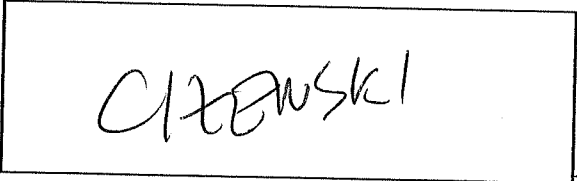


Physics 227 – Final Exam  
Tuesdya, December 20, 2016, 4:00 - 7:00 PM  
CAC Gym (A-J), CAC Gym Annex (K-R) and  
Scott Hall 135 (S-Z)

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Your exam code

SIGN HERE

1. Use a #2 pencil to mark entries on the answer sheet. *Enter the following ID information now, before the exam starts.*
2. In the section labeled NAME (Last, First, M.I.) enter your last name, then fill in the empty circle for a blank, then enter your first name, another blank, and finally your middle initial.
3. Under STUDENT # enter your 9-digit RUID Number.
4. Under CODE enter the exam code given above.
5. Enter 227 under COURSE. You do not need to write anything else on the answer sheet for now, but you may continue to read the instructions.
6. During the exam, you are allowed three (3) handwritten sheets of paper, 8.5 x 11 inches in size, handwritten on both sides. NO Calculators. NO Cell phones. NO smart watches.
7. The exam consists of 30 multiple-choice questions. For each multiple-choice question mark only **ONE** answer. There is no deduction of points for an incorrect answer, so even if you cannot work out the answer to a question, you should make an educated guess.
8. If you have questions or problems during the exam, you may raise your hand and a proctor will assist you. We will provide the value of physical constants that are needed. It is your responsibility to know the relevant equations.
9. You are not allowed to help any other student, ask for help from anyone but a proctor, change your seat without permission from a proctor or use any electronic device. Doing so will result in a zero score for the exam.
10. **When you are done with the exam, show your student ID to a proctor, hand in only this cover sheet and your answer sheet.**
11. Please sign above by the name sticker to indicate that you have read and understood these instructions.

/

**Possibly useful constants:**

$$\epsilon_0 = 1/\mu_0 c^2 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

$$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A} = 12.57 \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$$

$$c = \text{speed of light} = 3.00 \times 10^8 \text{ m/s}$$

$$-q_{\text{electron}} = q_{\text{proton}} = 1.602 \times 10^{-19} \text{ C}$$

$$m_{\text{electron}} = \text{electron mass} = 9.11 \times 10^{-31} \text{ kg}$$

$$m_{\text{proton}} = \text{proton mass} = 1.67 \times 10^{-27} \text{ kg}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Circumference of a circle  $= 2\pi r$ ; area of a circle is  $\pi r^2$

Surface area of a sphere  $= 4\pi r^2$ ; Volume of a sphere  $= \frac{4}{3}\pi r^3$

Surface area of a cylinder  $= 2\pi r h + 2\pi r^2$ ; Volume of cylinder  $= \pi r^2 h$

$$\sin(0^\circ) = \cos(90^\circ) = 0$$

$$\sin(90^\circ) = \cos(0^\circ) = 1$$

$$\sin(30^\circ) = \cos(60^\circ) = 1/2$$

$$\sin(60^\circ) = \cos(30^\circ) = \sqrt{3}/2$$

$$\sin(45^\circ) = \cos(45^\circ) = \sqrt{2}/2$$

$$\tan(30^\circ) = 1/\sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3}$$

$$\tan\theta = \sin\theta/\cos\theta$$

$$\cos\theta = 1/\sqrt{(1 + \tan^2\theta)}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \text{ except when } n = -1. \text{ For } n = -1, \int dx/x = \ln x$$

$$\frac{d}{dx}\sin(ax) = a\cos(ax)$$

$$\frac{d}{dx}\cos(ax) = -a\sin(ax)$$

$$\int \sin(ax) dx = -\cos(ax)/a$$

$$\int \cos(ax) dx = \sin(ax)/a$$

Some metric prefixes:

$$\text{f} = \text{femto} = 10^{-15}$$

$$\text{p} = \text{pico} = 10^{-12}$$

$$\text{n} = \text{nano} = 10^{-9}$$

$$\mu = \text{micro} = 10^{-6}$$

$$\text{m} = \text{milli} = 10^{-3}$$

$$\text{k} = \text{kilo} = 10^3$$

$$\text{M} = \text{mega} = 10^6$$

$$\text{G} = \text{giga} = 10^9$$

1. The electric field for a plane electromagnetic wave in vacuum is given by  $E = 2100 \text{ (N/C)} \times \sin[-0.6 \text{ m}^{-1} z + (0.01 \text{ rad/s})t] \hat{x}$ . In what direction does the wave propagate?

- a) The wave propagates in the  $+x$  direction.  
 b) The wave propagates in the  $-z$  direction.  
 c) The wave propagates in the  $+z$  direction.  
 d) The wave propagates in the  $-x$  direction.  
 e) The wave propagates in the  $+y$  direction.

of form  
 $E = A \sin(-kz + \omega t) \hat{x}$   
 $= -\sin(kz - \omega t) \hat{x}$   
 $\uparrow$   
 $+z \text{ direction}$

2. The intensity of a planar electromagnetic wave is  $I = 2 \text{ W/m}^2$ . What is the maximum value of the magnetic field  $B$  of this wave?

