Sheets, 3d Coulomb branches and Symplectic Duality

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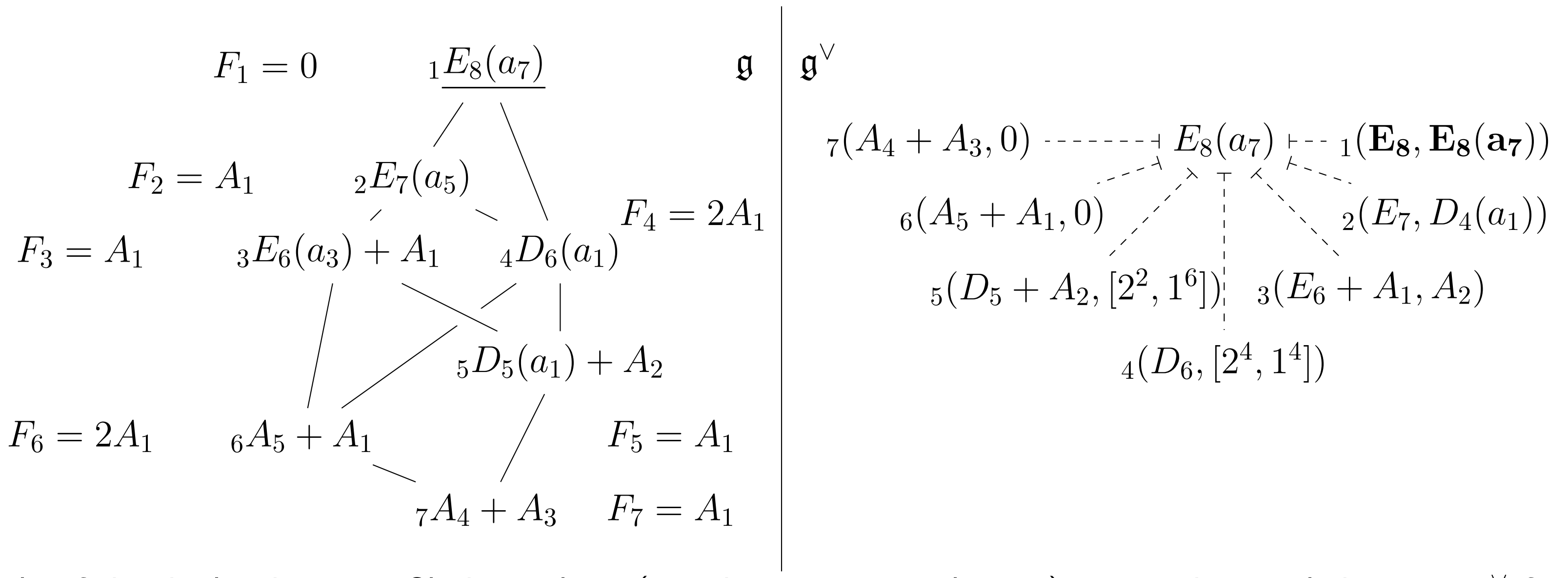
Slices | Special Sheets



An example of the duality between Slodowy slices (to orbits in a special piece) in $\mathfrak{g}$ and special sheets in $\mathfrak{g}^\vee$ for $\mathfrak{g} = E_8$.

3d $\mathcal{N} = 4$ theories

- A pair of symplectic singularities are naturally associated to every 3d $\mathcal{N} = 4$ superconformal field theory (SCFT) : The *Higgs branch* and the *Coulomb Branch*.
- A **necessary** requirement is that the two be **symplectic duals** of each other.
- How to identify such pairs ? One way is to try and mass deform the Coulomb branch. The mass deformed CB geometry is constrained by properties of **both** singular spaces.

The Gaiotto-Witten theories

- Let us focus on a particular class of 3d $\mathcal{N} = 4$ SCFTs that were introduced by Gaiotto-Witten.
- There is one such theory for every choice of a complex Lie algebra $\mathfrak{g}$ and a $\rho : \mathfrak{sl}_2 \to \mathfrak{g}$. They are called the $T^\rho[\mathfrak{g}]$ theories.
- By definition, the Higgs branch of a $T^\rho[\mathfrak{g}]$ is the **Slodowy slice** to the orbit $\mathcal{O}_{\rho(e)}$, where (e, f, h) is the $\mathfrak{sl}_2$ triple.
- The Coulomb branch is *some* adjoint orbit of $\mathfrak{g}^\vee$.
- **Question :** What is the Coulomb branch of $T^\rho[\mathfrak{g}]$?

Sheets

- Let $\mathcal{U}_d$ be the union of all adjoint orbits of a fixed dimension d in the complex Lie algebra g. The irreducible components of $\mathcal{U}$ are called **sheets**. The boundary of every sheet is a nilpotent orbit.
- They were originally studied by Borho, Kraft, Vogan. An analogous version for groups has been studied by Lusztig, Carnovale.

Special Sheets

Sheets in $\mathfrak{g}$ are classified by pairs $(\mathfrak{l}, \mathcal{O})$ where $\mathfrak{l}$ is a Levi subalgebra and $\mathcal{O}$ is a rigid nilpotent orbit in $\mathfrak{l}$.
Special Sheet : We define a special sheet to be those sheets for which the second entry in the pair $(\mathfrak{l}, \mathcal{O})$ is a special nilpotent orbit in $\mathfrak{l}$.

Slices-Sheets duality

- It turns out that **special sheets** in $\mathfrak{g}^\vee$ are exactly the **mass deformed Coulomb branches** of $T^\rho[\mathfrak{g}]$ theories. The massless limit is the nilpotent orbit at the boundary of the sheet [1].
- Special sheets are paired with Slodowy Slices (Higgs branches) according to a **Flavor condition**. Mathematicaly, this identifies the sheet Levi of the mass deformed CB with the Langlands dual of the Bala-Carter Levi of $\mathcal{O}_{\rho(e)}$.
- The resulting slices- (special) sheets duality can be viewed as a refinement of Spaltenstein-Barbasch-Vogan duality between nilpotent orbits.
- When $\mathcal{O}_{\rho(e)}$ is principal Levi type, the dual sheet is a **Dixmier sheet**. The corresponding $T^\rho[\mathfrak{g}]$ theories are **smoothable**.
- More generally, the **rigid theories** among $T^\rho[\mathfrak{g}]$ play an important role. This includes but is not limited to cases where the Coulomb branch is a rigid nilpotent orbit.
- This modified notion of rigidity implies that the dual to Slodowy slices are, in some cases, (sub)-sheets as opposed an entire sheet.
- Further conjectures in [2] suggests this duality is compatible with local system data on $\mathfrak{g}^\vee$ side and works of Sommers, Achar.

References

1. Balasubramanian, Aswin and Distler, Jacques, **"Masses, Sheets and Rigid SCFTs"**, arXiv:1810.10652
2. Chacaltana, Oscar, Jacques Distler, and Yuji Tachikawa, **"Nilpotent Orbits and Codimension-2 Defects of 6d $\mathbf{N} = (2, 0)$ Theories"**, International Journal of Modern Physics A 28 (2013).

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