Announcements

This week
- Homework due Thursday April 18, 11:59 pm
- Recitation quiz: HW8,L17+19 (Chapter 29); activity Chapter 30

Required on-line survey = 1% of your grade
Take survey TODAY (if not already)
https://rutgers.ca1.qualtrics.com/jfe/form/SV_3w1DABbPtzNxGzX

on your attitudes toward physics and textbook use. Taking the survey seriously and completing it will give you full credit for 1% of your grade. You can opt out of having your data used in the research, but you must complete the survey.

Final exam: Friday, May 10, 2019 4:00 to 7:00 PM in PLH
If you have 3 exams on May 10 OR 3 exams in a row, including Physics 227, contact Professor Cizewski with your entire exam schedule asap but not later than April 26 to request conflict exam. 30 multiple choice questions, ≈17 from Chapters 29-32, ≈13 cumulative Chapters 21-28. All exams are closed-book, no calculators or other electronic devices allowed.
• Changing magnetic flux in loop with time-dependent current \( i(t) \)

- Back EMF, that depends upon inductance (geometry) of the loop

- Self inductance \( L \) for \( N \)-loops:

- Inductor is a circuit element where potential difference depends upon
  - Direction of current flow
  - Sign of \( di/dt \)

- Inductors store energy of magnetic field

- EMF can be induced in a loop of conductors (e.g., solenoid) by changing \( \Phi_B \) in another loop: Mutual inductance

\[
\mathcal{E} = -\frac{d\Phi_B}{dt} = -L \frac{di}{dt}
\]

\[
L \equiv \frac{N\Phi_B}{i}
\]

\[
V_{ab} = V_a - V_b = L \frac{di}{dt}
\]

\[
U = \frac{1}{2} LI^2 \quad u_B = \frac{B^2}{2\mu_0}
\]

\[
\mathcal{E}_2 = -M_{1\rightarrow2} \frac{di_1}{dt}
\]

\[
M_{1\rightarrow2} = \frac{N_2\Phi_{1\rightarrow2}}{i_1}
\]
Lecture 21: April 17

Combining inductors in circuits with resistors and capacitors
Two identical concentric loops are arranged as shown in the Figure. Loop-1 has a steady current flowing through it (provided by a power supply). When the power is turned off, in what direction does the induced current flow in loop 2?

A. Current in loop 2 flows clockwise.

B. Current in loop 2 flows counterclockwise.

C. There is no induced current.

D. Something else happens.

E. The answer depends on what direction the current was flowing in loop 1.
Given circuit with battery, resistor R and inductor L

- At time \( t=0 \) the switch \( S_1 \) is closed
- Current \( i \) flows ccw, \( di/dt > 0 \)
- Loop rule
  \[
  \mathcal{E} - iR - L \frac{di}{dt} = 0
  \]

Rewrite so that can solve for \( i(t) \)

\[
\frac{di}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{L} i
\]

At instant \( S_1 \) closed, \( i(t=0)=0 \)

\[
\frac{di}{dt} = \frac{\mathcal{E}}{L}
\]

\( \left( \frac{di}{dt} \right)_{\text{final}} = \frac{\mathcal{E}}{L} - \frac{R}{L} I = 0 \)

A very long time after \( S_1 \) closed

At \( t=\infty \) \( di/dt=0 \)

\[
\frac{\mathcal{E}}{L} = \frac{R}{L} I \implies I = \frac{\mathcal{E}}{R} \]
Circuit with R-L; solve for $i(t)$

\[
\frac{di}{i - \left( \frac{\mathcal{E}}{R} \right)} = -\frac{R}{L} \, dt
\]

\[
\int_{a}^{b} \frac{dx}{x} = \ln \frac{b}{a}
\]
Circuit with resistor and inductor

\[
\frac{di}{dt} = \frac{E}{L} - \frac{R}{L}i
\]

