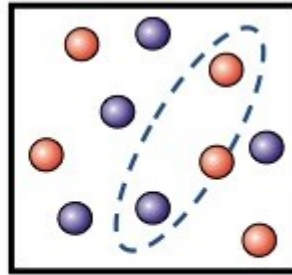


BEC



BCS

# BCS-BEC Crossover

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Solid State Physics I  
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# Outline

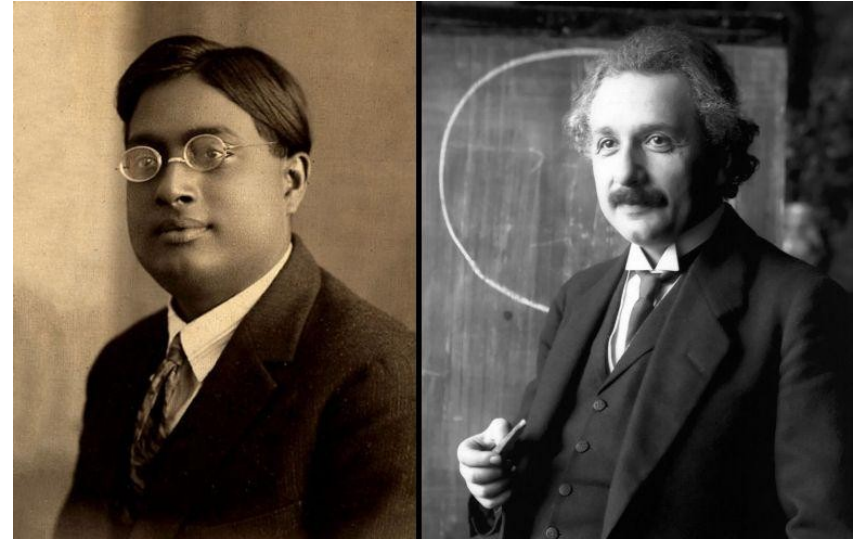
- Bose-Einstein Condensation (BEC)
- Bardeen Cooper Schrieffer Theory of Superconductivity (BCS)
- BCS-BEC Crossover
- Realization Of Crossover

# Bose-Einstein Condensation

# Introduction

- In 1924, Satyendra Bose discovered a way to distribute a collection of indistinguishable particles.

