Masses, Sheets and Rigid SCFTs

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Motivations

- Supersymmetric field theories are interesting laboratories for studying non-perturbative aspects of Quantum Field Theories.
- The maximally supersymmetric theory in four dimensions is $\mathcal{N} = 4$ SYM for a gauge group G. It is conjectured to obey a highly non-trivial duality (S-duality) relates strongly coupled G theory to a weakly coupled theory with gauge group $G^\vee$ (GNO/Langlands dual). This is arguably the 4d theory whose non-perturbative aspects are best understood.
- With lesser SUSY, we typically have less control and hence, it is more challenging to understand the non-perturbative dynamics.
- But, with lesser SUSY, we can have physics that is closer to the real world.

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Interesting intermediate case : 4d $\mathcal{N} = 2$ theories.

- The conventional way to build 4d $\mathcal{N} = 2$ theories would be to write SUSY Lagrangians using multiplets of the$\mathcal{N} = \in$ super-Poincare algebra : vector multiplets and hyper multiplets.
- A N=2 Vector multiplet is composed of a N=1 vector $(\tilde{\lambda}, A)$ and a N=1 chiral multiplet (ϕ, λ).
- A N=2 Hyper multiplet is composed of a N=1 chiral (ψ, η) and a N=1 anti-chiral $(\tilde{\psi}, \tilde{\eta})$.
- The Lagrangian contains potential terms for the scalars (ϕ, η)

$$\mathcal{L}_{N=2} \supset V(\phi, \eta) = \frac{1}{2}(D^2) + F^\dagger F$$

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Motivations

Classically, we have atleast two branches of vacua. One in which $\langle \phi \rangle = 0$ and the other in which $\langle \eta \rangle = 0$. In $\mathcal{N} = 2$ theories, these moduli spaces of vacua end up persisting in the quantum theory. These quantum moduli spaces of vacua are called

- The Higgs branch (the branch in which the $\langle \phi \rangle = 0$)
- The Coulomb branch (the branch in which $\langle \eta \rangle = 0$)

For Lagrangian theories, the Higgs branch is determined in a fairly canonical manner since the metric on it does not receive quantum corrections. On the other hand, the Coulomb branch metric receives highly non-trivial quantum corrections. So, it is more challenging to determine the metric on the Coulomb branch and the low energy EFT on the Coulomb branch.

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The low energy theory at a generic point of the Coulomb branch $\mathcal{B}$ is an interacting $U(1)^r$ theory where r is the rank of the gauge group.

Seiberg and Witten (1994) came up with a strategy to "solve" for the low energy EFT at a generic point $b \in \mathcal{B}$.

Their strategy combines constraints coming from SUSY and EM duality of the abelian theory. They were able to successfully carry out their strategy for specific examples : pure $SU(2)$ gauge theory and $SU(2)$ with $N_f = 1, 2, 3, 4$ (with and w/o masses).

It was extended to more examples in further works but progress in the higher rank cases was difficult.

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For example, the next obvious SCFT : the $SU(3), N_f = 6$ theory was studied in detail only in 2007!

In 2007, Argyres and Seiberg noted that the $SU(3), N_f = 6$ theory has a S-duality and it one of its duality frames, a $SU(2)$ gauging of the E_6 Minahan-Nemeschansky theory appeared as its "matter sector".

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One may wonder if it is possible to have a uniform construction of all of these Seiberg-Witten geometries. Answer : Yes, it is! This is the main motivation and underlying theme of the talk.

Many strategies exist in the literature. I will not be able to summarize all of them here.

A particular powerful strategy involves realizing these 4d $\mathcal{N} = 2$ theories from the 6d $(0, 2)$ theory. This is what I will focus on in the rest of my talk.

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Theory X

- For this talk, a particular family of six dimensional SCFTs will be important. These are the ones with $(0,2)$ SUSY in 6d.
- The full superconformal algebra is $osp(6,2|4)$. The R-symmetry algebra is $USp(4) \simeq so(5)$.
- The basic multiplet for this algebra is the abelian tensor multiplet. Using this multiplet, it is possible to construct free or interacting abelian theories.
- But, we are interested in non-abelian theories with $(0,2)$ SUSY. They do exist, but their existence was inferred by indirect means (by String/M-theory constructions). We are interested in these non-abelian theories.

