Announcements

• Homework – Tomorrow Jan 31 before 11:59 pm
• Recitation – first quiz Feb 1. Make sure you arrive on time. If arrive after collaborative starts, get zero for both quiz and collaborative. Must attend recitation for which registered, unless you get permission from Cizewski, in advance.
• Register your I clicker – home page has list of unregistered I clickers
Lecture 3 – January 30, 2019
Summary: Electric Charge & Field

- Properties of electric charge
  - Negative (electrons) and positive (nuclei of atoms)
  - Conductors vs insulators
- Coulomb’s Law
  \[ F_E = \frac{kq_1q_2}{r^2} \]
- Electric field: fields are important concept in physics
  \[ \vec{E} = \frac{\vec{F}_E}{q_0} \]
- Electric field lines: a graphical representation of electric fields
- Electric fields of simple configurations
  - Line of charge, electric dipole
- Electric dipoles: a simple configuration of 2 charges
Charged particles are fixed on grids having the same spacing. Each charge has the same magnitude $Q$ with signs given in the figure. Rank the magnitude of the electric field $E$, from strong to weak, at the location marked with an ‘x’.

A. $1 > 2 > 3 > 4$
B. $2 > 3 > 1 = 4$
C. $1 = 4 > 3 > 2$
D. $3 > 2 > 1 > 4$
Charged particles are fixed on grids having the same spacing. Each charge has the same magnitude $Q$ with signs given in the figure. Rank the magnitude of the electric field $E$, from strong to weak, at the location marked with an ‘x’.

A. $1 > 2 > 3 > 4$

B. $2 > 3 > 1 = 4$

C. $1 = 4 > 3 > 2$

D. $3 > 2 > 1 > 4$
What happens when electric dipole placed in uniform electric field?

For electric dipole with charges $\pm q$ and separated by distance $d$ have Electric Dipole Moment $\vec{p} = qd$

With direction from negative to positive charge

- Force on charges of electric dipole
- Torque on electric dipole $\vec{\tau} = \vec{r} \times \vec{F}$

$$\tau = rF \sin \phi$$

Perpendicular to both $\vec{r}$ and $\vec{F}$
When a dipole is placed in a uniform electric field \( E \), the net force is always zero.

- Force on \(+q\) is to the right
- Force on \( -q \) is to the left
- Total force = 0
Remembering torque

\[ \mathbf{\tau} = \mathbf{r} \times \mathbf{F} \]

\[ \tau = rF \sin \phi \]

- Pick a pivot = \( O \)
- Lever arm of length \( r \)
- At end of lever arm is the action of the force \( F \)
- Magnitude of torque = \( rF \sin \phi \)
- Direction of torque = right hand rule
In an E field: Torque on an electric dipole with dipole moment $p = qd$

- Can have a net torque – choose midpoint as pivot
  - Magnitude of torque on $-q$: $rF \sin \phi = \frac{d}{2} qE \sin \phi = \frac{p}{2} E \sin \phi$
  - Magnitude of torque on $+q$: $\frac{d}{2} qE \sin \phi = \frac{p}{2} E \sin \phi$

- Net torque rotating counterclockwise

\[
2 \frac{p}{2} E \sin \phi = pE \sin \phi
\]

\[
\vec{\tau} = \vec{p} \times \vec{E}
\]

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Net force = 0, Net torque ≠ 0 => rotation

- Potential energy $U$ of an electric dipole in $E$ field
  - Maximum when $p$ and $E$ anti-aligned
    - Dipole wants to flip so that $p$ and $E$ are aligned
  - Zero when $p$ and $E$ are perpendicular
  - Minimum when $p$ and $E$ are aligned

\[
U = -\mathbf{p} \cdot \mathbf{E}
\]

\[
U = pE \cos \phi
\]
Potential energy of electric dipole in E field

\[ U = -\vec{p} \cdot \vec{E} \]
\[ U = -pE \cos \phi \]

Dipole moment direction: from – to + charge.

http://gcse-math.co.uk/graphs/trigonometric-graphs
Summary: Electric Dipole in Electric Field

- Electric dipoles: a simple configuration of 2 opposite charges
- Dipole Moment $\vec{p} = q\vec{d}$
  - Magnitude: product of magnitude of charges and distance between the charges
  - Direction: from negative to positive charge
- When electric dipole in electric field
  - No net force on electric dipole
  - Net torque on electric dipole $\vec{\tau} = \vec{p} \times \vec{E}$
    - Rotation
- Electric dipole wants to align with the electric field
  - Minimizes potential energy of the dipole in E field
    $U = -\vec{p} \cdot \vec{E}$
Electric Flux & Gauss's Law
Gauss’s Law: Relates electric flux through a closed surface with the total charge enclosed

