Announcements

Reminder:

• Homework due Thursday March 8:
  • Chapter 25 + Chapter 26, sections 1-2
• Quiz on Friday March 9:
  • This week’s homework, Lectures 9+10 and 11 (today)
Summary Chapter 25a

- Deduced current \( I = \frac{dQ}{dt} = n|q|v_d A \)
- Vector current density; points in \( E \) direction \( \mathbf{J} = nq\mathbf{v}_d \)
- Material has a resistivity \( \rho = \frac{E}{J} \) w/ depend \( T \) \( \rho(T) = \rho_0[1 + \alpha(T - T_0)] \)
- Resistance of a material (conductor) depends on geometry and \( \rho \)
- For materials that obey Ohm’s law \( V = IR \) Cylindrical resistor \( R = \rho \frac{L}{A} \)
- Potential drop across (Ohmic) resistor \( \Delta V = -IR \)
- If resistors in series, equivalent resistance \( R_{eq} = \sum_i R_i \)
- Today: why equivalent resistance in series is the sum
Review from last lecture
Current in single-loop circuit (Ideal battery)

Going around loop clockwise

\[ V_a + \mathcal{E} - IR = V_a \]
\[ \mathcal{E} - IR = 0 \]
\[ I = \frac{\mathcal{E}}{R} \]

Loop Rule: The (algebraic) sum of the changes in potential around any loop of a circuit must be zero.

\[ \sum_i V_i = 0 \]

EMF Rule:
If go through an ideal EMF device in the direction of the EMF arrow (from − to +):

\[ \Delta V_{EMF} = +\mathcal{E} \]
Current in single-loop circuit (Ideal battery)

Going around loop counter clockwise

\[ V_a + \mathcal{E} - IR = V_a \]

\[ \mathcal{E} - IR = 0 \]

\[ I = \frac{\mathcal{E}}{R} \]

Loop Rule: The (algebraic) sum of the changes in potential around any loop of a circuit must be zero.

\[ \sum_i V_i = 0 \]

EMF Rule:
If go through an ideal EMF device in the direction of the EMF arrow (from $-$ to $+$):

\[ \Delta V_{EMF} = +\mathcal{E} \]

If go through an ideal EMF device in the direction opposite to the EMF arrow (from $+$ to $-$):

\[ \Delta V_{EMF} = -\mathcal{E} \]
Current in single-loop circuit (Ideal battery)

Going around loop clockwise

\[ V_a + \mathcal{E} - IR = V_a \]
\[ \mathcal{E} - IR = 0 \]
\[ I = \frac{\mathcal{E}}{R} \]

Loop Rule: The (algebraic) sum of the changes in potential around any loop of a circuit must be zero.

\[ \sum_i V_i = 0 \]

Resistance Rule:
If go through a resistor in the direction of the current:

\[ \Delta V_R = -IR \]
Current in single-loop circuit (Ideal battery)

Going around loop counter clockwise

\[ V_a + \mathcal{E} - IR = V_a \]
\[ \mathcal{E} - IR = 0 \]
\[ I \equiv \frac{\mathcal{E}}{R} \]

Loop Rule: The (algebraic) sum of the changes in potential around any loop of a circuit must be zero.

\( \sum_i V_i = 0 \)

Resistance Rule:
If go through a resistor in the direction of the current:
\( \Delta V_R = -IR \)
If go through a resistor opposite direction of the current:
\( \Delta V_R = +IR \)
• Real sources of emf actually contain some **internal resistance** $r$.

• The **terminal voltage** of the 12-V battery shown at the right is less than 12 V when it is connected to the light bulb.

• i.e., voltage drop $V_{ab}$ across the battery

• Depends upon intrinsic EMF $\varepsilon$ of the battery

• Minus the potential drop across the internal resistance $r$ of the battery

\[ V_{ab} = \varepsilon - Ir \]
Non-ideal Battery: with internal resistance \( r \)

Output voltage of the battery \( V \) 
= potential drop across the resistor(s)

\[
\mathcal{E} - Ir - IR = 0 \implies I = \frac{\mathcal{E}}{R + r}
\]

Note: 
If ideal battery (i.e., \( r = 0 \)), get result from previous slides
Non-ideal Battery: with internal resistance \( r \)

