

## Work & Energy, Circular Motion



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**Purpose:** To apply the Work-Energy Theorem to a real-life problem. To investigate the forces and accelerations associated with circular motion.

Apparatus: Loop De Loop apparatus, stainless steel ball, masking tape, timer, meter stick, Flying Cow hanging from string, spring scale (blue).

### Introduction

This lab consists of two parts, which will involve switching lab desks:

- You will predict (theoretical) the minimum height to which a stainless steel ball must be raised such that when it is released it rolls down an incline and into a circular loop and maintains contact with the loop during its circular motion. You will then find out the actual (experimental) height which enables it to do so and try to explain the discrepancy, if any, between theory and experiment.

- You will predict the tension in the string attached to a Flying Cow going around in a circle (dynamic). You will then measure the tension in the string indirectly using a spring scale, with the cow stationary (static), compare the results, and reconcile the number of forces identified in each case

### Theory (optional)

The Work-Energy Theorem States that

(in English)

The net *work* done on an object equals its change in kinetic energy.

(in Mathematics)

$$W = (KE)_f - (KE)_i \quad (1)$$

Recall that the kinetic energy of an object is

$$KE = \frac{1}{2}mv^2$$

The work done on an object is given by

$$W = (F \cos \theta) s$$

where  $s$  is the distance over which the force acts and  $\theta$  the angle between the force and the displacement. For a single mass dropping to the floor from height  $h$ , the only force doing work (causing motion) is the gravitational force on the small mass,  $mg$ , which acts over the distance  $h$ , the height from which it drops to the floor.

We can now expand on equation (1):

$$mgh = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad (2)$$

The quantity  $mgh$  is the potential energy the mass acquires as it is moved from the floor (zero potential) to its starting point ( $mgh$  potential). (Incidentally, we could also have assigned zero as the potential energy before the fall and  $-mgh$  the potential on the floor) You can therefore read equation (2) as “the potential energy of a falling small mass is converted to its kinetic energy”. Note that you can also read the equation **backwards**, as in “a small mass thrown in the air has its kinetic energy converted to potential energy”. In this way one form of energy is converted into the other, and possibly back again. For **conservative forces** (forces in which the work done is *independent of the path*, e.g. gravity), equation (2) can also be written as:

$$(KE)_i + (PE)_i = (KE)_f + (PE)_f \quad (3)$$

This is read as “total mechanical energy is conserved”. Mechanical energy is the sum of kinetic and potential energies. For **non-conservative** forces (forces in which the work done *is dependent on the path*, e.g. friction), the above equation does not hold. In such a situation energy is converted to a form that is not recoverable - one example is a plucked guitar string, in which potential energy is stored and converted to kinetic energy (motion of string) and sound energy (work done on surrounding air molecules). In this case energy is lost to sound and the string eventually stops vibrating.

Centripetal Acceleration is the acceleration that an object undergoes when it moves in uniform circular motion (constant speed). This is a vector, directed toward the center of the circle with magnitude given as  $a_c = v^2/R$ . Here  $v$  is the speed of the object and  $R$  is the radius of the circle. Imagine whirling a pail of water in a large vertical circle so that

the water doesn't spill out. If we examine the instant of time where the pail is at the top of the circle (the point of interest), Newton's Second Law for the water, modified to include centripetal acceleration, would be:

$$ma_c = F_{pw} + mg \quad \text{or} \quad m \frac{v^2}{R} = F_{pw} + mg$$

Here  $F_{pw}$ , the force of the pail on the water, which acts toward the center of circle, is denoted as positive. Gravitational force  $mg$ , which also acts toward the center of the circle, is also positive. The condition such that the water just barely maintains contact with the pail at the top of the circle can be expressed as  $F_{pw} = 0$ , so we get:

$$m \frac{v^2}{R} = mg \quad \text{or} \quad \frac{v^2}{R} = g$$

Note that you would either have to swing the pail with greater speed or a smaller radius circle if the force due to gravity was greater.

## Procedure

### I. Predict the height of release that just barely results in circular motion

Using your knowledge of (1) Conservation Of Energy and (2) Newton's 2<sup>nd</sup> Law, calculate the predicted *minimum* release height of the stainless steel ball that will result in it just barely executing a circular loop-de-loop, meaning that it just barely maintains contact with the track of the loop when it is at the top part of the loop.

The two points of greatest interest here are a) the point of release, where the ball initially only has potential energy (no kinetic energy), and b) the point at the top of the loop, where the ball has both some potential energy and some kinetic energy. *Hint: you will need to combine both the energy conservation equation and the force (Newton's 2<sup>nd</sup> Law) equation to arrive at answer, as all the initial potential energy does not turn into kinetic energy (there is still some potential energy at the top of the loop).*

When finished, show the results to the instructor then **write down your prediction (show all calculations) in the hand-in sheet.**

### II. Measure the actual height of release that just barely results in circular motion

Determine the actual release height by experiment, using trial and error, starting with your predicted release height, and **write this in your hand-in sheet.** Consider the two possible ways energy is lost in this experiment – friction (is the contact between the ball and track completely frictionless?) and losses due to rotational energy that we are ignoring by approximating the ball as a point particle (does the ball spin at all? If so, part of the energy goes into rotational kinetic energy, not just translational/center of mass kinetic energy). Note that we will not cover rotational dynamics until later this semester, so we will not figure that value into our calculations.

Calculate the percentage of energy lost in the experiment and **write this in your hand-in sheet**. Also, indicate the source of this energy loss (or energy conversion).

### III. Flying Cow in Circular Motion

The Flying Cow is suspended from the ceiling. Make sure its wings are fully extended (not folded back) and turn it on using the power switch below the neck. The wings should start flapping. Pull it out to the side, point it tangentially to the circular path you will try to make it execute, and give it a push. If it does not maintain a circle, try pushing it in the opposite direction. Once it settles into a steady circle, measure the radius of this circle using the meter stick and the speed using your watch's second hand or digit. A vertically placed meter stick and masking tape may help in marking off the radius on the lab table. If you don't have a watch, use Stopwatch.exe which is in the Lab Softwares folder on your lab PC's desktop. From this data, and the length of the string, calculate the tension in the string.

Now measure the tension directly by turning the Flying Cow. First turn it off and let it hang straight down. Then attach the blue spring scale's hook to the part where the string also hooks on. Holding the other end of the spring scale, horizontally pull it out to its radius when it was flying around in a circle. Note this reading down and calculate the tension in the string. You may find that reviewing past labs that dealt with vectors to be useful. Compare your result (static) to the result you obtained from the previous (dynamic) method. Explain the discrepancy and determine whether or not it is within experimental uncertainty. **Show all work in the hand-in sheet.**

### **Assessment and Presentation (Hand-in Sheet/Lab Notebook)**

Write the results from the three activities above in the hand-in sheet. Then answer these questions:

1. After you release the stainless steel ball along the inclined ramp of the loop-de-loop, at what point is it traveling the fastest?
2. If your predicted and experimental release heights are not close, what are the sources of the energy loss? What percentage of the initial potential energy is lost?
3. When the Flying Cow subtends an angle from the vertical, there is one horizontal force on it in the dynamic case but two horizontal forces on it in the static case. Explain why this is the case.