I. Review of electric dipoles

In recitation this week we will consider forces and torques on magnetic dipoles.

Consider an electric dipole consisting of charges $+q$ and $-q$ held at a separation $d$ and kept parallel to the Y-axis. There is a constant external electric field $\vec{E}$ in the space that is independent of the dipole charges. This independent electric field is uniform and points along the negative X direction.

Consult the homework problem “Torque on a Dipole in a Uniform Field” in Homework #1 (problem #9, tutorial).

1. Write down the electric dipole vector in terms of $q$, $d$, unit vector along X-axis and the unit vector along Y-axis.

2. Write down the forces on $+q$ and $-q$ due to the external electric field in terms of $q$, $\vec{E}$, unit vector along X-axis and the unit vector along Y-axis.

3. What is the net force on the dipole?

4. Imagine a rotation axis that intersects the dipole perpendicularly at its midpoint. How will the dipole rotate about this axis, clockwise or anticlockwise?

5. What is the magnitude of the torque acting on the $+q$ charge about this rotation axis? What is the magnitude of the torque acting on the $-q$ charge about this rotation axis?

6. Are the torques on $+q$ and $-q$ acting in the same direction or are they opposing each other? What is the magnitude of the net torque acting on the dipole due to the electric field? Write the net torque in terms of the electric dipole moment and the electric field.

7. Write down the most general vector equation relating the net torque on the electric dipole to the electric dipole moment and the external independent electric field. [Hint: refer to your textbook if you have trouble.]
II. Ampere’s Law.

Consider an infinitely long and infinitely thin straight current-carrying wire that carries a current $I$, as shown in Figure 1. The arrows on the wire show the direction of the current. Figure 2 shows the top view of the wire. Also imagine a circle around the wire whose plane is perpendicular to the wire, shown as the dashed line in Figures 1 and 2. [Note: If you encounter many issues with your understanding, look at Lecture 16 notes, and textbook Chapter 28 Section 6.]

1. Draw the directions of the magnetic fields at points B, C and D on Figure 2. [Hint: Use Biot-Savart Law or the right hand rule.]

2. Consider the tangents at the three consecutive points, B, C and D in Figure 2. How do the directions of the magnetic fields at those three points compare with the directions of the tangents?

Now imagine that you are a tiny ant crawling on the circular path shown in Figures 1 and 2.

3. When you (the ant) look at the wire from points B, C and D, would the wire and the current in the wire look the same or different?

4. Based on your answer in number 3, would the magnitude of the magnetic field felt at points B, C and D be the same or different?

5. If we translate the wire up or down along the direction of the wire does the current distribution change? [Hint: Remember, the wire is infinitely long.]

6. Imagine a rotational axis along the wire. If we rotate the wire about this axis in a clockwise or anticlockwise direction, will the current distribution change?

By now you may be convinced that this current distribution (a current-carrying infinite straight wire) possesses translational and rotational symmetry. According to the symmetry principle, the magnetic field produced by such a current distribution should also have the same symmetries. We can know the direction of the magnetic field produced by a current, assuming we know the direction of the current.
A well-known law that connects a static current distribution (static meaning the current is not changing with time) with the magnetic field it produces is Ampere’s Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} \quad (1)$$

The integral with a circle in the middle signifies a line integral along a closed loop or path. In this case, we will integrate the magnetic field along a closed circular path around the current-carrying wire. For example, the dashed circles drawn in Figures 1 and 2 are examples of such closed paths or loops. The right hand side of the equation consists of a constant called the magnetic permeability of free space ($\mu_0$) multiplied by the total current enclosed by the loop that we’re integrating over ($I_{\text{enclosed}}$). Let’s work through this process in the following steps:

1. Break the circle up in tiny little segments of length $dl$, so that each segment is a very good approximation for a point. Each line segment has the direction tangent to the curve at those points. Add up all of the line segments around the circular path in one direction until you arrive back at where you started.

2. Recall the direction of the magnetic fields on each of those tiny segments using the right hand rule.

3. For each segment calculate the dot product $\vec{B} \cdot d\vec{l}$. How does the direction of the magnetic field compare to the length segment: are they parallel, antiparallel, or perpendicular?

4. Finally, to evaluate the line integral of the magnetic field over the circle, add all the dot products that you have just calculated for each little segment.

Mathematically, steps 1 through 4 above are summarized by the following equation:

$$\oint \vec{B} \cdot d\vec{l} = \lim_{\Delta l \to 0} \sum_i (\vec{B}_i \cdot \Delta\vec{l}_i) \quad (2)$$