Announcements

**Register your I clicker** – for unregistered I clickers go to course home page:


Note: giving your I-clicker to another student to record answers is a violation of academic integrity and could result in separation of both of you from the university.

**First exam Monday Feb 25** in lecture 5:15-6:10

- Chapters 21-24 inclusive.
- If you have a conflict – let me know asap but not later than Feb 15
- If you would need extra time or other accommodations need to present letter from Office of Disabilities Services
• Defined the concept of electric flux through a surface, in analogy to flux of a fluid

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

• Gauss’s Law: total electric flux through a closed surface equals a constant times the total charge enclosed

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

• Used Gauss’s law to calculate electric fields for symmetric charge distributions (More examples in text; see Table page 743)
  • Infinite charged plane
  • Two oppositely charged conducting planes
  • Uniformly charged insulating sphere
  • Charged conducting sphere

• Demonstrated properties of electrical conductor
  • When excess charge placed on solid conductor, it is only outside of conductor
  • No electric fields inside of conductor
An **insulating** spherical shell of inner radius $a$ and outer radius $b$ is uniformly charged with a positive charge density. Which figure best depicts the radial component of the electric field $E_r(r)$?
Lecture 5 – February 6, 2019

Electrical Potential
Chapter 23 Goals: to solve electrical problems

• How to calculate the electric potential energy of a collection of charges.
• The meaning and significance of electric potential.
• How to calculate the electric potential that a collection of charges produces at a point in space.
• How to use equipotential surfaces to visualize how the electric potential varies in space.
• How to use electric potential to calculate the electric field.
Analogy to gravitational potential energy

- Want to solve mechanical/electrical problems more simply
  - Potential energy (rather than forces/fields)
Recall: Gravitational potential energy

Change in gravitational potential energy $\Delta U$

- Does not depend on path taken

$$\Delta U = -(\text{Work done by the gravitational force})$$

$$\Delta U = -W_{\text{gravity}}$$

General expression for $U$ at a point

$$U = -\frac{Gm_1m_2}{r}$$
Gravitational potential energy depends only on distance not on path taken.

$U = -\frac{G m_E m}{r}$

$U$ is always negative, but it becomes less negative with increasing radial distance $r$.

Property of conservative forces, including electrical force (Coulomb’s Law)
Electric potential energy of two point charges

- Electric potential energy $U$ only depends upon distance between the charges
  \[ U = \frac{kqq_0}{r} \]

- Same equation, no matter what sign the charges have

- $U$ defined to be ZERO when charges are infinitely far apart

- Unit of energy = Joule = N-m
  - $k=\frac{1}{4\pi \varepsilon_0} = 9 \times 10^9$ N-m$^2$/C$^2$
  - Electrical potential energy units: $(N\cdot m^2/C^2) \times C^2/m = N\cdot m$
Graphs of potential energy

If charges have the SAME sign
- Interaction is repulsive
- Electric potential energy is positive

\[ U = \frac{kq_1q_2}{r} \]

- \( U > 0 \)
- As \( r \to 0 \), \( U \to +\infty \).
- As \( r \to \infty \), \( U \to 0 \).

If charges have OPPOSITE signs
- Interaction is attractive
- Electric potential energy is negative

- \( U < 0 \)
- As \( r \to 0 \), \( U \to -\infty \).
- As \( r \to \infty \), \( U \to 0 \).
Potential energy of a test charge

- Force on the test charge
  \[ \vec{F} = q_0 \vec{E} \]

- Work done by the force to move object distance \( \Delta l \)
  \[ W = \vec{F} \cdot \Delta \vec{l} = q_0 \vec{E} \cdot \Delta \vec{l} = q_0 E \Delta l \cos \phi \]

- Work done by the force = \(-\) (change in potential energy)
  \[ W_{\text{electrical force}} = -\Delta U_{\text{electrical}} \]

- Energy conserved
  \[ \Delta U + \Delta K.E. = 0 \]
  - Change in potential energy + change in kinetic energy = 0
Potential energy change for + & - charges

+q_0 moves in direction of \( E \)
from point a to point b

- Field does positive work on +q_0
- U decreases
  (like falling in g)

+q_0 moves opposite to \( E \)
from point a to point b

- Field does negative work on +q_0
- U increases
  (like being raised in g)
Electric potential $V$ is electric potential energy per unit charge $U/q_0$

- Like field, potential is independent of the test charge
  \[ \vec{E} = \frac{\vec{F}}{q_0} \quad V = \frac{U}{q_0} \]

- Example: Field and potential associated with 2 large oppositely charged parallel plates

Uniform $\vec{E}$ field from + to – plates
- Potential $V$ depends upon distance $y$ from the bottom plate
- As move $q_0$ from $b$ to $a$, in direction opposite to $\vec{E}$, $V$ increases

\[ V = E \cdot y \]
Electric potential $V$ is electric potential energy per unit charge $U/q_0$

- Like field, potential is independent of the test charge

- Potential due to single point charge

$$V = \frac{kq}{r}$$

- Potential due to a collection of charges

$$V = k \sum_i \frac{q_i}{r_i}$$

Example: field from positive charge
- $V$ increases as move inward
- $V$ decreases as move outward
Consider system of charges. Charge \( q_3 = -q \) is brought from \( \infty \) so that it forms an equilateral triangle with charges \( q_1 = +q \) and \( q_2 = -q \). What is the potential energy \( U \) of charge \( q_3 \) in the field of charges \( q_1 \) and \( q_2 \)?