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- There is one such theory for every simply laced Lie algebra $j \in A, D, E$.
- The A_n series can be obtained as the theory on a stack of $M5$ branes.
- More generally, type II strings probing A,D,E type singularities give rise to the A, D, E series.
- The A_n, D_n theories can be studied/defined holographically.
- No settled name for this theory : Varyingly called "*The*$(0,2)$*theory*", the "$M5$ brane theory". More recently, "Theory $X[j]$".

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Class S theories

- Construction of class S(named for the Six-d origin) theories is done by formulating the 6d theory on $\mathbb{R}^{1,3} \times C_{g,n}$ (with a partial twist) and dimensionally reducing on C. We also insert certain 4d 1/2 BPS defects of the 6d theory (or co-dimension two defects) at the n punctures. [Gaiotto, Gaiotto-Moore-Neitzke].
- Non-perturbative dualities of class S have a geometric interpretation in terms of moving in the complex structure moduli space of C. This is a vast generalization of the case of $C = \mathbb{T}^2$ which corresponds to the S-duality of $\mathcal{N} = 4$ SYM.
- When we allow certain four dimensional modifications of the six dimensional theory (1/2 BPS co-dimension two defects) to sit on points in C, it changes the resulting theory in 4d.
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One of the defining virtues of class S theories is that their Seiberg-Witten solution is described using the Hitchin system for type j associated to the Riemann surface $C_{g,n}$. [Gaiotto-Moore-Neitzke]

The Hitchin system is a complex integrable system whose total space is the moduli space of solutions $\mathcal{M}_H$ to

$$F_A + [\phi, \phi^\dagger] = 0$$
$$\bar{\partial}_A \phi = 0$$

where A is a two dimensional j gauge field on C and ϕ is an adjoint valued one-form (the Higgs field).
An important property of $\mathcal{M}_H$ is that it is hyper-Kähler.

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There is the Hitchin map

$$\mu : \mathcal{M}_H \to \mathcal{B},$$

where $\mathcal{B}$ is parameterized by the Weyl invariant polynomials on $\mathfrak{h}(j)$, the Cartan subalgebra of $\mathfrak{j}$.
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Hitchin showed that $\mathcal{B}$ (called the base) is a Lagrangian subspace of $\mathcal{M}_H$ and that the generic fibers of the map μ are Lagrangian tori. This structure is the complex analog of real integrable systems (in Action-Angle variables) that are familiar in Hamiltonian dynamics. Hence, it is called a complex integrable system.

It turns out that for Class S theories, the base of the Hitchin system parameterizes the Coulomb branch of the 4d theory while the total space $\mathcal{M}_H$ parameterizes the Coulomb branch of the 4d theory formulated on a circle of radius R and taking R to be small.

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The SW curve is the spectral curve $\Sigma_b \equiv det(\lambda I - \phi) = 0$ and the SW differential is $\lambda dz\mid_C$.

The data near the punctures encodes the nature of the 4d defect
Two main classes of defects

- **Tame defects** : Cases where the Higgs one-form has a simple pole, $\phi = \frac{a}{z}dz + (\ldots)$, $a \in j$.
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For class S theories built out of tame defects, we obtain a super-conformal theory (SCFT) if we have no punctures (and $g \geq 2$) or if at every puncture, the residue *a* of the Higgs one form is a nilpotent element in the Lie algebra $\mathfrak{j}$.

Upto conjugacy, only the adjoint orbit in which *a* lies matters. There are a finite number of nilpotent orbits in any complex Lie algebra and these are classified (see for ex Collingwood-McGovern).

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This suggests that for the SCFTs built from tame defects, there is a natural class of relevant deformations corresponding to gauging the Flavor symmetries and giving VEVs to the scalars in the background vector multiplet.

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To be precise, we provide the solution to the problem of mass deforming the 3d $\mathcal{N} = 4$ theory obtained by dimensional reduction to three dimensions. In the rest of the talk, I will describe this solution.

Remark : Our solution builds on earlier work of [Chacaltana-Distler-Tachikawa] where the defects where studied in the massless limit. For a particular class of defects (that we call Smoothable defects), our recent work can be taken as an independent derivation of the results in [CDT].

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To describe the mass deformed geometries, I need to talk about a class of 3d $\mathcal{N} = 4$ SCFTs that are known as $T^\rho[J]$ theories (introduced by Gaiotto-Witten in their work on S-duality of boundary conditions in $\mathcal{N} = 4$).

They are relevant to us because these can be thought of as the theories arising from taking a single tame four dimensional defect and compactifying it on a circle and shrinking the radius of the circle to zero.

