**Objective:** To investigate the behavior of a rotating system subjected to internal torques and external torques. To apply knowledge gained from the linear momentum lab to its rotational analog. To see how total energy is a function of linear and rotational energy by re-visiting the loop-the-loop part of the Work, Energy & Circular Motion experiment.

**Apparatus:** Angular motion sensor, circular aluminum disc with pulleys of varying diameter on other side, meter stick, 20g mass on a string, flexible goose neck mass.

**Introduction:**

A few weeks ago you investigated the interaction of a system of two masses without any external forces, namely the conservation of momentum of two colliding carts. In this lab you will investigate a similar "collision", but there will be no linear motion, only rotational. Below are some useful equations you may remember from lecture (all scalar).

Definition of angular velocity:
\[ \omega = \frac{v}{r} \]

Moment of Inertia of a point mass about an axis of rotation \( r \) away:
\[ I_m = mr^2 \]
Moment of Inertia of a circular disc about a perpendicular axis through its center:
\[ I_{\text{disk}} = \frac{1}{2}mr^2 \]

Moment of Inertia of a rod about an axis through its center (perpendicular to its length):
\[ I_{\text{(rod, center)}} = \frac{1}{12}ml^2 \]

Definition of angular momentum:
\[ L = I\omega \]

Kinetic energy of rotation:
\[ KE_{\text{rotational}} = \frac{1}{2}I\omega^2 \]

Total energy of an object undergoing both translational and rotational motion:
\[ KE_{\text{total}} = KE_{\text{translational}} + KE_{\text{rotational}} + PE_{\text{translational}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 + mgh_{\text{CM}} \]

Newton's Law in rotational form (tau is the external torque; alpha is the \textit{angular acceleration}):
\[ \tau = I\alpha \]

Equation of motion in rotational form (constant angular acceleration):
\[ \theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2 \]

Time-independent version of equation above:
\[ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta \]

In addition, the Moment of Inertia of an object of known \( I_0 \), when taken about another axis parallel to the axis used to calculate the known \( I \) (when the two axes are a distance \( h \) away) is:
\[ I = I_0 + mh^2 \]

This is also known as the \textit{Parallel Axis Theorem}.

You will perform two experiments real experiments and one video analysis activity:

1. Gently drop a non-rotating rod (which can either be straight or bent into an approximate circle) onto a circular disc rotating at low angular momentum. The disc is permanently mounted on an angular sensor which will give output angular position and speed to Logger Pro. This is the rotational equivalent of the linear collision you
performed with the two Pasco carts on the Pasco track. In this experiment the system (disc + rod) experiences no external torques, just as the two carts experienced no external forces; all interactions are internal.

2. Use a small mass, connected to a pulley with a string, to rotate the circular disc. Here you are using the same equipment as in (1) above, except that you have re-oriented the setup 90 degrees. This is the rotational equivalent of a force causing a previously stationary mass to accelerate linearly. In this experiment there is an external torque, just as a cart that is pulled by a mass hanging over a pulley experienced a force.

3. Analyze three video clips, variations of an experiment you performed a few weeks ago – the loop-the-loop experiment, measure the relative speeds of the ball at various points of the motion, and determine the relative amounts of linear and rotational energy in the motion. The three balls in the videos will have different coefficients of friction.

Procedure

1. Gently drop a non-rotating rod onto a circular disc (no external torques)

1. Open the Logger Pro file Angular Momentum Conservation.cmbl, which is a Logger Pro template file in the same folder as this write-up. Scroll through the different pages by clicking on the arrows to the left and right of page selector near the upper left hand corner: Page 1, Page 2, Page 3. On Page 3 you will see the data table. Double click on "L rod" and examine the way it is calculated (currently $I_{rod,center} = \frac{1}{12} ml^2$). Look at the handwritten mass value on the aluminum circular disc, and change this in Logger Pro if necessary. Note that the 1/12 coefficient will have to be changed if you alter the shape of the rod from its straight shape. Also measure the length of the rod, and change that parameter in Logger Pro if needed.

2. Make sure the apparatus is positioned as in the photo below. The plane of the circular disc should be horizontal, with the red toothed "catch" facing upwards:
3. Go back to Page 1. Press the Collect button in Logger Pro and practice spinning the disc at low angular speed (you must spin counterclockwise; software is configured for CCW spin) and gently dropping the straight rod onto the red toothed catch. It should be centered - the middle of the rod should land on the center of the disc. Look at the resulting graph and visually check if your results make sense. You should see something like the photo below:
Plots, with all data, of rod being dropped onto disk. Note that on some sensors, clockwise rotation corresponds to decreasing angle (downward slope on Theta, upward jump on Omega. If this is the case, go to Page 3 in Logger Pro, double-click on the Omega (Angular Velocity) column, and put a minus sign in front of the expression in the Equation box.