- a)  $B = 2\mu_0$  in units of T  
 b)  $B = 2\epsilon_0$  in units of T  
 c)  $B = 2/c$  in units of T  
 d)  $B = (2\mu_0/c)^{1/2}$  in units of T  
 e)  $B = 2(\mu_0/c)^{1/2}$  in units of T

$I = \langle \vec{S} \rangle = \langle \frac{1}{\mu_0} \vec{E} \times \vec{B} \rangle$   
 $= \frac{1}{2} \frac{1}{\mu_0} E_0 B_0; E_0 = \frac{B_0 c}{\epsilon_0}$   
 $= \frac{1}{2} \frac{c}{\mu_0} B_0^2 \Rightarrow B = \sqrt{\frac{2I\mu_0}{c}}$

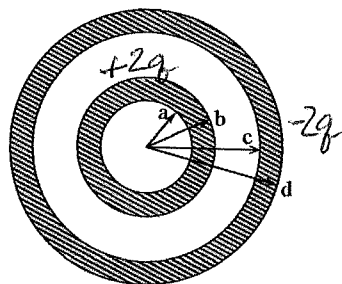
3. A toroidal inductor is created when a long solenoid that is much longer than its radius is bent into a circle so that its ends meet. (Its external shape is then that of a very skinny doughnut.) The solenoid cross sectional area is  $A$ , its length is  $D$  and it is wound with  $N$  turns of wire. What is the inductance  $L$  of this toroidal inductor?

- a)  $L = \mu_0 N^2 A / 2\pi D$   
 b)  $L = \mu_0 N^2 A / D^2$   
 c)  $L = \mu_0 N^2 A / D$   
 d)  $L = \mu_0 N A / D$   
 e)  $L = \mu_0 N A / 2\pi D$

$\oint \vec{B} \cdot d\vec{l} = B 2\pi R = \mu_0 I_{\text{enclosed}}$   
 $= \mu_0 N I$   
 $\Phi_B = (B \cdot A) N = \mu_0 N^2 I A / D$   
 $L = \frac{\Phi_B}{I} = \mu_0 N^2 A / D$

4. A small conducting spherical shell with inner radius  $a$  and outer radius  $b$  is concentric with a larger conducting spherical shell with inner radius  $c$  and outer radius  $d$ . The inner shell has a total charge  $+2q$ , and the outer shell has a total charge  $-2q$ . What is the total charge  $Q_1$  on the inner surface of the large shell and the total charge  $Q_2$  on the outer surface of the large shell?

- a)  $Q_1 = +2q$  and  $Q_2 = +2q$   
 b)  $Q_1 = -2q$  and  $Q_2 = -2q$   
 c)  $Q_1 = +2q$  and  $Q_2 = 0$   
 d)  $Q_1 = -2q$  and  $Q_2 = 0$   
 e)  $Q_1 = 0$  and  $Q_2 = -2q$



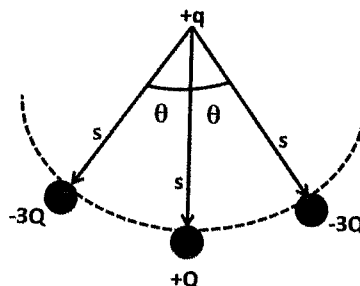
surface  
 outer  
 inner  
 shell

Inside of outer shell is conductor  
 $Q_2 = Q_{\text{outer}} = +2q - 2q = 0 = Q_{\text{enclosed}}$   
 Surface

$Q_{\text{inner}} = 0 = +2q - 2q$   
 Conductor  
 $Q_1 = -2q$   
 inner surface  
 outer shell

5. In the figure, at the top is a charge  $+q$  and you are given 2 outer charges  $-3Q$  and an inner charge  $+Q$ . At what angle  $\theta$  is the net force  $F$  on the positive charge  $+q$  at the top equal to zero? (Assume  $0^\circ \leq \theta \leq 90^\circ$ .)

- a) The force  $F$  can never be equal to zero.  
 b)  $F = 0$  when  $\theta = 0^\circ$ .  
 c)  $F = 0$  when  $\theta = 90^\circ$ .  
 d)  $F = 0$  when  $\sin \theta = 1/6$ .  
 e)  $F = 0$  when  $\cos \theta = 1/6$ .

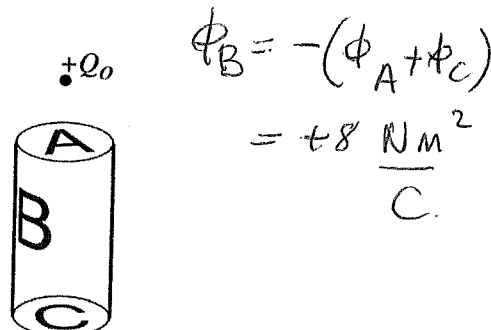


$\vec{F} = 0$  when  
 $F_x = 0 \rightarrow \text{all } \theta < 90^\circ$   
 $F_y = 0$   
 $= -Q + 2(3Q \cos \theta)$   
 $\Rightarrow \cos \theta = \frac{1}{6}$

6. Consider the flux through the Gaussian cylinder shown in the figure, with the top and bottom surfaces labeled A and C, respectively, and the curved side is surface B. If  $\Phi_A = -10 \text{ Nm}^2/\text{C}$  and  $\Phi_C = +2 \text{ Nm}^2/\text{C}$ , what is  $\Phi_B$ ?

$$\Phi_{\text{tot}} = 0 = \frac{Q_{\text{enclosed}}}{\epsilon_0} = \Phi_A + \Phi_B + \Phi_C$$

- a) Not enough information is provided to determine  $\Phi_B$ .  
 b)  $\Phi_B = +8 \text{ Nm}^2/\text{C}$   
 c)  $\Phi_B = -8 \text{ Nm}^2/\text{C}$   
 d)  $\Phi_B = +12 \text{ Nm}^2/\text{C}$   
 e)  $\Phi_B = -12 \text{ Nm}^2/\text{C}$



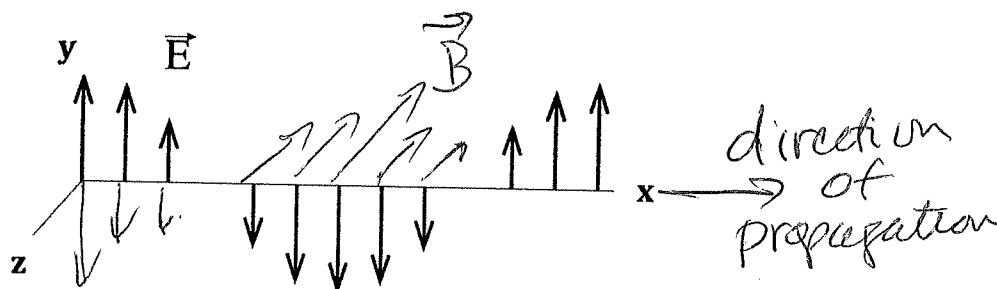
$\Phi_B = -(\Phi_A + \Phi_C)$   
 $= +8 \frac{\text{Nm}^2}{\text{C}}$

7. A circular conducting ring with radius  $r_0$  lies in the  $xy$ -plane in a region of a uniform magnetic field  $B(t) = B_0[1 - 3t^2 + 2t^3]\hat{k}$  where  $B_0$  is constant. What is the magnitude of the induced EMF  $\mathcal{E}$  in the ring?