Output voltage of the battery \( V \) = potential drop across the resistor(s)

\[
\varepsilon - Ir - IR = 0 \implies I = \frac{\varepsilon}{R + r}
\]

\[
I = \frac{12 \text{ V}}{(4 + 2) \Omega} = 2 \text{ A}
\]
Single loop circuit: Resistors in series

\[ V_{\text{applied}} = \mathcal{E} = IR_1 + IR_2 \]

\[ I = \frac{\mathcal{E}}{R_1 + R_2} \]

\[ R_{\text{equivalent}} = R_1 + R_2 = \sum_{i} R_i \]

\[ V_{\text{applied}} = \mathcal{E} = IR_{\text{equivalent}} \]

\[ I = \frac{\mathcal{E}}{R_{\text{equivalent}}} \]

Resistors in Series
Single loop circuit: Resistors in series

- What have we seen?
  - In example of battery with internal resistance \( r \neq 0 \), same current went through external resistor \( R \) as went through \( r \)

- Summary:
  - When a potential difference \( V \) is applied across resistances connected in series, identical current flows through the resistors.
  - Sum of potential differences = applied potential difference
  - Resistors connected in series can be replaced with an equivalent resistor that has same current \( I \) and same total potential difference \( V \) as actual resistors

\[
\sum_{i} V_{R_i} = V_{\text{applied}}
\]

\[
V_{\text{applied}} = IR_{\text{equivalent}}
\]
Think about the charge carrier

- Goes “downhill” from higher $V_a$ to lower $V_b$
- Dissipates energy $eV_{ab}$ into environment
  - Additional kinetic energy
  - Thermal (heat) or light energy

Consider charge $dq$ passing a circuit element in time $dt \Rightarrow$ power dissipated

$$P = \frac{dq V_{ab}}{dt} = V_{ab} I$$

If the circuit element is a resistor:

$$V = IR$$

$$P = VI = RI^2 = \frac{V^2}{R}$$
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in series as shown.

Across which bulb is there the greatest voltage drop?

A. the 60-W light bulb
B. the 120-W light bulb
C. the 240-W light bulb
D. All three light bulbs have the same voltage drop.
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D. All three light bulbs have the same voltage drop.
Lecture 11: Chapter 26

Resistors in circuits & Kirchoff’s Rules
Single-loop circuit: resistors + 2 EMFs

\[ \mathcal{E}_1 - I r_1 - I R_1 - I r_2 - \mathcal{E}_2 - I R_2 = 0 \]

\[ \Rightarrow I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + R_1 + r_2 + R_2} \]

\[ I = \frac{12 \text{ V} - 4 \text{ V}}{(2 + 3 + 4 + 7) \Omega} = 0.5 \text{ A} \]
Two junctions: a and b – analogous to water pipe

- Charge is conserved; water entering pipe = water out of pipe
- Current into the junction must equal current out of junction
Two junctions: a and b

- Charge is conserved
- Current into the junction must equal current out of junction
- Kirchhoff’s junction rule $\sum I_i = 0$
- Or $\sum I_{in} = \sum I_{out}$
Two junctions: a and b

- Charge is conserved
- Current into the junction must equal current out of junction

Kirchhoff’s junction rule \[ \sum_{i} I_i = 0 \]

Kirchhoff’s loop rule \[ \sum_{i} V_i = 0 \]
Multiloop circuits: resistors in parallel

\[ \sum_{i} V_i = 0 \]

Loop rule:
Voltage drops across the resistors are the same

Junction rule:

\[ \sum_{i} I_i = 0 \implies I = I_1 + I_2 + I_3 \]

When resistors in parallel:

\[ \frac{1}{R_{eq}} = \sum_{i} \frac{1}{R_i} \]
Kirchhoff’s Rules Summary

- If resistors in series, equivalent resistance: \( R_{eq} = \sum R_i \)

- If resistors in parallel, equivalent resistance: \( \frac{1}{R_{eq}} = \sum \frac{1}{R_i} \)

- Kirchhoff’s Loop rule: \( \sum V_i = 0 \)

- Potential drop across resistor in direction of current: \( \Delta V = -IR \)

- Analogy: completing a hike where go up a mountain, then down into a valley and end up where you started

- Kirchhoff’s Junction rule: \( \sum I_i = 0 \) or \( \sum I_{in} = \sum I_{out} \)

- Analogy: fluid in a pipe – what comes in, must go out

- From conservation of charge (current)
Three identical resistors, each of resistance $R$, are connected as shown. What is the equivalent resistance of this arrangement of three resistors?

