Announcements

This week:

• Homework 6 due Thursday March 28:
  • Chapter 26 sections 3-5 + Chapter 27
• Recitation on Friday March 29: Chapter 27+28.
• Quiz on Friday March 29:
  • Homework 6, Lectures 12, 13 and 14
Summary Chapter 27

- Properties of magnets, and how magnets interact with each other.
- Visualizing magnetic field lines.
- No magnetic monopoles
- Magnetic flux thru closed surface
- Analyzing magnetic forces on moving charged particles
- Application: cyclotron motion
- Force on current carrying wires
- Current carrying loops:
  - Magnetic moment
  - Torque, potential energy in B-field
- DC Motor

\[ \Phi_B = \oint B \cdot d\vec{A} = 0 \]

\[ \vec{F} = q\vec{v} \times \vec{B} \quad R = \frac{mv}{qB} \]

\[ \vec{F} = I\vec{\ell} \times \vec{B} \]

\[ \vec{\mu} = I\vec{A} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \quad U = -\vec{\mu} \cdot \vec{B} \]
Lecture 15: March 25 Chapter 28a

Sources of magnetic fields
Electrostatics=>Magnetostatics

**Review of electrostatics**

- Electric force on a charged particle: \( \vec{F} = q \vec{E} \)

- Electric flux through a closed surface:
  \[
  \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}
  \]

- Electric dipoles: \( \vec{p} = q \vec{d} \)
  \[
  \vec{\tau} = \vec{p} \times \vec{E} \quad U = -\vec{p} \cdot \vec{E}
  \]

- Electric work done during a closed loop (circuit):
  \[
  \oint \vec{E} \cdot d\vec{l} = 0
  \]

**Magnetostatics analogy**

- Magnetic force on charged particle: \( \vec{F} = q \vec{v} \times \vec{B} \)

- Magnetic flux through a closed surface:
  \[
  \Phi_B = \oint \vec{B} \cdot d\vec{A} = 0
  \]

- Magnetic dipoles: \( \vec{\mu} = I \vec{A} \)
  \[
  \vec{\tau} = \vec{\mu} \times \vec{B} \quad U = -\vec{\mu} \cdot \vec{B}
  \]

- Work done by magnetic force around a closed loop?
  \[
  \oint \vec{B} \cdot d\vec{l} = ??
  \]
Chapter 28: Sources of magnetic fields

- Calculating magnetic fields
  - Single moving charged particle
  - Straight current-carrying wire
  - Current-carrying wire bent into a circle
- Forces between current carrying wires
- Calculating magnetic fields from current distributions => Ampere’s Law
  - How to get a uniform magnetic field
Magnetic field of a moving charge

A moving charge generates magnetic field that depends on
- velocity of the charge
- distance from the charge
- Direction: right hand rule
  - Cross product

\[ B(r) = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \mathbf{r}}{r^2} = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \mathbf{r}}{r^3} \]

\[ \mu_0 = 4\pi \cdot 10^{-7} \, \frac{Ns^2}{C} = 4\pi \cdot 10^{-7} \, \frac{T \cdot m}{A} \]

\( \mu_0 \) is vacuum permeability
A moving charge generates magnetic field that depends on:
- velocity of the charge
- distance from the charge
- Direction: right hand rule

\[ \vec{B}(r) = \frac{\mu_0 q}{4\pi} \left( \vec{v} \times \vec{r} \right) = \frac{\mu_0 q}{4\pi} \left( \vec{v} \times \vec{r} \right) \]

\[ \mu_0 = 4\pi \cdot 10^{-7} \frac{Ns^2}{C} = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A} \]

\( \mu_0 \) is vacuum permeability
A positive point charge is moving directly toward point $P$. The magnetic field that the point charge produces at point $P$

A. points from the charge toward point $P$.

B. points from point $P$ toward the charge.

C. is perpendicular to the line from the point charge to point $P$.

D. is zero.

E. The answer depends on the speed of the point charge.

\[ \vec{B}(r) = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^2} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3} \]
Magnetic field of a current element