- a)  $\mathcal{E} = B_0[-6t + 6t^2]$   
 b)  $\mathcal{E} = \text{zero}$   
 c)  $\mathcal{E} = B_0\pi r_0^2[-6t + 6t^2]$   
 d)  $\mathcal{E} = 2B_0\pi r_0^2[-6t + 6t^2]$   
 e)  $\mathcal{E} = B_0\pi r_0^2[1 - 3t^2 + 2t^3]$

$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)(\pi r_0^2)$   
 $\frac{dB}{dt} = [-6t + 6t^2]B_0$   
 $\mathcal{E} = B_0\pi r_0^2(-6t + 6t^2)$

8. The electric field  $E$  of an electromagnetic wave traveling in the positive  $x$  direction is illustrated in the figure. For the magnetic field, what are the direction and the phase relative to the electric field at a point where the electric field is in the negative  $y$  direction?



- a) The  $\vec{B}$  field is in the  $-z$  direction and in phase with the  $\vec{E}$  field.  
 b) The  $\vec{B}$  field is in the  $+z$  direction and  $90^\circ$  out of phase with the  $\vec{E}$  field.  
 c) The  $\vec{B}$  field is in the  $-z$  direction and  $90^\circ$  out of phase with the  $\vec{E}$  field.  
 d) The  $\vec{B}$  field is in the  $+z$  direction and in phase with the  $\vec{E}$  field.  
 e) The  $\vec{B}$  field is in the  $+y$  direction and in phase with the  $\vec{E}$  field.
9. An electromagnetic wave in vacuum traveling in the  $+x$  direction generated by a variable source initially has an angular frequency  $\omega$  and a maximum electric field  $E_{max}$  in the  $+y$  direction. If the period of the wave is doubled, what could be the equation of the resulting magnetic field component of the wave?

- a)  $\vec{B} = (cE_{max}) \cos(kx/2 - \omega t/2) \hat{z}$   
 b)  $\vec{B} = (E_{max}/c) \cos(kx - \omega t) \hat{x}$   
 c)  $\vec{B} = (E_{max}/c) \cos(kx/2 - \omega t/2) \hat{y}$   
 d)  $\vec{B} = (E_{max}/c) \cos(kx/2 - \omega t/2) \hat{z}$   
 e)  $\vec{B} = (cE_{max}) \cos(kx - \omega t/2) \hat{y}$

$$v = \text{constant} = c = \frac{\omega}{k}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T \rightarrow 2T \Rightarrow \omega \rightarrow \frac{\omega}{2} \Rightarrow k \rightarrow \frac{k}{2}$$

5

10. A current of 4.0 A is running through a series  $LR$  circuit containing a  $2.0\ \Omega$  resistor and a 10 mH inductor. If the emf driving the current is suddenly switched off, which expression can be used to calculate the time it takes for the current to drop down to 1.0 A?  $\Rightarrow$  decreasing

- a)  $I = I_0 e^{-t/\tau}$  where  $I_0 = 4.0\text{ A}$  and  $\tau = 20\text{ ms}$ .  
 b)  $I = I_0 e^{-t/\tau}$  where  $I_0 = 1.0\text{ A}$  and  $\tau = 5\text{ ms}$ .  
 c)  $I = I_0 e^{-t/\tau}$  where  $I_0 = 4.0\text{ A}$  and  $\tau = 5\text{ ms}$ .  
 d)  $I = I_0(1 - e^{-t/\tau})$  where  $I_0 = 4.0\text{ A}$  and  $\tau = 5\text{ s}$ .  
 e)  $I = I_0 e^{-t/\tau}$  where  $I_0 = 4.0\text{ A}$  and  $\tau = 5\text{ s}$ .

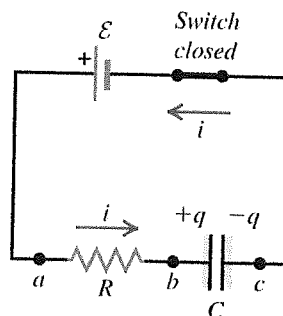
$$I = I_0 e^{-t/\tau}$$

$$I_0 = 4\text{ A}$$

$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}\text{ H}}{2\ \Omega} = 5 \times 10^{-3}\text{ s} = 5\text{ ms}$$

11. In the figure, which is the correct expression for the time dependence of the charge  $q$  in an  $R-C$  circuit with EMF  $V$  when the capacitor is charging?  $\Rightarrow$  function

- a)  $q = Q_f(1 - \exp(-t/\tau))$  where  $\tau = 1/RC$  and  $Q_f = CV$   
 b)  $q = Q_f(1 - \exp(-t/\tau))$  where  $\tau = RC$  and  $Q_f = CV$   
 c)  $q = Q_f(\exp(-t/\tau))$  where  $\tau = RC$  and  $Q_f = V/C$   
 d)  $q = Q_f(\exp(-t/\tau))$  where  $\tau = 1/RC$  and  $Q_f = V/C$   
 e)  $q = Q_f(1 - \exp(-t/\tau))$  where  $\tau = RC$  and  $Q_f = V/C$



$$q = Q_f(1 - \exp(-t/\tau)) \quad \tau = RC \quad Q_f = CV$$

12. Consider a source charge  $Q = +5\text{ C}$ , fixed in place at the origin that creates an electric field in the surrounding region. You bring a small test charge  $q = -5\mu\text{C}$ , from infinite distance to a distance  $r = 5\text{ meters}$  from the source charge. How much work  $W$  is done by the electric field on the test charge in bringing the test charge from infinite distance to a distance  $r$  away from the source charge? [Answer in terms of  $k = 1/(4\pi\epsilon_0)$ ]