\[
\frac{\frac{di}{dt}}{i - \left(\frac{E}{R}\right)} = -\frac{R}{L} \frac{dt}{d} \\
\int_{0}^{i} \frac{di'}{i' - \left(\frac{E}{R}\right)} = -\int_{0}^{t} \frac{R}{L} dt'
\]

\[
\ln[i - \left(\frac{E}{R}\right)]_{0}^{i} = \ln\left[\frac{i - \left(\frac{E}{R}\right)}{-\left(\frac{E}{R}\right)}\right] = -\frac{R}{L} t
\]

\[
\frac{i - \left(\frac{E}{R}\right)}{-\left(\frac{E}{R}\right)} = e^{-\left(\frac{R}{L}\right)t} \Rightarrow i = \frac{E}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t}\right)
\]

\[
\frac{di}{dt} = \frac{E}{L} e^{-\left(\frac{R}{L}\right)t}
\]
Circuit with resistor and inductor

Given circuit with battery, resistor $R$ and inductor $L$

- At time $t=0$ the switch $S_1$ is closed

\[
\mathcal{E} - iR - L \frac{di}{dt} = 0 \quad \frac{di}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{L} i
\]

In general $i(t)$ and $\frac{di}{dt}$

\[
\begin{align*}
  i &= \frac{\mathcal{E}}{R} \left( 1 - e^{-t/\tau} \right) \\
  \frac{di}{dt} &= \frac{\mathcal{E}}{L} e^{-t/\tau}
\end{align*}
\]

At $t=0$ $S_1$ closed, $i=0$

At $t=\infty$ (very long time after $S_1$ closed)

\[
I = \frac{\mathcal{E}}{R} \quad \left( \frac{di}{dt} \right)_{\text{final}} = 0
\]

\[
\tau = \frac{L}{R}
\]

\[
I \left( 1 - \frac{1}{e} \right)
\]

\[
O \quad t = \tau = \frac{L}{R}
\]
Energy in R-L circuit

Instantaneous rate at which EMF delivers energy to the circuit \( P = \mathcal{E}i \)

Instantaneous rate at which energy is dissipated in resistor and stored in inductor

\[
P = \mathcal{E}i = i^2 R + L \frac{di}{dt}
\]

Power supplied by EMF is dissipated in resistor and stored in inductor

**Steady state, \( t\gg0 \), Final current**

\[
I = \frac{\mathcal{E}}{R}
\]

- Power delivered to the circuit \( P = \mathcal{E}I \)
- Power dissipated in the resistor \( P_R = I^2 R \)
- Energy stored in the inductor \( U_L = \frac{1}{2} LI^2 \)
Given circuit with battery, resistor R and inductor L

- Switch $S_1$ has been closed for a long time

\[ i = \frac{\mathcal{E}}{R} = I_0 \quad (\frac{di}{dt})_{\text{final}} = 0 \]

Now close switch $S_2$ and open switch $S_1$
Given circuit with battery, resistor R and inductor L

- Switch $S_1$ has been closed for a long time
  \[ i = \frac{\mathcal{E}}{R} = I_0 \quad (\frac{di}{dt})_{\text{final}} = 0 \]

Now close switch $S_2$ and open switch $S_1$

\[ iR + L \frac{di}{dt} = 0 \]

At instant $S_2$ closed, $i = I_0$, $\frac{di}{dt} < 0$

A very long time after $S_2$ closed

\[ i = 0 \quad (\frac{di}{dt})_{\text{final}} = 0 \]
\[ \frac{di}{dt} = -\frac{R}{L}i \]
\[ \frac{di}{i} = -\frac{R}{L}dt \]
\[ \int_{i_0}^{i'} \frac{di'}{i'} = -\int_{0}^{t} \frac{R}{L}dt' \]
\[ \ln\frac{i}{I_0} = -\frac{R}{L}t \]
\[ i = I_0 e^{-\left(\frac{R}{L}\right)t} = I_0 e^{-t/\tau} \]
Given circuit with battery, resistor $R$ and inductor $L$

- Switch $S_1$ has been closed for a long time
  \[ i = \frac{\mathcal{E}}{R} = I_0, \quad \left( \frac{di}{dt} \right)_{\text{final}} = 0 \]