Instead of a single moving charge, consider current carrying wire

B field from superposition of charge in each segment

\[ \vec{B}(r) = \frac{\mu_0}{4\pi} q_{\text{seg}} \frac{\vec{v} \times r}{r^2} \]

Charge of carriers in wire segment volume=Adℓ

\[ q_{\text{seg}} = \sum_i q_i = neA d\ell \]

\[ I = nev_d A \quad \Rightarrow \quad q_{\text{seg}} \vec{v}_d = I d\ell \]

\[ d\vec{B}(r) = \frac{\mu_0}{4\pi} I \frac{d\ell \times r}{r^2} \]
Magnetic field of a current element

Instead of a single moving charge, consider current carrying wire

Superposition of each segment

\[ \vec{B}(r) = \frac{\mu_0}{4\pi} q_{\text{seg}} \frac{\vec{v} \times \vec{r}}{r^2} \]

\( \vec{B}(r) \) proportional to \( 1/r^2 \)

Similar to \( E \) field of a point charge

Direction: current element \( I d\vec{\ell} \times \vec{r} \)

Law of Biot-Savart
Example: Magnetic Field of circular current carrying loop

B field at center of circular current carrying loop of radius R?

Contribution of a small portion of the loop:

\[
\delta \vec{B}(r) = \frac{\mu_0}{4\pi} I \frac{\vec{d}\ell \times \hat{r}}{R^2}
\]

\[
\vec{d}\ell \times \hat{r} \text{ all in same direction and } \perp
\]

\[
\sum \delta B(r) = \frac{\mu_0}{4\pi} I \frac{\sum d\ell}{R^2} \quad \sum d\ell = 2\pi R
\]

Field at center of loop

\[
B(0) = \frac{\mu_0 NI}{2\pi R}
\]

If N turns in loop, B is N times stronger
In the figure an irregular loop of wire carrying a current lines in the plane of the paper. Suppose that the loop is distorted into some other shape while remaining in the same plane. Point P is still within the loop. Which of the following is a TRUE statement concerning this situation?

A. The magnetic field at point P will always lie in the plane of the paper.

B. It is possible that the magnetic field at point P is zero

C. The magnetic field at point P will not change in magnitude when the loop is distorted.

D. The magnetic field at point P will not change in direction when the loop is distorted.

E. None of the other statements is true.
**Magnetic field of a straight current-carrying conductor**

\( \textbf{B} \) field a distance \( x \) from long wire

Consider a small element of wire \( d\ell \)

Choose origin at point \( P \)

\[
\textbf{d}\textbf{B}(r) = \frac{\mu_0}{4\pi} \frac{I \, d\ell \times (-\hat{r})}{r^3}
\]

\( d\textbf{B} \) points into the page

To solve for \( \textbf{B} \) need to express all variables in terms of angle \( \phi \)

To calculate \( \textbf{B} \): \(-90^\circ \leq \phi \leq +90^\circ \)

At point \( P \), the field \( d\textbf{B} \) from \( d\ell \) points into the page

Total field \( \textbf{B} \) from the entire conductor also points into the page
**Magnetic field of a straight current-carrying conductor**

$B$ field a distance $x$ from wire

Consider a small element of wire $d\ell$

Choose origin at point P

$d\vec{B}(r) = \frac{\mu_0 I}{4\pi} \frac{d\ell \times (-\hat{r})}{r^3}$

$d\vec{B}$ points into the page

To solve for $B$ need to express all variables in terms of angle $\phi$

What is $r$ in terms of $\phi$?