A. $3R$
B. $2R$
C. $3R/2$
D. $2R/3$
E. $R/3$
Three identical resistors, each of resistance $R$, are connected as shown. What is the equivalent resistance of this arrangement of three resistors?

A. $3R$

B. $2R$

C. $3R/2$

D. $2R/3$

E. $R/3$
Sign conventions for Kirchhoff’s Rules

Travel through EMF from neg (−) to pos (+) terminal

Travel through EMF from pos (+) to neg (−) terminal

Determine direction of current $I$ through resistor $R$

Travel through $R$ in direction opposite to $I$

Travel through $R$ in direction of $I$
Example: Complex circuit; what are currents & $R_{eq}$

Three loops, several junctions

3 equations with 3 unknowns: $I_1$ $I_2$ $I_3$

Loop 1

\[ V - I_1 (1\Omega) - (I_1 - I_3)(1\Omega) = 0 \]

Loop 2

\[ V - I_2 (1\Omega) - (I_2 + I_3)(2\Omega) = 0 \]

Loop 3

\[ -I_1 (1\Omega) - I_3 (1\Omega) + I_2 (1\Omega) = 0 \]

Loop 3 becomes

\[ -I_1 - I_3 + I_2 = 0 \]

\[ I_2 = I_1 + I_3 \]
Example: Complex circuit; what are currents & $R_{eq}$

Three loops, several junctions

Loop 1
$$V = I_1 (1\Omega) + (I_1 - I_3)(1\Omega)$$

Loop 2
$$V = I_2 (1\Omega) + (I_2 + I_3)(2\Omega)$$

New equation
$$I_2 = I_1 + I_3$$

Substitute $I_2$ into Loop 2
$$V = (I_1 + I_3)(1\Omega) + (I_1 + I_3 + I_3)(2\Omega)$$
$$V = I_1 (1\Omega + 2\Omega) + I_3 (1\Omega + 2 \cdot 2\Omega)$$
$$V = I_1 (3\Omega) + I_3 (5\Omega)$$
Example: Complex circuit; what are currents & $R_{eq}$

Three loops, several junctions

Loop 1

$V = I_1 (1\, \Omega) + (I_1 - I_3) (1\, \Omega)$

Rewritten Loop 2

$V = I_1 (3\, \Omega) + I_3 (5\, \Omega)$

Solve for $I_3$ in loop 1

$13V = (6A)(2\, \Omega) - I_3 (1\, \Omega)$

$I_3 = -1A$

$I_1 = 6A$

$I_2 = I_1 + I_3 = (6 - 1)A = 5A = I_2$
What is equivalent resistance?

\[ V = 13V = (I_1 + I_2)R_{eq} \]

\[ R_{eq} = \frac{13}{11} \Omega \]

\( I_1 = 6A \)
\( I_2 = 5A \)
\( I_3 = -1A \)

Not a simple series or parallel circuit

Note: If all of the resistors on the outside were the same

\( I_3 = 0 \) independent of middle resistance
In this circuit, what is the \( \varepsilon \) of the battery? Assume zero internal resistance for the battery.

A. \( \varepsilon = 4 \text{ V} \)
B. \( \varepsilon = 12 \text{ V} \)
C. \( \varepsilon = 8 \text{ V} \)
D. \( \varepsilon = 20 \text{ V} \)
E. Impossible to determine without knowing the value of R.
In this circuit, what is the $\varepsilon$ of the battery? Assume zero internal resistance for the battery.

A. $\varepsilon = 4\, \text{V}$
B. $\varepsilon = 12\, \text{V}$
C. $\varepsilon = 8\, \text{V}$
D. $\varepsilon = 20\, \text{V}$

E. Impossible to determine without knowing the value of $R$. 

**I clicker answer**
• Resistors in series
\[ R_{eq} = \sum_i R_i \]

• Resistors in parallel
\[ \frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \]

• Kirchoff’s Rules
  • Junction Rule
\[ \sum_i I_i = 0 \]
  • (Closed) Loop Rule
\[ \sum_i V_{\text{drops}} = 0 \]

Used these concepts to analyze complicated circuits with several resistors and several batteries.
Next lecture: Circuits with R and C