A. \( U = 0 \)

B. \( U = \frac{kq^2}{d} \)

C. \( U = -\frac{kq^2}{d} \)

D. \( U = \frac{k2q^2}{d} \)

E. \( U = -\frac{k2q^2}{d} \)
Potential difference and Electric field

• Move positive (test) charge in direction of E field, means electric force does work => potential (energy) decreases
• Potential energy difference $U_a - U_b$ equals the work done by the electric force to move a charge from point $b$ to point $a$.
• Potential difference $\Delta V = \Delta Q/q_0$
• Note: does not depend on path, only initial and final points

\[ U_a - U_b = q_0 (V_a - V_b) \]

Unit of electric potential $V$: Volt (V)
- Unit of electric field $E$: N/C=V/m
- Defines Volt = N-m/C
Application: accelerating atomic ions

Convenient unit of energy = electron Volt (eV)
- Energy of an electron (or proton) gained when accelerate through $\Delta V=1\ \text{V}$

Charge of electron = $1.6\times10^{-19}\ \text{C}$
1 eV = $1.6\times10^{-19}\ \text{Joules}$

Tandem Van de Graaff: Terminal in the MIDDLE
Application: accelerating atomic ions

Accelerating atomic ions with tandem Van de Graaff accelerator

- Charge the “terminal” with so much charge that $\Delta V = 5$ million volts
- Ion source: e.g., add an electron to carbon atom=>C$^{-1}$
  - Neutral Carbon: 6 protons (+), 6 electrons (-)
  - Accelerated to positive terminal
    - $\Delta U_1 = U_f - U_i = -5$ million electron volts = $-\Delta KE_1 = -(KE_f - KE_i)$
    - Now strip off 4 electrons=>C$^{+3}$ repelled by positive terminal
      - $\Delta U_2 = -3 \cdot 5$ MeV = -15 MeV = $-\Delta KE_2$
      - $KE_{final} = 20$ MeV
Potential difference and Electric field

• Recall relationship between Work done by a Force

\[ W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = -(U_b - U_a) = U_a - U_b \]
Potential difference and Electric field

- Recall relationship between Work done by a Force

\[ W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} q_0 \vec{E} \cdot d\vec{l} = -(U_b - U_a) = U_a - U_b \]

- Does not depend upon the path, only initial and final points

- For the electric field/potential

\[ \int_{a}^{b} \vec{E} \cdot d\vec{l} = \int_{a}^{b} E \cos \phi \, dl = V_a - V_b \]
Potential difference and Electric field

- Recall relationship between Work done by a Force

\[ W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} q_0 \vec{E} \cdot d\vec{l} = -(U_b - U_a) = (U_a - U_b) \]

- Does not depend upon the path, only initial and final points

\[ V_a - V_b = \int_{a}^{b} \vec{E} \cdot d\vec{l} = \int_{a}^{b} E \cos \phi \, dl \]

- If E from positive charge, move (positive test charge) in direction of E field, electric force does work => potential (energy) decreases

\[ V \text{ increases as you move inward.} \]

\[ V \text{ decreases as you move outward.} \]
Potential difference and Electric field

- Recall relationship between Work done by a Force
  \[ W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} q_0 \vec{E} \cdot d\vec{l} = -(U_b - U_a) = (U_a - U_b) \]

- Does not depend upon the path, only initial and final points
  \[ V_a - V_b = \int_{a}^{b} \vec{E} \cdot d\vec{l} = \int_{a}^{b} E \cos \phi \, dl \]

- If E from negative charge, move (positive test charge) in direction of E field, again electric force does work \(\Rightarrow\) potential (energy) decreases

![Diagram](image-url)
Example: Charged particle moving in uniform E field

Charged particle moving from point A to point B in a uniform electric field E

- What is the potential difference between points A and B?
Example: Charged particle moving in uniform E field

Charged particle moving from point A to point B in a uniform electric field E

- What is the potential difference between points A and B?

\[ V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} \]

\[ V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} = \vec{E} \cdot \int_A^B d\vec{l} \]

\[ = \vec{E} \cdot d \]

\[ = Ed \cos \theta \]
Two charges $q=+1\ \text{C}$ are 1 m apart and initially at rest. The charges are released. What is the net kinetic energy of the system when the charges are very far from one another? Recall $U(r) = \frac{kq_1q_2}{r}$.

A. $k_1\ \text{J}$  
B. $-k_1\ \text{J}$  
C. $k_2\ \text{J}$  
D. $-k_2\ \text{J}$  
E. 0
Summary

• Learned to calculate the electric potential (energy) of a collection of charges.

\[ V = k \sum_i \frac{q_i}{r_i} \]

• Learned the meaning of electric potential.
  • Relation between Work done by E field and change in Potential Energy

\[ W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l} = -(U_b - U_a) = (U_a - U_b) \]

• Learned how to calculate change in Electric Potential from moving test charge in Electric Field from a to b

\[ V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos \phi \, dl \]
Next lecture

Continuing calculating Electric Potentials