What is $d\ell$ in terms of $\phi$?
Magnetic field of a straight current-carrying conductor

de field a distance \( x \) from a small element of wire \( \text{d} \ell \)

Choose origin at point P

\[
\text{d} \vec{B}(r) = \frac{\mu_0 I}{4\pi} \frac{\text{d} \ell \times (-\hat{r})}{r^3}
\]

\( \text{d} \vec{B} \) points into the page

To solve for \( \vec{B} \) need to express all variables in terms of angle \( \phi \)

\[
\tan \phi = \frac{y}{x} \implies y = x \tan \phi
\]

\[
dy = d \ell = x \sec^2 \phi d\phi = \frac{x d \phi}{\cos^2 \phi}
\]

\[
\cos \phi = \frac{x}{r} \implies r = \frac{x}{\cos \phi}
\]

\[
r = \sqrt{x^2 + y^2}
\]
Magnetic field of a straight current-carrying conductor

\( \mathbf{dB} \) field a distance \( x \) from a small element of wire \( d\ell \)

Choose origin at point P

\( \mathbf{dB} \) points into the page

\[
\mathbf{d}\mathbf{B}(r) = \frac{\mu_0}{4\pi} I \frac{d\ell \times (-r)}{r^2}
\]
Magnetic field of a straight current-carrying conductor

d\vec{B} field a distance x from a small element of wire d\ell
Choose origin at point P

$d\vec{B}(r) = \frac{\mu_0 I}{4\pi} \frac{d\ell \times (-\hat{r})}{r^3}$

$d\vec{B}$ points into the page
To solve for $B$ need to express all variables in terms of angle $\phi$

$d\vec{B}(r) = \frac{\mu_0 I}{4\pi} \frac{d\ell \times (-\hat{r})}{r^2}$

$d\vec{B}(r) = \frac{\mu_0 I}{4\pi} \frac{d\ell \times (-\hat{r})}{r^3} = \frac{\mu_0 I}{4\pi} I\left(\frac{xd\phi}{\cos^2 \phi}\right)\cos \phi \left(\frac{\cos^2 \phi}{x^2}\right) = \frac{\mu_0 I \cos \phi d\phi}{4\pi x}$
Magnetic field of a straight current-carrying conductor

To solve for $B$ need variables in terms of $\phi$

To calculate $B$: $-90^\circ \leq \phi \leq +90^\circ$

$$\int dB(r) = \frac{\mu_0}{4\pi x} I \int_{\phi=\frac{-\pi}{2}}^{\phi=\frac{\pi}{2}} \cos \phi \, d\phi$$

$$d\vec{B}(r) = \frac{\mu_0}{4\pi x} I \cos \phi \, d\phi$$
Magnetic field of a straight current-carrying conductor

To solve for $B$ need variables in terms of $\phi$

To calculate $B$: $-90^\circ \leq \phi \leq +90^\circ$

$$\int dB(r) = \frac{\mu_0}{4\pi x} I \int_{\phi=-\pi/2}^{\phi=+\pi/2} \cos \phi \, d\phi$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi x} [1 - (-1)] = \frac{\mu_0 I}{2\pi x}$$

Total field $B$ from the entire conductor given by right hand rule: thumb in direction of current, hand in direction of $B$
Consider a wire bent in the hairpin shape. The wire carries a current $I$. What is the approximate magnitude of the magnetic field at point $a$?

Apply superposition: the net field at point $a$ is superposition of three $B$ fields produced by the current — the semi-circle plus two straight wires.

Semi-circle: $B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$  
(1/2 of the field of a circular loop)

Top wire: $B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{4\pi R}$  
(1/2 of the field of an infinite wire)

Bottom wire: $B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{4\pi R}$  
(1/2 of the field of an infinite wire)
Example

Consider a wire bent in the hairpin shape. The wire carries a current I. What is the approximate magnitude of the magnetic field at point a?

Apply superposition: the net field at point a is superposition of three B fields produced by the current — the semi-circle plus two straight wires.