- What is electric flux? How do I calculate?
- Simple derivation of Gauss’s Law
- How determine amount of charge within a closed surface by examining the electric field on the surface
- How to use Gauss’s law to calculate the electric field due to a symmetric charge distribution
- How to use Gauss’s Law to understand where charge is located on a charged conductor
Recall fluid – related to flux

- Consider flow tube with changing cross-sectional area.
- The **continuity equation** for an incompressible fluid is
  \[ A_1 v_1 = A_2 v_2 = \text{constant} \]
- First component = AREA
  - Magnitude = area of a surface
  - Area vector points outwards
What is area vector?

Analogous to displacement and distance (vector)

- Magnitude and direction
- Define +direction

Area vector

- Magnitude (area of a surface) and direction
- Define +direction:
  - Pointing out, away from surface
  - Perpendicular to surface at every point

http://www.animalplanet.com/tv-shows/too-cute/photos/hedgehogs/
https://www.pinterest.com/pin/161637074101438386/
Recall fluid – related to flux

• Consider flow tube with changing cross-sectional area.

• The **continuity equation** for an incompressible fluid is

\[ A_1 v_1 = A_2 v_2 = \text{constant} \]

• Analogous to **electric flux** through a surface
  • What comes in, must come out
  • Velocity \( \leftrightarrow \) Electric field
  • Flux maximum when \( E \) and \( A \) are parallel

\[ \Phi_E = EA \cos \theta = \vec{E} \cdot \vec{A} \]

• Area vector points outwards
**Concept of Flux**

\[
\Phi_E = \mathbf{E} \cdot \mathbf{A} = E A \cos \phi
\]

- Applies to any vector field, such as electric field, or field of velocities in a fluid, or gravitational field.
- Area vector points outward!

In general for any vector field (not just electric), the flux of the field through a small element of surface \( d\mathbf{A} \) can be calculated.

If \( \mathbf{E} \) and \( \phi \) are constant, then only integrate over the area.

\[
\Phi = \int \mathbf{E} \cdot d\mathbf{A} = E A \cos \phi
\]
I clicker: What is the electric flux?

Which statement is TRUE about electric flux?

A. Flux of (a) = EA; Flux of (b) = Zero
B. Flux of (a) = Zero; Flux of (b) = EA
C. Flux of (b) = EA; Flux of (c) = Zero
D. Flux of (b) = Zero; Flux of (c) = EA \cos \theta

$$\Phi_E = EA \cos \theta = \vec{E} \cdot \vec{A}$$
I clicker: What is the electric flux? - solution

Which statement is TRUE about electric flux?

A. Flux of (a) = EA; Flux of (b) = Zero
B. Flux of (a) = Zero; Flux of (b) = EA
C. Flux of (b) = EA; Flux of (c) = Zero
D. Flux of (b) = Zero; Flux of (c) = EA \cos \theta

(a): Zero – E and A perpendicular
(b): EA - E and A parallel
(c): EA \cos \theta – angle between E and A is \theta
Example: What is flux through hemisphere?

Given a hemisphere of cross sectional area $A=\pi r^2$. A uniform electric field passes through the hemisphere. What is the electric flux through the top part?

Total flux through entire surface = 0

- Flux through the bottom = $-\text{flux through the top half}$

$$\Phi_{\text{total}} = \Phi_{\text{bottom}} + \Phi_{\text{top}}$$

$$\Phi_{\text{bottom}} = \vec{E} \cdot \vec{A} = -E(\pi r^2)$$

$$\Phi_{\text{top}} = -\Phi_{\text{bottom}} = E(\pi r^2)$$

$$\Phi_{\text{top}} = E\pi r^2$$
Flux through sphere surrounding point charge

\[ d\Phi = \vec{E} \cdot d\vec{A} \]

\[ \Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = E \int_{\text{surface}} dA \]

- Electric field from point charge
  \[ E = \frac{q}{4\pi\varepsilon_0 r^2} \]

- Surface area of sphere
  \[ A = 4\pi r^2 \]

- Electric flux
  \[ \Phi = \int \vec{E} \cdot d\vec{A} = \left( \frac{q}{4\pi\varepsilon_0 r^2} \right) (4\pi r^2) = \frac{q}{\varepsilon_0} = \Phi \]
Gauss’s Law

- The total electric flux through a closed surface is equal to the total charge enclosed divided by $\varepsilon_0$
  \[ \Phi_E = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

- Surface can be complicated
  - pick a piece of the surface $dA$ and calculate flux $d\Phi$:
  \[ d\Phi_E = \vec{E} \cdot d\vec{A} \]