- a)  $W = +k(5 \times 10^{-6})\text{ J}$   
 b)  $W = -k(5 \times 10^{-6})\text{ J}$   
 c)  $W = +k(1 \times 10^{-6})\text{ J}$   
 d)  $W = -k(1 \times 10^{-6})\text{ J}$   
 e)  $W = +k(25 \times 10^{-6})\text{ J}$

$$-\Delta U = -(U_f - U_i) = W$$

$$U_i = 0$$

$$U_f = \frac{kqQ}{r}$$

$$W = -\frac{kqQ}{r} = -k \left[ \frac{(-5 \times 10^{-6})(+5)}{5} \right] = +k(5 \times 10^{-6})\text{ J}$$

13. A cylindrical wire has a resistance  $R_1$  and resistivity  $\rho$ . If its length and diameter are BOTH cut in half, what will be its resistance  $R_2$ ?

a)  $R_2 = R_1$   
 b)  $R_2 = 4R_1$   
 c)  $R_2 = R_1/4$   
 d)  $R_2 = R_1/2$   
 e)  $R_2 = 2R_1$

$$R_1 = \frac{\rho L}{\pi \left(\frac{D}{2}\right)^2} \quad R_2 = \frac{\rho L/2}{\pi (D/4)^2}$$

$$R_2/R_1 = \frac{\rho L/2}{\pi (D/4)^2} \div \frac{\rho L}{\pi (D/2)^2} = 2 \Rightarrow R_2 = 2R_1$$

14. Point charges  $q_1 = -5 \text{ nC}$  and  $q_2 = +5 \text{ nC}$  are separated by a distance  $d = 1.0 \text{ mm}$ , forming an electric dipole. The charges are in a uniform electric field whose direction has an angle of  $\theta = 30^\circ$  with the line connecting the charges. If the torque exerted on the dipole has magnitude  $\tau = 10 \times 10^{-9} \text{ N}\cdot\text{m}$ , what is the magnitude of the electric field  $E$ ?

- a)  $E = 4000 \text{ N/C}$   
 b)  $E = 4000\sqrt{3} \text{ N/C}$   
 c)  $E = 2000 \text{ N/C}$   
 d)  $E = 2000\sqrt{3} \text{ N/C}$   
 e)  $E = 4 \text{ N/C}$

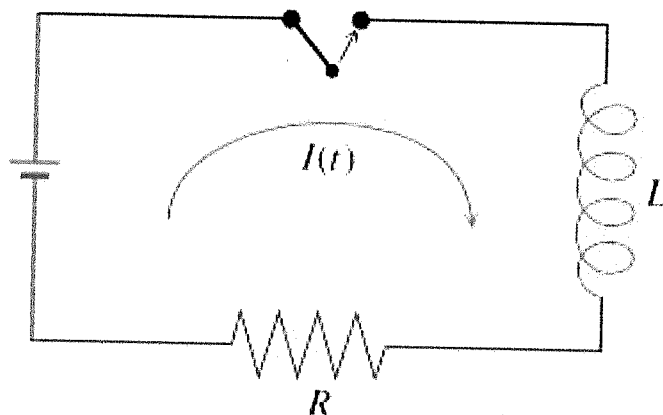
$$\vec{\tau} = \vec{p} \times \vec{E} = qdE \sin 30^\circ$$

$$E = \tau / qd \sin 30^\circ$$

$$= \frac{10 \times 10^{-9} \text{ N}\cdot\text{m}}{(5 \times 10^{-9} \text{ C})(10^{-3} \text{ m})(\frac{1}{2})}$$

$$= 4000 \text{ N/C}$$

15. The figure displays a series circuit containing a resistor of resistance  $R$  and an inductor of inductance  $L$  connected to a source of EMF  $\varepsilon$  with negligible internal resistance. The wires are also assumed to have zero resistance. Immediately after the switch is closed, which of the following are correct for the voltage drop across the inductor  $V_L$ , the voltage drop across the resistor,  $V_R$ , and the initial current  $I_{initial}$ ?



$$I_{initial} = 0$$

$$\Rightarrow V_R = 0$$

$$V_L = \varepsilon$$

- a)  $V_L = \varepsilon$ ,  $V_R = \varepsilon$ ,  $I_{initial} = \varepsilon/R$
  - b)  $V_L = \varepsilon$ ,  $V_R = \varepsilon$ ,  $I_{initial} = 0$
  - ☒ c)  $V_L = \varepsilon$ ,  $V_R = 0$ ,  $I_{initial} = 0$
  - d)  $V_L = \varepsilon/2$ ,  $V_R = \varepsilon/2$ ,  $I_{initial} = \varepsilon/R$
  - e)  $V_L = 0$ ,  $V_R = \varepsilon$ ,  $I_{initial} = \varepsilon/R$
16. Which of the following is TRUE?
- ☒ a) We can think of the energy stored in a parallel plate capacitor as being stored in the electric field between the plates.
  - b) Energy cannot be stored in a vacuum. *False.*
  - c) Electric fields require a dielectric, and do not develop in a perfect vacuum. *False.*
  - d) The electric field  $\vec{E} = \vec{F}_0/q_0$  measured at a point in space diverges as the test charge  $q_0$  is sent to zero. *False.*
  - e) In general, electric field and electric potential are directly proportional to one another. *False.*



17. A plane sinusoidal electromagnetic wave in air has an electric field amplitude of  $E_0$ . What average force  $F$  does this radiation exert on a totally absorbing surface with area  $A$  perpendicular to the direction of propagation?

a)  $F = \text{zero}$ , because area is perpendicular to the direction of propagation.