$$N = \sum_j \frac{1}{e^{\beta(\epsilon_j - \mu)} - 1}$$

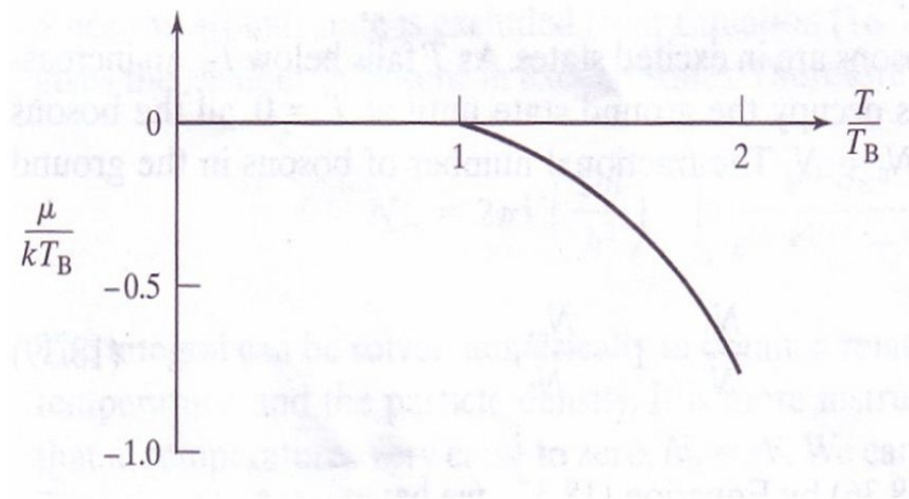


(Left) Wikimedia Commons; (Right) Ferdinand Schmutzer via Wikimedia Commons

- Together with Einstein, they formed the basis for Boson statistics

# Bose-Einstein Condensation

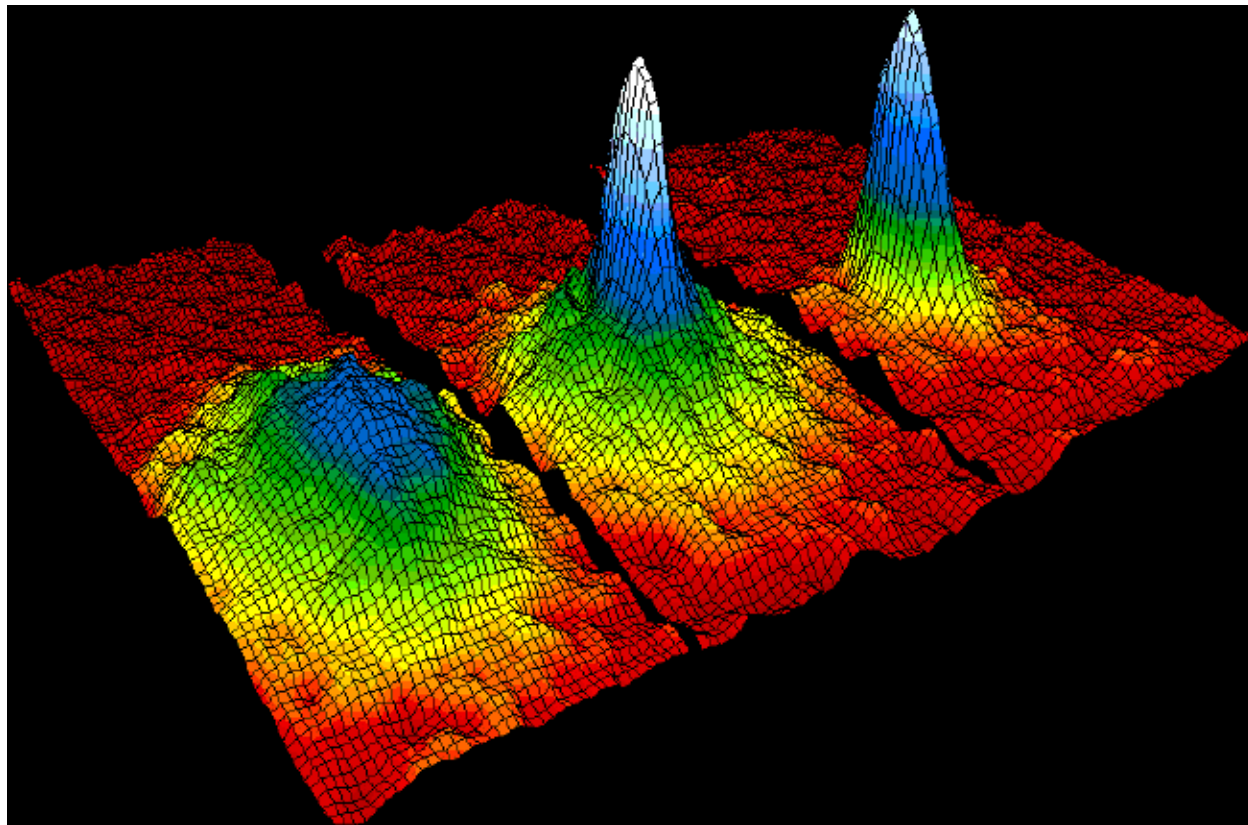
- Main achievement of the theory was the idea of Bose-Einstein Condensation



- As temperature decreases, so does the chemical potential until it reaches zero

# Bose-Einstein Condensation

- This leads to a macroscopic occupation of the lowest energy state



Wikimedia Commons

# BCS Theory

# Introduction

- In 1957, Bardeen, Cooper, and Schrieffer proposed the BCS Theory of superconductivity
- The model described an attractive interaction between electrons

