This week:

• No Homework
• Required recitation on Friday April 12: Chapter 29.
  • No quiz

**Required on-line survey** = 1% of your grade
You are required to complete an on-line [survey](https://rutgers.ca1.qualtrics.com/jfe/form/SV_3w1DABbPtzNxGzX) on your attitudes toward physics and textbook use. Taking the survey seriously and completing it will give you full credit for 1% of your grade. You can opt out of having your data used in the research, but you must complete the survey. Take the survey TODAY.
Chap 29B: Motional EMF

Wednesday April 10
Faraday’s Law

Induced EMF in a closed loop = - time rate of change of magnetic flux through the loop.

Lenz’s Law

Induced current has direction such that the magnetic field due to the induced current \textbf{opposes the change} in the magnetic flux that induces the current.
A bar magnet falls through a conducting ring. Which plot of the current vs. time is correct? The positive values of the current correspond to the direction shown in the Figure.
Faraday’s law of induction

- When the magnetic flux through a single closed loop changes with time
  - Induced emf that can drive a current around the loop
  - Direction: induced B opposes change in $\Phi_B$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Change in magnetic flux:
  - Change in magnetic field
  - Change in area
  - Change in angle between B and A
  - Any combination of these
  - Not restricted to when $\vec{B} \parallel \vec{A}$

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$$
Circular wire coil – seen edge on
B is uniform and decreasing vs t
➢ Induce B that opposes change in $\Phi_B$
➢ Induced EMF as shown

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt} \cdot \vec{A} = \frac{dB}{dt} A \cos \phi$$
Circular wire coil – seen edge on

\[ \epsilon = \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \frac{\mathrm{d}(\vec{B} \cdot \vec{A})}{\mathrm{d}t} = -BA \frac{\mathrm{d}\cos\phi}{\mathrm{d}t} \]

\[ \epsilon(t) = +BA \sin\phi \]
Rectangular loop rotated w/ constant angular speed $\omega$ about axis.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(\vec{B} \cdot \vec{A})}{dt} = -BA \frac{d\cos\omega t}{dt}$$

$$\mathcal{E}(t) = +BA\omega \sin \omega t$$

Slip rings connected to circuit
Current varies sinusoidally
Example: simple alternator

Rectangular loop rotated w/ constant angular speed $\omega$ about axis. With $N$ turns

\[
\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(\vec{B} \cdot \vec{A})}{dt} = -NBA \frac{d\cos \omega t}{dt}
\]

\[
\varepsilon(t) = +NBA\omega \sin \omega t
\]

Slip rings connected to circuit
Current varies sinusoidally

Loop (seen end-on)

\[
\varepsilon_{\text{max}} = BA\omega
\]

\[
\Phi_B
\]

$t$
Example: DC generator

Rectangular N loops rotated w/ constant angular speed $\omega$ about axis.
Split rings (commutator) connected to circuit
Reverses connections when EMF reverses
$\Rightarrow$ Absolute value of EMF

$$|\mathcal{E}| = NBA\omega |\sin \omega t|$$

$\Rightarrow$ Average value of $\sin \omega t = 2/\pi \Rightarrow \mathcal{E}_{av} = \frac{NBA\omega}{\pi}$
Example: DC generator $<\sin \omega t>$

Rectangular N loops rotated w/ constant angular speed $\omega$ about axis.
Split rings (commutator) connected to circuit
Reverses connections when EMF reverses
⇒ Absolute value of EMF

$$|\mathcal{E}| = NBA\omega |\sin \omega t|$$

⇒ Solving Average value of $\sin \omega t$
Example: area is changing

- B field into the page
- Conducting loop + sliding bar with resistance R

If bar moves to right $\Delta A > 0$, $\Delta \Phi_B > 0$
  - Induced B
  - Induced EMF

\[
\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(\vec{B} \cdot \vec{A})}{dt} = -B \frac{dA}{dt}
\]

\[
\mathcal{E} = -BL \frac{v dt}{dt} = -BLv
\]

In general motional EMF

\[
d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{\ell}
\]

\[
\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}
\]
The rectangular loop of wire is being moved to the right at constant velocity. A constant current $I$ flows in the long straight wire in the direction shown. The current induced in the loop is

A. clockwise and proportional to $I$.
B. counterclockwise and proportional to $I$.
C. clockwise and proportional to $I^2$.
D. counterclockwise and proportional to $I^2$.
E. zero.
The rectangular loop of wire is being moved to the right at constant velocity. A constant current $I$ flows in the long straight wire in the direction shown. Why no induced (motional) current/EMF?

