String Theory Homework #4
due Feb 13, 2006

1) In the text, the action

$$S = -mc \int_{\mathcal{P}} ds,$$

where $\mathcal{P}$ is a timelike path through spacetime and $(ds)^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$, is evaluated by considering $x^\mu(s)$, where $s = \int ds$ is the proper time of the particle travelling along the path.

The path might more generally be described by an arbitrary parameter λ as long as λ increases in a smooth and monotone fashion as we traverse the path. Write the action in this form and consider a variation $\delta x^\mu(\lambda)$ and the corresponding variation δS. Use Hamilton's principle[1] to find the equations of motion for $x^\mu(\lambda)$. Explain whether or not these equations determine $x^\mu(\lambda)$ in terms of some initial conditions, and why.

2) Zwiebach Problem 5.7

3) Zwiebach Problem 6.5

4) Zwiebach Problem 6.7

[1]Hamilton's principle, or the principle of stationary action, says that the variation of the action vanishes along the solutions to the equations of motion. I always get confused because Hamilton's principle gives the Lagrange equations of motion, not the equations of motion in Hamiltonian form.