4} \text{ m/s} \]

Remarkably low!
Drift velocity and start of current flow

- Drift velocity in copper wire $v_d \approx 10^{-4}$ m/s
- Velocity of current carriers $\approx “instantaneous” \approx 10^6$ m/s
- Analogy water tank: water droplet movement vs start of flow
Drift velocity and start of current flow

- Drift velocity in copper wire $v_d = 0.074 \text{ mm/s}$
- Velocity of current carriers $\approx \text{“instantaneous”} \approx 10^6 \text{ m/s}$
- Analogy water tank: water droplet movement vs start of flow
  - Open spigot at 3 => water flows
  - Water droplet at 1: takes a while to come out
• We can define a vector current density that includes the direction of the drift velocity:

\[ \vec{J} = nq\vec{v}_d \]

• The vector current density is always in the same direction as the electric field, no matter what the signs of the charge carriers are.
Two copper wires of different diameter are joined end-to-end, and a current flows in the wire combination. What is kept constant as the current flows in wire A to wire B?

A. The electric field $E$.

B. The drift velocity $v_d$.

C. The current density $J$.

D. The current $I$.

E. None of the above.
Resistivity

The **resistivity** of a material is the ratio of the electric field in the material to the current density it causes:

\[
\rho = \frac{E}{J}
\]

- Units: Ohm-meter = \(\Omega\cdot m\) = \((\text{Volt/m})/(\text{Ampere/m}^2)\) = V\cdot m/A

- The *conductivity* is the reciprocal of the resistivity.

- In general, resistivity depends upon temperature

\[
\rho(T) = \rho_0 \left[1 + \alpha(T - T_0)\right]
\]

Where \(\rho_0\) is resistivity at reference temperature \(T_0\).
Resistors where \( I \propto V \)

- When apply a voltage across a material and the current increases linearly
  - material obeys Ohm’s Law (material is ohmic)

\[
V = IR
\]

Where resistance = \( R \)

Units: Ohm, \( \Omega = V/A \)

- For a cylindrical conductor
  - \( R \) depends upon length \( L \) and cross sectional area \( A \)

\[
R = \rho \frac{L}{A}
\]

Where \( \rho \) is property of the material, independent upon dimensions
PhET: Resistance in a wire


\[ R = \frac{\rho L}{A} \]

- \( \rho \): resistivity
- \( L \): length
- \( A \): area

Resistance = 0.67 ohm

\( \rho \): 0.50 \(\Omega\) cm
\( L \): 10.00 cm
\( A \): 7.50 cm\(^2\)

Resistance in a Wire
• Current flows from higher to lower electric potential (direction of $E = \text{direction of } J$)

• Potential drops when current travels through a resistor

$$\Delta V = -IR$$
Nonohmic resistors

• Not all electronic devices obey Ohm’s law, i.e.,
  • the relationship of voltage to current may not be a direct proportion
  • the relationship of voltage to current may be different for the two directions of current.

• Example: diode

\[
\begin{align*}
I & \\
V & \\
O & \\
\end{align*}
\]

In the direction of positive current and voltage, \( I \) increases nonlinearly with \( V \).

In the direction of negative current and voltage, little current flows.
Some materials show a phenomenon called **superconductivity**. As the temperature decreases, the resistivity $\rho$ at first decreases smoothly, like that of any metal.

Below a certain critical temperature $T_c$ a phase transition occurs and the resistivity $\rho$ suddenly drops to zero.

Once a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field.
Which resistor obeys Ohm’s Law?

\[ V = IR \]

<table>
<thead>
<tr>
<th>Device</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Device 1</td>
<td>2.00</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>6.75</td>
</tr>
<tr>
<td></td>
<td>4.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Device 2</td>
<td>2.00</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>4.00</td>
<td>2.80</td>
</tr>
</tbody>
</table>
Need a complete circuit to establish a current

Given a piece of conductor (not part of circuit) with resistivity $\rho$ and establish an electric field $\vec{E}_1$  
$\Rightarrow$ Current with current density $\vec{J} = \frac{\vec{E}_1}{\rho}$  
$\Rightarrow$ Positive charge R end  
  Negative charge L end

These charges set up a field $\vec{E}_2$ going in opposite direction  
$\Rightarrow$ Very quickly $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0 \Rightarrow \vec{J} = 0$

Need (electromotive) force to increase potential energy of charges in the loop to maintain a current
Just as a water fountain requires a pump, an electric circuit requires a source of *electromotive force* to sustain a steady current.
Electromotive force and circuits

- **Electromotive force** (abbreviated **EMF**, symbol $\mathcal{E}$)
  - Makes current flow from lower to higher potential
  - **Source of emf** = circuit device that provides emf (e.g., battery)
  - Note: EMF is not a force but energy/unit charge
- **Unit**: Volt = Joule/Coulomb
- Typical flashlight battery has an emf of 1.5 V;
  - Battery does 1.5 J of work on every Coulomb of charge that passes through it.
- Most common source: electric generators
  - Source of EMF converts one form of energy (e.g., chemical or mechanical or solar) into electric potential energy

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Electromotive Force: Ideal battery

**Ideal battery**
- Electric field $E$ between + and - terminals
- No energy loss (dissipation) inside battery $\Leftrightarrow$ internal resistance $= 0$

EMF $\mathcal{E}$

$$\mathcal{E} = \frac{dW}{dq}$$

$\mathcal{E}$ of an EMF device is the work per unit charge that the device does in moving charge from its low-potential (-) terminal to its high-potential (+) terminal.

Ideal 12-V battery, no internal resistance $\Rightarrow$ $\Delta V$ between its terminals always 12 V
Real EMF device, internal resistance $\neq 0$ $\Rightarrow$ when connected in circuit $\Delta V \neq \text{EMF}$
Electrons in an electric circuit pass through a resistor. The wire has the same diameter on each side of the resistor.

Compared to the drift speed of the electrons before entering the resistor, the drift speed of the electrons after leaving the resistor is

A. faster.

B. slower.

C. the same.

D. not enough information given to decide
Summary Lecture 9

- Deduced current = time rate of change of charge flow

\[ I = \frac{dQ}{dt} = n|q|v_d A \]

- Vector current density; points in \( E \) direction

\[ \vec{J} = nq\vec{v}_d \]

- Material has a resistivity

That can depend upon temperature:

\[ \rho(T) = \rho_0 [1 + \alpha(T - T_0)] \]

- Resistance of a material (conductor) depends of \( L, A, \) and \( \rho \)

\[ R = \rho \frac{L}{A} \]

- For materials that obey Ohm’s law

\[ V = IR \]

\[ \Delta V = -IR \]
Next Lecture

Energy and Power in electric circuits