b)  $F = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 A$

c)  $F = \epsilon_0 E_{\text{max}}^2 A$

d)  $F = \frac{1}{2} \epsilon_0 E_{\text{max}} A$

e)  $F = \epsilon_0 E_{\text{max}} A$

For totally absorbing surface

$$P_{\text{rad}} = \frac{I}{c} = \frac{F}{A}$$

$$\Rightarrow F = \left( \frac{I}{c} \right) (A) = \frac{1}{2} \epsilon_0 c E_0^2 A = \frac{1}{2} \epsilon_0 E_0^2 A$$

18. An inductor with  $L = 10 \text{ mH}$  is connected across an ac source that has a voltage amplitude  $V_0 = 40 \text{ V}$ . What value for the frequency  $f$  of the source results in a current amplitude of  $I_0 = 4 \text{ A}$ ?

a)  $f = 500/\pi \text{ Hz}$

b)  $f = 5/\pi \text{ Hz}$

c)  $f = 500 \text{ Hz}$

d)  $f = 100/\pi \text{ Hz}$

e)  $f = 100/\pi \text{ Hz}$

$$V_0 = \omega L I \Rightarrow \omega = \frac{V_0}{L I} = \frac{40 \text{ V}}{(10 \times 10^{-3} \text{ H})(4 \text{ A})}$$

$$f = \frac{500}{\pi} \text{ Hz}$$

19. An air-filled parallel-plate capacitor has plate area  $A$  and plate separation  $d$ . The capacitor is connected to a battery that creates a constant voltage  $V$ . The energy stored in the capacitor is  $U_1$ . While the capacitor stays connected to the battery, a dielectric plate with thickness  $d$  and dielectric constant  $K$ , is slowly moved into the capacitor until it fills only the left half of the space between the plates. What is the new energy  $U_2$  now stored in the capacitor?

a)  $U_2 = U_1/2$

b)  $U_2 = K U_1$

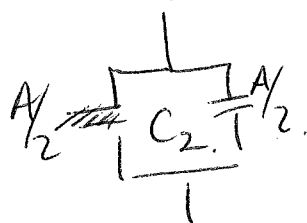
c)  $U_2 = (K+1) U_1$

d)  $U_2 = (K+1) U_1/2$

e)  $U_2 = U_1/[2(K+1)]$

$\Rightarrow V = \text{constant}$

$$U_1 = \frac{1}{2} \left( \frac{\epsilon_0 A}{d} \right) V^2 \quad U_2 = \frac{1}{2} C_2 V^2 = \frac{1}{2} \left( \frac{\epsilon_0 A}{2d} \right) (K+1) V^2$$



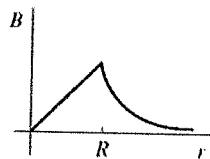
$$C_2 = C_L + C_R$$

$$= \frac{\epsilon_0 A}{2d} K + \frac{\epsilon_0 A}{2d}$$

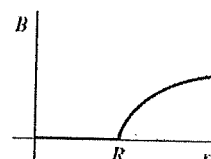
$$= \frac{\epsilon_0 A}{2d} (K+1)$$

$$\frac{U_2}{U_1} = \frac{\frac{1}{2} \left( \frac{\epsilon_0 A}{2d} \right) (K+1) V^2}{\frac{1}{2} \left( \frac{\epsilon_0 A}{d} \right) V^2} = \frac{K+1}{2}$$

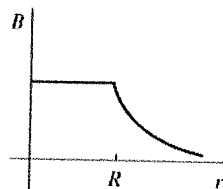
20. A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius  $R$  carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude  $B$  of the magnetic field produced by this current as a function of the distance  $r$  from the central axis?



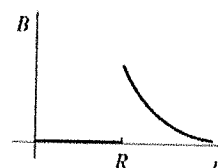
(1)



(2)

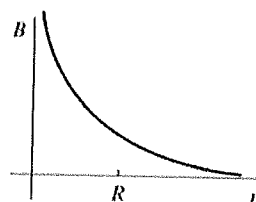


(3)

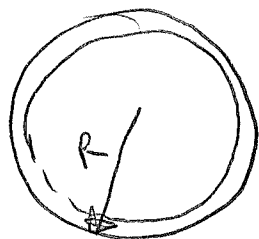


(4)

- a) Graph (3) best represents  $B$  as a function of  $r$ .  
 b) Graph (5) best represents  $B$  as a function of  $r$ .  
 c) Graph (4) best represents  $B$  as a function of  $r$ .  
 d) Graph (2) best represents  $B$  as a function of  $r$ .  
 e) Graph (1) best represents  $B$  as a function of  $r$ .



(5)