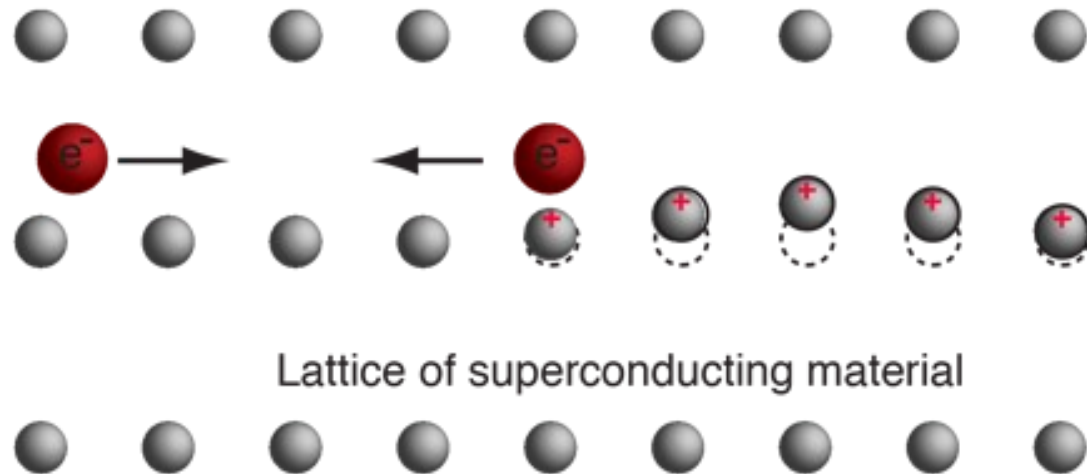


$$\mathcal{H} = \sum_k \epsilon(k) \left( c_{k,\uparrow}^\dagger c_{k,\uparrow} + c_{-k,\downarrow}^\dagger c_{-k,\downarrow} \right) - g \sum_{k,k'} c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger c_{-k',\downarrow} c_{k',\uparrow}$$



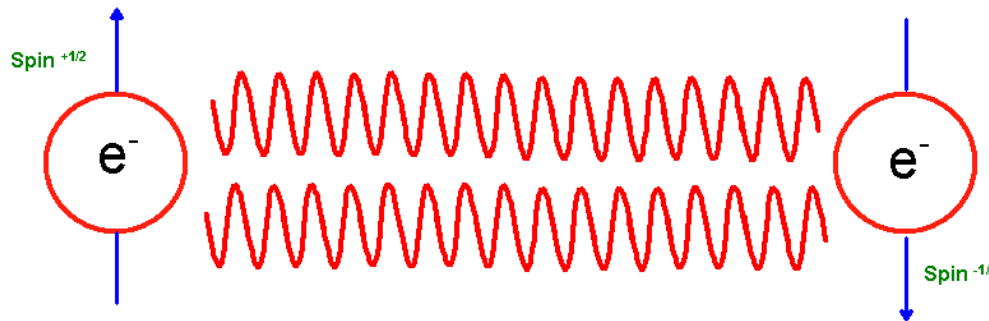
# BCS Theory

- We can understand this attraction through polarization of the lattice



# BCS Theory

- If we consider s-wave pairing between electrons, opposite spin and momenta are paired

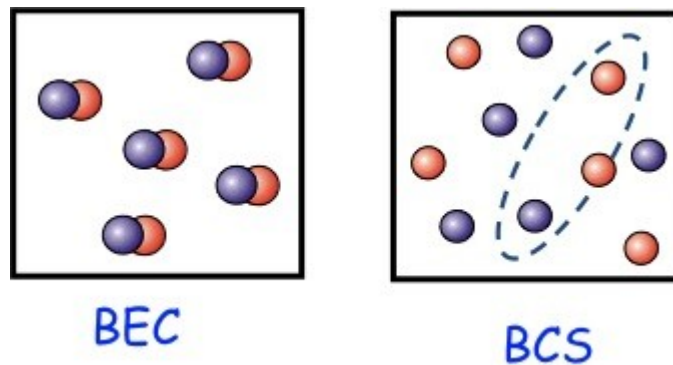


- The pair behaves like a boson and at low temperature it can condense and behave collectively

# BCS-BEC Crossover

# BCS-BEC Crossover

- What happens if we tune the interaction strength between atoms?



- Is there a smooth crossover between BEC and BCS?

# BCS-BEC Crossover

- BEC wave function

$$|\Psi_{BEC}\rangle = N \exp\left(\sum_k \phi_k c_k^\dagger c_{-k}^\dagger\right) |0\rangle$$

- BCS wave function

$$|\Psi_{BCS}\rangle = \prod_k (u_k + v_k c_k^\dagger c_{-k}^\dagger) |0\rangle$$

- Wave functions are actually the same!