Now close switch $S_2$ and open switch $S_1$

\[ i = I_0 e^{-t/\tau} \]

At instant $S_2$ closed, \( i = I_0 \), \( di/dt < 0 \)

A very long time after $S_2$ closed

\[ i = 0, \quad \left( \frac{di}{dt} \right)_{\text{final}} = 0 \]

\[ \tau = \frac{L}{R} \]
Circuit with resistor and inductor

\[ \tau = \frac{L}{R} \]

Close switch \( S_1 \), current increases vs time

\[ i = \frac{\mathcal{E}}{R} \left( 1 - e^{-t/\tau} \right) \]
Circuit with resistor and inductor

\[ \tau = \frac{L}{R} \]

Open switch \( S_2 \), close switch \( S_1 \), current decreases vs time

[Diagram of the circuit with a resistor and an inductor, showing the current flowing through the circuit]

\[ i = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{\tau}}\right) \]

\[ i = I_0 e^{-\frac{t}{\tau}} \]

Switch \( S_2 \) is closed at \( t = 0 \).
Circuit with resistor and inductor

After $S_2$ closed

Energy stored in inductor

$$U_L(t) = \frac{1}{2} Li(t)^2$$

Current in the circuit decreases exponentially

$$i(t) = I_0 e^{-t/\tau} = I_0 e^{-t/\tau}$$

Energy stored in inductor vs time

$$U_L(t) = \frac{1}{2} Li(t)^2 = \frac{1}{2} LI_0^2 e^{-2Rt/L}$$

$$U_L(t) = U_0 e^{-t/\tau'} \quad \tau' = \frac{L}{2R}$$

$$\tau = \frac{L}{R}$$

$$t = \frac{L}{R}$$
An inductance $L$ and a resistance $R$ are connected to a source of emf as shown. When switch $S_1$ is closed, a current begins to flow. The final value of the current is

A. directly proportional to $RL$.
B. directly proportional to $R/L$.
C. directly proportional to $L/R$.
D. directly proportional to $1/(RL)$.
E. independent of $L$. 
Circuit with inductor and capacitor

Circuit with capacitor and inductor (no resistance)

- Two elements that can store energy
- Energy flows back and forth between C and L
- Harmonic oscillations in $q(t)$ and $i(t) = \frac{dq}{dt}$

Initially capacitor completely charged

$q(t=0) = Q_m$  \quad $i(t=0) = 0$

In general

Electric field energy stored in capacitor

$$U_E = \frac{1}{2} \frac{q^2}{C}$$

Magnetic field energy in inductor

$$U_B = \frac{1}{2} Li^2$$
Circuit with inductor $U_B = \frac{1}{2}Li^2$ and capacitor $U_E = \frac{1}{2}q^2/C$

Circuit with capacitor and inductor (no resistance)
- Two elements that can store energy
- Energy flows back and forth between C and L
- Harmonic oscillations in $q(t)$ and $i(t) = dq/dt$

Initially capacitor completely charged

$q(t=0)=Q_m \quad i(t=0)=0$
Circuit with inductor $U_B = \frac{1}{2} Li^2$ and capacitor $U_E = \frac{1}{2} \frac{Q^2}{C}$

Circuit with capacitor and inductor (no resistance)

• Two elements that can store energy
• Energy flows back and forth between C and L
• Harmonic oscillations in $q(t)$ and $i(t) = dq/dt$

Initially capacitor completely charged

$q(t=0) = Q_m$  $i(t=0) = 0$

All energy stored in capacitor

Now close the switch => capacitor discharges
Circuit with inductor $U_B = \frac{1}{2}Li^2$ and capacitor $U_E = \frac{1}{2}q^2/C$

Capacitor discharging
Initial current $i=0$
di/dt = maximum
- Induced EMF
- Induced current