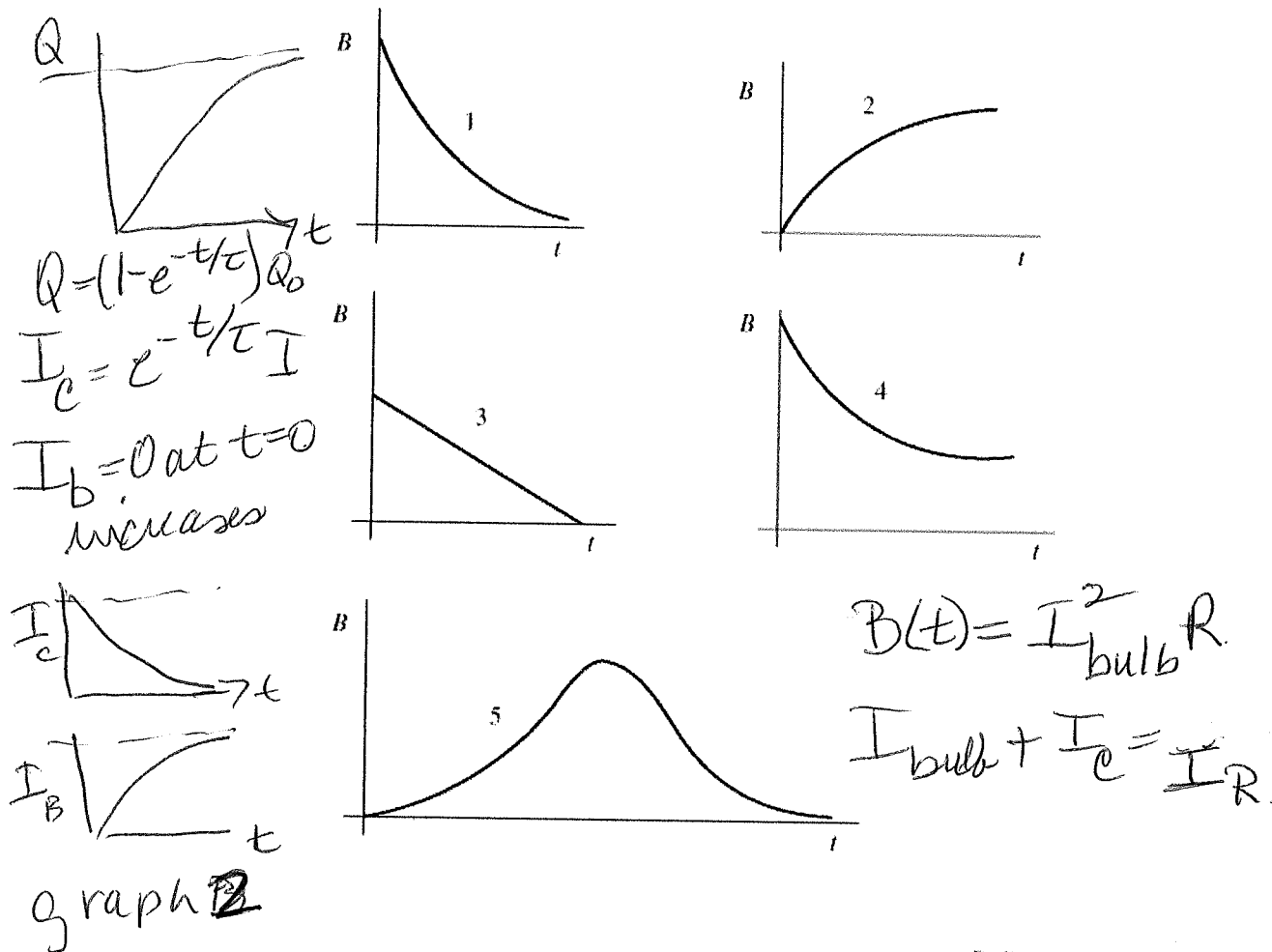


$$0 < r < R \quad I_{\text{enclosed}} = 0 \\ \Rightarrow B(r) = 0$$

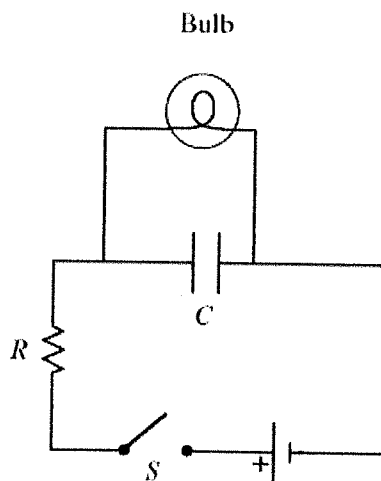
$$r > R \quad B(2\pi r) = \mu_0 I \\ B = \frac{\mu_0 I}{2\pi r} \propto \frac{1}{r}$$

graph (4) best describes

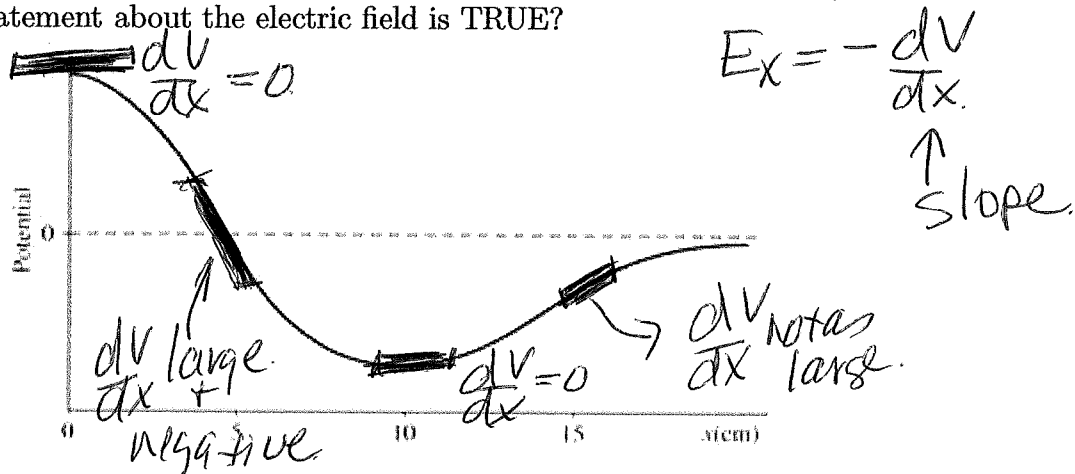
21. A light bulb is connected in the circuit shown in the bottom figure with the switch  $S$  open and the capacitor uncharged. The battery has no appreciable internal resistance. Which one of the following graphs best describes the brightness  $B(t)$  of the bulb as a function of time  $t$  after closing the switch?



- a) Graph 3 best describes the brightness  $B(t)$ .
- b) Graph 1 best describes the brightness  $B(t)$ .
- c) Graph 5 best describes the brightness  $B(t)$ .
- d) Graph 4 best describes the brightness  $B(t)$ .
- e) Graph 2 best describes the brightness  $B(t)$ .**



22. The electric potential as a function of  $x$  is shown in the graph in the figure. What statement about the electric field is TRUE?