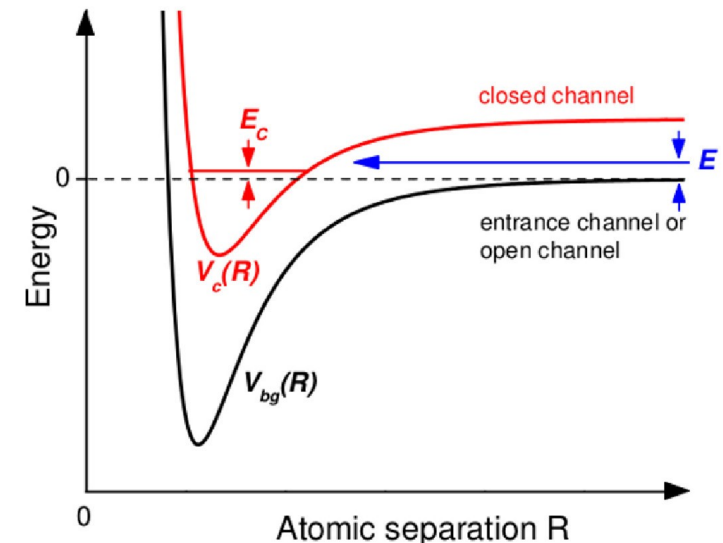
# BCS-BEC Crossover

- The BEC and BCS regimes are described by the same wavefunction that evolves smoothly
- This is a crossover not a transition
- Based on a mean-field theory so does it hold in experiment?

# Realization Of Crossover

# Realization Of Crossover

- Interactions between atoms can be tuned using a Feshbach resonance
- The interaction strength is characterized by the s-wave scattering length



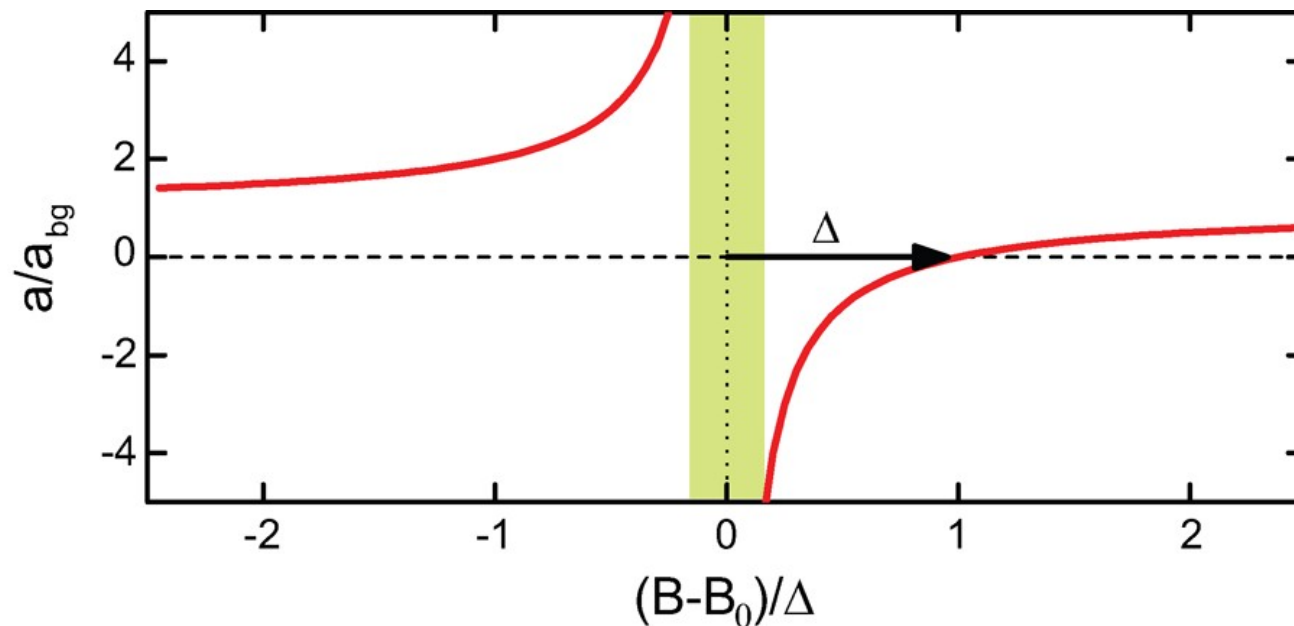
Feshbach Resonances in Ultracold Gases  
– Chin, et al. 2009