Semi-circle: \( B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R} \) (1/2 of the field of a circular loop)

Top wire: \( B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{4\pi R} \) (1/2 of the field of an infinite wire)

Bottom wire: \( B(r) = \left( \frac{1}{2} \right) \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{4\pi R} \) (1/2 of the field of an infinite wire)

\[
B(r) = \frac{\mu_0 I}{4R} + 2\left( \frac{\mu_0 I}{4\pi R} \right) = \frac{\mu_0 I}{4R} \left( 1 + \frac{2}{\pi} \right)
\]
Interaction between 2 current carrying wires

Magnetic field due to $I_1$ at wire $I_2$

$$\vec{B}(r) = \frac{\mu_0 I_1}{2\pi r}$$

Magnetic force on wire $I_2$

$$\vec{F} = I_2\vec{\ell} \times \vec{B}$$

$$\frac{F}{L} = I_2B = I_2\frac{\mu_0 I_1}{2\pi r}$$

Force/unit length between current carrying wires

- Attract: when parallel currents
- Repel: when anti-parallel currents
Interaction between 2 current carrying wires

Magnetic field due to $I_1$ at wire $I_2$

$$\vec{B}(r) = \frac{\mu_0 I_1}{2\pi r}$$

Magnetic force on wire $I_2$

$$\vec{F} = I_2 \ell \times \vec{B}$$

$$\frac{F}{L} = I_2 B = I_2 \frac{\mu_0 I_1}{2\pi r}$$

Force/unit length between current carrying wires

- Attract: when parallel currents
- Repel: when anti-parallel currents
A wire consists of two straight sections with a semicircular section between them. If current flows in the wire as shown, what is the direction of the magnetic field at $P$ due to the current?

A. to the right  
B. to the left  
C. out of the plane of the figure  
D. into the plane of the figure  
E. misleading question — the magnetic field at $P$ is zero
Electrostatics

- Electric flux through a closed surface:
  \[ \Phi_E = \oint E \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

- Electric work done during a closed loop (circuit):
  \[ \oint E \cdot d\vec{l} = 0 \]

- Only in electrostatics

Magnetostatics analogy

- Magnetic flux through a closed surface:
  \[ \Phi_B = \oint B \cdot d\vec{A} = 0 \]

- Work done by magnetic force around a closed loop?
  \[ \oint B \cdot d\vec{l} \neq 0 \]

- In general
Pick a ("Amperian") loop

Direction of circulation:
right hand rule for direction of current

\( \mathbf{B} \) field a distance \( r \) from straight wire with current

\[
\mathbf{B}(r) = \frac{\mu_0 I}{2\pi r}
\]

\[
\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{\ell} = B(r) \oint d\ell = \frac{\mu_0 I}{2\pi r} (2\pi r)
\]

\[
\oint \mathbf{B} \cdot d\mathbf{\ell} = \mu_0 I
\]

Circulation of \( \mathbf{B} \) field \( \neq 0 \)
if current enclosed by loop \( \neq 0 \).
Pick a ("Amperian") loop

Direction of circulation:
right hand rule for direction of current

$B$ field a distance $r$ from straight wire with current

$$\vec{B}(r) = \frac{\mu_0 I}{2\pi r}$$

Pick another loop that does not enclose the current carrying wire

$$\Delta \ell = \varphi r$$

$$\oint \vec{B}(\vec{r}) \cdot d\vec{\ell} = 0 + \frac{\mu_0 I}{2\pi} \varphi + 0 - \frac{\mu_0 I}{2\pi} \varphi = 0$$

For a loop that does not enclose any current, the circulation is 0.
Circulation of B for current carrying wire

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \]

Ampere’s Law
Circulation of B field = current enclosed \( \times \mu_0 \)

Next lecture:
Use Ampere’s Law to calculate B fields of symmetric objects
Chapter 28: Summary today

- Calculating magnetic fields
  - Single moving charged particle
    \[ \vec{B}(r) = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^3} \]
  - Straight current-carrying wire
    \[ \vec{B}(r)_{\text{line}} = \frac{\mu_0 I}{2\pi r} \]
  - Current-carrying wire bent into a circle
    \[ \vec{B}(0)_{\text{loop}} = \frac{\mu_0 I}{2R} \]
- Forces between current carrying wires
  - Attract: parallel
  - Repel: anti-parallel
- Ampere’s Law: \[ \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I \]