When capacitor fully discharged
di/dt=0
Current is maximum $I_m$
All energy stored in inductor
- Charges capacitor

\[ E = U_B + U_E \]
\[ U_B = 0 \]
\[ U_E = \frac{1}{2} \frac{Q_{\text{max}}^2}{C} \]

\[ E = U_B + U_E \]
\[ U_B = \frac{1}{2} \frac{I_{\text{max}}^2}{L} \]
\[ U_E = 0 \]
Circuit with inductor $U_B = \frac{1}{2} L i^2$ and capacitor $U_E = \frac{1}{2} \frac{q^2}{C}$

Capacitor fully charged, but opposite to first stage
All energy stored in C
Capacitor discharges
⇒ $\frac{di}{dt} = \text{max}$
⇒ EMF and $i$ induced
Circuit with inductor \( U_B = \frac{1}{2} L i^2 \) and capacitor \( U_E = \frac{1}{2} \frac{q^2}{C} \)

\[
E = U_B + U_E

U_B = 0

U_E = \frac{1}{2} \frac{Q_{\text{max}}^2}{C}
\]

\[
E = U_B + U_E

U_B = \frac{1}{2} I_{\text{max}}^2

U_E = 0
\]

\[
E = U_B + U_E

U_B = 0

U_E = \frac{1}{2} \frac{Q_{\text{max}}^2}{L}
\]

\[
E = U_B + U_E

U_B = \frac{1}{2} I_{\text{max}}^2

U_E = 0
\]
Circuit with inductor \( U_B = \frac{1}{2} Li^2 \) and capacitor \( U_E = \frac{1}{2} \frac{q^2}{C} \)

Complete cycle. Energy transferred from capacitor to inductor then back again to capacitor, back to inductor, back to capacitor, etc.

\[
E = U_E + U_B = \frac{(q(t))^2}{2C} + \frac{L(i(t))^2}{2}
\]
Circuit with inductor $U_B = \frac{1}{2} Li^2$ and capacitor $U_E = \frac{1}{2} \frac{q^2}{C}$

Kirchhoff’s loop equation

\[
0 = -L \frac{di}{dt} - \frac{q(t)}{C} = -L \frac{d^2 q(t)}{dt^2} - \frac{q(t)}{C}
\]

Solution

where $\phi_0$ phase at $t=0$.

\[
\frac{d^2 q(t)}{dt^2} + \frac{1}{LC} q(t) = 0
\]

\[
q(t) = Q_m \cos(\omega t + \phi_0)
\]

\[
\omega = \sqrt{\frac{1}{LC}} = \frac{2\pi}{T}
\]

\[
i(t) = \frac{dq(t)}{dt} = -\omega Q_m \sin(\omega t + \phi_0)
\]

\[
i(t) = -I_m \sin(\omega t + \phi_0)
\]

http://www.kshitij-iitjee.com/oscillatory-LC-circuit/
Circuit with inductor \( U_B = \frac{1}{2} Li^2 \) and capacitor \( U_E = \frac{1}{2} \frac{q^2}{C} \)

\[
q(t) = Q_{\text{max}} \cos(\omega t + \phi_0)
\]

\[
i(t) = \frac{dq(t)}{dt} = -I_{\text{max}} \sin(\omega t + \phi_0)
\]

\[
U_E = \frac{(q(t))^2}{2C} = \frac{Q_{\text{max}}^2}{2C} \cos^2(\omega t)
\]

\[
U_B = \frac{L(i(t))^2}{2} = \frac{L I_{\text{max}}^2}{2} \sin^2(\omega t)
\]

\[
U_{E-\text{max}} (i = 0) = U_{B-\text{max}} (q = 0) \Rightarrow \frac{Q_{\text{max}}^2}{2C} = \frac{L I_{\text{max}}^2}{2}
\]

\[
E = \frac{Q_{\text{max}}^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{\text{max}}^2}{2C}
\]

http://www.kshitij-iitjee.com/oscillatory-LC-circuit/
Circuit with inductor $U_B = \frac{1}{2}Li^2$ and capacitor $U_E = \frac{1}{2}q^2/C$