# Realization of Crossover

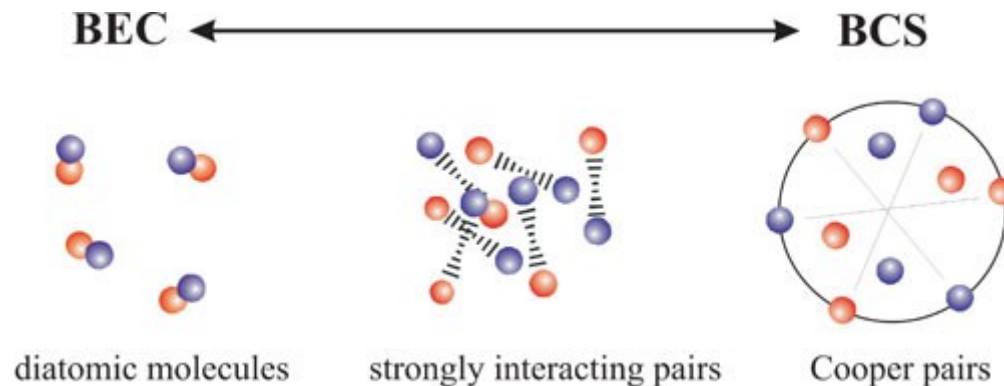
- The field dependent scattering length is given by

$$a(B) = a_{BG} \left( 1 - \frac{\Delta_B}{B - B_0} \right)$$



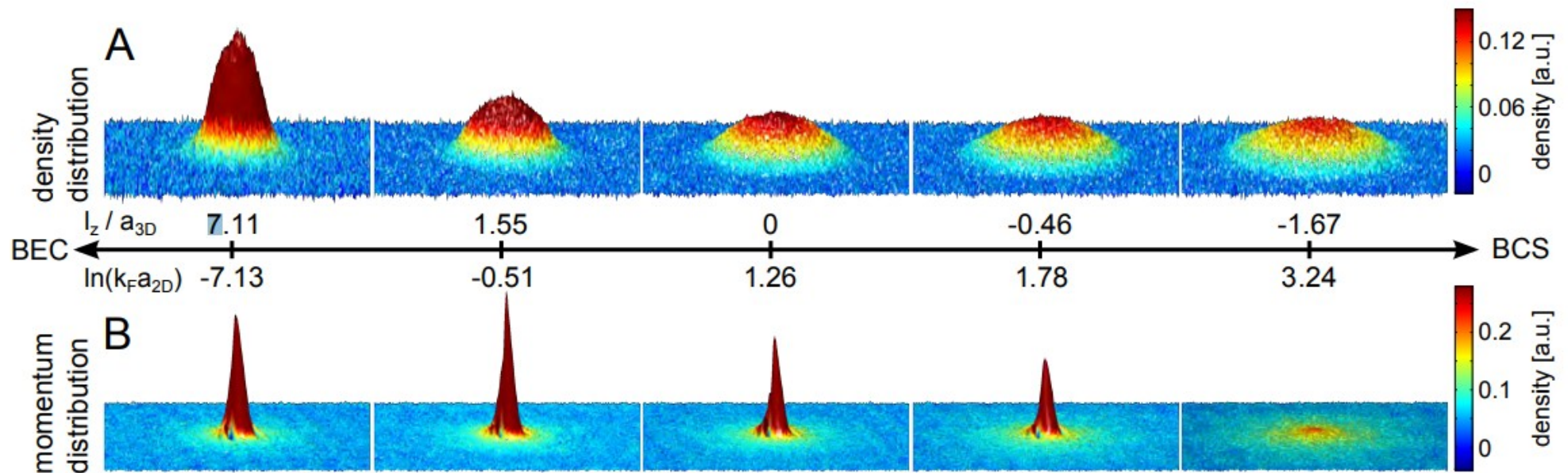
# Realization of Crossover

- Tuning this interaction leads to three regimes:
  - BEC  $a > 0$
  - Unitary  $a \rightarrow \text{inf}$
  - BCS  $a < 0$



