Announcements

• Register your I clicker – home page has list of unregistered I clickers

• Optional: Peer coaching sessions
  • Learn new study strategies, develop time management
  • Register https://rlc.rutgers.edu/peercoaching

• First exam Monday Feb 25 in lecture 5:15-6:10
  • Chapters 21-24 inclusive.
  • If you have a conflict – let me know asap but not later than Feb 15
  • If you would need extra time or other accommodations need to present letter from Office of Disabilities Services
What is electric flux? How do I calculate?

• Measure of amount of electric field passing through a region of area A
• Depends upon the magnitude of electric field E
• Depends upon the magnitude of area A (or small area dA)
• Depends upon the angle between $\vec{E}$ and $\vec{A}$

• In general $d\Phi = \vec{E} \cdot d\vec{A} = E \, dA \cos \phi$
• When E is constant over the entire area, and angle between E and dA does not change

$$\Phi = \int \vec{E} \cdot d\vec{A} = E \cos \phi \int dA$$

$$\Phi = EA \cos \phi$$
Review of Lecture 3 - Gauss’s Law

• The total electric flux through a closed surface is equal to the total charge enclosed divided by \( \varepsilon_0 \)
  \[
  \Phi_E = \frac{q_{\text{enclosed}}}{\varepsilon_0}
  \]

• Surface can be complicated => so pick a piece of the surface \( dA \) and calculate flux \( d\Phi \):
  \[
  d\Phi_E = \vec{E} \cdot d\vec{A}
  \]

• Calculate the flux over the entire closed surface
  \[
  \Phi_E = \oint d\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}
  \]
Gauss’s Law is powerful; can calculate fields more simply

Example 1:
E from infinite line of charge
Charge per unit length = \( \lambda \)
Gaussian surface = cylinder around the line

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = E(2\pi rL)
\]

\[
E = \frac{\lambda L}{\varepsilon_0} \frac{1}{2\pi rL} = \frac{\lambda}{2\pi \varepsilon_0 r} = E
\]

\[
q_{\text{enclosed}} = \lambda L
\]
Gauss’s Law is powerful; can calculate fields more simply

Example 2:

E from infinite plane sheet of charge

Charge per unit area = \( \sigma = \frac{Q}{A} \)

\[ q_{\text{enclosed}} = \sigma A \]

- Gaussian surface = cylinder that goes through the plane

\[ \Phi_{\text{total}} = 2EA = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \]

\[ E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} = E \]

E only depends upon \( \sigma \) NOT on distance
Consider a closed Gaussian surface. Which of the following statements is **FALSE**?

A. If there is no net charge enclosed by the surface, then the total electric flux through the surface is zero.

B. If the electric field is zero everywhere on the surface, then there can be no net charge enclosed by the surface.

C. If the total electric flux through the surface is zero, then the total charge enclosed by the surface is zero.

D. If the electric field is zero everywhere on the surface, then the total electric flux through the surface is zero.

E. If the total electric flux through the surface is zero, then the electric field must be zero everywhere on the surface.
Lecture 4 – February 4, 2019

Gauss's Law
Applications and Conductors
Electric field and charged conductors

- Conductors: “free” electron – bound to the conductor as a whole, can wander through the material
- Assume electrical conductor (any shape) in electrostatic equilibrium, i.e., any net charges have to remain fixed in position
- Will show following conditions characterize conductors:
  - At any point inside the conductor E=0
  - Any net charge on the conductor is entirely on the surface
  - E is perpendicular to the surface at each point on the exterior surface
  - E is strongest at those points on the surface at which net charge is most heavily concentrated
Showing: At any point inside the conductor $E=0$

- Suppose that $E$ was not zero at some point within the conductor
  - Place a free electron at this point
  - It would feel a force => electron accelerated
  - Contradiction! – we had assumed electrostatic equilibrium

- Electric field inside conductor identically equal to zero
Electric field and charged conductors in electrostatic equilibrium

Showing: Any net charge on the conductor is entirely on the surface – use Gauss’s Law for arbitrarily shaped conductor.

E=0 inside of conductor (just showed this)
- E=0 at every point on this Gaussian surface
- Net flux $\Phi_E=0$ through this surface
- No charge enclosed => any charge must be on the outside
Electric field and charged conductors in electrostatic equilibrium

**Proving:** Any net charge on the conductor is entirely on the surface and \( E \) is perpendicular to the surface at each point on the exterior surface – use Gauss’s Law for arbitrarily shaped conductor.

**Surface of conductor**

- Cylindrical Gaussian surface, small so that faces of cylinder parallel to “flat” surface
- Endcap inside: \( E=0 \)
- Endcap outside: \( E=\sigma/\varepsilon_0 \), points out, perpendicular to surface

Note: if \( E \) had a component tangential to surface, the free charges would move along the surface

- **Contradiction** – assume electrostatic equilibrium
- No tangential component
Properties of conductors:

- At any point inside the conductor \( E = 0 \)
- \( E \) is perpendicular to the surface at each point on the exterior surface
- Any net charge on the conductor is entirely on the surface
- \( E \) is strongest at those points on the surface at which net charge is most heavily concentrated
- Remember: \( \vec{E} = \frac{\vec{F}}{q_{\text{positive test}}} \)
Summary: Electric field and charged conductors

Used Gauss’s Law to show the following conditions characterize conductors:

- At any point inside the conductor E=0
- Any net charge on the conductor is entirely on the surface
- E is perpendicular to the surface at each point on the exterior surface
- E is strongest at those points on the surface at which net charge is most heavily concentrated
A point charge +Q is placed inside a neutral, hollow, spherical conductor. As the charge is moved around inside, the electric field outside