- Calculate the flux over the entire closed surface
  \[ \Phi_E = \oint d\Phi_E = \oint \vec{E} \cdot d\vec{A} \]
Gauss’s Law

• The total electric flux through a closed surface is equal to the total charge enclosed divided by $\varepsilon_0$

$$\Phi_E = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

• Surface can be complicated => so pick a piece of the surface $dA$ and calculate flux $d\Phi$:

$$d\Phi_E = \vec{E} \cdot d\vec{A}$$

• Calculate the flux over the entire closed surface

$$\Phi_E = \oiint d\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$
Gauss’s Law is powerful; can calculate fields more simply

\[ \Phi_E = \oint E \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Example:
Calculate \( E \) from infinite line of charge
Charge per unit length = \( \lambda \)

- Gaussian surface = (coaxial) cylinder
  Length = \( L \); radius = \( r \)
- \( E \) only points away from the line
  - End caps \( E_{\perp} = 0 \implies \) no flux
  - \( E \) points away from the line
    - Parallel to the surface element \( dA \)
    - Only depends upon \( r \)
      \( \Rightarrow \) Constant over the surface
Gauss’s Law is powerful; can calculate fields more simply.

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Example:
Calculate \( E \) from infinite line of charge.
Charge per unit length = \( \lambda \)

- Gaussian surface = (coaxial) cylinder
  - Length = \( L \); radius = \( r \)

\[ \Phi_E = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0} \]

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = E(2\pi rL) \]

\[ E = \frac{\lambda L}{\varepsilon_0} \times \frac{1}{2\pi rL} = \frac{\lambda}{2\pi \varepsilon_0 r} = E \]

Same result as we got last lecture but a lot less work!
Gauss’s Law is powerful; can calculate fields more simply

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Example:
Calculate \( E \) from infinite plane sheet of charge
Charge per unit area = \( \sigma = \frac{Q}{A} \)

\[ q_{\text{enclosed}} = \sigma A \]

- Gaussian surface = cylinder that goes through the plane
- \( E \) vector is out of the plane (not in the plane) and parallel to \( A \) vector, away from surface
  \[ \Phi_{\text{left}} = \Phi_{\text{right}} = \vec{E} \cdot \vec{A} = EA \]

Solve for \( E \)

\[ \Phi_{\text{total}} = 2EA = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \]
Gauss’s Law is powerful; can calculate fields more simply

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{\text{charge enclosed}}{\varepsilon_0} \]

Example:
Calculate E from infinite plane sheet of charge
Charge per unit area = \( \sigma = \frac{Q}{A} \)

\[ q_{\text{enclosed}} = \sigma A \]

- Gaussian surface = cylinder that goes through the plane
- E vector is out of the plane (not in the plane) and parallel to A vector, away from surface

\[ \Phi_{\text{total}} = 2EA = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \]

\[ E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} = E \]

E only depends upon \( \sigma \)
NOT on distance
**Gauss’s Law:** If know $E$ can calculate $q_{\text{enclosed}}$

Example: homework problem “Flux through a cube”

- Given electric field, e.g.,

  $$\vec{E} = (a + bx)\hat{i} + C\hat{j}$$

- Can calculate charged enclosed via

  $$\sum_{\text{all faces}} \Phi_{E} = \sum_{\text{each face } i} \vec{E}_i \cdot \vec{A}_i = \frac{q}{\varepsilon_0}$$

➤ Solve for $q$
I clicker question

Two point charges, \( +q \) and \( -q \), are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?

A. surface \( A \)  
B. surface \( B \)  
C. surface \( C \)  
D. surface \( D \)  
E. both surface \( C \) and surface \( D \)
Two point charges, +q and −q, are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?

A. surface A  
B. surface B  
C. surface C  
D. surface D  
E. both surface C and surface D  

Because no net charge enclosed in surfaces C and D
Summary of Lecture 3

Dipoles and in electric field: Forces, torque, potential energy

\[ \vec{\tau} = \vec{p} \times \vec{E} \quad U = -\vec{p} \cdot \vec{E} \]

Gauss’s Law: Relates electric flux through a closed surface with the total charge enclosed

- What is electric flux? How do I calculate?
- Simple derivation of Gauss’s Law

\[ \Phi_E = EA \cos \theta = \vec{E} \cdot \vec{A} \]

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Next Lecture: E fields and conductors:

- How determine amount of charge within a closed surface by examining the electric field on the surface
- How to use Gauss’s law to calculate the electric field due to a spherical charge distribution
- How to use Gauss’s Law to understand where charge is located on a charged conductor