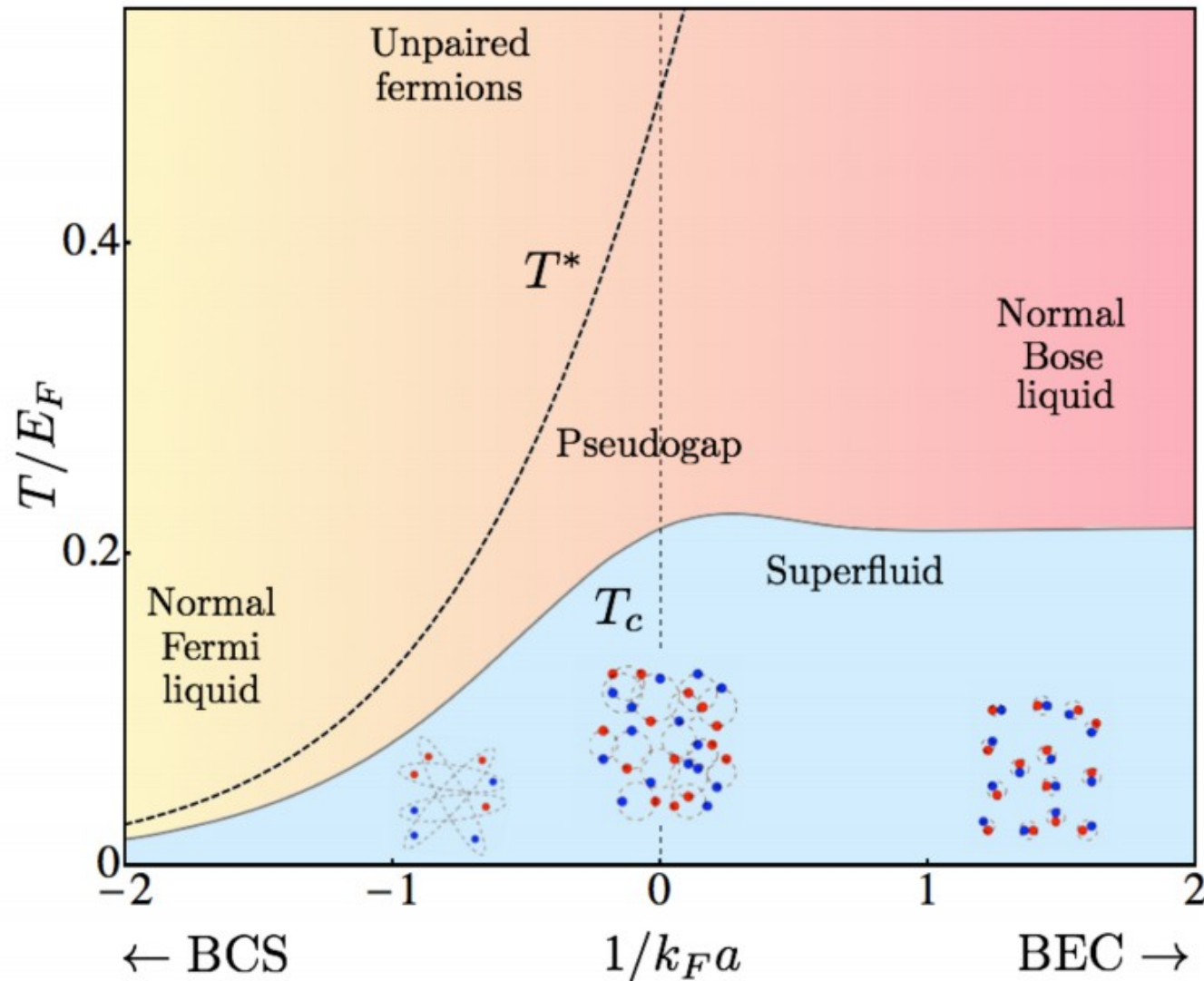
- Interesting behavior in the crossover/unitary regime – interactions are the strongest

# Realization of Crossover



Observation of Pair Condensation in the Quasi-2D BEC-BCS Crossover  
– MG Ries, et al. 2015

# Realization of Crossover



# Summary

- There is a similarity between BCS and BEC physics that can be described by a smooth crossover
- This can be realized experimentally using ultra cold atomic gases whose interactions are tuned through a Feshbach resonance
- The Unitary Fermi Gas regime where the scattering length diverges allows for the study of strongly interacting systems

# References

- Feshbach Resonances in Ultracold Gases – C. Chin, et al., 2009
- BCS-BEC Crossover and the Unitary Fermi Gas – M. Randeria, E. Taylor, 2014
- Statistical Mechanics – Pathria, Beale, 2011
- Theory of Superconductivity – Bardeen Cooper Schreiffer, 1957