I. Self-Inductance of a small circular loop.

Consider a single loop of radius $a$, with a counter-clockwise current $I$, as shown in the figure to the right. Assume that the loop wire is infinitely thin and the radius $a$ is very small. The effect of assuming a small loop radius is that one can take the magnetic field inside the loop to be approximately constant and equal to the magnetic field at the center of the loop.

1. Recall from Chapter 28, equation 28.17, that the magnetic field at the center of the loop is $\frac{\mu_0 I}{2a}$ and is pointed out of the page. Using the assumption that the loop has a small enough radius to treat the magnetic field inside the loop as constant and equal to the value at the center, calculate the magnitude of the magnetic flux through the loop.

2. Assume the current $I$ is a function of time $I(t)$ and $\frac{dI(t)}{dt} \neq 0$. By changing the current in the loop we change the magnetic field due to $I(t)$. This in turn induces an EMF. If everything else in the problem other than the current remains unchanged, calculate the magnitude of the induced EMF in the loop. Is the induced EMF directly or inversely proportional to $\frac{dI(t)}{dt}$? [Hint: Use Faraday’s law.]

3. In the relationship between the induced EMF in the loop and $\frac{dI(t)}{dt}$, the proportionality constant is called the self-inductance of the loop. Using your result from question 2, what is the self-inductance of the loop shown in the figure?

4. The answer to question 3 reveals that the self-inductance of the loop in this scenario depends only on the geometry (size and shape) of the loop. The following scenarios describe different loop geometries and currents flowing around each loop. Circle all of the scenarios that have a DIFFERENT self-inductance than the circular loop of radius $a$ from question 3.

(a) A square loop of side $a$ carrying current $I$. (c) A circular loop of radius $2a$ carrying a current $I$.

(b) A circular loop of radius $a$ carrying a current $3I$. (d) A circular loop of radius $a$ carrying a current $I$ that increases twice as fast.