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The embedding ρ identifies a pure Nahm pole boundary condition in $\mathcal{N} = 4$ SYM with gauge group J. For the present discussion, let us take J to be simply laced, but more general choices are possible. If y is a co-ordinate on a half-line with $y = 0$ being the boundary, the Nahm pole bc takes the following form

$$\frac{dX^i}{dy} = \epsilon^{ijk}[X^j, X^k], X^i = \frac{\tau^i}{z},$$

where $\tau^1 = \rho(e) + i\rho(f), \tau^2 = \rho(e) - i\rho(f), \tau^3 = \rho(h)$ and X^i are three of the six scalars in $\mathcal{N} = 4$.

When J is classical, it is possible to realize the Nahm pole BCs using a configuration of D3 and D5 branes in type IIB [Diaconescu, Gaiotto-Witten]. The dual brane configurations involve D3 and NS5 branes and lead to brane realizations of some $T^\rho[J]$ theories.

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How are properties of the $T^\rho[J]$ theories related to the properties of the tame defects ?

- $T^\rho[J]$ has a Higgs branch which is nothing but the moduli space of solutions to Nahm's equations. Depending on the choice of ρ, this moduli space could have a non-trivial continuous hyper-Kähler isometry F. Let the associated complex Lie algebra be $\mathfrak{f}$. This is the flavor symmetry associated to the tame defect.
- $T^\rho[J]$ has a Coulomb branch which is the closure of a nilpotent orbit in the complex Lie algebra $\mathfrak{j}$. This nilpotent orbit is the one appearing in the Hitchin system as the residue of the Higgs field.

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- If we were dealing with $T^\rho[G]$ theories with G non-simply laced, then the Coulomb branch will be a nilpotent orbit in $\mathfrak{g}^\vee$. Our paper handles these cases as well, but let us ignore them for the moment.
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Tame defects and $T^\rho[J]$ theories

What does a mass deformation do to a 3d $\mathcal{N} = 4$ theory ? On the Higgs branch, it can lift certain directions in (or the entire) Higgs branch.

On the Coulomb branch :

1. We expect it to deform the Coulomb branch such that $[\Omega]^{CB}_{m \neq 0} \propto m_i$, where m_i are the mass parameters.
2. In the limit of zero masses, we have that $[\Omega]^{CB}_{m \to 0} = [\Omega]^{CB}_{m=0}$. In particular, we want $\dim(CB_{m\neq 0}) = \dim(CB_{m=0})$.
3. By superconformal representation theory, we also have that the number of mass parameters is constrained by the Flavor symmetry acting on the Higgs branch. More accurately, we require that $m_i \in \mathfrak{h}(\mathfrak{f})$. We call this the Flavor condition.

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Mass-Like Deformations vs Mass Deformations

Our strategy involves finding deformations of the Hitchin system that obey conditions (1), (2) and (3). The first two are naturally imposed using data that is transparent in the Coulomb branch geometry. The third condition, on the other hand, is something that ties together the Coulomb and Higgs branch data. In our work, we found it convenient to impose them in steps.

Step 1 : We identify deformations of the Hitchin system that obey conditions (1) and (2). We call such deformations mass-like deformations.

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Our strategy involves finding deformations of the Hitchin system that obey conditions (1), (2) and (3). The first two are naturally imposed using data that is transparent in the Coulomb branch geometry. The third condition, on the other hand, is something that ties together the Coulomb and Higgs branch data. In our work, we found it convenient to impose them in steps.

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To carry out our **Step 1**, it is helpful to introduce the idea of a sheet in a Lie algebra.

Consider the union $\mathcal{U}_d$ of all adjoint orbits of fixed complex dimension d. This can be a rather complicated space. Take its irreducible components. These components are known as sheets. Sheets are somewhat well known to those studying geometric representation theory but they appear to not be well known in the physics literature. Our paper includes a rather longish introduction to this theory. I will only review some highlights here.

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Sheets :

- There are are a finite number of sheets in any complex Lie algebra.
- The boundary of any sheet is a nilpotent orbit. If a sheet has non-nilpotent orbit, then it has an infinite number of them. So, a typical sheet will have the following schematic form :

$$\tilde{a} = \tilde{a}_{ss} + \tilde{a}_n, [\tilde{a}_{ss}, \tilde{a}_n] = 0$$

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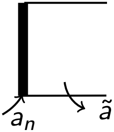

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Sheets : There are three types of sheets

1. Sheets that contain semi-simple elements. These are called Dixmier Sheets. The nilpotent orbits that occur at the boundary of Dixmier sheets are called Richardson orbits.
2. Sheets that contain just a single nilpotent orbit. These are called Rigid Sheets. The corresponding nilpotent orbit is called a rigid nilpotent orbit. Ex : The smallest non-zero orbit (minimal nilpotent orbit) in Lie algebras of types other than A is always a rigid orbit. This is also the one instanton moduli space in a 4d gauge theory with gauge group J.
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Sheets : How are sheets classified ?