\[ d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l} \]

\[ B(r)_{\text{line}} = \frac{\mu_0 I}{2\pi r} \]
Faraday’s Law & Induced electric fields

- Long thin solenoid of area $A$
- Magnetic field inside of solenoid
- Surround solenoid with conducting wire loop
- If current in solenoid increasing
  - $\Phi_B$ inside of loop increasing
  - Induced EMF
  - Induced Current $I'$ in loop w/ resistance $R$

\[
\mathcal{E} = -\frac{d}{dt} \Phi_B = -\mu_0 nA \frac{dI}{dt} = I'R
\]

\[
\vec{B} = \mu_0 n I
\]
Faraday’s Law & Induced electric fields

- Induced EMF
  \[ \mathcal{E} = -\frac{d}{dt} \Phi_B = -\mu_0 n A \frac{dI}{dt} = I'R \]

- Induced Current I’

- Consider a charged particle being forced to go around this wire loop
  - The force ≠ magnetic force
  - No magnetic field outside of the solenoid

- Must be electric force from induced electric field
  - Caused by induced EMF from changing magnetic flux

- Consider total work being done on this charge q going around the loop
  \[ W = q \oint \vec{E} \cdot d\vec{\ell} \neq 0 = q \mathcal{E} = -q \frac{d}{dt} \Phi_B \]
Faraday’s Law & Induced electric fields

• Induced EMF
\[ \mathcal{E} = -\frac{d}{dt} \Phi_B = -\mu_0 n A \frac{dI}{dt} = I' R \]

• Induced Current \( I' \)

• Work \( \neq 0 \)

• Changing magnetic flux always generates an electric field

\[ -\frac{d}{dt} \Phi_B = \oint \vec{E} \cdot d\vec{l} = \mathcal{E} \quad \text{loop} \]

Does not contradict what observed in electrostatics
Electrostatics vs Electrodynamics

**Electrostatics**
- E fields from static charges are conservative
  - No net work done in transporting charge around a closed loop
    \[ \oint \vec{E} \cdot d\vec{l} = 0 \]
  - Can associate a potential V with conservative E field
- E fields from static charges
  - Originate on positive charges
  - End on negative charges

**Electrodynamics**
- E fields from changes in \( \Phi_B \) are not conservative
  - Work done in transporting charge around a closed loop
    \[ \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B \]
  - Cannot associate a potential V with non-conservative E field
- E fields from changing \( \Phi_B \)
  - Form closed loops
What is happening (sometimes)?: Eddy currents

Changing magnetic flux in a loop

- Induced EMF and current
  \[ \mathcal{E} = -\frac{d}{dt} \Phi_B = \oint \vec{E} \cdot d\ell \_	ext{loop} \]

In bulk material

- Circulating currents of metal rotating through a B field
- Eddy Currents
  - Lenz’s Law: direction of Eddy currents must oppose change in magnetic flux
- Repulsive force

[Image of eddy currents]

What is happening (sometimes)?: Eddy currents

- Changing magnetic flux in a loop
  - induced EMF and current
  \[
  \mathcal{E} = -\frac{d}{dt} \Phi_B = \oint_{\text{loop}} \vec{E} \cdot d\vec{\ell}
  \]

When enter B-field region
- Increase \( \Phi_B \Rightarrow \bigcirc \ B_{\text{induced}} \)
- Current/EMF counter-clockwise
  - Force opposes direction of motion

When leave B field region
- Decrease \( \Phi_B \Rightarrow \bigotimes \ B_{\text{induced}} \)
- Current/EMF clockwise
  - Force opposes direction of motion

What is happening (sometimes)?: Eddy currents

In bulk (solid) material
- Circulating currents of metal rotating through a B field
- Eddy Currents
  - Lenz’s Law: direction of Eddy currents must oppose change in magnetic flux
- Repulsive force

In slotted material
- Eddy Currents can’t connect w/ each other
- Much less repulsive force
In what order do the disk race through a magnetic field? (Ignore changes in the moments of inertia. Assume the B field is a constant and the same size as the disks.)

A. 1, 2, 3, 4
B. 4, 3, 2, 1
C. 1, 4, 3, 2
D. 2, 3, 4, 1
A flexible loop of wire lies in a uniform magnetic field $B$ directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

A. is zero because the magnetic field is time-independent.
B. flows upward through resistor $R$.
C. flows downward through resistor $R$.
D. does not flow through resistor $R$. 
Faraday’s Law

\[ \mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \]

Induced EMF in a closed loop = - time rate of change of magnetic flux through the loop.

E fields from changing $\Phi_B$ form closed loops

Lenz’s Law
Induced current has direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

Special case: Motional EMF

\[ \mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \]

closed loop