A. is zero and does not change
B. is non-zero but does not change
C. is zero when centered but changes
D. is non-zero and changes
Electric field from (isolated) large conducting sheet

- $+\sigma$ charge/unit area; outside surfaces
- Magnitude $E_L = E_R = \frac{\sigma}{2\varepsilon_0}$

Electric field from (isolated) large conducting sheet

- $-\sigma$ charge/unit area; outside surfaces
- Magnitude $E_L = E_R = \frac{\sigma}{2\varepsilon_0}$

Adopted from Halliday & Resnick
Electric field from (2) conducting plates

Field between 2 oppositely charged conducting plates

Bring the two sheets together

- The charges move, attracted to each other, to inner surfaces
- Charge on each plate $\pm 2\sigma$
- $E$ only between the plates
- No electric field outside of plates – from Gauss’s Law

Adopted from Halliday & Resnick
Realistic situation and considerations

For conducting plates
- When bring the plates together the charges move
- Can’t just add the fields from when the plates were separated
- Example 22.8: another approach with same answer

For insulating plates
- The charges won’t move when brought together
- Can add the fields from when plates separated

Realistic situation: finite size sheets
- Fringing fields

When distance between plates $<<$ Area of plates
- Ignore fringing fields
- Field lines between the plates $\approx$ straight, parallel lines
Electric fields from spherical insulators & conductors
E fields for solid spherical insulator vs. conductor

I. Uniformly charged (insulating) sphere

Uniformly charged sphere: (volume) charge density \( \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} \)

Electric field when \( r > R \) = Electric field from point charge

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2}
\]
E fields for solid spherical insulator vs. conductor

I. Uniformly charge (insulating) sphere

Uniformly charged sphere: (volume) charge density = \( \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3} \pi R^3} \)

Electric field when \( r < R \)

\[
q_{\text{enclosed}} = \rho V = \frac{Q}{\frac{4}{3} \pi R^3} \left( \frac{4}{3} \pi r^3 \right) = Q \frac{r^3}{R^3}
\]

Use Gauss's Law: \( \Phi_{\text{Electric}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \)
E fields for solid spherical insulator vs. conductor

I. Uniformly charge (insulating) sphere

Uniformly charged sphere: (volume) charge density = \( \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3} \pi R^3} \)

Electric field when \( r < R \)

\[ \Phi = \oint E \cdot dA = EA = E(4\pi r^2) = \frac{q_{enclosed}}{\varepsilon_0} = Q \frac{r^3}{\varepsilon_0 R^3} \]

\[ \Phi_{\text{Electric}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0} \]

Solve for \( E \)
E fields for solid spherical insulator vs. conductor

I. Uniformly charge (insulating) sphere

Uniformly charged sphere: (volume) charge density = \( \rho = \frac{Q}{V} = \frac{Q}{4/3 \pi R^3} \)

Electric field when \( r < R \)

\[ q_{\text{enclosed}} = \rho V = \frac{Q}{4/3 \pi R^3} \left( 4/3 \pi r^3 \right) = Q \frac{r^3}{R^3} \]

\[ \Phi = EA = E (4\pi r^2) = \frac{q_{\text{enclosed}}}{\varepsilon_0} = Q \frac{r^3}{\varepsilon_0 R^3} \]

\[ E = Q \frac{r^3}{\varepsilon_0 R^3} \frac{1}{(4\pi r^2)} = \frac{Qr}{4\pi \varepsilon_0 R^3} \]
E fields for solid spherical insulator vs. conductor

1. Uniformly charge (insulating) sphere

Electric field when \( r > R \)

\[
E(R) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}
\]

Electric field when \( r < R \)

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3}
\]
Charged conducting sphere with $+Q$

Electric field when $r<R$ is zero

- inside of conductor – no charges

Electric field when $r>R = \text{Electric field from point charge}$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$
E fields for solid spherical insulator vs. conductor

Although objects have the same radius R

- Fields for r<R very different
  - E increases vs. r for insulator; No E field for conductor
- Electric fields for r>R are the same

Mathematical expressions:

- For solid spherical insulator:
  \[ E(R) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2} \]
  \[ E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{r^2} \]

- For solid spherical conductor:
  \[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{R^2} \]
  \[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]
Consider an uncharged conducting sphere, with a cavity inside. We insert a 5-µC charge into the cavity. Which is the correct statement about the charge on the inner (QI) and outer (QO) surfaces of the conductor?

A. QI = 0 µC, QO = 0 µC.

B. QI = -5 µC, QO = -5 µC.

C. QI = -5 µC, QO = +5 µC.

D. QI = +5 µC, QO = -5 µC.

E. QI = +5 µC, QO = +5 µC.
Summary Chapter 22

• Defined the concept of electric flux through a surface, in analogy to flux of a fluid
  \[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

• Gauss’s Law: total electric flux through a closed surface equals a constant times the total charge enclosed
  \[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

• Used Gauss’s law to calculate electric fields for symmetric charge distributions (More examples in text; see Table page 743)
  • Infinite charged plane
  • Two oppositely charged conducting planes
  • Uniformly charged insulating sphere
  • Charged conducting sphere

• Demonstrated properties of electrical conductor
  • When excess charge placed on solid conductor, it is only outside of conductor
  • No electric fields inside of conductor
Next lecture – Chapter 23

Solving electricity problems with ENERGY rather than Forces/Fields