- Sheets in any complex Lie algebra $\mathfrak{j}$ are classified by a pair $(\mathfrak{l}, \mathcal{O})$, where $\mathfrak{l}$ is a Levi-subalgebra of $\mathfrak{j}$ and $\mathcal{O}$ is a rigid nilpotent orbit in $\mathfrak{l}$.
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- So, to enumerate all sheets, we first enumerate all nilpotent orbits and then find the sheets attached to each of those nilpotent orbits.

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Sheets :

- A dramatic simplification occurs in type A : Every nilpotent orbit occurs at the boundary of a unique sheet! This is no longer true in other Cartan types.
- Example : In the D_4, the nilpotent orbit $[3^2, 1^2]$ occurs at the boundary of two sheets : $(A_2, 0)$ and $(A_1 + D_2, 0)$.
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Now, back to the **Hitchin System** :

- Let us start with a Hitchin system on a punctured disc with a tame nilpotent singularity at the puncture. Let this nilpotent orbit be $\mathcal{O}_H$ ("The Hitchin label ").
- Now, let us deform the residue a of the Higgs field from being nilpotent to being a generic orbit in a sheet attached to $\mathcal{O}_H$
- Every such deformation obeys the conditions (1) and (2) for being a mass-like deformation. It is also possible to show that these are all the (local) mass-like deformations.
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To identify the actual Mass deformations, we need to additionally impose the flavor condition. We do this in the following way.

- First, we associate a second nilpotent orbit to the tame defect. This is done by taking the embedding ρ and using the Jacobson-Morozov theorem to identify a corresponding nilpotent orbit. Let us denote this nilpotent orbit to be $\mathcal{O}_N$ ("The Nahm label") .
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Specifying which of the pairs $(\mathfrak{l}, \mathcal{O})$ (these label sheets) end up obeying the flavor condition is a bit technical and I don't have time to explain it in detail. This is done in the paper. But, I will give a quick summary :

- Associated to $\mathcal{O}_N$, there is a Levi subalgebra called its Bala-Carter Levi. It is the Levi subalgebra in which $\mathcal{O}_N$ is a distinguished nilpotent orbit. There is a standard algorithm to find the BC Levi of any nilpotent orbit. Let us denote this Levi by $\mathfrak{l}_{BC}$.
- The pairs $(\mathfrak{l}, \mathcal{O})$ that obey the Flavor condition always obey $\mathfrak{l} = \mathfrak{l}^{\vee}_{BC}$
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Summary

In the paper, we write out in detail the sheet data for every mass deformed tame defect in the exceptional Lie algebras and for every tame defect in some low rank classical cases. We also give the general algorithm in the classical cases.
What did we learn ?

- The $T^{\rho}[J]$ theories fall into three deformation classes : **Smoothable**, **Malleable** and **Rigid**.
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Further Questions/Applications

- I briefly mentioned that some of the $T^{\rho}[J]$ theories admit brane descriptions. These lead to interesting Quiver gauge theories which flow in the IR to these SCFTs.
- But, such UV Lagrangian starting points are known only for certain $T^{\rho}[J]$ theories (studied for example by [Hanany et al]). When J is $SU(N)$, such UV Lagrangians are always available.
- When UV Lagrangians exist, can they detect the deformation type of the IR SCFT to which it flows to ?
- Are these any other methods that see the deformation type of the SCFTS ? For example, one can easily obtain "large N" sequences where every theory in the sequence is a Rigid SCFT. So, the property of being Rigid can survive at large N. Can it then be detected using a AdS_4 dual ?

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- The $T^{\rho}[J]$ theories were in fact originally introduced by Gaiotto-Witten in this context.
- A closely related subject is the study of Symplectic Duality. This was initiated by Braden-Licata-Proudfoot-Webster in the math literature.
- In physics terms, Symplectic Duality is a statement about how Higgs and Coulomb branches of 3d $\mathcal{N} = 4$ theories get paired up.
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