- (a) The electric field is zero at  $x = 0$ , its magnitude is at a maximum at  $x = 5$  cm, and the electric field is directed to the right at  $x = 5$  cm.
- b) The electric field is zero at  $x = 0$ , its magnitude is at a maximum at  $x = 5$  cm, and the electric field is directed to the left at  $x = 5$  cm.
- c) The electric field is zero at  $x = 0$  cm, its magnitude is at a maximum at  $x = 15$  cm, and the electric field is directed to the right at  $x = 15$  cm.
- d) The electric field is zero at  $x = 5$  cm, its magnitude is at a maximum at  $x = 0$ , and the electric field is directed to the right at  $x = 0$ .
- e) The electric field is zero at  $x = 10$ , its magnitude is at a maximum at  $x = 5$  cm, and the electric field is directed to the left at  $x = 5$  cm.
23. At what rate would the current in a 100-mH inductor have to change to induce an EMF of magnitude 100 V in the inductor?

- a) The rate should be 10 A/s
- (b) The rate should be 1000 A/s
- c) The rate should be 0.1 A/s
- d) The rate should be 1 A/s
- e) The rate should be 100 A/s
- Handwritten calculations:
- $$\mathcal{E} = L \frac{dI}{dt}$$
- $$\frac{dI}{dt} = \frac{\mathcal{E}}{L} = \frac{100 \text{ V}}{100 \times 10^{-3} \text{ H}} = 10^3 \frac{\text{A}}{\text{s}}$$

24. You are given a reactance of an inductor  $X_L = 100 \Omega$ , reactance of a capacitor  $X_C = 10 \Omega$ , and frequency = 100 Hz. Which of the following is correct?

- a) The inductance  $L = 2/\pi$  H
- b) The capacitance  $C = 5/\pi$  F
- c) The inductance  $L = 1$  H
- d) The capacitance  $C = 10^{-3}$  F
- (e) The capacitance  $C = 10^{-3}/2\pi$  F

Handwritten calculations:

$$2\pi f = \omega = (100)(2\pi) \text{ rad/s}$$

$$X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{100}{(100)(2\pi)} = \frac{1}{2\pi} \text{ H}$$

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{(2\pi)(100)(10)} = \frac{1}{2\pi} \times 10^{-3} \text{ F}$$

12

25. You are given four series  $R - L - C$  circuits where  $R$  is the resistance,  $L$  is the inductance and  $C$  is the capacitance of the circuits. Each circuit is driven at its resonance frequency by a  $1000 V_{rms}$  AC power supply. Rank these circuits on the basis of their maximum current  $I_{max}$ .

Circuit A:  $R = 1000 \Omega$ ,  $C = 0.5 \mu F$ ,  $L = 2.0 H$   $I_A \approx 1 A$

Circuit B:  $R = 500 \Omega$ ,  $C = 4.0 \mu F$ ,  $L = 1.0 H$   $I_B \approx 2 A$

Circuit C:  $R = 500 \Omega$ ,  $C = 1.0 \mu F$ ,  $L = 1.5 H$   $I_C \approx 2 A$

Circuit D:  $R = 2000 \Omega$ ,  $C = 4.0 \mu F$ ,  $L = 1.0 H$   $I_D \approx \frac{1}{2} A$

- a)  $I_D = I_A > I_C > I_B$   
 b)  $I_C > I_B > I_D > I_A$   
 c)  $I_B = I_C > I_A > I_D$   
 d)  $I_A > I_B > I_C > I_D$   
 e)  $I_D > I_A > I_B = I_C$

$$I_{max} \propto \frac{V_{rms}}{R}$$

26. A charge is accelerated from rest through a potential difference  $V_1$  and then enters a uniform magnetic field oriented perpendicular to its path. The field deflects the particle into a circular arc of radius  $R_1$ . If the accelerating potential is tripled to  $V_2 = 3V_1$ , what will be the radius  $R_2$  of the circular arc?

- a)  $R_2 = R_1/\sqrt{3}$   
 b)  $R_2 = 9R_1$   
 c)  $R_2 = R_1\sqrt{3}$   
 d)  $R_2 = 3R_1$   
 e)  $R_2 = R_1/9$

