Project #1

Note: This project is to be worked on in three groups, which we will discuss now.

Each group should meet soon to lay out the steps necessary to complete the project, and divide up the work. Unlike ordinary homework, where though communication is encouraged, each individual is expected to work through all parts him/her-self, in a project it is acceptable for each member to have only read through and understood each part, without actually having worked through each part individually. But everyone is responsible for it being correct and clear.

The project is to be written up consistently, coherently, and neatly, preferably on a computer, even more preferably in TeX or LaTeX. Each member is responsible for proofreading before submission.

The project uses generalized curvilinear coordinates in two related problems.

1 A charged conductor in the shape of an oblate ellipsoid, a flying saucer with
\[ \left( \frac{x}{A} \right)^2 + \left( \frac{y}{A} \right)^2 + \left( \frac{z}{B} \right)^2 = 1, \] (1)
with \( A > B \), sits in otherwise empty space.

   (a) Find the electrostatic potential \( V(\vec{r}) \) everywhere outside the conductor, assuming \( V = V_0 \) on the conducting surface, and \( V \to 0 \) at infinity. This is to be done analytically, using the appropriate generalized coordinates.

   (b) From this general case, take the limit \( B \to 0 \). From the large distance limit of the potential, extract the net charge on the conductor, and hence find the capacitance of a thin disk of radius \( a \).
(c) Now repeat for a prolate ellipsoid, a cigar-shaped given by Eq.(1) with $A < B$. Again find the electrostatic potential everywhere outside the ellipsoid. You may find the keywords “ellipsoidal coordinates” useful.