It is critical to remember that the only thing that the instrumentation will directly measure is the angular position and speed of the rotary sensor (connected to the disc); the angular velocity of the rod should be the same as the disc after the collision since they spin in unison. Before the collision the rod's angular speed (and the resulting calculated angular momentum and rotational kinetic energy) should be zero; the software displays a non-zero value since the software algorithm assumes that the rod always spins with the disc. You will then have to eliminate these false data points by highlighting the L rod and KE rod points in Logger Pro and going to Edit--> Strike Through Data Cells. Note that the points you will be striking through should correspond to the time interval before the rod and disc have settled into a fairly constant (yet slowly dropping) angular speed. Examine the numbers in the data table for L rod - they will help you choose where to put the strike through cutoff by showing periods of mostly constant values. After you strike through the pre-collision and collision data point, your graph should appear as follows:
Plots, with pre-dropping data removed, of rod being dropped onto disk. Note that on some sensors, clockwise rotation corresponds to decreasing angle (downward slope on Theta, upward jump on Omega. If this is the case, go to Page 3 in Logger Pro, double-click on the Omega (Angular Velocity) column, and put a minus sign in front of the expression in the Equation box.

Notice that before the collision, only the disc has angular velocity, momentum; and kinetic energy the rod's should be zero. After the collision, all three quantities are present in both rod and disc.

For your lab report (there is no hand-in sheet), print out your graph as above and record the following on the graph: total energies (initial i, final f, ratio f/i); total angular momenta(i, f, f/i). Note that initial refers to just before the collision, and final refers to just after the collision. Use your experience from Linear Momentum to select the appropriate points.

4. Now bend the rod into a roughly circular shape and repeat the experiment. For the red to retain a circular shape, you'll have to tape the ends together with a single, short strip of tape oriented along the length of the rod – no need to wind the tape around the diameter of the rod (which would make it difficult for the next class to unwind). See photo on next page:
Don't forget to change parameters – as you did in the Linear Momentum lab by double clicking on the data column header in Logger Pro - for rod mass, rod radius (when bent in circle; use average between long radius and short radius) and moment of inertia coefficient (1/12 for straight rod, 1 for circular rod) in Logger Pro. Print out a graph and record values, as above.

2. Use a small mass to rotate the circular disc (external torque)

1. Open up the Logger Pro template file Constant Torque Rotational Motion.cml. Unscrew the angular sensor/disc from the ring stand and re-orient it so that the plane of the disc is now vertical (see photo below). Anchor a thread or string through a pulley slot and wrap a thread or string with small weight suspended around one of the drive pulleys. The free end of the string must be the first thing to wrap around the pulley. Also, the string must be wrapped tightly enough not to slip. Then the external torque exerted is $mgR_{\text{pulley}}$ (0.025m for the larger pulley; remember to change this in the parameters if necessary. The smaller pulley will have a radius of 0.0125m). This being constant (we
neglect any variation in earth gravitational pull with distance from earth center), the constant angular acceleration equations apply (see equations in introduction). These have the same form as those for constant force in translational acceleration. In the photo below, the string is wrapped around the larger (0.025m radius) pulley:

![String wrapped around pulley](image)

The drop mass PE is set to zero by subtraction of the time-zero height. (This is accomplished with the two data columns “theta sub” (subset) and “init theta”.

(This is not a “before and after” collision test. We are interested in the continuous time interval from time zero until dropping drive mass hits. We are not interested in any data after drop mass hits the table. Strike through subsequent data only if it makes you feel better.)

2. Practice winding the drive mass string around the pulley and collecting data. The picture and LP graph below show the operation and result, for 20 gram drop mass on larger pulley.
NOTE: You would expect potential energy to decrease as the 20g mass falls; if Logger Pro shows the opposite, simply wind the string around the pulley in the opposite direction.

The 20 gm weight hit the table at about 1.6 seconds; algorithms fail for later data (the PE no longer changes). Most of the drive mass gravitational PE (red) is converted to disk rotational KE (blue); the KE of the falling drive mass is relatively very small (both theoretically and experimentally). Total energy (green) appears to decrease slightly.