$$\frac{1}{2} m v_1^2 = q V_1 \Rightarrow v_1 = \sqrt{\frac{2qV_1}{m}}$$

$$\frac{1}{2} m v_2^2 = q(3V_1) \Rightarrow v_2 = \sqrt{\frac{2q(3V_1)}{m}}$$

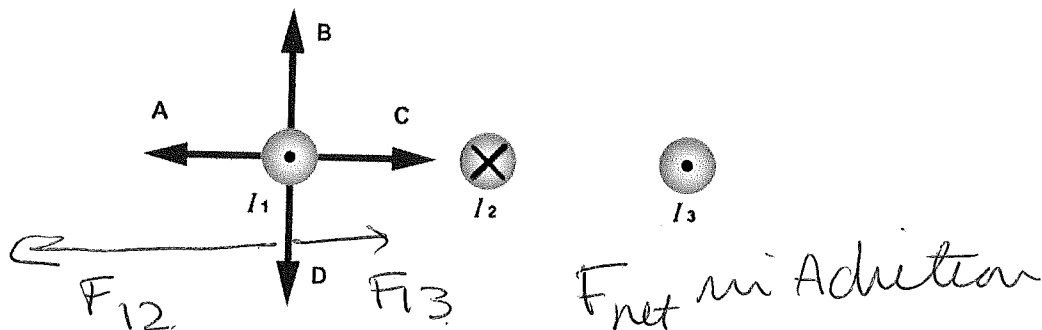
$$v_2 = \sqrt{3} v_1$$

$$\frac{m v_1^2}{R_1} = q v_1 B \Rightarrow R_1 = \frac{m v_1}{q B}$$

$$\frac{m v_2^2}{R_2} = q v_2 B \Rightarrow R_2 = \frac{m v_2}{q B}$$

$$\frac{R_2}{R_1} = \frac{v_2}{v_1} = \sqrt{3} \Rightarrow R_2 = R_1 \sqrt{3}$$

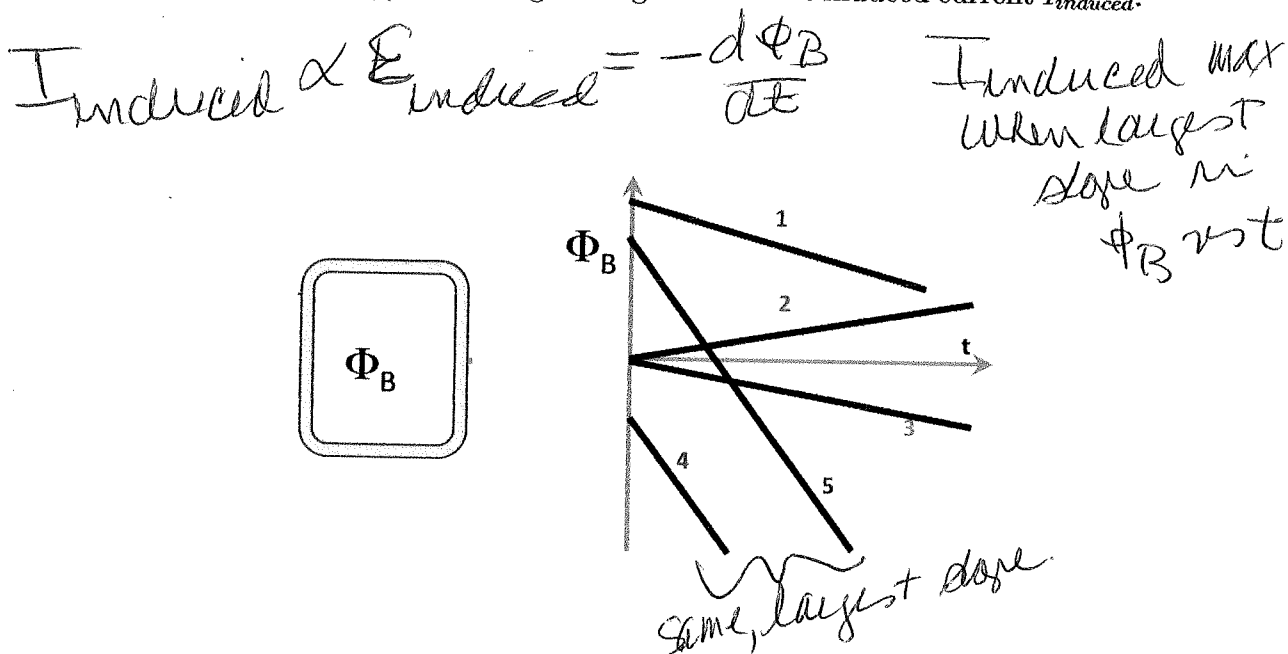
27. The figure shows three long, parallel current-carrying wires. The magnitudes of the currents are equal and their directions are indicated in the figure. Which of the arrows drawn near the wire carrying current  $I_1$  correctly indicates the direction of the magnetic force acting on that wire?



- a) The magnetic force on the wire carrying current  $I_1$  is zero.
- ☒ b) The direction of the magnetic force on the wire carrying current  $I_1$  is in direction A.
- c) The direction of the magnetic force on the wire carrying current  $I_1$  is in direction C.
- d) The direction of the magnetic force on the wire carrying current  $I_1$  is in direction B.
- e) The direction of the magnetic force on the wire carrying current  $I_1$  is in direction D.

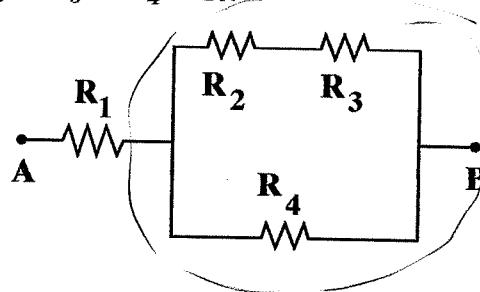
Currents in same direction  $\rightarrow$   $F$  attractive  
 Currents in opposite direction  $\rightarrow$   $F$  repulsive  
 $F \propto \frac{1}{\text{distance}}$

28. The flux of the magnetic field in the square wire loop on the left varies as a function of time as shown in the figure on the right. Which dependence(s) of  $\Phi_B(t)$  correspond(s) to the largest magnitude of the induced current  $I_{\text{induced}}$ .



- a) The magnitude of  $I_{\text{induced}}$  is largest for curve 5.  
 b) The magnitude of  $I_{\text{induced}}$  is largest for curve 1.  
 c) The magnitude of  $I_{\text{induced}}$  is largest for curve 2 and curve 3.  
 d) The magnitude of  $I_{\text{induced}}$  is largest for curve 4 and curve 5.  
 e) The magnitude of  $I_{\text{induced}}$  is largest for curve 4.
29. What is the equivalent resistance (measured in  $\Omega$ ) of the segment of a circuit in the figure if  $R_1 = 2R$  and  $R_2 = R_3 = R_4 = R$ ?

- a)  $5R/3$   
 b)  $8R/3$   
 c)  $3R/8$   
 d)  $5R$   
 e)  $5R/2$

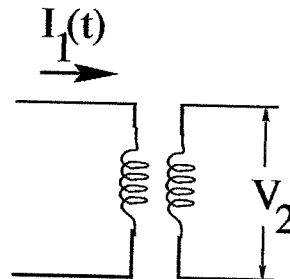


$$\frac{1}{R_{11}} = \frac{1}{2R} + \frac{1}{R} = \frac{1+2}{2R} \Rightarrow R_{11} = \frac{2R}{3}$$

$$R_{\text{net}} = 2R + \frac{2R}{3} = \frac{6+2}{3} R = \frac{8}{3} R$$

30. Two coils are close to each other, with one connected to a current source which produces a current of  $I_1(t) = 3t^2 + 5$  amps, where  $t$  is expressed in seconds. If at time  $t = 3$  seconds the voltage  $V_2 = 0.18$  V, what is the mutual inductance  $M$  of the two coils?

- a)  $M = 5.6$  mH
- ☒ b)  $M = 10$  mH
- c)  $M = 20$  mH
- d)  $M = 30$  mH
- e)  $M = 60$  mH



$$V = -M \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{d}{dt}(3t^2 + 5) = 6t$$

$$M = V / 6(t=3s) = \frac{18 \times 10^{-2}}{18} = 1 \times 10^{-2} = 10 \times 10^{-3} \text{ H}$$