$U_E = \frac{(q(t))^2}{2C} = \frac{Q_{\text{max}}^2}{2C} \cos^2(\omega t)$

$U_B = \frac{L(i(t))^2}{2} = \frac{LI_{\text{max}}^2}{2} \sin^2(\omega t)$

$E = \frac{Q_{\text{max}}^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{\text{max}}^2}{2C}$

$\omega = \sqrt{\frac{1}{LC}} = \frac{2\pi}{T}$
An inductor (inductance $L$) and a capacitor (capacitance $C$) are connected as shown. If the values of both $L$ and $C$ are doubled, what happens to the time required for the capacitor charge to oscillate through a complete cycle?

A. Time becomes 4 times longer.
B. Time becomes twice as long.
C. Time is unchanged.
D. Time becomes $1/2$ as long.
E. Time becomes $1/4$ as long.

$$\omega = \sqrt{\frac{1}{LC}} = \frac{2\pi}{T}$$
Kirchhoff’s loop equation after C charge, switch S closed

\[ 0 = -iR - L \frac{di}{dt} - \frac{q(t)}{C} \]

\[ 0 = -R \frac{dq(t)}{dt} - L \frac{d^2 q(t)}{dt^2} - \frac{q(t)}{C} \]

\[ \frac{d^2 q(t)}{dt^2} + \frac{R}{L} \frac{dq(t)}{dt} + \frac{1}{LC} q(t) = 0 \]

Solution for weak damping: \( \frac{1}{LC} \gg \frac{R^2}{4L^2} \)

\[ q(t) = Q_m \exp\left(-\frac{Rt}{2L}\right) \cos(\omega't + \phi_0) \]
Circuits with L-R-C: Damping of oscillations

Kirchhoff’s loop equation

\[ 0 = -iR - L \frac{di}{dt} - \frac{q(t)}{C} = -R \frac{dq(t)}{dt} - L \frac{d^2 q(t)}{dt^2} - \frac{q(t)}{C} \]

\[ \frac{d^2 q(t)}{dt^2} + \frac{R}{L} \frac{dq(t)}{dt} + \frac{1}{LC} q(t) = 0 \]

Solution for weak (under) damping: \( \frac{1}{LC} \gg \frac{R^2}{4L^2} \)

\[ q(t) = Q_m \exp\left(-\frac{Rt}{2L}\right) \cos(\omega't + \phi_0) \]

\[ \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \]

Cosine function
Amplitude decreases exponentially
Circuits with L-R-C: Damping of oscillations

Solution for weak (under) damping: \( \frac{1}{LC} >> \frac{R^2}{4L^2} \)

\[ q(t) = Q_m \exp\left(-\frac{Rt}{2L}\right)\cos(\omega't + \phi_0) \]

\[ \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \]

Solution for critical damping, R large

Solution for strong (over) damping, R very large
Summary Chapter 30

- Changing magnetic flux in loop with time-dependent current $i(t)$
  - Back EMF, that depends upon inductance (geometry) of the loop
  - Self inductance of one loop and mutual inductance of two loops
- Inductor is a circuit element where $V_{ab}$
- Inductors store energy of magnetic field
- L-R circuits
  - Growth and decay of $i(t)$ is exponential
- L-C circuits
  - Electric and magnetic energy exchanged between $\omega = \sqrt{1/LC}$
- L-R-C circuit
  - Damped oscillations $\omega' = \sqrt{1/\left(\frac{L}{C} - \frac{R^2}{4L^2}\right)}$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L \frac{di}{dt} = -M \frac{di}{dt}$

$L \equiv \frac{N\Phi_B}{i}$

$M \equiv \frac{N_2\Phi_{1\rightarrow 2}}{i}$

$V_{ab} = V_a - V_b = L \frac{di}{dt}$

$U = \frac{1}{2}LI^2$

$u_B = B^2 \frac{L/2}{2\mu_0}$

$\tau = \frac{L}{R}$
Lecture 22: April 22

Alternating Current Circuits