I. A subway train at rest accelerates from the station at a constant rate of $2.0 \, \text{m/s}^2$ for half the distance to the next station, then slows down at the same rate for the second half of the journey. The total distance between stations is 800 m.

a) Determine the maximum speed of the train.

\[
\alpha = 2 \, \text{m/s}^2 \quad V_f^2 = V_0^2 + 2a\Delta x
\]

$\Delta x, \quad 400 \, \text{m}$

$V_0 = 0$

$V_f = ?$

$t = ?$

$V_f = 40 \, \text{m/s}$

\[
V_{\text{max}} = \frac{40 \, \text{m/s}}{2 \, \text{m/s}^2} = 20 \, \text{s}
\]

b) Graph the velocity as a function of time on the axes provided below.

\[
v_f = v_0 + \alpha t
\]

\[
t_{1/2} = \frac{\Delta V}{\alpha}
\]

\[
t_{1/2} = \frac{40 \, \text{m/s}}{2 \, \text{m/s}^2} = 20 \, \text{s}
\]

c) Graph the position as a function of time on the axes provided below.
II. You're entering in the big boat race, and you're trying to select the proper engine. Specifically, you need an engine that can make your boat speed up quickly. Some engines push harder on the water to propel the boat (i.e. produce more thrust), but they also tend to be heavier. Note: The contest rules mandate that all boats must weigh 500kg, NOT including the engine.

(a) Make a force diagram for each of Ozzy's Outboards' boats and label each diagram as we have been doing in class. Consider the engine to be part of the boat.

(b) Create a speeding up index for each engine that can help you decide which company builds the engines that can speed your boat up most quickly. SHOW ALL WORK.

Remember the rules: (1) Bigger is better
(2) Each company has one index that characterizes both if its motors.

\[
\text{Melvin's Marine Motors} = \frac{108}{500} = 0.18
\]

\[
\text{INDEX} = \frac{\text{FORCE}}{\text{MASS}}
\]

\[
\text{Ozzy's Outboards} \quad \frac{145}{580} = 0.25
\]

\[
\text{Engine Mass:} \quad 50 \quad \text{Engine Thrust:} \quad 110
\]

\[
\text{Engine Mass:} \quad 50 \quad \text{Engine Thrust:} \quad 110
\]

\[
\text{INDEX} = \frac{160}{640} = 0.25
\]

\[
\text{Engine Mass:} \quad 140 \quad \text{Engine Thrust:} \quad 160
\]

\[
\text{Engine Mass:} \quad 25 \quad \text{Engine Thrust:} \quad 525
\]

\[
\text{INDEX} = \frac{105}{525} = 0.2
\]

\[
\text{Engine Mass:} \quad 25 \quad \text{Engine Thrust:} \quad 105
\]

(c) If the engine mass is in kg and the thrust is in Newtons, what are the units of the index? \(\frac{N}{\text{kg}}\)

(d) Suppose another company, ACME motors, makes a motor with an index of 0.3. What does the number 0.3 tell you about ACME's motors?

For each kg of boat mass the engine provides 1 N of push.
3. A rock is thrown straight upward from an unknown height above the ground with an initial speed of 10 m/s. It strikes the ground 5.0 s later. Determine the initial height of the rock above the ground.

\[ \Delta y = V_y t + \frac{1}{2}at^2 \]

\[ \Delta y = 72.5 \text{ m} \]

\[ a = -9.8 \text{ m/s}^2 \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>14 m</td>
</tr>
<tr>
<td>b)</td>
<td>73 m</td>
</tr>
<tr>
<td>c)</td>
<td>125 m</td>
</tr>
<tr>
<td>d)</td>
<td>80 m</td>
</tr>
<tr>
<td>e)</td>
<td>50 m</td>
</tr>
</tbody>
</table>

4. A bicyclist moves with constant acceleration from \( x = 20 \text{ m} \) to \( x = 32 \text{ m} \) in 2 s. Her speed at the end is 7.0 m/s. How much did she accelerate?

\[ \Delta x = 12 \text{ m} \]

\[ t = 2 \text{ s} \]

\[ V_f = 7 \text{ m/s} \]

\[ V_o = ? \]

\[ a = ? \]

\[ a = \frac{V_f - V_o}{t} = \frac{2 \text{ m/s}}{2 \text{ s}} = 1 \text{ m/s}^2 \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>1.0 m/s²</td>
</tr>
<tr>
<td>b)</td>
<td>3.5 m/s²</td>
</tr>
<tr>
<td>c)</td>
<td>2.0 m/s²</td>
</tr>
<tr>
<td>d)</td>
<td>3.0 m/s²</td>
</tr>
<tr>
<td>e)</td>
<td>She moves at a constant velocity, so ( a = 0 )</td>
</tr>
</tbody>
</table>

5. A bullet is fired through a board, 0.18 m thick, with its line of motion perpendicular to the face of the board. If it enters with a speed of 450 m/s and emerges with a speed of 220 m/s, what is the bullet’s acceleration as it passes through the board?

\[ \Delta x = 0.18 \text{ m} \]

\[ V_f = 220 \text{ m/s} \]

\[ V_o = 450 \text{ m/s} \]

\[ a = ? \]

\[ a = \frac{V_f^2 - V_o^2}{2 \Delta x} = \frac{(220^2 - 450^2)}{(2)(0.18)} \]

\[ a = -428,000 \text{ m/s}^2 \]

\[ a = -430 \text{ km/s}^2 \]
6. Two forces are the only forces acting on a 3.0-kg object which moves with an acceleration of 3.0 m/s² in the positive x direction. If one of the forces acts in the positive x direction and has a magnitude of 12 N, what is the magnitude of the other force?

   a) 6 N
   b) 9 N
   c) 3 N
   d) 18 N
   e) 24 N

\[ F = \frac{ma}{\alpha} = \frac{3 \times 12}{3} = 9 N \]

7. An object's position is described by \( x(t) = (3t + 4t^2 - t^3) \) meters. Which of the following statements is TRUE?

   a) The net force on the object is constant.
   b) The object starts from rest.
   c) The object moves with a constant velocity.
   d) The acceleration of the object at \( t=1 \) s is +2 m/s².
   e) The object always moves in the positive direction.

\[ a(t) = \frac{dv}{dt} = 3 + 8t - 3t^2 \]

8. Two students sit in identical office chairs facing each other. Bob has a mass of 95 kg, while Jim has a mass of 77 kg. Bob places his bare feet on Jim's knees, as shown to the right. Bob then suddenly pushes outward with his feet, causing both chairs to start moving. In this situation, while Bob's feet are in contact with Jim's knees,

   a. neither student exerts a force on the other.
   b. Bob exerts a force on Jim, but Jim doesn't exert any force on Bob.
   c. each student exerts a force on the other, but Jim exerts the larger force.
   d. each student exerts a force on the other, but Bob exerts the larger force.
   e. each student exerts the same amount of force on the other.