(Note, drive mass PE would be linear if plotted vs. height h, but this is a time plot.)

If the string slips while the drive mass falls, assumptions are invalid!

3. Print the graphs for at least one good example, as in the first experiment (collision). Label the points where the mass was released and when it hit the table. Enter numerical values, and ratios of experimental to theoretical, for quantities listed above.

4. Calculate (from theory), and write on the graph, the following quantities:
   a) The torque exerted on the disk by the drive mass
   b) The moment of inertia of the disk
   c) The calculated angular acceleration of the disk
   d) The change in theta (angle) and omega (angular velocity) from the time the drive mass is released to when it hits the table. You may find these equations useful:
Kinematics equation without time variable \[ \omega^2 = \omega_0^2 + 2 \alpha \Delta \theta \]
Relationship between drop height and change in angle \[ \Delta h = r \Delta \theta \]

5) Compare your value for angular acceleration (part d in step 4 above) to your experimental value (values from the Logger Pro plots/tables) and give your analysis on how close (or far) these values are. Remember that you can obtain the angular acceleration value from the angular velocity vs. time plot, just as you did with linear acceleration and the linear velocity vs. time plot.

3. Video Analysis of the Loop-the-loop (your instructor may decide to omit this section)

1. Individually open the files Stainless Steel.mov, Rubber.mov, Rough Aluminum.mov, slow-motion videos filmed at 420 FPS (frames per second) which are in the same experiment folder as this write-up. IMPORTANT: You must drag the file and drop it onto the QuickTime icon in the experiment folder; otherwise you will not see the time code. You can play and pause the video using controls in the Quick Time player, or with the space bar on the keyboard. You can also step through the video frame-by-frame using the Right Arrow key (forward) and the Left Arrow key (backward). Press Apple-I (Command-I) to bring up the “Inspector” which will display the time code (the Current Time in the video – displayed in Days, Hours, Minutes, Seconds, Hundredths of seconds); this will let you precisely measure time intervals. Expand the video so that the control bar does not obstruct your view of any part of the loop or ramp.

These three will show the motion of three balls of different coefficients of friction falling down the loop, with some experiencing more rotation (more rotational energy, rather than translational energy) than others. **ASSUME THAT ALL THREE BALLS HAVE THE SAME MASS.** Note that the rubber ball has the most “grip” and the steel ball has the least; in other words, \( \mu_{\text{rubber}} > \mu_{\text{aluminum}} > \mu_{\text{steel}} \).

2. Note down the times (seconds and hundredths of seconds, as in 06:84) of these events within each video; enter in hand-in sheet; note that these are video playing times rather than actual experiment times (slowed down by a factor of fourteen to 30 FPS):

<table>
<thead>
<tr>
<th>Event Description</th>
<th>Stainless Steel ball</th>
<th>Rubber ball</th>
<th>Rough Aluminum ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Release of Ball</td>
<td>00.00</td>
<td>00.00</td>
<td>00.00</td>
</tr>
<tr>
<td>Ball at Loop 6 O’clock (bottom)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ball at Loop 3 O’clock (right)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ball at Loop 12 O’clock (top)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ball at Loop 9 O’clock (left)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ball at Loop 6 O’clock (2nd time)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Based on your times above, fill in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Stainless Steel ball</th>
<th>Rubber ball</th>
<th>Rough Aluminum ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time taken to descend ramp</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time between 3 and 9 O'clock (top)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time between 6 O'clock (2nd) and end</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Based on the two above, answer these questions in the hand-in sheet. Remember that kinetic friction always *opposes* the direction of motion, while static friction can often *aid* it – just like the static friction between a tire and the ground to help push a car forward:

a) Which ball descended the ramp the quickest? Why was it the quickest?
b) Which ball descended the ramp the slowest? Why was it the slowest?
c) Which ball had the highest speed coming out of the loop (horizontal exit)? Why was it the fastest there?
d) Which ball had the lowest speed coming out of the loop (horizontal exit)? Why was it the lowest there?
e) Which ball lost the most energy to friction? How can you tell?
f) Which ball lost the least energy to friction? How can you tell?
g) To which type of friction was the energy lost?

5. The release heights for the three balls in order to just execute a loop the loop were, from highest to lowest:

a) Rough Aluminum ball (3 cm on ramp scale where lower number equals greater height)
b) Stainless Steel ball (8 cm on ramp scale)
c) Rubber ball (16 cm on ramp scale)

From this data, which ball lost the most energy to friction? Is your answer consistent with your answer to question 4e above?